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THE USE OF RESETTABLE INTEGRATORS OR CONVENTIONAL FILTERS IN TIME-DIVISION DATA TRANSMISSION SYSTEMS

MAY 1966

B. M. Hadfield
R. T. Herman

Prepared for

DIRECTORATE OF AEROSPACE INSTRUMENTATION

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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FOREWORD

This report was originally intended to record a simple derivation of a noise reduction formula found, without proof, in 1963 during work on Telemetry Systems Standardization. The proof included in this report was derived by R. T. Herman, after a literature search that disclosed no other.

It is an essential feature of the overall telemetry program that the limits, bounds, and areas of application of such formulas to system designs be established clearly, so as to lead to design and application criteria. These critieria would then be used to compare current and advanced telemetry system designs with the present and future requirements. This comparison would ultimately lead to recommendations for standard telemetry systems.

In considering the application of this noise reduction method, comparisons with more conventional methods were made, and several offshoots concerning the general design and performance of time-division systems became apparent. Since consideration of these is probably more important than anything else, it was felt that this document should be expanded in order to establish some guidelines.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

C. V. Horrigan

C. V. HORRIGAN
Acting Director
Aerospace Instrumentation

ABSTRACT

This report establishes the "effective noise power bandwidths" and "noise reduction factors" of a resettable integrator and of conventional passive lowpass filters. It then discusses the use of the resettable integrator with its necessary sample and hold circuit, as compared to the use of a passive lowpass filter which follows the receiver demodulator in time-division data transmission systems, such as telemetry links. It is concluded that since the overall lowpass bandwidth is determined by the conventional output lowpass filter necessary for removing the sampling "noise" (quantum steps), then any other filter device is redundant, provided the demodulator does not sensibly alter the statistical properties of the input random noise.

As a germane and highly important side issue, a more logical method for selecting the sampling rate in time-division systems is also discussed.

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SECTION I

INTRODUCTION

In recent publications, the statement has been made that the signal-to-noise (S/N) power value can be "improved"* by a factor of 2WT in time-division communication systems, where W is the lowpass bandwidth of the data channel and T is the duration of the channel time-slot. To achieve this improvement, an integration circuit is used on the demodulated, unfiltered pulse output over the period T. The integrated output is then sampled and held on another circuit to allow the integrator to be reset and to operate on the next channel period T whenever this occurs.† The implication of the statement is that such a technique is a great improvement on conventional techniques.

We have been unable to find a proof of the statement, although such may exist. This report will give a proof based on certain obligatory assumptions concerning the input and the demodulation process. It will also discuss the effects of departure from these assumptions, will compare the resettable integrators and conventional filters, and will indicate certain design requirements of time-division systems, concerned with the data source sampling rates and the inevitable degradation of the data.

It should be stated that the process itself is not novel and has been used for some time in data-reduction applications.

---

*"Noise Reduction Factor." In this report we shall refer to this factor rather than to the "S/N Improvement Factor," because the device does, in fact, reduce the noise.

†That is, generally after a quiescent period $(\tau - T)$, during which other time-slots are in use, and where τ is the frame period.

The immediate problem is to verify the signal-to-noise (S/N) power ratio "improvement"* given by Equation (1),

$$\frac{S_o}{N_o} = 2WT \frac{S_i}{N_i} , \tag{1}$$

where W is the given input lowpass bandwidth having a noise power level N_i, and S_i is the mean (d. c.) power level of the input signal over the sample period, T, for which the integrator integrates. The output noise level is denoted by N_o, and the signal power level is specified by S_o. This is shown diagrammatically in Figure 1. (In general, $S_o = S_i$, since a perfect

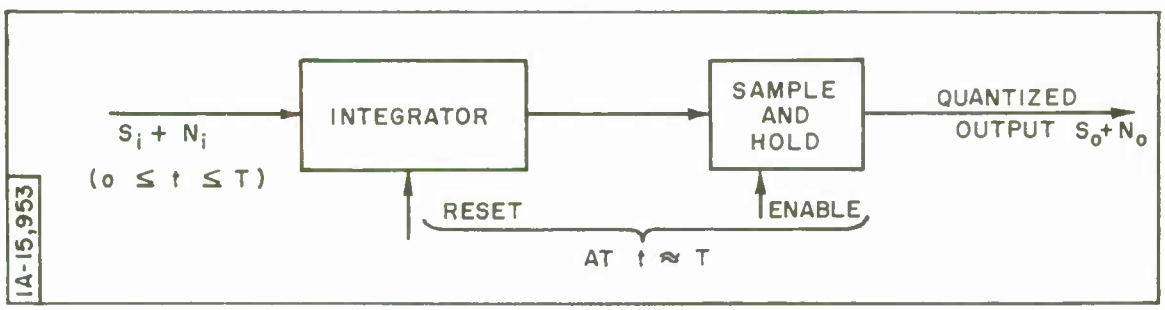

Figure 1. Specific Resettable Integration Process

integration process is non-dissipative. The terminal integrated value is held indefinitely, or until the onset of the next time-slot resets the whole device. The d. c. power levels are S_o and S_i. See also Section III.)

*See footnote, page 1.

The general representation of the essential S/N process under discussion is shown in Figure 2. The sample and hold elements should not in themselves affect the S/N ratio established over any given period T (but see Section V).

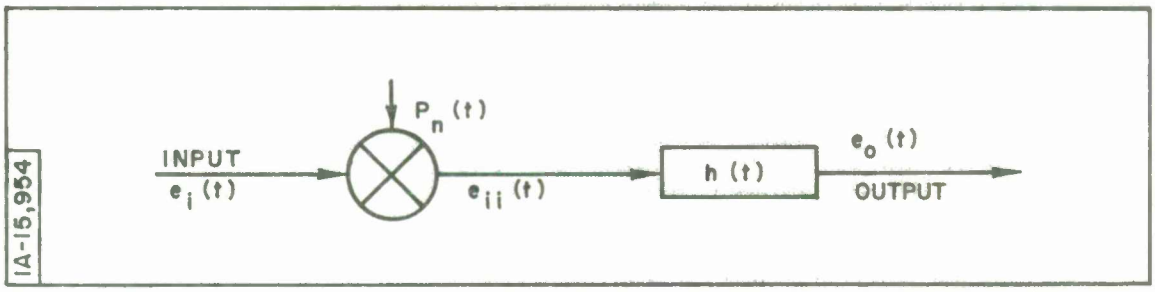

Figure 2. Generalized Process

$P_n(t)$ represents a local enabling function (switching, commutation, etc.) for a particular channel that selects this channel input, $e_i(t)$, as the desired signal of time-length T which has a mean d. c. value with superimposed noise. The integrator network unit impulse response is denoted by h(t) (see Figure 3).* In Figure 3, the ordinate value 1/T and the abcissa time T are chosen so as to give a normalized (unity) forcing function.

The time interval T is the channel time-slot duration or sample period. The transmitted data level is conventionally held constant over this period, since this is an analog time-division data transmission system. At the

*See Appendix .

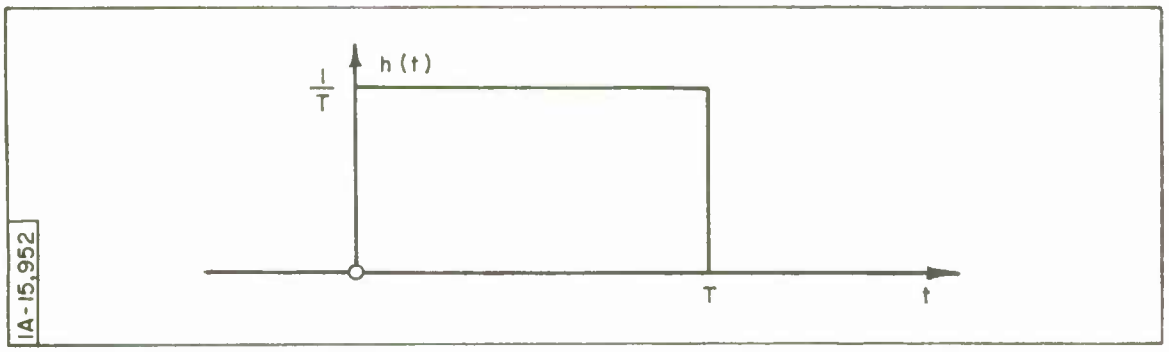

Figure 3. Integrator Unit Impulse Response

receiver, the integrator output is sampled "instantaneously" at the end of every T seconds after the initiation of integration and transferred to a hold circuit. This allows the integrator to be reset and to be available to operate similarly on the next received data sample of length T .* The data value is given by the successive (constant) amplitudes of the hold circuit output from each successive period T . The steps between the succession of constant levels are subsequently removed by an output lowpass filter.

---

*In a multichannel, sequential, time-division system, this will occur $\tau - T$ seconds later on a given channel, where $1/\tau$ is the frame rate. There are usually τ/T equal time-slot channels. However, there is at least one sampling system where all channels are sampled simultaneously. Then, there is no interval $\tau - T$, that is, $\tau = T$.

SECTION II

THE EFFECTIVE NOISE POWER BANDWIDTH OF THE RESETTABLE INTEGRATOR

The necessary steps in this process are:

1. establish the normalized voltage transfer function with frequency of the integrator operating over time T , and take its square as denoting the normalized power transfer function; and

2. integrate the normalized power transfer function from zero to infinite frequency and divide by the value of the power transfer function at zero frequency.

(Note: The second step assumes that the power spectrum of the input is flat to infinite frequency; that is, the input noise is "white." See also general proofs of effective noise bandwidth.* This assumption and the above steps are conventional in all general analyses of S/N values based on effective noise power bandwidths.)

It should be clearly understood that the effective noise power bandwidth calculated in this manner is that bandwidth (from zero frequency) of "white" noise which would produce the same output noise power. This does not mean that the noise output of the device is "white"; the output is a noise spectrum defined by the voltage transfer function. This must be kept in mind when evaluating the "noise-reducing" properties of any device. This is the reason for stating in detail the steps in the calculation.

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*J. J. Freeman, Principles of Noise, New York, John Wiley & Sons, 1958.

THE VOLTAGE AND POWER TRANSFER FUNCTIONS OF THE RESETTABLE INTEGRATOR

This integrator operates only over time T, or, more accurately, over successive periods T of a given channel.* The integrator is reset to zero at or before the start of each new period, T. Its transfer function must, therefore, be dependent on T, as well as on the input frequency. To find this transfer function $H(\omega)$, we use the Fourier transform involving the unit impulse response $h(t)$, as follows:

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt \,, \tag{2}$$

where ω is the frequency in radians/sec. From Figure 3,

$$h(t) = \frac{1}{T} \,, \quad (0 \leq t \leq T) \,. \tag{3}$$

Hence,

$$H(\omega) = \int_{0}^{T} \frac{1}{T}\, e^{-j\omega t}\, dt$$

$$= \frac{1}{j\omega t} \left[1 - e^{-j\omega T} \right] \tag{4}$$

$$\therefore\ H(\omega) = \frac{1}{\omega T} \left[\sin \omega T + j\,(\cos \omega T - 1 \right] \,. \tag{5}$$

---

*These may be separated by longer periods $\tau - T$, where τ is the frame period.

The magnitude of $H(\omega)$ is given by:

$$|H(\omega)| = \left| \frac{\sqrt{\sin^2 \omega T + (\cos \omega T - 1)^2}}{\omega T} \right|$$

$$= \left| \frac{\sqrt{2(1 - \cos \omega T)}}{\omega T} \right|$$

$$= \left| \frac{\sin \omega T/2}{\omega T/2} \right| . \tag{6}$$

As regards the phase angle, Θ ,

$$\Theta = \tan^{-1}\left[\frac{(\cos \omega T - 1)}{\sin \omega T}\right] = \tan^{-1} - \left[\tan\left(\frac{\omega T}{2}\right)\right] = \frac{-\omega T}{2} \tag{7}$$

and

$$|H(\omega)|^2 = \left[\frac{\sin \omega T/2}{\omega T/2}\right]^2 . \tag{8}$$

These voltage and power transfer functions are seen in Figure 4. For convenience, the horizontal axis is shown in terms of fT , where $\omega = 2\pi f$, because the zero values occur at $\omega T/2 = \pi$, 2π, 3π, etc.

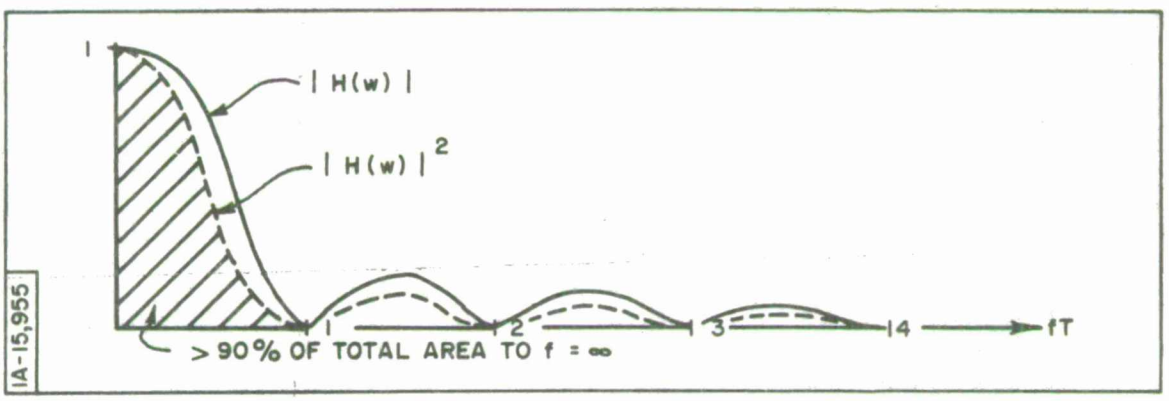

Figure 4. Normalized Voltage and Power Transfer Functions

These response curves are analogous to those for passive lowpass filters of the m-derived type;* the 3-db point for $|H(\omega)|$ is at about $fT = 0.43$, or the "cut-off" frequency is about $0.43/T$. From this analogy, it is now apparent how the device reduces the noise bandwidth: it acts as a lowpass filter. A discussion of this filter characteristic in relation to passive filters, the action of the sample and hold circuit, and other practical matters will be found in Sections V and VI.

THE EFFECTIVE NOISE POWER BANDWIDTH

As stated above, the effective noise power bandwidth, W_e, is given by integration of the normalized power transfer function from zero to infinite frequency, and its division by its value at zero frequency, as follows:

$$W_e = \frac{1}{2\pi} \frac{\int_0^\infty |H(\omega)|^2 \, d\omega}{|H(0)|^2} . \tag{9}$$

From Equation (8),

$$\begin{aligned} \int_0^\infty |H(\omega)|^2 d\omega &= \int_0^\infty \left[\frac{\sin \omega T/2}{\omega T/2}\right]^2 d\omega \\ &= \frac{2}{T} \int_0^\infty \left[\frac{\sin x}{x}\right]^2 dx \\ &= \frac{\pi}{T} . \end{aligned} \tag{10}$$

---

*That is, where there are infinite attenuation "slots" in the finite attenuation region above the "cut-off" frequency (see also Figure 5, full-line curve).

From Equation (6),

$$|H(\omega)| = \frac{\sin \omega T/2}{\omega T/2} \rightarrow 1, \text{ when } \omega \rightarrow 0 .$$

$$\therefore \; |H(0)| = 1 . \tag{11}$$

Combining Equations (9), (10), and (11), the effective noise power bandwidth, W_e, is given by

$$W_e = \frac{\pi}{T} \frac{1}{2\pi} = \frac{1}{2T} . \tag{12}$$

A MINIMUM INPUT CRITERION FOR THE EFFECTIVE NOISE POWER BANDWIDTH, 1/2T

The above bandwidth, $1/2T$, is that bandwidth (from zero frequency) of "white" noise which would give the same output noise power, when the input is "white" noise extending from zero to infinite frequency.

More than 90 percent of the area found by the integral of $|H(\omega)|^2$ from zero to infinity lies within the area of $|H(\omega)|^2$ up to $fT = 1$ (see also Figure 4). Hence, the effective noise bandwidth is still given by $1/2T$ (to a few percent), if the input "white" noise lowpass bandwidth extends to at least $1/T$.

SECTION III

THE S/N "IMPROVEMENT" FORMULA* AND THE NOISE REDUCTION FACTOR

As mentioned above, provided the input "white" noise power bandwidth, W, is equal to or greater than $1/T$, then the effective output "white" noise power bandwidth of the integrator is $1/2T$. Since both input and output noise spectra are "white," the input and output noise powers are proportional to their bandwidths over the period T. Hence,

$$\left[\frac{\text{Output Noise Power}}{\text{Input Noise Power}}\right]_0^T = \frac{N_o}{N_i} = \frac{1/2T}{W} = \frac{1}{2WT} \quad . \tag{13}$$

Now, as stated in Section I, if S_i is defined as the mean (d. c.) power level of the input signal over the period T, then the output power level S_o, is the same; the integration process being nondissipative. On this basis, then,

$$\left[\frac{S_o}{N_o}\right]_0^T = 2WT \left[\frac{S_i}{N_i}\right]_0^T \quad . \tag{14}$$

This proves the original formula stated in Equation (1), at least for the time-slot periods T, and hence for any sampling system with contiguous channel time-slots.

For (given) channel time-slots that are separated to allow for sequential handling of many such channels, the formula is still substantially true using the given device. The sample and hold feature maintains the $S_o + N_o$ value

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*See footnote, page 1.

sampled at the end of a time-slot T, over the intervening quiescent $S_i + N_i$ period, while the other time-slots are handled elsewhere. The fact that the S_o/N_o ratio may not be quite the same as is meant by the preceding proof is dealt with in Section IV. Because of the practical necessity for a sample and hold technique following a perfect integrator, and in order to use the output, this effect is the same whether the channel time-slots are contiguous or separated, since it occurs at the instant of sampling, i.e., at the end of any given time-slot T.

Hence, we have proved the general applicability of Equation (1) using the power value of the signal and noise, whether the time-slots for a given channel are contiguous or not. However, we will now show that as a result of the sample and hold technique, the practical S_o/N_o is not quite the same thing; again, this is independent of time-slot contiguity. We will also discuss whether the device as a whole really has anything to offer. As a lead-in to this, we will first discuss the general $|S_o = S_i|$ assumption made above.

COMMENTS ON THE $S_o = S_i$ ASSUMPTION

In the general analysis of problems involving "white" noise, it is usual and convenient to operate in terms of the noise power, on the assumption that somewhere both noise and signal are going to be dissipated in a resistor. Then, depending on the particular circumstances, a certain S/N power ratio means something in terms of performance.

Since noise power is used to calculate the effective bandwidth, then S/N can only be stated in terms of signal power. The result is given above in Equation (14).

However, most practical applications involve the detection or measurement of the signal at a particular voltage level, that is, at a particular time in the received signal waveform due to a given input.

This device is no exception, since the integrator waveform* is sampled at the end of the period T. Thus, if the practical S_o/N_o is desired, it is necessary to measure the noise waveform amplitude superimposed on the maximum value of the signal ramp voltage, that is, at the end of the period T (see also Section V).

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* In general, a ramp with superimposed noise.

SECTION IV

GENERAL COMMENTS ON THE RESETTABLE INTEGRATOR

THE EFFECT OF DEMODULATION PROCESS ON THE NOISE SPECTRUM

The above analysis assumes implicitly that the noise power density spectrum over the input bandwidth W is unaffected by the selection or demodulation process,* which is necessary to obtain a given channel signal having a mean value over the period T. Where the demodulation process is of a switching type, this assumption is probably true, provided the intervals between switching are not very small compared to T; or, alternatively, that the intervals are much longer than $1/W$, where W is the input noise bandwidth.

However, in practice, the validity of this assumption must be confirmed for the particular application.

THE EFFECTIVE NOISE POWER BANDWIDTH AND THE IDEAL SAMPLING RATE

When this device is applied in a time-division system, the time T would be equal to or less than the period, τ, between samples of the analog data source. In order that errors due to aliasing shall not occur in the signal output, the highest signal frequency component (of significant amplitude) that may be sampled is $1/2\tau$. A sampling rate of $1/\tau$ is then ideal.

This device has an effective noise bandwidth of $1/2T$. This quantity is equal to or greater than the maximum permissible signal frequency. However,

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*This may be as simple as pure rectification, or as complicated as waveform correlation.

the actual noise bandwidth is larger than this, although not more so than with other forms of filtering (see Section V).

The fact that this device has an effective noise bandwidth which is equal to or greater than the highest signal frequency component that may be transmitted in a time-division system is not unique, as has been claimed elsewhere; it is merely a consequence of its operation. In general, the choice of sampling rate in time-division systems is a contentious matter, which will be discussed in Section VII. It is worth remarking here, however, that not only is it difficult to insure that no significant signal component of frequency higher than $1/2\tau$ is sampled, but it is even more difficult to remove the sampling "noise" at the receiver output without seriously reducing the lowpass frequency response of the overall system.

SECTION V

COMPARISON WITH CONVENTIONAL FILTERS

As mentioned before, the peculiar characteristics of this filter are attributable to two facts:

1. it performs a perfect integration process ; and
2. the integration is over a finite time T starting from a zero value.

In order to compare its performance with more conventional types of filters, $H(\omega)$, or the voltage transfer characteristic (Figure 5), has been redrawn, using log-log paper. Also shown is the effective noise power bandwidth, $1/2T$, or $fT = 0.5$.

It will be seen that the primary passband below $fT = 1$ is characterized by a sharp roll-off. This, however, does not indicate an oscillatory mode, as it would with a conventional filter, because the phase angle response is linear (see Equation 7). The step-function response is, of course, a ramp of constant slope over the time T . The subsidiary maxima above $fT = 1$ follow a - 6 db/octave envelope curve. The effective noise power bandwidth of $fT = 0.5$ is obviously a good equivalent.

CONVENTIONAL FILTER CHARACTERISTICS AND THEIR EFFECTIVE NOISE POWER BANDWIDTHS

For convenience, the Butterworth filter is used here. With this filter, the independent variable is n , which is the number of poles or reactances. It is understood that the reactances are alternately capacitive and inductive, shunt and series, respectively, composing a ladder network which operates between characteristic source and load resistances. Since this is a passive

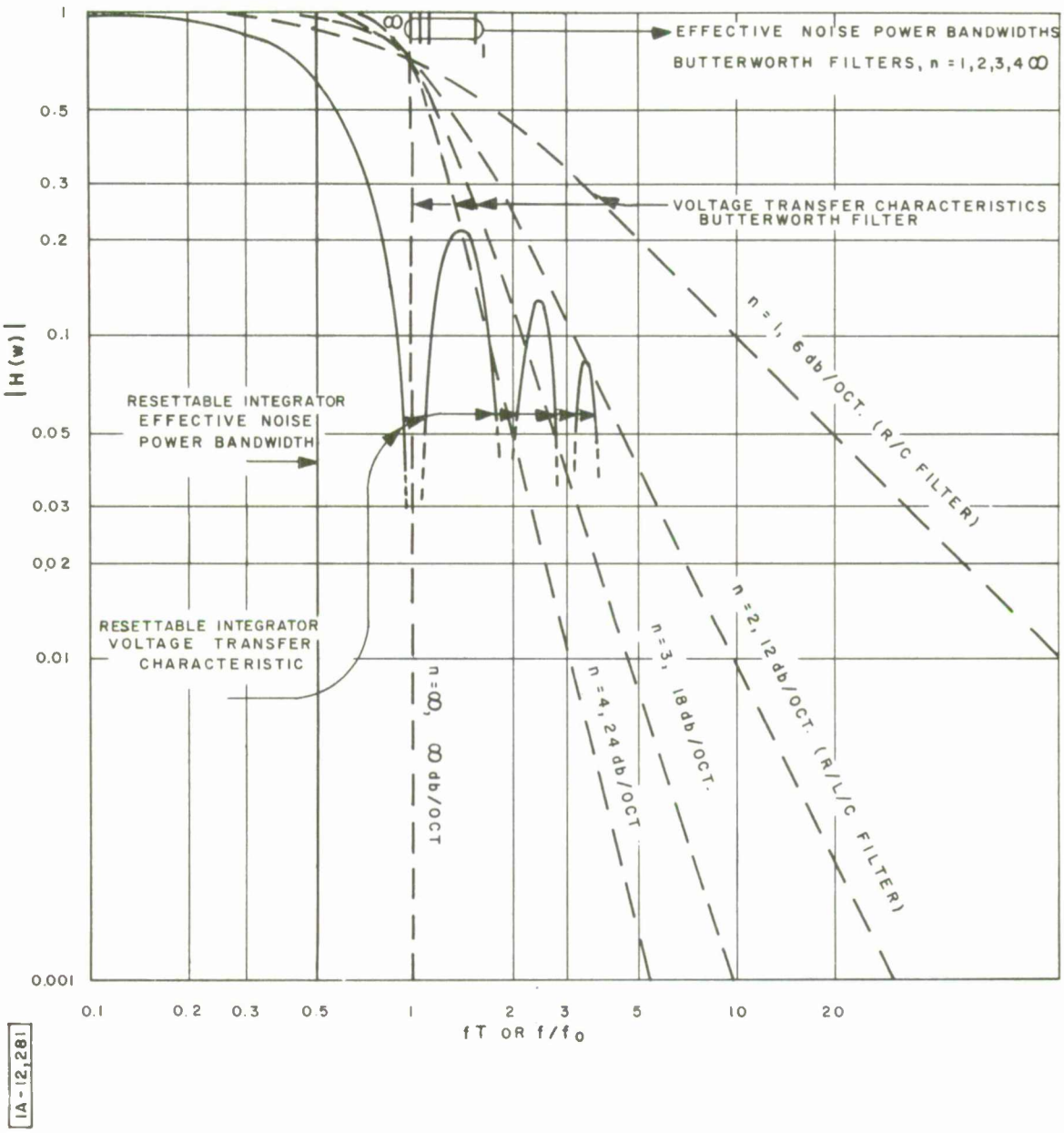

Figure 5. Normalized Voltage Transfer Characteristics of Various Filters

linear filter, the fact that the input signal mean level may change every successive time-slot T is immaterial from the point of view of the alternating component of the noise output. However, as noted in Section IV, the following discussion does depend on the statistical properties of the input noise remaining unaltered by the signal demodulation process.

Using the method discussed in Section II, we have, in general, for the Butterworth filter, an effective noise power bandwidth, W_e, given by

$$W_e = \frac{1}{2\pi} \frac{\int_0^\infty |H(\omega)|^2 d\omega}{|H(0)|^2} . \tag{9}$$

The voltage transfer function for a Butterworth filter is given by:

$$|H(\omega)| = \left[\frac{1}{1 + \left(\omega/\omega_o\right)^{2n}} \right]^{1/2} , \tag{15}$$

where ω is the input frequency, ω_o is the half-power frequency (both in radians/sec.), and n = 1, 2, 3, ...

Since $|H(0)| = 1$, by inspection, then

$$W_e = \frac{1}{2\pi} \int_0^\infty \frac{1}{1 + \left(\omega/\omega_o\right)^{2n}} d\omega \tag{16}$$

$$\therefore \; W_e = f_o \int_0^\infty \frac{1}{1 + x^{2n}} dx , \tag{17}$$

where $x = \omega/\omega_o$ and $f_o = \omega_o/2\pi$. Using integral tables, we have

$$W_e = f_o \frac{\pi}{2n \sin(\pi/2n)} . \tag{18}$$

Hence, for $n = 1, 2, 3, 4, \ldots$.

h	W_e
1	$1.57\ f_o$
2	$1.1\ f_o$
3	$1.05\ f_o$
4	$1.02\ f_o$
∞	f_o

The voltage transfer characteristics of the Butterworth filter are shown for $n = 1, 2, 3, 4,$ and infinity, using broken lines, in Figure 5. Their effective noise power bandwidths of 1.57, 1.1, 1.05, 1.02, and 1 times f_o are also shown. The abscissa are in terms of f_o , the 0.707 (or half-power) point, which is characteristically at $f_o = 1$ for all n values. It will be seen that the "roll-off" rates are 6, 12, 18, 24, and ∞ db/octave.

By inspection, it is seen that if f_o is made equal to $1/2T$, i.e., the Butterworth curves are shifted over to the left by a ratio of 0.5 , then, for $n = 4$ (or higher), the significant transfer characteristics become similar to those for the resettable integrator. Thus, the effective noise power bandwidths tend rapidly to fT (or f/f_o) = 0.5 .

From this, we may conclude there is no essential difference between the noise power reduction qualities of the (resettable) integrator and those of the Butterworth filter of four poles or more, provided the relationship $f_o = 1/2T$ exists. However, there is a difference in the utility of the signal component output.

THE SIGNAL WAVEFORM OUTPUT OF THE RESETTABLE INTEGRATOR AND OF A CONVENTIONAL FILTER

The important practical characteristics of either device in time-division systems are:

1. the slope of the signal voltage component; and
2. the magnitude of the noise component with respect to the signal component.

These are taken at the time of measurement and/or transfer to the output circuit (usually at the end of the time-slot period, T).

In Section V, we have shown that a simple $(n \geq 4)$ passive filter can have an effective noise power bandwidth, within 10 percent, of the resettable integrator, provided $f_o = 1/2T$. Furthermore, the voltage transfer curves are not so remarkably different as to lead to any marked differences in the noise voltage waveform. Hence, it is reasonable to conclude that the noise magnitude will be substantially the same. (Note: In this comparison, we are ignoring circuit design and component differences for the moment, because of the conclusions drawn later.)

The technical utility of the resettable integrator output depends on the magnitude of the signal component with respect to the noise component and its slope at the sampling time, i.e., at the end of the period T .

The resettable integrator exhibits a constant slope output with time, derived from the mean (constant) d.c. value of the demodulated signal applied

to it over the time T .* The conventional filter exhibits a quasi-raised-cosine type of output with time, the periodic time being $1/f_o$. These are demonstrated in Figure 6, using small quiescent periods between the time slots, T ,

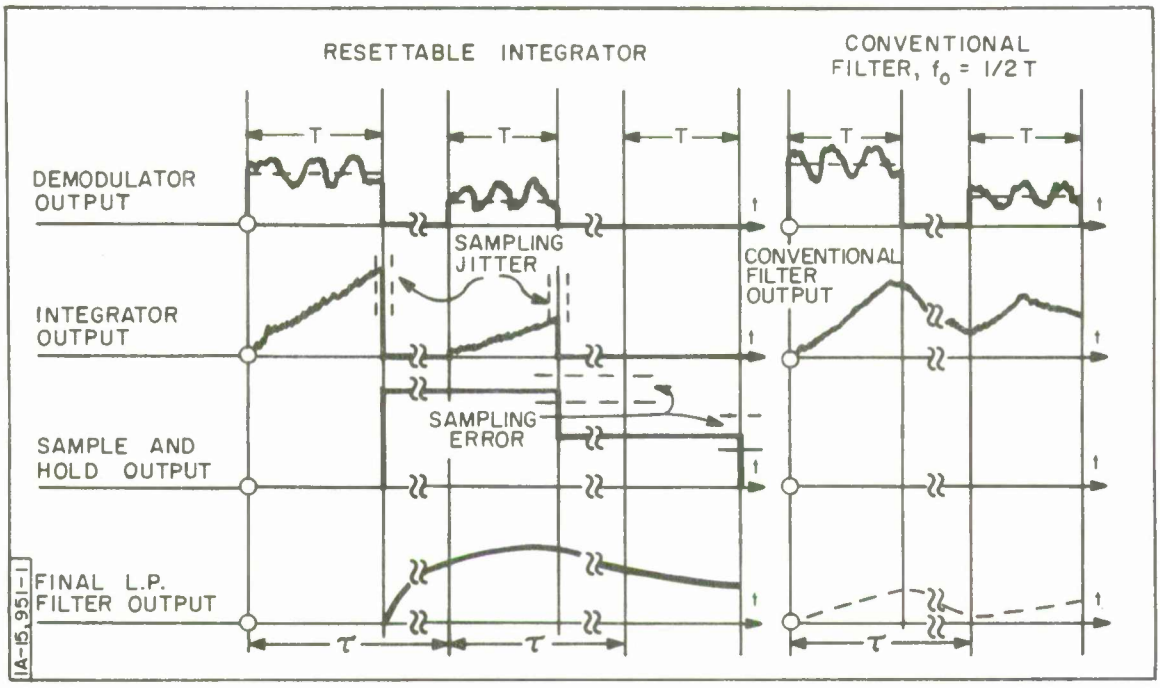

Figure 6. Typical Operating Waveforms

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*The output of the demodulator can always be separated into a mean d. c. level plus an alternating component over the time T . The mean d. c. level is due to the desired signal, plus any demodulation bias, plus any mean value of the noise over the time T . The a. c. component is due to any transient signal component (due to band-limiting prior to the demodulator), unbalance in the demodulator, and input noise. The transient signal component may cause a small initial perturbation in the linear ramp output with time of the integrator.

in order to clarify the latter. Hence, the output of the filter attains its maximum value, and has zero slope, at $1/2\, f_o$. But if $f_o = 1/2T$, in order to obtain the same effective noise power bandwidth (see Section V), the conventional filter attains its maximum output at the end of the sample period, T , and has zero slope (see Figure 6).

Hence, at the end of the time slot T , we have the same S/N voltage ratio on both types of filters, but a finite slope in the case of the resettable integrator and a zero slope in the case of the conventional filter. Now there are always errors in sampling due to jitter and to the finite width of the sampling gate. These errors will apply to the resettable integrator, but they will not apply in the case of the conventional filter, because there is no need to perform the sample and hold functions which are essential with the resettable integrator (see also Figure 6). It should be noted that as no prior samples have been assumed, the final lowpass filter output cannot be an accurate measure of the input on the first sample. But this is not necessarily a serious disadvantage (see Sections VI and VII). The final lowpass filter output has been shown dotted for the conventional filter case, because Section VI will show that the conventional filter elements can also be designed for use as the final filter.

Two other points can also be made from Figure 6.

1. The resettable integrator sampled output has amplitude errors both as regards the signal and noise values, due to sampling time-jitter. The fact that there is no noise on the "hold" output pulse amplitudes does not mean they are free from noise. These amplitudes include that value of noise existing at the instant of sampling.

2. An additional output lowpass filter is necessary for the resettable integrator (see also Section VI).

From the above, we may conclude that the equivalent $\left(f_o = 1/2T\right)$ conventional filter is not inferior to the resettable integrator in performance. Section VI will show that this filter is also superior in its components and complexity.

SECTION VI

THE OVERALL OPERATION OF THE RECEIVING LOWPASS CIRCUIT

In order that the output shall approximate the original analog waveform which was sampled at the data source, the sampling steps of the resettable integrator output have to be removed by another (final output) lowpass filter. This must be of conventional type.

This output lowpass filter must greatly attenuate the quantizing or sampling frequency, $1/2\tau$, and all higher frequencies. Referring to Figure 5 for the general Butterworth transfer characteristics, the f_o, or -3 db frequency of the final output filter must be $1/20\tau$, $1/9\tau$, $1/6\tau$, etc., for $n = 2, 3, 4$, respectively, if the sampling steps are to be reduced to about 1 percent of their original value. On the other hand, if the reduction of the sampling steps to about 10 percent of the input value is considered enough, then f_o must be $1/6.33\tau$, $1/4.2\tau$, $1/3.5\tau$, for $n = 2, 3, 4$, respectively, and so on.

The choice of this lower-valued lowpass output filter is a matter of opinion. It depends on whether the successive data samples vary widely (as will be the case with the minimum theoretical sampling rate); how much inaccuracy due to sampling "noise" the designer-user of the data link is prepared to tolerate in the output; and how much degradation of the higher signal frequency components is permissible.

The point that we wish to establish here is that the overall receiver transfer characteristic from the demodulator to the usable output has a much smaller lowpass bandwidth than $1/2T$, due to the final output filter necessary for removing the sampling noise. In these circumstances, it seems reasonable to apply the conventional output filter directly to the output of the demodulator.

In this way, the input "white" noise, any demodulator switching "noise," and the quantitizing "noise" are all simultaneously reduced to the minimum possible values. In addition, some critical (active) circuit components have been eliminated without demanding any more from the output lowpass filter design.

Thus, the other question as to whether the components, power, size, weight, etc., of the resettable integrator are less as compared to a conventional filter does not arise. A conventional final output filter has to be there anyway. Section V has shown that it might as well be applied directly to the demodulated output.

The only question remaining in the design of the lowpass output circuit of the data channel is the choice of the output filter -3db frequency, in terms of the number of filter components and the inevitable degradation of the original data source waveform that is inherent in any practical sampling time-division method.

GENERAL COMMENTS ON THE OVERALL DATA CHANNEL BANDWIDTH AND ITS S/N RATIO

From the foregoing, it can be seen that the output filtering needed to minimize noise due to multiplex (time-division) signaling techniques determines the S/N of the system (where N is now the vestigial signal techniques noise plus random nonsignal noise). This approach is generally applicable to all such data transmission systems, with one exception.

This exception concerns the demodulation process in any system that uses a carrier for the data amplitude, including systems where a coded pulse sequence denotes the data amplitude.

If the demodulation process produces significant, unwanted components and/or alters the statistical nature of the random nonsignal noise, then band-

width restriction prior to the demodulator, i.e., bandpass filters, in addition to the lowpass output filter may be superior in overall S/N. However, these cases are rare or can be made so using modern techniques.

Hence, in general, it is permissible to state that the necessary output filtering to reduce time-division signal handling technique noise without seriously affecting signal information determines the S/N. In this connection, for example, bandpass filters in a frequency-division system seldom determine the output noise, but they always determine the channel separation characteristics. Bandpass filters could be dispensed with, if other ways could be found for selecting the carriers of the channels together with their modulations, which also operate linearly over the total range of inputs due to all the channels.

SECTION VII

THE CHOICE OF THE SAMPLING RATE IN TIME-DIVISION SYSTEMS

The subject matter of this section is germane to this paper on the resettable integrator, because the performance of this device is rigidly related to the sample period, T. As noted in Section IV, it happens that the effective noise power (lowpass) bandwidth of the device is the same as that frequency from the data source which can be sampled at a $1/T$ rate without aliasing. This does not mean that aliasing of higher ("unimportant") frequencies from the data source will not occur, and that the device does not have any influence on this problem. The aliasing/sampling rate problem can only be settled at the data source. A method will be suggested for resolving the aliasing problem which is more logical than the present "rule-of-thumb." The latter method uses a sampling rate up to five times the highest data frequency of interest.

If the overall lowpass transfer response of a given data source at the sampling point is known, and if the designer-user of the receiver output data is willing to stipulate the vestigial level of this source response that will not affect the received data content or accuracy if it is aliased, then the required sampling rate is stipulated for the given data source. It is twice the frequency from the data source that gives the stipulated vestigial, or fractional, value that can be permitted to be aliased without ill effect.

Clearly, this definition compels attention to be paid to the data source roll-off lowpass response, which is as it should be. For example, if the roll-off is at -6 db/octave and if the maximum aliased fractional output value is 1 percent, then the required sampling rate is 200 times the nominal highest

data frequency (i.e., the half-power frequency of the data source).* To give the conventional sampling rate of five times the highest data frequency of interest for a 1 percent or less permitted aliasing level would require a data source roll-off rate of more than 24 db/octave.†

The trade-off choice is clearly up to the designer-user. In general, he must find data sources with inherently high roll-off attenuation rates; or he must use lowpass filters before sampling the data source; or he must put up with a high sampling rate or with aliasing. It is not good practice to assume that a vestigial response of the data source will not occur sufficiently often, or that such response does not matter. Furthermore, the above method of stipulating the sampling rate not only prevents excessive aliasing errors, but it also usually results in a more simple and effective design for the final output filter that removes the sampling noise.**

However, it is necessarily a corollary of the above, that most time-division systems must have inherently poorer S/N ratios than frequency-division systems handling the same data. There is usually no aliasing problem to consider in frequency-division systems, because there are many half-cycles (samples) of carrier frequency per highest data source frequency, even if the vestigial output condition is considered. Hence, the channel transmitter/

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* Using the normalized Butterworth lowpass filter curves of Figure 5, for convenience, a 1 percent response at a roll-off rate of 6 db/octave ($n = 1$) occurs at $f/f_0 = 100$. Therefore, the minimum sampling rate must be 200 f_0.

† Inversely, f/f_0 would now be 2.5 on Figure 5, which needs $n > 4$, or a roll-off $>$24 db/octave to attain a 1 percent response.

** The sampling rate usually has to be much higher than the source -3 db frequency, and hence the f/f_0 value for the channel output filter is automatically large. Thus, fewer poles are needed.

reception bandwidths can be limited only to those that accommodate the highest desired data frequency (according to the modulation method and desired accuracy). Consequently, a frequency division channel always has similar, desired signal and noise bandwidths. A time-division channel may have a much larger noise bandwidth than the desired signal if the roll-off rate of the data source is small and a high degree of protection against aliasing frequencies in the roll-off region is desired.

SECTION VIII

SUMMARY AND CONCLUSIONS

1. The effective noise power bandwidth of a resettable integrator inserted after the (perfect) demodulator in a time-division system is given by 1/2T, where T is the channel time-slot period. On a power transfer basis, the output S/N is 2 WT times the input, where W is the input bandwidth of "white" noise ($\geq 1/T$).

2. The effective noise power bandwidth of a conventional passive filter can be made substantially the same as the above (with very few reactances) provided the cutoff frequency is made equal to 1/2T.

3. The practical S/N value of the equivalent, conventional passive filter is superior to the resettable integrator, because its output does not contain the jitter and gate length variations of the resettable integrator that occur at the end of the sampling period.

4. Therefore, the use of an equivalent, conventional passive filter applied to the demodulator output avoids the necessity for sample and hold techniques.*

5. However, the same conventional filter can also be designed for a lower cutoff frequency which will minimize the sampling steps. On the other hand, the resettable integrator needs not only the sample and hold circuits but also has to follow these by a conventional filter of identical (lower cutoff frequency) design for minimizing the

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*However, these may be used to improve the channel gain

sampling steps to the same degree. In other words, the resettable integrator and its hold and sample circuits are all redundant.

6. In time-division systems, the channel noise bandwidth and signal response are always ultimately determined by the sampling rate necessary to avoid aliasing errors.

7. In time-division systems, the choice of sampling rate should be based on the data source vestigial response that is permissible without aliasing errors, whether or not such response is deemed to occur sufficiently often. At least, the aliasing error probability should be used to guide the choice.

8. The overall S/N of a frequency-division system is almost always better than a time-division system carrying the same desired highest data frequency, because it rarely has an aliasing problem.

9. This report also leads to further work in several areas.
   a. A general analysis of data source (transducer) response curves is needed to determine whether groups can be formed having similar roll-off rates and the derivation of practical sampling rate versus -3 db frequencies for such groups. Arising out of this, there should be recommendations for improving the roll-off rates of certain classes of transducers.
   b. An analysis is required of the inherent degradation of the data source analog waveform incurred by the sampling and interpolation processes in time-division systems, as used in telemetry, and applying the data found above.

c. An analysis is required concerning the effect of various demodulation processes on the statistical distribution of received noise over the frequency spectrum. Recommendations should be made on preferred processes.

d. An analysis, in practical terms, of the noise errors likely to occur using sample and hold receiving methods is needed.

e. Expansion of the present work would enable practical comparison of overall S/N of time-division and frequency-division telemetry systems.

APPENDIX

PROOF OF THE RESETTABLE INTEGRATOR h(t)

The fact that the average over T can be represented by the h(t) of Figure 3 can be seen as follows: from communication theory, we know that

$$e_o(t) = \int_{-\infty}^{\infty} h(x)\, e_{ii}(t-x)\, dx \quad ,$$

and if $h(x) = h(t) = 1/T$, $(0 \leq t \leq T)$, then

$$e_o(t) = \frac{1}{T} \int_{0}^{T} e_{ii}(t-x)\, dx \quad .$$

Let $y = t - x$, and $dy = -dx$. Then

$$e_o(t) = \frac{1}{T} \int_{t-T}^{t} e_{ii}(y)\, dy \quad .$$

Thus, h(t) performs a continuous integration over a range of time equal to T .

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Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* The MITRE Corporation Bedford, Massachusetts	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

THE USE OF RESETTABLE INTEGRATORS OR CONVENTIONAL FILTERS IN TIME-DIVISION DATA TRANSMISSION SYSTEMS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

N/A

5. AUTHOR(S) *(Last name, first name, initial)*

Hadfield, Bertram M.
Herman, Ronald T.

6. REPORT DATE May 1966	7a. TOTAL NO. OF PAGES 40	7b. NO. OF REFS 3
8a. CONTRACT OR GRANT NO. AF19(628)-5165 b. PROJECT NO. 705C c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) ESD-TR 65-409 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* TM-3836; TM-3836, Rev. 1	

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Directorate of Aerospace Instrumentation, Electronic Systems Division, L. G. Hanscom Field, Bedford, Mass.

13. ABSTRACT

This report establishes the "effective noise power bandwidths" and "noise reduction factors" of a resettable integrator and of conventional passive lowpass filters. It then discusses the use of the resettable integrator with its necessary sample and hold circuit, as compared to the use of a passive lowpass filter which follows the receiver demodulator in time-division data transmission systems, such as telemetry links. It is concluded that since the overall lowpass bandwidth is determined by the conventional output lowpass filter necessary for removing the sampling "noise" (quantum steps), then any other filter device is redundant, provided the demodulator does not sensibly alter the statistical properties of the input random noise.

As a germane and highly important side issue, a more logical method for selecting the sampling rate in time-division systems is also discussed.

DD FORM 1 JAN 64 1473

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Integrators						
Time-Division Techniques						

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