PRELIMINARY FLUTTER ANALYSIS

VOLUME III   CONTROL SURFACES

LIFT FAN FLIGHT RESEARCH AIRCRAFT PROGRAM

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PRELIMINARY FLUTTER ANALYSIS
VOLUME III - CONTROL SURFACES

XV-5A Lift Fan
Flight Research Aircraft
Contract DA 44-177-TC-715

December 1965

ADVANCED ENGINE & TECHNOLOGY DEPARTMENT
GENERAL ELECTRIC COMPANY
CINCINNATI, OHIO 45215
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1.0 SUMMARY

The overall preliminary theoretical flutter analysis of the Ryan XV-5A Lift Fan Aircraft is presented in three volumes, each under separate cover. This is Volume III which reports the investigation of the flutter characteristics of the control surfaces. Volumes I and II detail the analysis of the wing and empennage, respectively.

Equations of motion and stability equations are formulated using lumped spring-mass idealization and incompressible strip theory aerodynamics; control system elasticity and inertia are taken into account. One altitude only, sea level, is investigated. Solutions of the stability equations are obtained, giving flutter frequencies and complex damping coefficients required to maintain flutter as functions of flutter velocities for various values of stiffness parameter, both with and without aerodynamic balancing (centering) of the control surface.

Both a symmetric and an antisymmetric analysis of the aileron-aileron flight tab - aileron control system are made. An analysis of the rudder - rudder trim tab - rudder control system is included and a symmetric analysis of the horizontal stabilizer - elevator - elevator control system is also made.

On the basis of preliminary input information used in the analyses herein, increased stiffness of the aileron flight tab restraint is required (the symmetric aileron condition is critical), and the stiffness of the rudder trim tab actuator should be such that the uncoupled rigid body tab rotation frequency is at least 50 cps, in order to insure a flutter-free airplane within the established flight envelope. The antisymmetric aileron condition and elevator are flutter-free.
This report presents a portion of the preliminary flutter analysis of the U.S. Army XV-5A Lift Fan Research Aircraft. Volumes I and II of the preliminary flutter analysis are concerned with the wing and with the empennage, respectively. The report herein is Volume III.

The XV-5A is a V/STOL aircraft designed for research flight testing of the General Electric X353-5 lift fan propulsion system. A three-view drawing and a cutaway drawing of the aircraft are presented in Volume I. The XV-5A also features high subsonic conventional flight operation. It has a basic design gross weight of 9200 pounds, a limit dive speed of 500 KEAS (q = 850 psf) and corresponding 0.90 maximum Mach number. Thus, only one altitude, sea level, is investigated herein.

The analysis is based on incompressible strip theory aerodynamics with a lumped spring-mass system idealization, taking into account control system elasticity and inertia.

The purpose of the analysis is to calculate flutter velocities where they exist, for various values of stiffness parameters that reflect design unknowns, both with and without aerodynamic balancing (centering) of the control surfaces. Comparison of results with limit dive speed requirements then affords a basis for design changes to ensure a flutter-free aircraft, or for assurance that current design is flutter safe.

The report is subdivided into two main parts, Sections 3.0 and 4.0. Section 3.0 presents a general approach to the analysis method, developed for a control surface - flight tab system, both symmetric and antisymmetric, and also developed for a symmetric parent surface - control surface system. Equations of motion and of stability are formulated in general form, the aerodynamics are explained, as are mass terms, stiffness terms, coordinates, and basic assumptions, and the general characteristics of the solutions are discussed.

Section 4.0 shows the application of the method of approach to the specific cases at hand:

1. The aileron and aileron flight tab system (Both symmetric and antisymmetric aileron analyses are made.)
2. The rudder and rudder tab system
3. The horizontal stabilizer and elevator system (Only a symmetric analysis is made, since antisymmetric elevator flutter is clearly not critical; this conclusion is discussed in Section 4.0.)

In Section 4.0, for each of the three cases above, schematics of the control system, of the aerodynamic balancing arrangement, and of the idealization are given, and plan views of the surfaces are shown. Values of the numerous input parameters are given, and results are presented both tabularly and graphically. The results are interpreted, and conclusions are drawn.
3.0 METHOD OF APPROACH

The equations of motion for a flutter investigation of a control surface and flight tab system, which applies to the lateral control system (CTOL mode) of the XV-5A aircraft, are developed for both a symmetric and antisymmetric analysis. In general, the procedure follows the development of the governing equations, as presented in Reference 1. The same equations of motion also apply to the directional control system (CTOL mode) of the XV-5A aircraft, since the rudder trim tab is one of zero gearing and thus may be treated as a flight tab with nonexistent connections between it and the pedal.

A similar development, for the symmetric case only but with different components, (a parent surface and a control surface) is the case of the longitudinal system (CTOL mode) of the XV-5A aircraft, and is presented separately.

Specific details applicable to each individual analysis are discussed in Section 4.0
3.1 CONTROL SURFACE - FLIGHT TAB SYSTEM, SYMMETRIC AND ANTSYMMETRIC

The degrees of freedom considered are:

\[ \beta = \text{Control surface rotation} \]

\[ \delta = \text{Control surface flight tab rotation, relative to the control surface} \]

\[ \gamma = \text{Control stick or pedal rotation} \]

Lagrange's equations of motion for the system governed by the above degrees of freedom are:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\beta}} \right) + \frac{\partial U}{\partial \beta} = Q_\beta \\
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\delta}} \right) + \frac{\partial U}{\partial \delta} = Q_\delta \\
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\gamma}} \right) + \frac{\partial U}{\partial \gamma} = 0
\]

where

- \( T \) is the kinetic energy of the system
- \( U \) is the potential energy (internal strain energy) of the system
- \( Q_\beta, Q_\delta \) are the generalized aerodynamic forces applied to the system

The kinetic energy of the system is

\[
T = \frac{1}{2} \left( I_\beta \dot{\beta}^2 + 2 \dot{\beta} \dot{\delta} p_{\beta \delta} + I_\delta \dot{\delta}^2 \right) + \frac{1}{2} (J \dot{\gamma}^2)
\]

where

- \( I_\beta \) = Mass moment of inertia of control surface including tab, about control surface hinge line
- \( I_\delta \) = Mass moment of inertia of flight tab about its hinge line
\[ P_{\beta \delta} = \text{Mass product of inertia of flight tab about control surface hinge line and tab hinge line} \]

\[ = I_{\delta} + S_{\delta} l_{t} \]

where:

\[ S_{\delta} = \text{Static unbalance of flight tab about its hinge line} \]

\[ l_{t} = \text{Distance between control surface and tab hinge lines, measured perpendicular to control surface hinge line} \]

\[ J = \text{Mass moment of inertia of control stick or pedal, including linkages, about control stick or pedal axis of rotation} \]

The potential energy of the system is derived from the displacements of concentrated springs within the control surface - flight tab systems. A symmetric or antisymmetric analysis is determined by the springs that participate in the motion of the control surface and flight tab. For present purposes, the potential energy is, in general terms,

\[ U = \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} S_{ij} q_{i} q_{j} \]

where

\[ q_{i}, q_{j} = \text{The degrees of freedom considered: } \beta, \delta, \gamma \]

\[ S_{ij} = \text{Constants generated through the rates of the concentrated springs and linkage arms through which they act. These constants are the elements of a stiffness matrix.} \]

The virtual work done by the air forces is

\[ \delta W_{i} = Q_{i} \delta q_{i} \]

where

\[ \delta W_{i} = \text{The virtual work for the } i\text{th degree of freedom} \]
\[ Q_1 = \text{The total generalized aerodynamic force in the } i^{th} \text{ degree of freedom} \]
\[ \delta q_1 = \text{The virtual displacement of the } i^{th} \text{ degree of freedom} \]

The symbol \( \delta \) means "a variation of".

Consider the following displacements:

\[ d_1 = \text{Angle of rotation of control surface about its hinge line (plus trailing edge down)} \]
\[ d_2 = \text{Angle of rotation of flight tab about the tab hinge line, relative to the control surface (plus trailing edge down)} \]

and let

\[ \begin{bmatrix} d \end{bmatrix} = [\varphi] [q] \]  \hspace{1cm} (7)

where

\[ [\varphi] = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix} \] \hspace{1cm} (8)

The displacements expressed by Equation (7) represent rigid body displacements about the hinge lines of the control surface and flight tab. These displacements normally are about swept axes, and we require rotations about axes perpendicular to the free stream. Therefore,

\[ \begin{bmatrix} \beta' \\ \delta' \end{bmatrix} = \begin{bmatrix} \cos \psi_{c} & 0 \\ 0 & \cos \phi_{t} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \] \hspace{1cm} (9)

where:

\[ \beta' = \text{Angle of rotation of control surface about an axis perpendicular to the free stream} \]
\[ \delta' = \text{Angle of rotation of flight tab about an axis perpendicular to the free stream} \]
\[ \psi_{c} = \text{Sweepback angle of control surface hinge line} \]
\( \psi_{L_t} = \) Sweepback angle of flight tab hinge line

Combining Equations (7), (8) and (9),

\[
\begin{bmatrix}
\beta' \\
\delta'
\end{bmatrix} = \begin{bmatrix}
\cos \psi_{L_c} & 0 \\
0 & \cos \psi_{L_t}
\end{bmatrix} \begin{bmatrix}
1.0 \\
1.0
\end{bmatrix} \begin{bmatrix}
\beta \\
\delta
\end{bmatrix}
\]

(10)

Using two-dimensional oscillatory aerodynamic coefficients and in the rotation of References 2 and 3 (Section 6.0), the aerodynamic moments acting on the control surface and flight tab, per unit span are

\[
\begin{bmatrix}
T' \\
Q'
\end{bmatrix} = \frac{\pi \rho \omega^2 b^4 \cos \psi_{c/4}}{2} \begin{bmatrix}
T\beta & T\delta \\
Q\beta & Q\delta
\end{bmatrix} \begin{bmatrix}
\beta' \\
\delta'
\end{bmatrix}
\]

(11)

where

- \( \rho = \) Mass density of air at altitudes under consideration
- \( \omega = \) Circular frequency of oscillation
- \( b = \) Half-chord length
- \( \psi_{c/4} = \) Sweepback angle of quarter-chord line of parent surface
- \( T\beta, T\delta, Q\beta, Q\delta = \) Oscillatory aerodynamic coefficients (nondimensional) in the rotation of References 2 and 3, in general, complex numbers. They contain no terms to account for the effects of aerodynamic balance; i.e., nose sections are assumed simply rounded. Corrections to account for the internal balancing system for the control surfaces used in the XV-5A aircraft are discussed on Page 11.

From the principle of virtual work, the work done by the moments of Equation (11) in undergoing a virtual displacement is

\[
\delta W = \int_0^S (T' \delta \beta' + Q' \delta \delta') \, dy
\]

(12)
where $S$ is the semispan parallel to the $y$-axis which is perpendicular to the free stream. Primes indicate "in the stream direction".

In matrix notation

$$
\delta W = \int_0^S \begin{vmatrix} 
\delta \beta' & \delta \delta' 
\end{vmatrix} \begin{bmatrix} T' \\ Q' \end{bmatrix} \, dy
$$

(13)

Since, from Equation (10) we have

$$
\begin{bmatrix} 
\beta' \\
\delta' 
\end{bmatrix} = \begin{bmatrix} 
\cos \psi_{w,c} & 0 \\
0 & \cos \psi_{w,t} 
\end{bmatrix} \begin{bmatrix} 
\beta \\
\delta 
\end{bmatrix}
$$

and therefore

$$
\begin{bmatrix} 
\delta \beta' \\
\delta \delta' 
\end{bmatrix} = \begin{bmatrix} 
\cos \psi_{w,c} & 0 \\
0 & \cos \psi_{w,t} 
\end{bmatrix} \begin{bmatrix} 
\delta \beta \\
\delta \delta 
\end{bmatrix}
$$

or, transposing,

$$
\begin{bmatrix} 
\delta \beta' & \delta \delta' 
\end{bmatrix} = \begin{bmatrix} 
\cos \psi_{w,c} & 0 \\
0 & \cos \psi_{w,t} 
\end{bmatrix}
$$

(14)

Combining Equations (11), (13) and (14), and noting that $\beta$, $\delta$ and their variations are not functions of $y$, the virtual work is

$$
\delta W = \int_0^S \begin{vmatrix} 
\pi \rho \omega_2 \cos \psi_{c/4} & 0 \\
\cos \psi_{w,c} & \cos \psi_{w,t} 
\end{vmatrix} \begin{bmatrix} 
T' \\
Q' 
\end{bmatrix} \, dy
$$

or,

$$
\begin{bmatrix} 
T' \\
Q' 
\end{bmatrix} = \begin{bmatrix} 
\cos \psi_{w,c} & 0 \\
0 & \cos \psi_{w,t} 
\end{bmatrix} \begin{bmatrix} 
\beta \\
\delta 
\end{bmatrix}
$$

(15)
therefore, since \( \delta W = \sum_{i=1}^{3} \delta W_i = \sum_{i=1}^{3} Q_i \delta q_i = Q_\beta \delta \beta + Q_\delta \delta \delta + 0 \cdot \delta \gamma \)

\[
= \begin{bmatrix} \delta \beta \\ \delta \delta \end{bmatrix} \begin{bmatrix} Q_\beta \\ Q_\delta \end{bmatrix}
\]

the column of two-dimensional, generalized aerodynamic forces is

\[
\begin{bmatrix} Q_\beta \\ Q_\delta \end{bmatrix} = \pi \rho \omega^2 \cos \frac{\psi}{4} \int_{S} \begin{bmatrix} \cos \psi_{c} \\ 0 \\ 0 \cos \psi_{c} \end{bmatrix} \begin{bmatrix} T_\beta \\ T_\delta \end{bmatrix} d\gamma
\]

\[
\begin{bmatrix} \cos \psi_{c} \\ 0 \\ \cos \psi_{c} \end{bmatrix} \int_{S} \begin{bmatrix} T_\beta \\ T_\delta \end{bmatrix} d\gamma
\]

\[
(16)
\]

These generalized forces are the right-hand sides of the Lagrange's equations of motion, Equations (1) and (2).

We next treat the required correction to the generalized force \( Q_\beta \) as a result of internal aerodynamic balance of the control surface, maintaining zero aerodynamic balance for the flight tab. Using References 1 and 4, the incremental two-dimensional aerodynamic moment acting on the control surface, per unit span is

\[
\Delta T' = -\pi \rho \omega^2 b^4 \cos \frac{\psi}{4} \begin{bmatrix} \bar{A} \\ \bar{B} \end{bmatrix} \begin{bmatrix} \beta' \\ \delta' \end{bmatrix}
\]

\[
(17)
\]

where

\[
\bar{A} = F_{BB} K \left( \frac{\Delta C_{H_c}}{\delta c} \right) \text{Exp} C(c, g, k)
\]

\[
\bar{B} = \left( \frac{\Delta C_{H_c}}{\delta c} \right) 3
\]
\[ \mathbf{f} = F_{BB} K \frac{\left( \frac{\Delta C_{H e} \delta}{t/3} \right)}{\left( \frac{\Delta C_{H e} \delta}{t/3} \right)} \text{Exp. (c, d, k)} \]

and

\( F_{BB} \) is an auxiliary function defining the internal aerodynamic balance geometry and is equal to [c(e' - e'') - \frac{(e' - e'')(e' + e'')}{2}]. (See Figure 3.)

\( K \) is a pressure recovery factor to account for leakage across the internal balance arrangement. \( \left( \frac{\Delta C_{H e} \delta}{c} \text{Exp.} \right) \), \( \left( \frac{\Delta C_{H e} \delta}{c} \text{Exp.} \right) \) are control surface, steady-state hinge moment derivatives, experimental and theoretical, respectively (aerodynamic balance contribution through control surface deflection).

\( \left( \frac{\Delta C_{H e} \delta}{c} \text{Exp.} \right) \), \( \left( \frac{\Delta C_{H e} \delta}{c} \text{Exp.} \right) \) are control surface, steady-state hinge moment derivatives, experimental and theoretical, respectively (aerodynamic balance contribution through flight tab deflection).

\( C(c, g, k) \) is a dimensionless coefficient similar to \( T_{\delta} \) (Equation (11)) and is a function only of airfoil and control surface geometry.

\( D(c, d, k) \) is a dimensionless coefficient similar to \( T_{\delta} \) (Equation (11)) and is a function only of airfoil, control surface and flight tab geometry.

The elements of the row matrix in Equation (17) are, in general, complex numbers. Now, again using the principle of virtual work, the work done by the internal balance forces in undergoing a virtual displacement is

\[ \delta W_{\beta} = \int_{0}^{t} \Delta T' \delta \beta' \, dy \]  

(18)

where \( t \) is the semispan of the internal aerodynamic balance arrangement (i.e., simple nose overhang) parallel to the \( y \) axis, which axis is perpendicular to the free stream.
Combining Equations (10), (14), (17) and (18), the virtual work of internal balance forces is written:

\[
\delta W_{\beta} = -\delta \beta \left[ \pi \rho \omega^2 \cos \frac{c}{4} \cos \psi_c \int_0^t b^4 \begin{bmatrix} \bar{A} & \bar{B} \end{bmatrix} \right]
\begin{bmatrix}
\cos \psi_{c} & 0 \\
0 & \cos \psi_{t}
\end{bmatrix} dy \begin{bmatrix} \beta \\
\delta
\end{bmatrix}
\]

(19)

Since \( \delta W_{\beta} = \Delta Q_{\beta} \delta \beta \) in this case of correction for control surface internal aerodynamic balance, the incremental two-dimensional generalized aerodynamic force to be added to \( Q_\beta \) in Equation (16) is

\[
\Delta Q_{\beta} = -\pi \rho \omega^2 \cos \frac{c}{4} \cos \psi_c \int_0^t b^4 \begin{bmatrix} \bar{A} & \bar{B} \end{bmatrix} \right]
\begin{bmatrix}
\cos \psi_{c} & 0 \\
0 & \cos \psi_{t}
\end{bmatrix} dy \begin{bmatrix} \beta \\
\delta
\end{bmatrix}
\]

(20)

Addition of the increment \( \Delta Q_{\beta} \) from Equation (20) to \( Q_\beta \) in Equation (16) gives the case of the XV-5A aileron and rudder systems, namely, internal aerodynamic balance for the control surface and no aerodynamic balance for the tab.

The generalized aerodynamic forces are expressed simply in 3x1 matrix form as

\[
\begin{bmatrix}
Q_\beta \\
Q_\delta \\
0
\end{bmatrix} = \omega^2 \begin{bmatrix}
I_1'_{11} & I_1'_{12} & 0 \\
I_2'_{21} & I_2'_{22} & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix} \beta \\
\delta \\
\gamma
\end{bmatrix}
\]

(21)

where the \( I'_{ij} \) elements, in general complex numbers contain both the contributions of the basic surface (control aft of the hinge line) and of the internal aerodynamic balance (given by \( \Delta Q_{\beta} \)). \( Q_\beta \) and \( Q_\delta \) are now corrected values.
For convenience of calculations, the I' matrix of Equation (21) is decomposed into the sum of two matrices, the first containing the terms due to the basic surfaces (control surface and unbalanced flight tab), and the second containing the terms due to the internal aerodynamic balancing arrangement of the control surface.

Thus, Equation (21) becomes

\[
\begin{pmatrix}
Q_\beta \\
Q_\delta \\
0
\end{pmatrix}
= \omega^2
\begin{bmatrix}
A_{11} & A_{12} & 0 \\
A_{21} & A_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
- \begin{bmatrix}
B_{11} & B_{12} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
\beta \\
\delta \\
\gamma
\end{pmatrix}
\]

(22)

where

\[
A_{11} = \pi \rho \cos \psi_c / 4 \int_{\text{Control Surface}} b^4 \cos^2 \psi_c \cos \psi_t \left( \frac{C \delta}{h_c} \right) \exp \left( \frac{T_\beta dy}{3} \right)
\]

\[
A_{12} = \pi \rho \cos \psi_c / 4 \int_{\text{Tab}} b^4 \cos^2 \psi_c \cos \psi_t \left( \frac{C \delta}{h_c} \right) \exp \left( \frac{T_\delta dy}{3} \right)
\]

\[
A_{21} = \pi \rho \cos \psi_c / 4 \int_{\text{Tab}} b^4 \cos^2 \psi_c \cos \psi_t \left( \frac{C \delta}{h_c} \right) \exp \left( \frac{Q_\beta dy}{3} \right)
\]

\[
A_{22} = \pi \rho \cos \psi_c / 4 \int_{\text{Tab}} b^4 \cos^2 \psi_c \cos \psi_t \left( \frac{C \delta}{h_c} \right) \exp \left( \frac{Q_\delta dy}{3} \right)
\]
B_{11} = \pi \rho \cos \psi_{c}/4 \int_{\text{Aero. Bal.}} b^4 \cos^2 \psi_{c} F_{BB} K \left( \frac{\Delta C_{H c}}{\delta_{c}} \right) \text{Exp.} C(c, g, k) dy

B_{12} = \pi \rho \cos \psi_{c}/4 \int_{\text{Aero. Bal.}} b^4 \cos \psi_{c} \cos \psi_{t} F_{BB} K \left( \frac{\Delta C_{H c}}{\delta_{t}} \right) \text{Exp.} D(c, d, k) dy

Note that the basic surface expressions \( A_{11}, A_{12}, A_{21} \) and \( A_{22} \) have been modified by hinge moment correction factors \( C_{H c}/\text{Exp.}/ (C_{H c}/\text{Exp.}) \), etc. in order that these terms be consistent with the expressions for the internal aerodynamic balance arrangement.

In order to evaluate the elements of \([A]\) and \([B]\) given above, the control surface, flight tab and internal balance are divided into equal spanwise segments appropriate to the system under investigation, and all integration is performed numerically using the trapezoidal rule.

The equations of motion for a control surface-flight tab flutter analysis are written in the familiar \( F = ma \) form by substituting Equations (4), (5) and (21) into Lagrange's Equations (1), (2), and (3).

Expressing the results in matrix form,

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \rho \beta \delta & 0 \\
0 & 0 & J
\end{bmatrix}
\begin{bmatrix}
\ddot{\beta} \\
\ddot{\delta} \\
\ddot{\gamma}
\end{bmatrix}
+
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\begin{bmatrix}
\beta \\
\delta \\
\gamma
\end{bmatrix}
-
\omega^2
\begin{bmatrix}
I_{11} & I_{12} & 0 \\
I_{21} & I_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\beta \\
\delta \\
\gamma
\end{bmatrix}
=
0
\]

(23)
We assume harmonic motion \( q_i = q_i^0 e^{i\omega t} \) and introduce a reference spring rate \( \bar{K} \), which is merely a scaling factor for computing convenience. Further, we formulate the variable \( \Omega \), which is the basic unknown,

\[
\Omega = \frac{\bar{K}}{\omega^2} (1 + ig)
\]

(24)

Note that \( \Omega \) introduces a damping force that is proportional to displacement and in phase with the velocity, with \( g \) being a measure of this damping force, and is the so-called complex damping coefficient.

Substitution of \( q_i = q_i^0 e^{i\omega t} \) and Equation (24) into Equation (23) gives

\[
\begin{bmatrix}
I & P & 0 \\
\overline{P} & \overline{I} & 0 \\
0 & 0 & J
\end{bmatrix}
\begin{bmatrix}
\Omega \\
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\begin{bmatrix}
I'_{11} & I'_{12} & 0 \\
I'_{21} & I'_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\beta' \\
\delta' \\
\gamma'
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

(25)

where it is now understood that the \( S_{ij} \) elements of the matrix derived from the potential energy expression are divided by the scaling factor \( \bar{K} \).

Upon expansion of the characteristic determinant, the determinant of the summed 3 x 3 matrix in Equation (25), there results a stability equation of the form:

\[
A\Omega^3 + B\Omega^2 + C\Omega + D = 0
\]

(26)

where \( A \) is real and \( B, C \) and \( D \) are, in general, complex.

The degree of the above equation depends implicitly on the type of analysis, i.e. symmetric or antisymmetric, being performed.

The solution of Equation (26) gives, in general, the complex roots \( \Omega \) from which \( g \) and \( \omega \) are obtained. Negative \( g \) is the stable state, and positive \( g \) is the unstable (flutter) state. \( \omega \) is the circular flutter frequency. \( [I'] \) in Equation (25) is chosen for several prescribed values of the dimensionless parameter \( V/b_0 \omega \), the reduced velocity. \( b_0 \) is some reference half chord. Thus, solutions of Equation (26) give plots of \( g \) vs. \( \omega \), and, by substitution of \( \omega \) into corresponding \( V/b_0 \omega \) values, plots of
g vs V are obtained. These are the basic stability relationships. The smallest nonzero value of V at g = 0 is the flutter velocity, V_f. Also, cross-plots of ω vs V are useful, showing modal coalescence to the unstable state. Because of the three (at most) complex roots Ω from Equation (26), for each analysis, there are no more than three branches to the g - V and ω - V plots.

Setting [I'] = [0] and g = 0 in Equation (25), the uncoupled natural frequency f_β = ω_β/2π, control surface rotation, may be calculated, for interpretive purposes, from

\[ I_β - \frac{K}{ω_β^2} S_{11} = 0 \]  

(27)

The uncoupled natural frequency f_δ = ω_δ/2π, flight tab rotation, may be calculated from

\[ I_δ - \frac{K}{ω_δ^2} S_{22} = 0 \]  

(28)

and the uncoupled natural frequency f_γ = ω_γ/2π, control stick or pedal rotation, from

\[ J - \frac{K}{ω_γ^2} S_{33} = 0 \]  

(29)
3.2 PARENT SURFACE-CONTROL SURFACE SYSTEM, SYMMETRIC

The degrees of freedom considered are

\[ \alpha = \text{Parent surface rotation} \]
\[ \beta = \text{Control surface rotation relative to parent surface} \]
\[ \gamma = \text{Control stick rotation} \]

The general analysis herein is applied in Paragraph 4.3 to the horizontal stabilizer-elevator system of the XV-5A aircraft. All of the general analysis made in Paragraph 3.1 applies for the system under consideration here, with some important differences:

1. The generalized coordinates \( \beta \) and \( \delta \) in Paragraph 3.1 are replaced by the generalized coordinates \( \alpha \) and \( \beta \), respectively, \( \alpha \) referring to the generalized coordinate "parent surface rotation". The generalized coordinate "tab rotation" does not apply to the present case, since there is no tab.

2. The kinetic energy, \( T \), of the system is given by an equation of form similar to Equation (4) in Paragraph 3.1, but the definitions of terms are slightly modified and the symbol \( J \) is replaced by \( J_0 \) with a different definition:

\[
T = \frac{1}{2} \left( I_\alpha \dot{\alpha}^2 + 2 \dot{\alpha} \dot{\beta} P_{\alpha \beta} + I_\beta \dot{\beta}^2 \right) + \frac{1}{2} (J_0 \dot{\gamma}^2) \tag{30}
\]

where

\[ I_\alpha = \text{Mass moment of inertia of half the parent surface, including the half of entire control surface, about the pivot axis of the parent surface} \]
\[ I_\beta = \text{Mass moment of inertia of one control surface about its hinge line. It is assumed that the complete system contains two control surfaces, a right-hand and a left-hand one} \]
\[ P_{\alpha \beta} = \text{Mass product of inertia of one control surface about parent surface pivot axis and control surface hinge line} \]
\[ = I_\beta + S_\beta \ell \]
where:

\[ S = \text{Static unbalance of one control surface about its hinge line} \]

\[ l = \text{Streamwise distance between parent surface pivot axis and control surface hinge line} \]

\[ J_0 = \text{Effective mass moment of inertia of half the control circuit and control stick, about control stick axis of rotation} \]

3. The generalized forces and, in particular, the aerodynamic matrix \([F']\) in Equation (23) are reformulated to take into account normal-to-plane motion of the parent surface.

The virtual work done by the air forces is as before:

\[ \delta \omega_i = Q \delta q_i \quad i = 1, 2, 3 \]

Consider the following displacements at a representative station:

- \( d_1 = \) Deflection of the elastic axis of the parent surface in the \( z \) (normal-to-plane) direction (plus down)
- \( d_2 = \) Angle of rotation of parent surface about its elastic axis (plus leading edge up)
- \( d_3 = \) Angle of rotation about an axis perpendicular to the elastic axis of the parent surface (plus when tip of surface is depressed); \( d_3 \) is a bending displacement
- \( d_4 = \) Angle of rotation of control surface about its hinge line, relative to the parent surface (plus trailing edge down)

And let

\[ \{d\} = [\varphi] \{q\} \quad \text{as before} \quad (31) \]

where

\[ \{q\} = \{\alpha\}_{\beta} \quad \text{here} \]
and

\[
\begin{bmatrix}
\phi_{11} & 0 \\
\phi_{21} & 0 \\
\phi_{31} & 0 \\
0 & \varphi_{41}
\end{bmatrix}
\]  \hspace{1cm} (32)

Note that there are no generalized aerodynamic force components due to \( \gamma \).

From the geometry and definition of the displacements \( d_i \),

\[
\begin{align*}
\phi_{11} &= (1.0) g \\
\phi_{21} &= (1.0) \cos \psi_{e.a.} \\
\phi_{31} &= (1.0) \sin \psi_{e.a.} \\
\phi_{41} &= 1.0
\end{align*}
\]

where

\[
\begin{align*}
g &= \text{Distance from parent surface pivot axis to its elastic axis} \\
& \quad \text{in the stream direction (plus aft of pivot axis); this} \\
& \quad \text{dimensional variable is not the same as expressed in} \\
& \quad \text{Equation (22)}
\end{align*}
\]

\[
\begin{align*}
\psi_{e.a.} &= \text{Sweepback angle of parent surface elastic axis}
\end{align*}
\]

Substituting element values into Equation (32),

\[
\begin{bmatrix}
g & 0 \\
\cos \psi_{e.a.} & 0 \\
\sin \psi_{e.a.} & 0 \\
0 & 1.0
\end{bmatrix}
\]  \hspace{1cm} (33)

Since the deflections \( d_i \), at this point, are deflections relative to the elastic axis (except for \( d_4 \), which is relative to the control surface hinge line) we transform the four displacements to axes parallel and perpendicular to the free stream at the representative station.
Let
\[
\begin{align*}
d_1' &= \text{Vertical displacement of the parent surface at its elastic axis} \\
d_2' &= \text{Angle of rotation of parent surface about an axis perpendicular to the free stream, through the elastic axis of the parent surface (pitch angle of parent surface)} \\
d_3' &= \text{Angle of rotation of parent surface about an axis parallel to the free stream, through the elastic axis of the parent surface (roll angle of parent surface)} \\
d_4' &= \text{Angle of rotation of control surface about an axis perpendicular to the free stream, through the control surface hinge line; the rotation being relative to } d_2' \text{ (pitch angle of control surface rotation relative to parent surface)}
\end{align*}
\]

From the geometry and definitions of the displacements $d_1'$, 

\[
\begin{pmatrix}
d_1' \\
d_2' \\
d_3' \\
d_4'
\end{pmatrix} = \begin{pmatrix}
1.0 & 0 & 0 & 0 \\
0 & \cos \psi & \sin \psi & 0 \\
0 & -\sin \psi & \cos \psi & 0 \\
0 & 0 & 0 & \cos \psi_{hc}
\end{pmatrix}
\begin{pmatrix}
d_1' \\
d_2' \\
d_3' \\
d_4'
\end{pmatrix}
\]

where $\psi_{hc}$ = sweepback angle of the control surface hinge line. Since the air forces are written for displacements of the quarter chord, maintaining rotations relative to free stream axes, we transform the vertical displacement at the elastic axis to that at the quarter-chord line, still at the same representative station.

Let
\[
\begin{align*}
h &= \text{Vertical displacement of parent surface at the quarter-chord line. } h \text{ is a linear combination of } d_1' \text{ and } d_2'. \\
\alpha' &= \text{Angle of rotation of parent surface about axis perpendicular to free stream direction, axis passing through the quarter chord point (pitch angle of parent surface); thus } \alpha' = d_2'
\end{align*}
\]
\[ \beta' = \text{Angle of rotation of control surface about axis perpendicular to free stream direction, relative to parent surface; axis passing through the hinge line (pitch angle of control surface relative to parent surface); thus } \beta' = \frac{d'}{4} \]

Note that neither \( h \), \( \alpha' \) nor \( \beta' \) is a function of \( d' \).

The required transformation is

\[
\begin{pmatrix}
 h \\
 \alpha' \\
 \beta'
\end{pmatrix} =
\begin{bmatrix}
1.0 & -a & 0 & 0 \\
0 & 1.0 & 0 & 0 \\
0 & 0 & 0 & 1.0
\end{bmatrix}
\begin{pmatrix}
d'_{1} \\
d'_{2} \\
d'_{3} \\
d'_{4}
\end{pmatrix}
\] (35)

where \( a = \text{streamwise distance from quarter-chord line to elastic axis at representative station.} \)

Combining Equations (31), (32), (33), (34) and (35), and multiplying out gives

\[
\begin{pmatrix}
 h \\
 \alpha' \\
 \beta'
\end{pmatrix} =
\begin{bmatrix}
g-a & 0 \\
1.0 & 0 \\
0 & \cos \psi_c
\end{bmatrix}
\begin{pmatrix}
\alpha \\
\beta
\end{pmatrix}
\] (36)

Equation (36) could have been written out directly from considerations of the rigid body degrees of freedom involved; the preceding was developed from the XV-5A empennage flutter analysis and used herein for continuity.

Using two-dimensional aerodynamic oscillatory coefficients in the notation of References 2 and 3 and assuming, at this point, zero aerodynamic balance for the control surface, the aerodynamic forces acting per unit span are

\[
\begin{pmatrix}
 L'b \\
 M' \\
 T'
\end{pmatrix} =
\begin{pmatrix}
\pi \rho \omega^2 b^4 \cos \psi_c/4 \\
 L_h \alpha \beta \\
 M_h \alpha \beta \\
 T_h \alpha \beta
\end{pmatrix}
\begin{pmatrix}
h/b \\
\alpha' \\
\beta'
\end{pmatrix}
\] (37)

From the principle of virtual work,

\[
\delta W = \int_{0}^{S} \left[ (L'b) \delta \left( \frac{h}{b} \right) + (M' \delta \alpha' + T' \delta \beta') \right] dy
\] (38)
where $S$ is again the semispan parallel to the $y$-axis, which is perpendicular to the free stream. Primes indicate "in the stream direction".

In matrix notation:

$$
\delta W = \int_0^S \left( \delta \left( \frac{h}{b} \right) \right) \delta \alpha' \delta \beta' \left[ \begin{array}{c}
L' b \\
M' \\
T'
\end{array} \right] dy
$$

Nondimensionalizing the $h$ deflection and applying a variation in the variables of Equation (36), we have

$$
\begin{bmatrix}
\delta \left( \frac{h}{b} \right) \\
\delta \alpha' \\
\delta \beta'
\end{bmatrix} =
\begin{bmatrix}
g - a \\
0 \\
1.0 \\
0 \\
0 \cos \psi_c
\end{bmatrix}
\begin{bmatrix}
\delta \alpha \\
\delta \beta
\end{bmatrix}
$$

or, transposing,

$$
L \delta \left( \frac{h}{b} \right) \delta \alpha' \delta \beta' =
L \delta \alpha \delta \beta
\begin{bmatrix}
g - a \\
0 \\
1.0 \\
0 \cos \psi_c
\end{bmatrix}
$$

Combining Equations (36), (37), (39) and (40) and noting that $\alpha$ and $\beta$ and their variations are not functions of $y$, the virtual work is

$$
\delta W = \delta \alpha \delta \beta
\begin{bmatrix}
\pi \rho \omega^2 \cos \psi_c / 4 \\
\int_0^S b^4 \left[ \begin{array}{c}
g - a \\
0 \\
1.0 \\
0 \cos \psi_c
\end{array} \right] dy
\end{bmatrix}
$$

Since $\delta W = L \delta \alpha \delta \beta \left\{ Q_\alpha \right\}$, from the basic definition of virtual work, the column of two-dimensional, generalized aerodynamic forces is
\[
\begin{pmatrix}
Q_α \\
Q_β
\end{pmatrix} = \pi \rho \omega^2 \cos \psi_c/4 \int_0^b 4 \begin{bmatrix}
g-a & 1.0 & 0 \\
0 & 0 & \cos \psi_b_c
\end{bmatrix}
\begin{bmatrix}
L_h & L_α & L_β \\
M_h & M_α & M_β \\
T_h & T_α & T_β
\end{bmatrix}
\begin{bmatrix}
g-a/b & 0 \\
1.0 & 0 \\
0 & \cos \psi_b_c
\end{bmatrix}
\]

These generalized forces are the right-hand sides of Lagrange's equations of motion for the system at hand.

These equations are:

\[
\frac{\partial \frac{\partial T}{\partial \alpha}}{\partial t} + \frac{\partial U}{\partial \alpha} = Q_α
\] (43)

\[
\frac{\partial \frac{\partial T}{\partial \beta}}{\partial t} + \frac{\partial U}{\partial \beta} = Q_β
\] (44)

\[
\frac{\partial \frac{\partial T}{\partial \gamma}}{\partial t} + \frac{\partial U}{\partial \gamma} = 0
\] (45)

where \( \gamma \) is the degree of freedom "control stick rotation". We next treat the correction to the generalized force \( Q_β \) as a result of the internal aerodynamic balance arrangement of the control surface. The XV-5A elevator system is of this type, since it uses a simple nose overhang with a curtain seal.

Again using References 1 and 4, the incremental, two-dimensional aerodynamic moment acting on the control surface, per unit span, is

\[
\Delta T' = -\pi \rho \omega^2 b^4 \cos \psi_c/4 \begin{bmatrix}
\bar{A} & \bar{B} & \bar{C}
\end{bmatrix} \begin{bmatrix}
h/b \\
\alpha' \\
\beta'
\end{bmatrix}
\] (46)
where

\[
\bar{A} = F_{BB} K \begin{pmatrix} \frac{\Delta C_{Hc}}{\Delta C_{Hc}} \end{pmatrix}\text{Exp.} \quad A(c, k)
\]

\[
\bar{B} = F_{BB} K \begin{pmatrix} \frac{\Delta C_{Hc}}{\Delta C_{Hc}} \end{pmatrix}\text{Exp.} \quad B(c, a, k)
\]

\[
\bar{C} = F_{BB} K \begin{pmatrix} \frac{\Delta C_{Hc}}{\Delta C_{Hc}} \end{pmatrix}\text{Exp.} \quad C(c, g, k)
\]

and

\[F_{BB}\] and \(K\) are the same as defined in Paragraph 3.1

\[
\begin{pmatrix} \frac{\Delta C_{Hc}}{\Delta C_{Hc}} \end{pmatrix}_{\text{Exp.}}, \begin{pmatrix} \frac{\Delta C_{Hc}}{\Delta C_{Hc}} \end{pmatrix}_{3}\text{ are control surface steady-state hinge}
\]

moment derivatives, experimental and theoretical, respectively (aerodynamic balance contribution through angle of attack)

\[
\begin{pmatrix} \frac{\Delta C_{Hc}}{\Delta C_{Hc}} \end{pmatrix}_{\text{Exp.}}, \begin{pmatrix} \frac{\Delta C_{Hc}}{\Delta C_{Hc}} \end{pmatrix}_{3}\text{ are the same as defined in Paragraph 3.1}
\]

\(A(c, k)\) is a dimensionless coefficient similar to \(T_h\) (Equation (37)) and is a function only of airfoil and control surface geometry. \(B(c, a, k)\) is a dimensionless coefficient similar to \(T_\alpha\) (Equation (37)) and is a function only of airfoil and control surface geometry (the parameter "a" being equal to -0.5 for this analysis, different from that expressed in Equation (36))

\(C(c, g, k)\) is the same as defined in Paragraph 3.1

The elements of the row matrix in Equation (46) are, in general, complex numbers. Using again the principle of virtual work, the work done by the internal balance forces in undergoing a virtual displacement is
\[ \delta W = \int_{0}^{l} (\Delta T') \delta \beta' \, dy \]

where \( l \) is the semispan of the internal aerodynamic balance arrangement (i.e., simple nose overhang), as before.

The work may be written in the equivalent form:

\[ \delta W = \int_{0}^{l} \left[ \delta \left( \frac{b}{h} \right) \delta \alpha' \delta \beta' \right] \begin{bmatrix} 0 \\ 0 \\ \Delta T' \end{bmatrix} \, dy \]

Combining Equations (36), (40), (46) and (47), the virtual work of internal balance forces is

\[ \delta W = 2\pi \omega \cos \psi_{c/4} \int_{0}^{b} \begin{bmatrix} \frac{g-a}{b} & 1.0 & 0 \\ 0 & 0 & \cos \psi_{\Phi_{c}} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.0 & 0 \\ \cos \psi_{c} \end{bmatrix} dy \]

Since \( \delta W = \Delta Q_{\beta} \delta \beta' \), in this case of correction for the control surface internal aerodynamic balance, the incremental, two-dimensional, generalized aerodynamic force to be added to \( Q_{\beta} \) in Equation (42) is

\[ \Delta Q_{\beta} = -\pi \omega \cos \psi_{c/4} \int_{0}^{b} \begin{bmatrix} \frac{g-a}{b} & 1.0 & 0 \\ 0 & 0 & \cos \psi_{\Phi_{c}} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.0 & 0 \\ \cos \psi_{c} \end{bmatrix} dy \]

\[ \begin{bmatrix} \frac{g-a}{b} & 0 \\ 1.0 & 0 \\ \cos \psi_{c} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \]
From Equations (42) and (49) we write the generalized aerodynamic forces, in 3x1 matrix form, as follows:

\[
\begin{bmatrix}
Q_\alpha \\
Q_\beta \\
0
\end{bmatrix} = \omega^2
\begin{bmatrix}
A_{11} & A_{12} & 0 \\
A_{21} & A_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix}
\]

(generalized aerodynamic force, no aerodynamic balance)

\[(50)\]

\[
\begin{bmatrix}
0 \\
\Delta Q_\beta \\
0
\end{bmatrix} = -\omega^2
\begin{bmatrix}
B_{21} & B_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix}
\]

(generalized aerodynamic force, aerodynamic balance increment)

\[(51)\]

where, by expanding out Equations (42) and (49)

\[
A_{11} = \pi \rho \cos \frac{\psi}{4} \int_{\text{Parent Surface}} b^4 \left[ L_h \left( \frac{g-a}{b} \right)^2 + (L_\alpha + M_h) \left( \frac{g-a}{b} \right) + M_\alpha \right] dy
\]

\[
A_{12} = \pi \rho \cos \frac{\psi}{4} \int_{\text{Control Surface}} b^4 \cos \psi_c \left[ L_\beta \left( \frac{g-a}{b} \right) + M_\beta \right] dy
\]

\[
A_{21} = \pi \rho \cos \frac{\psi}{4} \int_{\text{Control Surface}} b^4 \cos \psi_c \left[ \left( \frac{C_{Hc\alpha}}{C_{Hc\delta}} \right) \exp \left[ T_h \left( \frac{g-a}{b} \right) + T_\alpha \right] \right] dy
\]

\[
A_{22} = \pi \rho \cos \frac{\psi}{4} \int_{\text{Control Surface}} b^4 \cos^2 \psi_c \left[ \left( \frac{C_{Hc\delta}}{C_{Hc\delta}} \right) \exp T_\beta \right] dy
\]
\[
B_{21} = \pi \rho \cos \frac{\psi}{4} \int_{Aero. Bal.} b^4 \cos \psi_{\alpha_c} F_{BB} K \left( \frac{\Delta C_{H_{c\alpha}}}{\Delta C_{H_{c\alpha}}^3} \right) \exp \left[ A(c, k) \left( \frac{g-a}{b} \right) \right] + B(c, a, k) dy
\]

\[
B_{22} = \pi \rho \cos \frac{\psi}{4} \int_{Aero. Bal.} b^4 \cos^2 \psi_{\alpha_c} F_{BB} K \left( \frac{\Delta C_{H_{d/\alpha}}}{\Delta C_{H_{d/\alpha}}^3} \right) \exp \left[ C(c, g, h) dy \right]
\]

Note that the basic surface expressions \(A_{21}\) and \(A_{22}\) have been modified by hinge moment correction factors \(\left( \frac{C_{H_{c\alpha}}}{C_{H_{c\alpha}}^3} \right)\), etc. in order that these terms be consistent with the expressions for the internal aerodynamic balance arrangement.

The remaining portions of the analysis parallel the procedure treated in Paragraph 3.1, with changes in generalized coordinates as discussed in this paragraph. Note also that \([I'] = [A] - [B].\)
4.0 DISCUSSION AND RESULTS

4.1 AILERON AND AILERON FLIGHT TAB

A schematic of the aileron and aileron flight tab system is shown in Figure 1. A plan view of the wing, outer panel, aileron, and flight tab, showing the geometry of the analysis, is given in Figure 2, and the aerodynamic balancing arrangement is shown in Figure 3.

Parameters

Table 1 lists the aerodynamic parameters required to evaluate the oscillatory forces as defined in References 2, 3 and 4 and in Equation (22).

The inertia parameters are:

\[ I_\beta = 4.80598 \text{ lb.-in.-sec.}^2 \]
\[ I_\delta = 0.03108 \text{ lb.-in.-sec.}^2 \]
\[ S_\delta = -0.00086 \text{ lb.-sec.}^2 \text{ (nose heavy)} \]
\[ t_t = 14.974 \text{ in.} \]
\[ \Delta I_\delta = 0.10834 \text{ lb.-in.-sec.}^2 \text{ (see page 34 for definition)} \]
\[ J = 1.50690 \text{ lb.-in.-sec.}^2 \]

The uncoupled aileron rotation natural frequencies, a critical parameter for this preliminary analysis, are: \( f_\beta = 15, 20, \) and \( 30 \text{ cps} \).

Symmetric Analysis

For the purpose of a symmetric flutter analysis, the system is idealized as shown in Figure 4.

Since symmetric motion of the ailerons is contrary to the normal operation of the system, the control stick is assumed locked; hence the stick rotation freedom is not imposed in the symmetric analysis. It is further assumed that there is no mechanical gearing between flight tab and aileron. Note that the restraint of the power boost for the control system is represented herein by a simple spring.
Figure 2 Wing Outer Panel - Plan View
Figure 3  Aileron Aerodynamic Balancing Arrangement

Figure 4  Idealization of Symmetric Aileron System
The degrees of freedom considered are

\[ \beta = \text{Aileron rotation} \]
\[ \delta = \text{Aileron flight tab rotation, relative to the aileron} \]

The potential energy of the system is

\[ U = \frac{1}{2} k c \left( \frac{r_5}{r_2} \beta - \frac{r_1}{r_2} \frac{r_3}{r_4} \frac{r_5}{r_2} \delta \right)^2 + \frac{1}{2} K p \beta^2 \]  \hspace{1cm} (52)

where

\[ k_c = \text{Linear spring restraint offered by control circuit, i.e., effective spring of half the control circuit from Point A to floating link at aileron hinge line} \]
\[ K_p = \text{Rotational spring restraint offered by power actuator} \]
\[ r_1, r_2, r_3, r_4, r_5 = \text{Linkage arm lengths} \]

(See Figure 4 for a schematic representation of \( k_c \), \( K_p \) and the \( r \)'s.)

Calculating the potential energy terms that are part of Lagrange's equations of motion (Equations (1) and (2)) by taking partial derivatives of Equation (52).

\[ \frac{\partial U}{\partial \beta} = k_c \left( b_1 \beta - b_4 \delta \right) + K_p \beta \]

and

\[ \frac{\partial U}{\partial \delta} = k_c \left( -b_4 \beta + b_2 \delta \right) \]

where

\[ b_1 = r_5^2 \]
\[ b_2 = \left( \frac{r_1}{r_2} \frac{r_3}{r_4} \frac{r_5}{r_2} \right)^2 \]
\[ b_4 = \frac{r_1}{r_2} \frac{r_3}{r_4} \frac{r_5}{r_2} \]

33
In matrix notation, the potential energy (spring) terms are,

\[
\begin{bmatrix}
    b_1 k + K p & -b_4 k_c \\
    -b_4 k_c & b_2 k_c
\end{bmatrix}
\begin{bmatrix}
    \beta \\
    \delta
\end{bmatrix} = [s] \{q\}
\]

and Equation (25), then, is

\[
\begin{bmatrix}
    I^\beta & P \beta \delta \\
    P \delta \beta & I^\delta
\end{bmatrix}
\begin{bmatrix}
    b_1 k + K p & -b_4 k_c \\
    -b_4 k_c & b_2 k_c
\end{bmatrix}
+ \begin{bmatrix}
    I'_1 & I'_2 \\
    I'_2 & I'_3
\end{bmatrix}
\begin{bmatrix}
    \beta \\
    \delta
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0
\end{bmatrix}
\]

For the symmetric case, \( \overline{I}_\delta \) is the mass moment of the inertia of the tab about its hinge line \( I_\delta \) plus the effective mass moments of inertia \( \Delta I_\delta \) of half the control circuit (from Point A to tab) about the tab hinge line. The circuit members are assumed to move with the tab, not with the aileron nor independently, in this condition.

Upon expansion of the characteristic determinant of Equation (53), the following stability equation is obtained:

\[
A \Omega^2 + B \Omega + C = 0
\]

where \( A \) is real, and \( B \) and \( C \) are complex.

Parameters for the Symmetric Analysis

The linkage parameters are

\[
\begin{align*}
    r_1 &= 2.50 \text{ in.} \\
    r_2 &= 2.26 \text{ in.} \\
    r_3 &= 3.40 \text{ in.} \\
    r_4 &= 2.50 \text{ in.} \\
    r_5 &= 2.50 \text{ in.}
\end{align*}
\]
The spring constants are

\[ k_c = 5373 \text{ lb./in.} \]

\[ K = 9109; 42,312; \text{ and } 137,178 \text{ lb.-in./rad., corresponding to } f_\beta = 15, 20 \text{ and } 30 \text{ cps, respectively.} \]

\[ \bar{K} = 10,000 \text{ lb.-in./rad.} \]

The value for \( k_c \) was calculated from design values for the aileron flight control tab system. The values for \( K \) were determined in using Equation 27, for the various values of the uncoupled aileron frequency \( f_\beta \). The scaling constant \( \bar{K} \) was assigned its value arbitrarily, for computing convenience.

**Results**

Tables 2 through 7 and Figures 5 through 10 show the effects of varying the aileron uncoupled frequency \( f_\beta \) and aerodynamic balance. Figure 11 shows a cross-plot of sea-level flutter velocity \( V_f \) versus \( f_\beta \) for the symmetric aileron condition.

**Conclusions**

In order to clear the flight envelope to 500 knots \( +15\% \) (575 knots) at sea level, as required, examination of the tables and curves shows that the uncoupled tab frequency \( f_\delta \) must be made greater than indicated (51.9 cps) either with or without aerodynamic balancing for the aileron. Since \( f_\delta \) is not a parameter of the analysis, quantitative requirements for \( f_\delta \) are not available. The coupled, predominantly aileron mode (the lower-frequency branch) is stable for all \( f_\beta \) values investigated. The symmetric flutter condition occurs in the coupled, predominantly tab mode (the higher-frequency branch). \( f_\delta \), particularly the value of \( k_c \), would strongly affect the flutter mode.

The uncoupled aileron frequency \( f_\beta \) has little effect on the flutter frequency, but the flutter velocity decreases with increasing \( f_\beta \). This effect is understandable in that the predominantly aileron mode coalesces to the predominantly tab mode with increasing \( f_\beta \), thereby increasing the coupling between the two modes. Increased coupling means a lower flutter velocity. Stating the above conclusions in another way, the (coupled) frequencies of the two branches should be separated more. Since the flutter mode consists predominantly of tab motion, the best way to secure increased separation is to increase \( f_\delta \), which in turn is best increased.
by an increased value of $k_c$, the circuit stiffness. Further analysis should be made on calculated refinements to the circuit stiffness as the design develops and on the basis of tested values of uncoupled aileron and tab natural frequencies.

Aerodynamic balance of the aileron has little effect on the flutter frequency; however, the cases with balanced aileron ($B\neq 0$) yield slightly higher flutter velocities, by about 30 knots, than the cases with unbalanced aileron ($B=0$) for the same $f_\beta$ value.
TABLE 1

AILERON - AILERON FLIGHT TAB AERODYNAMIC PARAMETERS

Aileron - Aileron Flight Tab Geometry:

<table>
<thead>
<tr>
<th>Sta. No.</th>
<th>B. L. in.</th>
<th>b in.</th>
<th>c</th>
<th>$e'$</th>
<th>$e''$</th>
<th>g*</th>
<th>$F_{BB}$</th>
<th>d</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>102.40</td>
<td>55.120</td>
<td>0.626</td>
<td>0.559</td>
<td>0.498</td>
<td>0.708</td>
<td>0.00590</td>
<td>0.906</td>
<td>0.906</td>
</tr>
<tr>
<td>2</td>
<td>111.77</td>
<td>50.742</td>
<td>0.602</td>
<td>0.533</td>
<td>0.473</td>
<td>0.682</td>
<td>0.00590</td>
<td>0.901</td>
<td>0.901</td>
</tr>
<tr>
<td>3</td>
<td>121.13</td>
<td>46.364</td>
<td>0.574</td>
<td>0.502</td>
<td>0.443</td>
<td>0.654</td>
<td>0.00589</td>
<td>0.894</td>
<td>0.894</td>
</tr>
<tr>
<td>4</td>
<td>130.50</td>
<td>41.986</td>
<td>0.539</td>
<td>0.464</td>
<td>0.407</td>
<td>0.619</td>
<td>0.00587</td>
<td>0.887</td>
<td>0.887</td>
</tr>
<tr>
<td>5</td>
<td>130.50</td>
<td>41.986</td>
<td>0.539</td>
<td>0.464</td>
<td>0.407</td>
<td>0.619</td>
<td>0.00587</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>140.17</td>
<td>37.919</td>
<td>0.501</td>
<td>0.422</td>
<td>0.368</td>
<td>0.581</td>
<td>0.00579</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>149.83</td>
<td>33.852</td>
<td>0.454</td>
<td>0.370</td>
<td>0.318</td>
<td>0.534</td>
<td>0.00566</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>159.50</td>
<td>29.785</td>
<td>0.393</td>
<td>0.303</td>
<td>0.255</td>
<td>0.473</td>
<td>0.00548</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>169.16</td>
<td>25.718</td>
<td>0.314</td>
<td>0.216</td>
<td>0.172</td>
<td>0.394</td>
<td>0.00519</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>178.83</td>
<td>21.652</td>
<td>0.204</td>
<td>0.130</td>
<td>0.130</td>
<td>0.284</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Aileron, internal - simple nose overhang (curtain seal)
Flight Tab, none - radius nose

Hinge Moment Corrections:

\[
\left( \frac{C_H}{c} \right)^3 \quad \text{Exp.} = 1.0 \quad ; \quad \left( \frac{C_H}{\delta} \right)^3 \quad \text{Exp.} = 1.0
\]
Pressure Recovery Factor:

\[ K = 1.0 \]

General:

Reference half-chord \( (b_0) = 55.120 \) in.

\[
\cos \psi_{c/4} = 0.87114 \\
\cos \psi_{c} = 0.99871 \\
\cos \psi_{t} = 0.99657
\]

Altitude - Sea Level \( (\rho = 0.114626 \times 10^{-6} \text{ lb.-sec}^2 \text{-in.}^{-4}) \)
TABLE 2
SOLUTIONS OF STABILITY EQUATIONS
(Symmetric Analysis)

Plot: Figure 5

Uncoupled Natural Frequency, Aileron Rotation: $f_\beta = 15$ cps

Spring Constants:
- $k_c = 5373$ lb./in.
- $K_p = 9109$ lb.-in./rad.

Aerodynamics: Basic Surface with Aerodynamic Balance ($B \neq 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_0}$</th>
<th>$f_1$ (cps)</th>
<th>$V_1$ (Knots)</th>
<th>$g_1$</th>
<th>$f_2$ (cps)</th>
<th>$V_2$ (Knots)</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>0.000</td>
<td>53.9</td>
<td>0.0</td>
<td>0.000</td>
</tr>
<tr>
<td>0.05</td>
<td>6.5</td>
<td>5.6</td>
<td>-0.032</td>
<td>53.8</td>
<td>46.0</td>
<td>-0.004</td>
</tr>
<tr>
<td>0.10</td>
<td>6.5</td>
<td>11.2</td>
<td>-0.064</td>
<td>53.8</td>
<td>92.0</td>
<td>-0.008</td>
</tr>
<tr>
<td>0.15</td>
<td>6.6</td>
<td>16.9</td>
<td>-0.097</td>
<td>53.8</td>
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<td>-0.011</td>
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<td>-0.014</td>
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<tr>
<td>0.75</td>
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<td>-1.280</td>
<td>54.2</td>
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<td>-</td>
<td>-</td>
<td>68.7</td>
<td>1174.1</td>
<td>0.434</td>
</tr>
</tbody>
</table>

$V_f = 463$ knots

Uncoupled Natural Frequency, Tab Rotation: $f_\delta = 51.9$ cps
TABLE 3
SOLUTIONS OF STABILITY EQUATIONS
(Symmetric Analysis)

Plot: Figure 6

Uncoupled Natural Frequency, Aileron Rotation: \( f_\beta = 20 \) cps

Spring Constants:
- \( k_c = 5373 \text{ lb./in.} \)
- \( K_p = 42,312 \text{ lb.-in./rad.} \)

Aerodynamics: Basic Surface with Aerodynamic Balance (\( B \neq 0 \))

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 )</th>
<th>( V_1 )</th>
<th>( g_1 )</th>
<th>( f_2 )</th>
<th>( V_2 )</th>
<th>( g_2 )</th>
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</thead>
<tbody>
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<td>.00</td>
<td>14.3</td>
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<td>.000</td>
<td>54.1</td>
<td>0.0</td>
<td>.000</td>
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<tr>
<td>.05</td>
<td>14.0</td>
<td>12.0</td>
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<td>46.1</td>
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<td>1198.3</td>
<td>.442</td>
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</table>

\( V_f = 432 \) knots

Uncoupled Natural Frequency, Tab Rotation: \( f_\delta = 51.9 \) cps
TABLE 4
SOLUTIONS OF STABILITY EQUATIONS
(Symmetric Analysis)

Plot: Figure 7

Uncoupled Natural Frequency, Aileron Rotation: \( f_\beta = 30 \) cps

Spring Constants:

\[
k_c = 5373 \text{ lb./in.} \\
K_p = 137,178 \text{ lb.-in./rad.}
\]

Aerodynamics: Basic Surface with Aerodynamic Balance (B ≠ 0)

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 )</th>
<th>( V_1 )</th>
<th>( g_1 )</th>
<th>( f_2 )</th>
<th>( V_2 )</th>
<th>( g_2 )</th>
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</thead>
<tbody>
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<td>.000</td>
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<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.05</td>
<td>25.0</td>
<td>21.4</td>
<td>-.032</td>
<td>54.4</td>
<td>46.5</td>
<td>-.004</td>
</tr>
<tr>
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<td>25.1</td>
<td>43.0</td>
<td>-.064</td>
<td>54.3</td>
<td>92.9</td>
<td>-.007</td>
</tr>
<tr>
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<td>25.4</td>
<td>65.0</td>
<td>-.098</td>
<td>54.3</td>
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<tr>
<td>.20</td>
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<td>87.7</td>
<td>-.134</td>
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<td>185.3</td>
<td>-.012</td>
</tr>
<tr>
<td>.50</td>
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<td>-.485</td>
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<td>-</td>
<td>-1.667</td>
<td>73.4</td>
<td>1255.3</td>
<td>.446</td>
</tr>
</tbody>
</table>

\( V_f = 356 \) knots

Uncoupled Natural Frequency, Tab Rotation: \( f_\delta = 51.9 \) cps
TABLE 5
SOLUTIONS OF STABILITY EQUATIONS
(Symmetric Analysis)

Plot: Figure 8

Uncoupled Natural Frequency, Aileron Rotation: \( f_\beta = 15 \) cps

Spring Constants:

\[
\begin{align*}
  k_c &= 5373 \text{ lb./in.} \\
  K_p &= 9109 \text{ lb.-in./rad.}
\end{align*}
\]

Aerodynamics: Basic Surface without Aerodynamic Balance \((B = 0)\)

<table>
<thead>
<tr>
<th>( \frac{1}{k_o} )</th>
<th>( f_1 )</th>
<th>( V_1 )</th>
<th>( g_1 )</th>
<th>( f_2 )</th>
<th>( V_2 )</th>
<th>( g_2 )</th>
</tr>
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<tr>
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<td>.000</td>
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<td>0.0</td>
<td>.000</td>
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<tr>
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<td>5.6</td>
<td>-.033</td>
<td>53.8</td>
<td>46.0</td>
<td>-.004</td>
</tr>
<tr>
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<td>11.2</td>
<td>-.067</td>
<td>53.8</td>
<td>92.0</td>
<td>-.008</td>
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<td>16.9</td>
<td>-.102</td>
<td>53.8</td>
<td>138.0</td>
<td>-.011</td>
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<td>184.0</td>
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<td>-</td>
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<td>.483</td>
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</table>

\( V_f = 434 \) knots

Uncoupled Natural Frequency, Tab Rotation: \( f_5 = 51.9 \) cps
TABLE 6
SOLUTIONS OF STABILITY EQUATIONS
(Symmetric Analysis)

Plot: Figure 9

Uncoupled Natural Frequency, Aileron Rotation: \( f_\beta = 20 \text{ cps} \)

Spring Constants:

\[
\begin{align*}
K_c &= 5373 \text{ lb./in.} \\
K_p &= 42,312 \text{ lb.-in./rad.}
\end{align*}
\]

Aerodynamics: Basic Surface without Aerodynamic Balance (\( B = 0 \))

<table>
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<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 )</th>
<th>( V_1 )</th>
<th>( g_1 )</th>
<th>( f_2 )</th>
<th>( V_2 )</th>
<th>( g_2 )</th>
</tr>
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<td>.000</td>
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<tr>
<td>.05</td>
<td>14.0</td>
<td>12.0</td>
<td>-.033</td>
<td>53.9</td>
<td>46.1</td>
<td>-.004</td>
</tr>
<tr>
<td>.10</td>
<td>14.1</td>
<td>24.0</td>
<td>-.067</td>
<td>53.9</td>
<td>92.2</td>
<td>-.008</td>
</tr>
<tr>
<td>.15</td>
<td>14.2</td>
<td>36.4</td>
<td>-.103</td>
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<td>138.3</td>
<td>-.011</td>
</tr>
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<td>184.2</td>
<td>-.014</td>
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<td>455.7</td>
<td>.015</td>
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<td>709.8</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>74.3</td>
<td>1269.9</td>
<td>.479</td>
</tr>
</tbody>
</table>

\( V_f = 404 \text{ knots} \)

Uncoupled Natural Frequency, Tab Rotation: \( f_\delta = 51.9 \text{ cps} \)
Plot: Figure 10

Uncoupled Natural Frequency, Aileron Rotation: $f_\beta = 30$ cps

Spring Constants:

\[ k_c = 5373 \text{ lb./in.} \]
\[ K_p = 137,178 \text{ lb.-in./rad.} \]

Aerodynamics: Basic Surface without Aerodynamic Balance ($B = 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_o}$</th>
<th>$f_1$ (cps)</th>
<th>$V_1$ (Knots)</th>
<th>$g_1$</th>
<th>$f_2$ (cps)</th>
<th>$V_2$ (Knots)</th>
<th>$g_2$</th>
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<td>.000</td>
<td>54.5</td>
<td>0.0</td>
<td>.000</td>
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<td>-.004</td>
</tr>
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<td>92.9</td>
<td>-.007</td>
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<tr>
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<td>65.0</td>
<td>-.104</td>
<td>54.3</td>
<td>139.2</td>
<td>-.010</td>
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<tr>
<td>.20</td>
<td>25.7</td>
<td>87.8</td>
<td>-.142</td>
<td>54.2</td>
<td>185.3</td>
<td>-.012</td>
</tr>
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<td>30.9</td>
<td>264.4</td>
<td>-.535</td>
<td>53.1</td>
<td>454.4</td>
<td>.053</td>
</tr>
<tr>
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<td>48.9</td>
<td>627.7</td>
<td>-2.127</td>
<td>58.1</td>
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<td>.313</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>76.9</td>
<td>1315.5</td>
<td>.461</td>
</tr>
</tbody>
</table>

$V_f = 320$ knots

Uncoupled Natural Frequency, Tab Rotation: $f_\delta = 51.9$ cps
Figure 5  g-V and f-V Plots, Symmetric Aileron, $f_\beta = 15$ cps, $B \neq 0$
Figure 6  g-V and f-V Plots, Symmetric Aileron, $f_\beta = 20$ cps, $B \neq 0$
Figure 7  g-V and f-V Plots, Symmetric Aileron, $f_\beta = 30$ cps, $B \neq 0$
Figure 8  g-V and f-V Plots, Symmetric Aileron, $f_\beta = 15$ cps, $B = 0$
Figure 9  g-V and f-V Plots, Symmetric Aileron, $f_\beta = 20$ cps, $B = 0$
Figure 10  g-V and f-V Plots, Symmetric Aileron, $f_B = 30$ cps, $B = 0$
Figure 11  \( V_f \) vs \( f_\beta \) Cross-plot, Symmetric Aileron
Antisymmetric Analysis

For the purpose of an antisymmetric flutter analysis, the system is idealized as shown in Figure 12.

![Figure 12 Idealization of Antisymmetric Aileron System](image)

The degrees of freedom considered are as follows:

- \( \beta \) = Aileron rotation
- \( \delta \) = Aileron flight tab rotation, relative to the aileron
- \( \gamma \) = Control stick rotation

It is still assumed that there is no mechanical gearing between flight tab and aileron and that the restraint of the power boost for the control system can be represented by a simple spring.

The potential energy of the system is

\[
U = \frac{1}{2} k_c (r_5 \beta - \frac{r_1}{r_2} \frac{r_3}{r_4} r_5 \delta + r_{23} \gamma)^2 + \frac{1}{2} K_p \beta^2 \tag{55}
\]
where

\[ k_c = \text{Linear spring restraint offered by control circuit; i.e., the effective spring of half the control circuit from control sticks to floating link at aileron hinge line} \]

\[ K_p = \text{Rotational spring restraint offered by power actuator} \]

\[ r_1, r_2, r_3, r_4, r_5, r_{23} = \text{Linkage arm lengths} \]

(See Figure 12 for a schematic representation of \( k_c, K_p \) and the \( r's \))

Taking partial derivative of Equation (55),

\[ \frac{\partial U}{\partial \beta} = k_c (a_1 \beta - a_4 \delta + a_5 \gamma) + K_p \beta \]

\[ \frac{\partial U}{\partial \delta} = k_c (-a_4 \beta + a_2 \delta - a_6 \gamma) \]

\[ \frac{\partial U}{\partial \gamma} = k_c (a_5 \beta - a_6 \delta + a_3 \gamma) \]

where

\[ a_1 = r_5^2 (= b_1 \text{ of the symmetrical case, page 33}) \]

\[ a_2 = \left( \frac{r_1}{r_2} \frac{r_3}{r_4} r_5 \right)^2 (= b_2) \]

\[ a_3 = r_{23} \]

\[ a_4 = \frac{r_1}{r_2} \frac{r_3}{r_4} r_5^2 (= b_4) \]

\[ a_5 = r_5 r_{23} \]

\[ a_6 = \frac{r_1}{r_2} \frac{r_3}{r_4} r_5 r_{23} \]
In matrix notation, then, the potential energy (spring) terms in Lagrange's equations of motion (Equations (1), (2), and (3)) are

\[
\begin{bmatrix}
  a_1 k_c + K_p & -a_4 k_c & a_5 k_c \\
  -a_4 k_c & a_2 k_c & -a_6 k_c \\
  a_5 k_c & -a_6 k_c & a_3 k_c
\end{bmatrix}
\begin{bmatrix}
  \beta \\
  \delta \\
  \gamma
\end{bmatrix} = [S] \{q\}
\]

and Equation (25) is

\[
\begin{bmatrix}
  I_\beta & P \delta & 0 \\
  P \delta & I_\delta & 0 \\
  0 & 0 & J
\end{bmatrix}
\begin{bmatrix}
  a_1 k_c + K_p & -a_4 k_c & a_5 k_c \\
  -a_4 k_c & a_2 k_c & -a_6 k_c \\
  a_5 k_c & -a_6 k_c & a_3 k_c
\end{bmatrix}
\begin{bmatrix}
  \beta \\
  \delta \\
  \gamma
\end{bmatrix} = [I']^{11} I'^{12} [0 \beta] = 0
\]

Upon expansion of the characteristic determinant of Equation (56), because of relationships between [S] elements in terms of the r's ([S] is singular), the following stability equation is obtained:

\[
A \Omega^2 + B \Omega + C = 0
\]

where A is real, and B and C are complex.

Parameters for the Antisymmetric Analysis

The linkage parameters, r through r_5, are the same as given previously for the symmetric case, page 34.

\[
r_{23} = 3.20 \text{ in.}
\]

The spring constants are:

\[
k_c = 911 \text{ lb./in.}
\]

\[
K_p = 39,496; 72,699; \text{ and } 167,565 \text{ lb.-in./rad. corresponding to } f_\beta = 15, 20 \text{ and } 30 \text{ cps, respectively.}
\]

\[
K = 10,000 \text{ lb.-in./rad.}
\]
The value for \( k_c \) was calculated from design values for the aileron-flight tab control system. The values for \( K_p \) were determined, using Equation 27, for the various values of the uncoupled aileron frequency \( f_\beta \). The scaling constant \( \bar{K} \) was assigned its value arbitrarily, for computing convenience.

**Results**

Tables 8 through 13 and Figures 13 through 18 show the effects of varying the aileron uncoupled frequency \( f_\beta \) and aerodynamic balance.

**Conclusions**

Examination of the analysis results show that antisymmetric aileron system flutter will not occur over the range of \( f_\beta \) and \( 1/k_0 \) values investigated, either with or without aerodynamic balance for the ailerons.
TABLE 8
SOLUTIONS OF STABILITY EQUATIONS
(Antisymmetric Analysis)

Plot: Figure 13

Uncoupled Natural Frequency, Aileron Rotation: \( f_\beta = 15 \) cps

Spring Constants:
\[
\begin{align*}
  k_c &= 911 \text{ lb./in.} \\
  K_p &= 39,496 \text{ lb.-in./rad.}
\end{align*}
\]

Aerodynamics: Basic Surface with Aerodynamic Balance (B ≠ 0)

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 ) ( \text{cps} )</th>
<th>( V_1 ) ( \text{Knots} )</th>
<th>( g_1 )</th>
<th>( f_2 ) ( \text{cps} )</th>
<th>( V_2 ) ( \text{Knots} )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>13.9</td>
<td>0.0</td>
<td>.000</td>
<td>103.8</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>0.05</td>
<td>13.6</td>
<td>11.6</td>
<td>-.031</td>
<td>102.6</td>
<td>87.7</td>
<td>-.029</td>
</tr>
<tr>
<td>0.10</td>
<td>13.6</td>
<td>23.3</td>
<td>-.061</td>
<td>103.4</td>
<td>176.9</td>
<td>-.060</td>
</tr>
<tr>
<td>0.15</td>
<td>13.7</td>
<td>35.1</td>
<td>-.093</td>
<td>104.9</td>
<td>269.0</td>
<td>-.094</td>
</tr>
<tr>
<td>0.20</td>
<td>13.8</td>
<td>47.1</td>
<td>-.126</td>
<td>107.0</td>
<td>365.9</td>
<td>-.133</td>
</tr>
<tr>
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<td>-.377</td>
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Uncoupled Natural Frequency, Tab Rotation: \( f_\delta = 102.5 \) cps
TABLE 9
SOLUTIONS OF STABILITY EQUATIONS
(Antisymmetric Analysis)

Plot: Figure 14

Uncoupled Natural Frequency, Aileron Rotation: $f_\beta = 20$ cps

Spring Constants

\[
k_c = 911 \text{ lb./in.}
\]
\[
K_p = 72,699 \text{ lb.-in./rad.}
\]

Aerodynamics: Basic Surface With Aerodynamic Balance ($B \neq 0$)

<table>
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<th>$\frac{1}{k_0}$</th>
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<th>$V_1$ \text{ Knots}</th>
<th>$s_1$</th>
<th>$f_2$ \text{ cps}</th>
<th>$V_2$ \text{ Knots}</th>
<th>$s_2$</th>
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</thead>
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<td>103.8</td>
<td>0.0</td>
<td>.000</td>
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<tr>
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<td>18.7</td>
<td>16.0</td>
<td>-.031</td>
<td>102.6</td>
<td>87.7</td>
<td>-.029</td>
</tr>
<tr>
<td>.10</td>
<td>18.7</td>
<td>32.0</td>
<td>-.061</td>
<td>103.5</td>
<td>176.9</td>
<td>-.060</td>
</tr>
<tr>
<td>.15</td>
<td>18.8</td>
<td>48.3</td>
<td>-.093</td>
<td>104.9</td>
<td>269.0</td>
<td>-.094</td>
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<td>107.0</td>
<td>365.9</td>
<td>-.132</td>
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<td>-</td>
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</table>

Uncoupled Natural Frequency, Tab Rotation: $f_\delta = 102.5$ cps
TABLE 10
SOLUTIONS OF STABILITY EQUATIONS
(Antisymmetric Analysis)

Plot: Figure 15

Uncoupled Natural Frequency, Aileron Rotation: $f_\beta = 30$ cps

Spring Constants:

\[
k_c = 911 \text{ lb./in.}
\]

\[
K_p = 167,565 \text{ lb.-in./rad.}
\]

Aerodynamics: Basic Surface with Aerodynamic Balance ($B \neq 0$)

<table>
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<tr>
<th>$k_0$</th>
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<th>$V_1$</th>
<th>$g_1$</th>
<th>$f_2$</th>
<th>$V_2$</th>
<th>$g_2$</th>
</tr>
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<td>[Knots]</td>
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<td>.000</td>
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<td>.000</td>
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<td>87.7</td>
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<tr>
<td>.15</td>
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<td>74.1</td>
<td>-.094</td>
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<td>-.093</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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</table>

Uncoupled Natural Frequency, Tab Rotation: $f_\phi = 102.5$ cps
TABLE 11

- SOLUTIONS OF STABILITY EQUATIONS

(Antisymmetric Analysis)

Plot: Figure 16

Uncoupled Natural Frequency, Aileron Rotation: \( f_\beta = 15 \text{ cps} \)

Spring Constants:

\[
\begin{align*}
  k_c &= 911 \text{ lb./in.} \\
  K_p &= 39,496 \text{ lb.-in/rad.}
\end{align*}
\]

Aerodynamics: Basic Surface without Aerodynamic Balance (B = 0)

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 \text{ cps} )</th>
<th>( V_1 \text{ Knots} )</th>
<th>( V_1 \text{ Knots} )</th>
<th>( V_2 \text{ Knots} )</th>
<th>( V_2 \text{ Knots} )</th>
</tr>
</thead>
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<td>0.0</td>
</tr>
<tr>
<td>.05</td>
<td>13.5</td>
<td>11.6</td>
<td>-0.032</td>
<td>102.6</td>
<td>87.7</td>
</tr>
<tr>
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<td>13.6</td>
<td>23.2</td>
<td>-0.065</td>
<td>103.4</td>
<td>176.9</td>
</tr>
<tr>
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<td>35.1</td>
<td>-0.098</td>
<td>104.9</td>
<td>269.0</td>
</tr>
<tr>
<td>.20</td>
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<td>-0.134</td>
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</tr>
<tr>
<td>.50</td>
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<td>132.3</td>
<td>-0.414</td>
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<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Tab Rotation: \( f_\theta = 102.5 \text{ cps} \)
### TABLE 12
SOLUTIONS OF STABILITY EQUATIONS

(Antisymmetric Analysis)

Plot: Figure 17

Uncoupled Natural Frequency, Aileron Rotation: \( f_\beta = 20 \) cps

Spring Constants:

\[
\begin{align*}
k_c &= 911 \text{ lb./in.} \\
K_P &= 72,699 \text{ lb.-in./rad.}
\end{align*}
\]

Aerodynamics: Basic surface without Aerodynamic Balance (\( B = 0 \))

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 ) ( \text{cps} )</th>
<th>( V_1 ) ( \text{Knots} )</th>
<th>( g_1 )</th>
<th>( f_2 ) ( \text{cps} )</th>
<th>( V_2 ) ( \text{Knots} )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>19.2</td>
<td>0.0</td>
<td>.000</td>
<td>103.8</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.05</td>
<td>18.7</td>
<td>15.9</td>
<td>-.032</td>
<td>102.6</td>
<td>87.7</td>
<td>-.029</td>
</tr>
<tr>
<td>.10</td>
<td>18.7</td>
<td>32.0</td>
<td>-.065</td>
<td>103.4</td>
<td>176.9</td>
<td>-.060</td>
</tr>
<tr>
<td>.15</td>
<td>18.8</td>
<td>48.3</td>
<td>-.099</td>
<td>104.9</td>
<td>269.0</td>
<td>-.094</td>
</tr>
<tr>
<td>.20</td>
<td>19.0</td>
<td>65.0</td>
<td>-.134</td>
<td>107.0</td>
<td>365.8</td>
<td>-.132</td>
</tr>
<tr>
<td>.50</td>
<td>21.4</td>
<td>182.6</td>
<td>-.414</td>
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<td>-.829</td>
</tr>
<tr>
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<td>-110.501</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Tab Rotation: \( f_\delta = 102.5 \) cps
TABLE 13

SOLUTIONS OF STABILITY EQUATIONS

(antisymmetric Analysis)

Plot: Figure 18

Uncoupled Natural Frequency, Aileron Rotation: $f_\beta = 30$ cps

Spring Constants:

\[ k_c = 911 \text{ lb./in.} \]
\[ K_p = 167,565 \text{ lb.-in./rad.} \]

Aerodynamics: Basic Surface without Aerodynamic Balance (B = 0)

<table>
<thead>
<tr>
<th>$\frac{1}{k_\circ}$</th>
<th>$f_1$</th>
<th>$V_1$</th>
<th>$g_1$</th>
<th>$f_2$</th>
<th>$V_2$</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.000</td>
<td>29.4</td>
<td>0.0</td>
<td>.000</td>
<td>103.8</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.05</td>
<td>28.6</td>
<td>24.5</td>
<td>-.033</td>
<td>102.7</td>
<td>87.8</td>
<td>-.029</td>
</tr>
<tr>
<td>.10</td>
<td>28.7</td>
<td>49.1</td>
<td>-.066</td>
<td>103.5</td>
<td>176.9</td>
<td>-.060</td>
</tr>
<tr>
<td>.15</td>
<td>28.9</td>
<td>74.1</td>
<td>-.100</td>
<td>104.9</td>
<td>268.9</td>
<td>-.093</td>
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<tr>
<td>.20</td>
<td>29.1</td>
<td>99.7</td>
<td>-.135</td>
<td>106.9</td>
<td>365.6</td>
<td>-.131</td>
</tr>
<tr>
<td>.50</td>
<td>32.9</td>
<td>281.2</td>
<td>-.415</td>
<td>150.4</td>
<td>1286.0</td>
<td>-.828</td>
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<tr>
<td>.75</td>
<td>42.9</td>
<td>550.2</td>
<td>-.976</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.00</td>
<td>415.5</td>
<td>7105.2</td>
<td>-34.56</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Tab Rotation: $f_\delta = 102.5$ cps
Figure 13 g-V and f-V Plots, Antisymmetric Aileron, $f_{\beta} = 15$ cps, $B \neq 0$
Figure 14  g-V and f-V Plots, Antisymmetric Aileron, $f_B = 20$ cps, $B \neq 0$
Figure 15  g-V and f-V Plots, Antisymmetric Aileron, \( f_\beta = 30 \text{ cps}, B \neq 0 \)
Figure 16  g-V and f-V Plots, Antisymmetric Aileron, $f_\beta = 15$ cps, $B = 0$
Figure 17  g-V and f-V Plots, Antisymmetric Aileron, \( f_B = 20 \) cps, B = 0
Figure 18  g-V and f-V Plots, Antisymmetric Aileron, $f_{\beta} = 30$ cps, $B = 0$
4.2 RUDDER AND RUDDER TRIM TAB

A schematic of the rudder and rudder trim tab system is shown in Figure 19. A plan view of the vertical tail, rudder and trim tab is shown in Figure 20, and the aerodynamic balancing arrangement is shown in Figure 21.

Analysis

For purposes of a flutter analysis, the system is idealized as shown in Figure 22. For the trim tab system at hand, no mechanical gearing exists between the rudder and tab; i.e., \( n = 0 \) (\( n \) is a gear ratio, trim tab rotation to rudder rotation \( \delta / \beta \)).

\[
\beta = \text{Rudder rotation} \\
\delta = \text{Rudder trim tab rotation, relative to the rudder} \\
\gamma = \text{Control pedal rotation}
\]

The potential energy of the system is

\[
U = \frac{1}{2} k_c (r_1 \beta - r_8 \gamma)^2 + \frac{1}{2} K_A \delta^2 \tag{58}
\]

where

\[
k_c = \text{Linear spring restraint offered by control circuit, i.e.} \\
\text{the effective spring of the complete control circuit from} \\
\text{control pedal to rudder quadrant}
\]

\[
K_A = \text{Rotational spring restraint offered by trim tab actuator}
\]

\[
r_1, r_8 = \text{Linkage arm lengths}
\]

(See Figure 22 for a schematic representation of \( k_c \), \( K_A \), \( r_1 \) and \( r_8 \).)

Calculating the potential energy terms that are part of Lagrange's equations of motion (Equations (1), (2) and (3)), by taking partial derivatives of Equation (58),
\[ \frac{\partial U}{\partial \beta} = k_c (a_1 \beta - a_2 \gamma) \]
\[ \frac{\partial U}{\partial \delta} = K_A \delta \]
\[ \frac{\partial U}{\partial \gamma} = k_c (-a_2 \beta + a_3 \gamma) \]

where
\[ a_1 = r_1^2 \]
\[ a_2 = r_1 r_8 \]
\[ a_3 = r_8^2 \]

In matrix notation, the potential energy (spring) terms are

\[
\begin{bmatrix}
  a_1 & k_c & 0 & -a_2 & k_c \\
  0 & K_A & 0 & 0 & 0 \\
  -a_2 & k_c & 0 & a_3 & k_c
\end{bmatrix}
\begin{bmatrix}
  \beta \\
  \delta \\
  \gamma
\end{bmatrix} = [S] \{q\}
\]

and Equation (25) is

\[
\begin{bmatrix}
  I_\beta & P \beta \delta & 0 \\
  P \delta \beta & I_\delta & 0 \\
  0 & 0 & J
\end{bmatrix}
\begin{bmatrix}
  a_1 k_c & 0 & -a_2 k_c \\
  0 & K_A & 0 \\
  -a_2 k_c & 0 & a_3 k_c
\end{bmatrix}
\begin{bmatrix}
  I_{11}' & I_{12}' & 0 \\
  I_{21}' & I_{22}' & 0 \\
  0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  \beta \\
  \delta \\
  \gamma
\end{bmatrix} = 0
\]

In the case at hand, J is the effective mass moment of inertia of the control circuit and pedal about the pedal pivot axis.

Upon expansion of the characteristic determinant of Equation (59), because of relationships between [S] elements in terms of the r's ([S] is singular), the following stability equation is obtained:

\[ A\Omega^2 + B\Omega + C = 0 \]

where A is real, and B and C are complex.
Figure 19  Schematic of Rudder System

Figure 20  Vertical Tail – Plan View
Figure 21  Rudder Aerodynamic Balancing Arrangement

Figure 22  Idealization of Rudder System
Parameters

Table 14 lists the aerodynamic parameters required to evaluate the oscillatory forces as defined in References 2, 3 and 4 and in Equation (22).

The inertia parameters are

\[ I_\beta = 2.31997 \text{ lb. -in. -sec.}^2 \]
\[ I_\delta = 0.02604 \text{ lb. -in. -sec.}^2 \]
\[ S_\delta = 0.00596 \text{ lb. -sec.}^2 \] (Tail heavy)
\[ f_t = 11.902 \text{ in.} \]
\[ J = 0.95234 \text{ lb. -in. -sec.}^2 \]

The uncoupled rudder trim tab rotation natural frequencies, a basic parameter for this preliminary analysis, are \( f_\delta = 20, 40 \) and \( 60 \text{ cps} \).

The linkage parameters are:

\[ r_1 = 5.5 \text{ in.} \]
\[ r_8 = 9.6 \text{ in.} \]

The spring constants are:

\[ k_c = 357 \text{ lb./in.} \]
\[ K_A = 411, 1645, \text{ and } 3702 \text{ lb.-in./rad.} \text{ corresponding to } f_\delta = 20, 40 \text{ and } 60 \text{ cps, respectively.} \]
\[ \bar{K} = 10,000 \text{ lb.-in./rad.} \]

The value for \( k_c \) was calculated from design values for the rudder-trim tab system. The values for \( K_A \) were determined using Equation 28, for the various values of uncoupled trim tab frequency \( f_\delta \). The scaling constant \( \bar{K} \) was assigned its value arbitrarily, for computing convenience.
Results

Tables 15 to 20 and Figures 23 to 28 show the effects of varying the rudder trim tab uncoupled frequency $f_5$ and aerodynamic balance. Figure 29 shows a cross-plot of sea-level flutter velocity $V_f$ versus $f_5$.

Conclusions

In order to clear the flight envelope to 500 knots + 15% (575 knots) at sea level, as required, examination of the tables and curves shows that the uncoupled trim tab frequency $f_5$ must be at least 59 cps for the aerodynamically unbalanced rudder ($\beta = 0$) and at least 50 cps for the aerodynamically balanced rudder ($\beta \neq 0$). Thus an increased $K_A$ value, trim tab actuator stiffness, is indicated. The coupled, predominantly rudder mode (the lower-frequency branch) is stable for all $f_5$ values investigated. The flutter condition occurs in the coupled, predominantly trim tab mode (the higher-frequency branch). $f_5$, and particularly the value of $K_A$, as expected, strongly influences both the flutter velocity and frequency. They both increase with increasing $f_5$. Further analysis should be made on calculated refinements to the circuit stiffness as the design develops and on the basis of tested values of uncoupled rudder and trim tab natural frequencies.

Aerodynamic balance of the rudder has considerable effect on the flutter velocity. Flutter velocities are substantially greater for the aerodynamically balanced rudder ($\beta \neq 0$) than for the unbalanced one ($\beta = 0$), for the same $f_5$ value. Figure 29 shows a quantitative comparison.
TABLE 14

RUDDER - RUDDER TRIM TAB AERODYNAMIC PARAMETERS

Rudder - Rudder Trim Tab Geometry:

<table>
<thead>
<tr>
<th>Sta. No.</th>
<th>W. L.</th>
<th>b</th>
<th>c</th>
<th>e'</th>
<th>e''</th>
<th>g</th>
<th>F_{BB}</th>
<th>d</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>118.63</td>
<td>50.449</td>
<td>0.640</td>
<td>0.579</td>
<td>0.511</td>
<td>0.690</td>
<td>0.00649</td>
<td>0.910</td>
<td>0.910</td>
</tr>
<tr>
<td>2</td>
<td>126.66</td>
<td>48.294</td>
<td>0.640</td>
<td>0.579</td>
<td>0.511</td>
<td>0.690</td>
<td>0.00652</td>
<td>0.910</td>
<td>0.910</td>
</tr>
<tr>
<td>3</td>
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<td>0.640</td>
<td>0.580</td>
<td>0.510</td>
<td>0.690</td>
<td>0.00658</td>
<td>0.910</td>
<td>0.910</td>
</tr>
<tr>
<td>4_{I}</td>
<td>142.72</td>
<td>43.984</td>
<td>0.640</td>
<td>0.580</td>
<td>0.510</td>
<td>0.690</td>
<td>0.00662</td>
<td>0.910</td>
<td>0.910</td>
</tr>
<tr>
<td>4_{O}</td>
<td>142.72</td>
<td>43.984</td>
<td>0.640</td>
<td>0.580</td>
<td>0.510</td>
<td>0.690</td>
<td>0.00662</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
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<td>41.329</td>
<td>0.640</td>
<td>0.581</td>
<td>0.510</td>
<td>0.690</td>
<td>0.00670</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>162.50</td>
<td>38.675</td>
<td>0.640</td>
<td>0.582</td>
<td>0.510</td>
<td>0.690</td>
<td>0.00679</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>172.39</td>
<td>36.021</td>
<td>0.640</td>
<td>0.582</td>
<td>0.509</td>
<td>0.690</td>
<td>0.00687</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>182.28</td>
<td>33.361</td>
<td>0.640</td>
<td>0.583</td>
<td>0.509</td>
<td>0.690</td>
<td>0.00698</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Aerodynamic Balance Arrangement:

Rudder, internal - simple nose overhang (curtain seal)

Trim Tab, none - radius nose

Hinge Moment Corrections:

\[
\begin{align*}
\left( \frac{C_H}{c_{\delta}} \right)_{\text{Exp.}} = 1.0 \\
\left( \frac{C_H}{c_{\delta}} \right)_{3}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{C_H}{c_{\delta}} \right)_{\text{Exp.}} = 1.0 \\
\left( \frac{C_H}{c_{\delta}} \right)_{3}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{C_H}{t_{\delta}} \right)_{\text{Exp.}} = 1.0 \\
\left( \frac{C_H}{t_{\delta}} \right)_{3}
\end{align*}
\]
TABLE 14 (Continued)

\[
\begin{pmatrix}
\Delta C_{Hc} \\
\Delta C_{Hc} / 3
\end{pmatrix}
\begin{pmatrix}
\text{c/Exp.} = 1.0 \\
\text{t/Exp.} = 1.0
\end{pmatrix}
\]

Pressure Recovery Factor:

\[ K = 1.0 \]

General:

Reference half chord \((b_0) = 51.960 \text{ in.}\)

\[
\cos \psi_{c/4} = 0.865999
\]

\[
\cos \psi_{\delta_c} = 0.965086
\]

\[
\cos \psi_{\delta_t} = 0.980775
\]

Altitude - sea level \((\rho = 0.114626 \times 10^{-6} \text{ lb-sec}^2 \text{-in.}^{-4})\)
TABLE 15
SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 23

Uncoupled Natural Frequency, Trim Tab Rotation: $f_\delta = 20$ cps

Spring Constants:

$$k_c = 357 \text{ lb./in.}$$

$$K_A = 411 \text{ lb.-in./rad}$$

Aerodynamics: Basic Surface with Aerodynamic Balance ($B \neq 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_o}$</th>
<th>$f_1$</th>
<th>$V_1$</th>
<th>$g_1$</th>
<th>$f_2$</th>
<th>$V_2$</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>0.0</td>
<td>.000</td>
<td>32.1</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>0.20</td>
<td>20.6</td>
<td>34.1</td>
<td>-.100</td>
<td>22.2</td>
<td>71.4</td>
<td>-.039</td>
</tr>
<tr>
<td>0.40</td>
<td>11.3</td>
<td>72.6</td>
<td>-.238</td>
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<td>139.4</td>
<td>-.028</td>
</tr>
<tr>
<td>0.60</td>
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<td>123.4</td>
<td>-.521</td>
<td>20.9</td>
<td>202.5</td>
<td>.140</td>
</tr>
<tr>
<td>0.80</td>
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<td>.603</td>
</tr>
<tr>
<td>1.00</td>
<td>20.7</td>
<td>334.4</td>
<td>-.827</td>
<td>33.0</td>
<td>532.3</td>
<td>2.039</td>
</tr>
<tr>
<td>1.20</td>
<td>97.5</td>
<td>1885.6</td>
<td>-75.178</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

$V_f = 170$ knots

Uncoupled Natural Frequency, Rudder Rotation: $f_\beta = 10.86$ cps
### TABLE 16

**SOLUTIONS OF STABILITY EQUATIONS**

Plot: Figure 24

Uncoupled Natural Frequency, Trim Tab Rotation: \( f_\delta = 40 \text{ cps} \)

Spring Constants:

\[
\begin{align*}
    k_c &= 357 \text{ lb./in.} \\
    K_A &= 1645 \text{ lb.-in/rad.}
\end{align*}
\]

Aerodynamics: Basic Surface with Aerodynamic Balance \((B \neq 0)\)

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 ) (Knots)</th>
<th>( g_1 )</th>
<th>( f_2 ) (Knots)</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>31.1</td>
<td>0.0</td>
<td>44.0</td>
<td>0.0</td>
</tr>
<tr>
<td>.20</td>
<td>10.7</td>
<td>34.5</td>
<td>-0.092</td>
<td>43.7</td>
</tr>
<tr>
<td>.40</td>
<td>11.5</td>
<td>71.4</td>
<td>-0.197</td>
<td>43.8</td>
</tr>
<tr>
<td>.60</td>
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<td>43.7</td>
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</tr>
<tr>
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<tr>
<td>1.40</td>
<td>42.9</td>
<td>967.3</td>
<td>-11.996</td>
<td>-</td>
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</tbody>
</table>

\( V_f = 446 \text{ knots} \)

Uncoupled Natural Frequency, Rudder Rotation: \( f_\beta = 10.86 \text{ cps} \)
TABLE 17  
SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 25

Uncoupled Natural Frequency, Trim Tab Rotation: $f_0 = 60$ cps

Spring Constants:

\[ k_c = 357 \text{ lb./in.} \]
\[ K_A = 3702 \text{ lb.-in./rad.} \]

Aerodynamics: Basic Surface with Aerodynamic Balance ($B \neq 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_0}$</th>
<th>$f_1$</th>
<th>$V_1$</th>
<th>$g_1$</th>
<th>$f_2$</th>
<th>$V_2$</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>31.4</td>
<td>0.0</td>
<td>.000</td>
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</table>

$V_1 = 700$ knots

Uncoupled Natural Frequency, Rudder Rotation: $f_\beta = 10.86$ cps
TABLE 18
SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 2G

Uncoupled Natural Frequency, Trim Tab Rotation: \( f_\phi = 20 \text{ cps} \)

Spring Constants:

\[
k_c = 357 \text{ lb./in.}
\]
\[
K_A = 411 \text{ lb.-in./rad.}
\]

Aerodynamics: Basic Surface without Aerodynamic Balance \((B = 0)\)

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 )</th>
<th>( V_1 )</th>
<th>( V_1 )</th>
<th>( f_2 )</th>
<th>( V_2 )</th>
<th>( g_2 )</th>
</tr>
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<tr>
<td>( \text{Knots} )</td>
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<td>( \text{Knots} )</td>
<td>( \text{Knots} )</td>
<td>( \text{Knots} )</td>
<td>( \text{Knots} )</td>
<td>( \text{Knots} )</td>
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<td>0.0</td>
<td>.000</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</table>

\( V_f = 138 \text{ knots} \)

Uncoupled Natural Frequency, Rudder Rotation: \( f_\beta = 10.86 \text{ cps} \)
TABLE 19

SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 27

Uncoupled Natural Frequency, Trim Tab Rotation: $f_0 = 40$ cps

Spring Constants:

\[ k_c = 357 \text{ lb./in.} \]
\[ K_A = 1645 \text{ lb.-in./rad.} \]

Aerodynamics: Basic Surface without Aerodynamic Balance ($B = 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_0}$</th>
<th>$f_1$ (\text{cps})</th>
<th>$V_1$ (\text{Knots})</th>
<th>$g_1$</th>
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<th>$V_2$ (\text{Knots})</th>
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$V_f = 379$ knots

Uncoupled Natural Frequency, Rudder Rotation: $f_\beta = 10.86$ cps
TABLE 20
SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 28

Uncoupled Natural Frequency, Trim Tab Rotation: $f_\delta = 60$ cps

Spring Constants:

\[
\begin{align*}
  k_c &= 357 \text{ lb./in.} \\
  K_A &= 3702 \text{ lb.-in./rad.}
\end{align*}
\]

Aerodynamics: Basic Surface without Aerodynamic Balance ($B = 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_o}$</th>
<th>$f_1$</th>
<th>$V_1$</th>
<th>$g_1$</th>
<th>$f_2$</th>
<th>$V_2$</th>
<th>$g_2$</th>
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</table>

$V_f = 587$ knots

Uncoupled Natural Frequency, Rudder Rotation: $f_\beta = 10.86$ cps
Figure 23  g-V and f-V Plots, Rudder, $f_0 = 20$ cps, $B \neq 0$
Figure 24  g-V and f-V Plots, Rudder, $f_0 = 40$ cps, $B \neq 0$
Figure 25  g-V and f-V Plots, Rudder, $f_0 = 60$ cps, $B \neq 0$
Figure 26  g-V and f-V Plots, Rudder, $f_\theta = 20$ cps, $B = 0$
Figure 27  g-V and f-V Plots, Rudder, $f_\theta = 40$ cps, $B = 0$
Figure 28  g-V and f-V Plots, Rudder, $f_\theta = 60$ cps, B = 0
\[ \Delta B = 0 \text{ (UNBALANCED RUDDER)} \]

- Limit Dive Speed

**Figure 29** \( V_f \) vs. \( f_\delta \) Cross-plot, Rudder
4.3 HORIZONTAL STABILIZER AND ELEVATOR

A schematic of the horizontal stabilizer and elevator system is shown in Figure 30. A plan view of the horizontal stabilizer and elevator is shown in Figure 31, and the aerodynamic balancing arrangement is shown in Figure 32.

Analysis

For purposes of a flutter analysis, the system is idealized as shown in Figure 33. Only the symmetric condition is treated here. Examination of the system shows that for the antisymmetric condition, a node exists at the centerline of symmetry. Thus, antisymmetric rotational motion of a representative station of the elevator and horizontal stabilizer are those of torsional cantilevers. The spring restraint is that of the elevator interconnect torque tube and the torque box material for these two inertial elements. There is no motion of the elevator control system since the elevator horn is located on the node (physically it is very near the aircraft centerline of symmetry at the vertical tail tip). Thus, the antisymmetric elevator system is extremely stiff relative to the symmetric one, while the inertias of the surfaces remain the same. For an antisymmetric analysis, the generalized coordinate $\gamma$, control stick rotation, would cease to be a consideration, and control circuit stiffness as well as horizontal stabilizer trim actuator stiffness (the actuator is also located at the node) would cease to be elements of the analysis; there would be no coupling to the control system. On the basis of these observations, the antisymmetric elevator flutter condition is clearly not critical in comparison to the far softer symmetric case.

The degrees of freedom considered for the symmetric analysis are:

- $\alpha =$ Horizontal stabilizer rotation
- $\beta =$ Elevator rotation, relative to horizontal stabilizer
- $\gamma =$ Control stick rotation

The potential energy of the system is

$$U = \frac{1}{2} k_c (-r_1 \beta - r_8 \gamma)^2 + \frac{1}{2} K_H \alpha^2$$

(61)
where \( k_c \) = Linear spring restraint offered by control circuit, i.e., the effective spring of half the control circuit from control stick to elevator horn.

\( K_H \) = Rotational spring restraint offered by horizontal stabilizer trim actuator, half of the actual spring, due to symmetry.

\( r_1, r_8 \) = Linkage arm lengths

(See Figure 33 for a schematic representation of \( k_c, K_H, r_1 \) and \( r_8 \).)

Calculating the potential energy terms that are part of Lagrange's equations of motion (Equations (43), (44) and (45)) by taking partial derivatives of Equation (61),

\[
\frac{\partial U}{\partial \alpha} = K_H \alpha
\]

\[
\frac{\partial U}{\partial \beta} = k_c(a_1 \beta + a_3 \gamma)
\]

\[
\frac{\partial U}{\partial \gamma} = k_c(a_3 \beta + a_2 \gamma)
\]

where \( a_1 = r_1^2 \)

\( a_2 = r_8^2 \)

\( a_3 = r_1 r_8 \)

In matrix notation, the potential energy (spring) terms, then, are

\[
\begin{bmatrix}
K_H & 0 & 0 \\
0 & a_1 k_c & a_3 k_c \\
0 & a_3 k_c & a_2 k_c
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix}
= [S] \{q\}
and Equation (25), with the changes in generalized coordinates discussed on page 18, becomes

\[
\begin{bmatrix}
I_\alpha & P_{\alpha\beta} & 0 \\
0 & I_\beta & 0 \\
0 & 0 & J_0
\end{bmatrix}
\begin{bmatrix}
K_H & 0 & 0 \\
0 & a_{k_c} & a_{k_c} \\
0 & a_{k_c} & a_{k_c}
\end{bmatrix}
+ \begin{bmatrix}
I'_{11} & I'_{12} & 0 \\
I'_{21} & I'_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix} = 0
\]

(62)

In the case at hand, because of symmetry, \(J_0\) is the effective mass moment of inertia of half the control circuit and control stick about the control stick axis of rotation.

Upon expansion of the characteristic determinant of Equation (62), because of relationships between \([S]\) elements in terms of the r's (\([S]\) is singular), the following stability equation is obtained:

\[
A\Omega^2 + B\Omega + C = 0
\]

where \(A\) is real, and \(B\) and \(C\) are complex.

**Parameters**

Table 21 lists the aerodynamic parameters required to evaluate the oscillatory forces as defined in References 2 and 4 and in Equations (50) and (51).

The inertia parameters are:

\[
\begin{align*}
I_\alpha &= 42.1394 \text{ lb.-in.-sec.}^2 \text{ (half horizontal stabilizer)} \\
I_\beta &= 0.8906 \text{ lb.-in.-sec.}^2 \text{ (half elevator)} \\
S_\beta &= -0.0153 \text{ lb.-sec.}^2 \text{ (half elevator) - (nose heavy)} \\
l &= 21.09 \text{ in.} \\
J_0 &= 1.4404 \text{ lb.-in.-sec.}^2
\end{align*}
\]

The uncoupled horizontal stabilizer rotation natural frequencies, basic parameters for this preliminary analysis, are \(f_\alpha = 20, 40 \text{ and } 60 \text{ cps.}\)
The linkage parameters are:

\[
\begin{align*}
    r_1 &= 4.0 \text{ in.} \\
    r_8 &= 5.125 \text{ in.}
\end{align*}
\]

The spring constants are:

\[
\begin{align*}
    k_c &= 1145 \text{ lb./in. (half system)} \\
    K_H &= 665,400; 2,661,800 \text{ and } 5,988,900 \text{ lb.-in./rad. corresponding to } f_\alpha = 20, 40 \text{ and } 60 \text{ cps, respectively.} \\
    \bar{K} &= 10,000 \text{ lb.-in./rad.}
\end{align*}
\]

The value for \( k_c \) was calculated from design values for the elevator system. The values of \( K_H \) were determined for the various values of uncoupled horizontal tail frequency \( f_\alpha \). The scaling constant \( \bar{K} \) was assigned its value arbitrarily, for computing convenience.

**Results**

Tables 22 through 27 and Figures 34 through 39 show the effects of varying the horizontal stabilizer uncoupled frequency \( f_\alpha \) and aerodynamic balance.

**Conclusions**

Examination of the analysis results shows that elevator system flutter will not occur over the range of \( f_\beta \) and \( 1/k_o \) values investigated, either with or without aerodynamic balance for the elevator.
Figure 30  Schematic of Elevator System

Figure 31  Horizontal Tail - Plan View
Figure 32  Elevator Aerodynamic Balancing Arrangement

Figure 33  Idealization of Elevator System
### TABLE 21

**HORIZONTAL STABILIZER - ELEVATOR**

**AERODYNAMIC PARAMETERS**

Horizontal Stabilizer - Elevator Geometry:

<table>
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<th>Sta. No.</th>
<th>B. L.</th>
<th>b</th>
<th>c</th>
<th>e'</th>
<th>e''</th>
<th>g</th>
<th>F BB in.</th>
<th>g in.</th>
<th>a in.</th>
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</table>

**Aerodynamic Balance Arrangement:**

Elevator, internal - simple nose overhang (curtain seal)

**Hinge Moment Corrections:**

\[
\begin{align*}
\frac{C_{H\alpha}}{C_{H\alpha} \text{ Exp.}} &= 1.0 ; \quad \frac{\Delta C_{H\alpha}}{\Delta C_{H\alpha} \text{ Exp.}} = 1.0 \\
\frac{C_{H\delta}}{C_{H\delta} \text{ Exp.}} &= 1.0 ; \quad \frac{\Delta C_{H\delta}}{\Delta C_{H\delta} \text{ Exp.}} = 1.0
\end{align*}
\]
TABLE 21 (Continued)

Pressure Recovery Factor:

\[ K = 1.0 \]

General:

Reference half chord \( (b_0) = 32.820 \text{ in.} \)

\[ \cos \psi_c/4 = 0.971554 \]

\[ \cos \psi_{R_c} = 1.00 \]

Altitude - sea level \( (\rho = 0.114626 \times 10^{-6} \text{ lb. -sec.}^2 \text{-in.}^-4) \)
TABLE 22

SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 34

Uncoupled Natural Frequency, Horizontal Stabilizer Rotation: $f_\alpha = 20$ cps

Spring Constants:

$$k_c = 1145 \text{ lb./in.}$$

$$K_H = 665,400 \text{ lb.-in./rad.}$$

Aerodynamics: Basic Surface with Aerodynamic Balance (B ≠ 0)

<table>
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<th>$\frac{1}{k_o}$</th>
<th>$f_1 \text{ cps}$</th>
<th>$V_1 \text{ Knots}$</th>
<th>$g_1$</th>
<th>$f_2 \text{ cps}$</th>
<th>$V_2 \text{ Knots}$</th>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>6.1</td>
<td>688.7</td>
<td>-.237</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Elevator Rotation: $f_\beta = 22.83$ cps
TABLE 23

SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 35

Uncoupled Natural Frequency, Horizontal Stabilizer Rotation: \( f_\alpha = 40 \text{ cps} \)

Spring Constants:

\[
k_c = 1145 \text{ lb./in.}
\]

\[
K_H = 2,661,800 \text{ lb.-in./rad.}
\]

Aerodynamics: Basic Surface with Aerodynamic Balance \((B \neq 0)\)

<table>
<thead>
<tr>
<th>(\frac{1}{k_0})</th>
<th>(f_1)</th>
<th>(V_1)</th>
<th>(g_1)</th>
<th>(f_2)</th>
<th>(V_2)</th>
<th>(g_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>32.3</td>
<td>0.0</td>
<td>.000</td>
<td>40.3</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.15</td>
<td>22.1</td>
<td>33.8</td>
<td>-.095</td>
<td>39.3</td>
<td>60.0</td>
<td>-.019</td>
</tr>
<tr>
<td>.25</td>
<td>22.3</td>
<td>56.9</td>
<td>-.162</td>
<td>39.2</td>
<td>99.7</td>
<td>-.031</td>
</tr>
<tr>
<td>.45</td>
<td>23.2</td>
<td>106.2</td>
<td>-.313</td>
<td>38.8</td>
<td>177.9</td>
<td>-.052</td>
</tr>
<tr>
<td>.65</td>
<td>24.8</td>
<td>164.1</td>
<td>-.515</td>
<td>38.3</td>
<td>253.5</td>
<td>-.065</td>
</tr>
<tr>
<td>1.00</td>
<td>31.2</td>
<td>317.6</td>
<td>-1.255</td>
<td>37.4</td>
<td>381.2</td>
<td>-.060</td>
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<tr>
<td>2.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>35.8</td>
<td>728.9</td>
<td>-.147</td>
</tr>
<tr>
<td>4.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>27.4</td>
<td>1116.7</td>
<td>-.283</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Elevator Rotation: \( f_\beta = 22.83 \text{ cps} \)
### TABLE 24

SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 36

Uncoupled Natural Frequency, Horizontal Stabilizer Rotation: $f_\alpha = 60$ cps

Spring Constants:

\[
k_c = 1145 \text{ lb./in.}
\]

\[
K_H = 5,988,900 \text{ lb.-in./rad.}
\]

Aerodynamics: Basic Surface with Aerodynamic Balance ($B \neq 0$)

<table>
<thead>
<tr>
<th>( \frac{1}{k_0} )</th>
<th>( f_1 )</th>
<th>( V_1 )</th>
<th>( g_1 )</th>
<th>( f_2 )</th>
<th>( V_2 )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>32.3</td>
<td>0.0</td>
<td>.000</td>
<td>60.3</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.15</td>
<td>22.2</td>
<td>33.8</td>
<td>-.093</td>
<td>58.8</td>
<td>89.8</td>
<td>-.020</td>
</tr>
<tr>
<td>.25</td>
<td>22.4</td>
<td>57.0</td>
<td>-.159</td>
<td>58.6</td>
<td>149.3</td>
<td>-.034</td>
</tr>
<tr>
<td>.45</td>
<td>23.2</td>
<td>106.3</td>
<td>-.305</td>
<td>58.2</td>
<td>266.4</td>
<td>-.059</td>
</tr>
<tr>
<td>.65</td>
<td>24.8</td>
<td>163.8</td>
<td>-.498</td>
<td>57.4</td>
<td>379.7</td>
<td>-.079</td>
</tr>
<tr>
<td>1.00</td>
<td>31.1</td>
<td>316.2</td>
<td>-.188</td>
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<td>565.5</td>
<td>-.087</td>
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<tr>
<td>2.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>53.1</td>
<td>1080.8</td>
<td>-.130</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Elevator Rotation: $f_\beta = 22.83$ cps
TABLE 25
SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 37

Uncoupled Natural Frequency, Horizontal Stabilizer Rotation: $f_\alpha = 20$ cps

Spring Constants:

\[
\begin{align*}
  k_c &= 1145 \text{ lb./in.} \\
  K_H &= 665,400 \text{ lb.-in./rad.}
\end{align*}
\]

Aerodynamics: Basic Surface without Aerodynamic Balance ($B = 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_0}$</th>
<th>$f_1$</th>
<th>$V_1$</th>
<th>$g_1$</th>
<th>$f_2$</th>
<th>$V_2$</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cps</td>
<td>Knots</td>
<td></td>
<td>cps</td>
<td>Knots</td>
<td></td>
</tr>
<tr>
<td>.00</td>
<td>32.5</td>
<td>0.0</td>
<td>.000</td>
<td>20.0</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.15</td>
<td>22.7</td>
<td>34.6</td>
<td>-.064</td>
<td>19.1</td>
<td>29.2</td>
<td>-.054</td>
</tr>
<tr>
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<td>57.7</td>
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<td>49.1</td>
<td>-.088</td>
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<tr>
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<td>90.4</td>
<td>-.133</td>
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<tr>
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<td>24.0</td>
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<td>20.0</td>
<td>132.3</td>
<td>-.137</td>
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<tr>
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<td>312.2</td>
<td>-1.318</td>
<td>19.8</td>
<td>201.9</td>
<td>-1.49</td>
</tr>
<tr>
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<td>-</td>
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<td>372.7</td>
<td>-2.30</td>
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<tr>
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<td>-</td>
<td>-</td>
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<td>-3.19</td>
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<tr>
<td>6.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10.6</td>
<td>645.3</td>
<td>-3.14</td>
</tr>
<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>8.3</td>
<td>674.6</td>
<td>-2.85</td>
</tr>
<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>6.2</td>
<td>690.1</td>
<td>-2.41</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Elevator Rotation: $f_\beta = 22.83$ cps
TABLE 26
SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 38

Uncoupled Natural Frequency, Horizontal Stabilizer Rotation: $f_\alpha = 40$ cps

Spring Constants:

\[ k_c = 1145 \text{ lb./in.} \]
\[ K_H = 2,661,800 \text{ lb.-in./rad.} \]

Aerodynamics: Basic Surface without Aerodynamic Balance (B = 0)

<table>
<thead>
<tr>
<th>$\frac{1}{k_0}$</th>
<th>$f_1$ (cps)</th>
<th>$V_1$ (Knots)</th>
<th>$g_1$</th>
<th>$f_2$ (cps)</th>
<th>$V_2$ (Knots)</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>32.3</td>
<td>0.0</td>
<td>.000</td>
<td>40.3</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.15</td>
<td>22.1</td>
<td>33.8</td>
<td>-.100</td>
<td>39.2</td>
<td>59.9</td>
<td>-.019</td>
</tr>
<tr>
<td>.25</td>
<td>22.4</td>
<td>57.0</td>
<td>-.170</td>
<td>39.2</td>
<td>99.7</td>
<td>-.031</td>
</tr>
<tr>
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<td>23.4</td>
<td>107.3</td>
<td>-.334</td>
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<td>178.0</td>
<td>-.052</td>
</tr>
<tr>
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<td>168.1</td>
<td>-.567</td>
<td>38.3</td>
<td>253.4</td>
<td>-.064</td>
</tr>
<tr>
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<td>350.5</td>
<td>1.601</td>
<td>37.6</td>
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<td>-.057</td>
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<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>35.8</td>
<td>729.1</td>
<td>-.162</td>
</tr>
<tr>
<td>4.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>27.5</td>
<td>1118.6</td>
<td>-.290</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Elevator Rotation: $f_\beta = 22.83$ cps
TABLE 27

SOLUTIONS OF STABILITY EQUATIONS

Plot: Figure 39

Uncoupled Natural Frequency, Horizontal Stabilizer Rotation: $f_\alpha = 60$ cps

Spring Constants:

\[ k_c = 1145 \text{ lb./in.} \]
\[ K_H = 5,988,900 \text{ lb.-in/rad.} \]

Aerodynamics: Basic Surface without Aerodynamic Balance ($B = 0$)

<table>
<thead>
<tr>
<th>$\frac{1}{k_0}$</th>
<th>( f_1 )</th>
<th>( V_1 )</th>
<th>( g_1 )</th>
<th>( f_2 )</th>
<th>( V_2 )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>32.4</td>
<td>0.0</td>
<td>.000</td>
<td>60.3</td>
<td>0.0</td>
<td>.000</td>
</tr>
<tr>
<td>.15</td>
<td>22.2</td>
<td>33.8</td>
<td>-.098</td>
<td>58.7</td>
<td>89.7</td>
<td>-.021</td>
</tr>
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<td>57.1</td>
<td>-.167</td>
<td>58.6</td>
<td>149.2</td>
<td>-.034</td>
</tr>
<tr>
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<td>107.4</td>
<td>-.327</td>
<td>58.1</td>
<td>266.3</td>
<td>-.059</td>
</tr>
<tr>
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<td>25.4</td>
<td>167.8</td>
<td>-.548</td>
<td>57.3</td>
<td>379.4</td>
<td>-.079</td>
</tr>
<tr>
<td>1.00</td>
<td>34.3</td>
<td>349.6</td>
<td>-1.525</td>
<td>55.6</td>
<td>565.8</td>
<td>-.080</td>
</tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>53.3</td>
<td>1084.5</td>
<td>-.147</td>
</tr>
</tbody>
</table>

Uncoupled Natural Frequency, Elevator Rotation: $f_\beta = 22.83$ cps
Figure 34  g-V and f-V Plots, Elevator, $f_\alpha = 20$ cps, $B \neq 0$
Figure 35  g-V and f-V Plots, Elevator, $\omega_n = 40$ cps, $B \neq 0$
Figure 36    g-V and f-V Plots, Elevator, $f_\alpha = 60$ cps, B $\neq 0$
Figure 37  g-V and f-V Plots, Elevator, \( f_\alpha = 20 \) cps, \( B = 0 \)
Figure 38  g-V and f-V Plots, Elevator, $f_\alpha = 40$ cps, $B = 0$
Figure 39  g-V and f-V Plots, Elevator, $f_\alpha = 60$ cps, $B = 0$
5.0 CONCLUSIONS AND RECOMMENDATIONS

Flutter analyses of all XV-5A control surfaces have been made, based on preliminary input information. Increased stiffness of the aileron control circuit (the symmetric aileron condition is critical) is required, and the stiffness of the rudder trim tab actuator should be such that the uncoupled rigid body tab rotational frequency is at least 50 cps in order to ensure a flutter-free airplane within the established flight envelope. The antisymmetric aileron condition and elevator are flutter-free.

It is recommended that further flutter analysis of all control surfaces be made on the basis of refinements to stiffness and inertia properties as the design develops, incorporating experimentally determined properties as they become available. It is recommended that the report herein be used as intended: for interpretive purposes as an aid to design development and not as a definitive prediction of control system flutter performance. Further, this report can and should be used in conjunction with the formulation of main surface flutter analyses (wing, and empennage, for example) to determine important control surface degrees of freedom that should be considered in the main surface analyses.
6.0 APPENDIX

6.1 REFERENCES


