HIGH-ALTITUDE REENTRY DRAG EFFECTS

JUNE 1966

S. Sharensen

Prepared for
DEPUTY FOR ENGINEERING & TECHNOLOGY
DIRECTORATE OF RADAR & OPTICS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Advanced Research Projects Agency
Project Defender
ARPA Order No. 596

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Project 8090
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THE MITRE CORPORATION
Bedford, Massachusetts
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FOREWORD

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REVIEW AND APPROVAL

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Project Officer
ABSTRACT

The effect of atmospheric drag on the trajectory of a reentry body is considered. The analysis is valid in the high altitude regime. Results are presented as the differential displacement and velocity from the vacuum trajectory for various values of ballistic coefficient.
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TABLE OF SYMBOLS AND UNITS

h = altitude - feet

ρ(h) = atmospheric density - pounds per cubic foot

S = distance along trajectory - feet

v = velocity along trajectory - feet per second

β = ballistic coefficient - pounds per square foot

θ_E = reentry angle

t = time - seconds
The differential displacement of a re-entering body due to atmospheric drag is considered in this report. The situation considered is shown below. The body enters the atmosphere at an angle $\Theta_E$ from the horizontal with an initial velocity $v_0$. The altitude, measured from the earth's surface, is $h$. The altitude $h_o$ is chosen large enough such that:

$$\rho(h_o) \approx 0$$  \hspace{1cm} (1)

where $\rho(h)$ is the atmospheric density as a function of altitude.

Neglecting the earth's curvature, the effect of earth gravity and atmospheric drag, the trajectory of the re-entering body is a straight line and the displacement along the trajectory is given by:

$$S = \frac{h_0 - h}{\sin \Theta_E}$$  \hspace{1cm} (2)
where

\[ S = v_0 t \] (3)

Including, now, the effect of drag (but not lift), the classical equation for deceleration due to drag is:

\[ - \frac{dv}{dt} = \frac{1}{2\beta} \rho(h) v^2 \] (4)

Here \( v \) is the instantaneous velocity of the vehicle and \( \beta \) the ballistic coefficient.

Constraining the analysis to the high altitude re-entry regime where changes in \( v \) are small, the drag force can be calculated by assuming \( v \approx v_0 \) in this regime.

From equations (2) and (3):

\[ dt = - \frac{dh}{v_0 \sin \theta_E} \] (5)

Substituting equation (4) for \( dt \), and setting \( v \approx v_0 \):

\[ \int_{v_0}^{v} dv = \int_{h_0}^{h} \frac{v_0}{2\beta \sin \theta_E} \rho(h) dh \] (6)

\[ v - v_0 = \frac{v_0}{2\beta \sin \theta_E} \int_{h_0}^{h} \rho(h) dh \] (7)

\[ v = v_0 + \frac{v_0}{2\beta \sin \theta_E} \int_{h_0}^{h} \rho(h) dh \] (8)
But:

\[ v = \frac{dS}{dt} \] (9)

Therefore:

\[ dS = v_0 \, dt + \frac{v_0 \, dt}{2 \beta \, \sin \theta_E} \int_{h_0}^{h} \rho(h) \, dh \] (10)

from equation (8). Substituting from equation (5):

\[ dS = v_0 \, dt - \frac{dh}{2 \beta (\sin \theta_E)^2} \int_{h_0}^{h} \rho(h) \, dh \] (11)

Integrating equation (11):

\[
\int_{S_0}^{S} dS = v_0 \int_{t_0}^{t} dt - \frac{1}{2 \beta (\sin \theta_E)^2} \int_{h_0}^{h} dh' \int_{h_0}^{h'} \rho(h') \, dh' \] (12)

But:

\[ S_0 = 0 \] (13)

at \( h = h_0 \). Thus

\[ S = v_0 (t - t_0) - \frac{1}{2 \beta (\sin \theta_E)^2} \int_{h_0}^{h} dh' \int_{h_0}^{h'} \rho(h') \, dh' \] (14)

Now, in the absence of the atmosphere, equation (4) reduces to:

\[ \frac{dv}{dt} = 0 \] (15)
and the displacement is simply:

\[ S' = v_0 (t - t_o) \]  \hspace{1cm} (16)

Subtracting (14) from (16) gives the differential displacement \( \Delta S \):

\[ \Delta S = S' - S = \frac{1}{2 \beta (\sin \theta_E)^2} \int_{h_o}^{h'} \int_{h_o}^{h'} \rho(h') \, dh' \, dh \]  \hspace{1cm} (17)

For comparison the velocity, \( \frac{d(\Delta S)}{dt} \), and the deceleration, \( \frac{d^2(\Delta S)}{dt^2} \), are derived:

\[ \frac{d(\Delta S)}{dt} = \frac{1}{2 \beta (\sin \theta_E)^2} \int_{h_o}^{h} \rho(h) \, dh \]  \hspace{1cm} (18)

From equations (2) and (3):

\[ \frac{dh}{dt} = - \sin \theta_E \frac{dS}{dt} \]  \hspace{1cm} (19)

or

\[ \frac{dh}{dt} = - \sin \theta_E \, v_0 \]  \hspace{1cm} (20)

Therefore:

\[ \frac{d(\Delta S)}{dt} = - \frac{v_0}{2 \beta \sin \theta_E} \int_{h_o}^{h} \rho(h) \, dh \]  \hspace{1cm} (21)

But:

\[ \int_{h_o}^{h} \rho(h) \, dh < 0 \]  \hspace{1cm} (22)
Therefore:

\[
\frac{d(\Delta S)}{dt} = \frac{v_o}{2\beta \sin \theta_E} \left| \int_{h_o}^{h} \rho(h) \, dh \right|
\]  

(23)

The deceleration is found from a second differentiation:

\[
\frac{d^2(\Delta S)}{dt^2} = \frac{v_o}{2\beta \sin \theta_E} \frac{dh}{dt} \rho(h)
\]  

(24)

or:

\[
\frac{d^2(\Delta S)}{dt^2} = -\frac{v_o^2}{2\beta} \rho(h)
\]  

(25)

The results of this analysis are presented in Figure 1, as \(\Delta S\) vs. \(h\) for three different values of \(\beta\). For comparison, \(\frac{d(\Delta S)}{dt}\) and \(\frac{d^2(\Delta S)}{dt^2}\) are presented in Figures 2 through 3. Since this analysis applies only in the region where the velocity change is small, the curves are terminated at the appropriate lowest altitude for the particular value of \(\beta\). The atmospheric density profile used is known in Figure 4*. The first and second integrals of the density profile are given in Figures 5 and 6.

S. Sharenson

SS/rjs

Figure 1

\( \Delta S \) vs Altitude  \( \theta = \) reentry angle  \( \beta = \) ballistic coeff. - PSF
Figure 2

\( \frac{d(\Delta S)}{dt} \) vs. Altitude \( \theta_E \) = Reentry Angle

5500 NM Trajectory \( \beta \) = Ballistic Coeff. - PSF
Figure 3

\[ \frac{d^2(\Delta s)}{dt^2} \] vs Altitude \[ \theta_E = \text{Reentry Angle} \]

5500 NM Trajectory \[ \beta = \text{Ballistic Coeff. - PSF} \]
Figure 4
ATMOSPHERIC DENSITY $- \rho(h)$
\[ \int_{h_0}^{h} \rho(h) \, dh \]

\[ \text{ALTITUDE - KFT} \]

**Figure 5**

\[ \int_{h_0}^{h} \rho(h) \, dh \cdot \frac{\text{LBS}}{\text{FT}^2} \]

IB-17.743
Figure 6

\[
\int_{h_0}^{h} \int_{h_0}^{h'} \rho(h') \frac{\text{LBS}}{\text{FT}} dh' dh
\]
High-Altitude ReEntry Drag Effects

Sharensen, Stanley

June 1966

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The effect of atmospheric drag on the trajectory of a reentry body is considered. The analysis is valid in the high altitude regime. Results are presented as the differential displacement and velocity from the vacuum trajectory for various values of ballistic coefficient.
Satellites
Reentry, at High Altitudes
Atmospheric Drag, Effects on Trajectory

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