

AD629082

MEMORANDUM  
RM-4854-PR  
JANUARY 1966

## ESTIMATING COST UNCERTAINTY USING MONTE CARLO TECHNIQUES

Paul F. Dienemann

*Code 1*

CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION			
Hardcopy	Microfiche		
\$3.00	\$0.50	55	pp as
ARCHIVE COPY			

*Code 1*

PREPARED FOR:  
UNITED STATES AIR FORCE PROJECT RAND

---

The RAND Corporation  
SANTA MONICA • CALIFORNIA

---

MEMORANDUM

RM-4854-PR

JANUARY 1966

ESTIMATING COST UNCERTAINTY USING  
MONTE CARLO TECHNIQUES

Paul F. Dienemann

This research is sponsored by the United States Air Force under Project RAND—Contract No. AF 49(638)-1700—monitored by the Directorate of Operational Requirements and Development Plans, Deputy Chief of Staff, Research and Development, Hq USAF. Views or conclusions contained in this Memorandum should not be interpreted as representing the official opinion or policy of the United States Air Force.

DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

---

The RAND Corporation

1700 MAIN ST. • SANTA MONICA • CALIFORNIA • 90406

Approved for release by the Clearinghouse for  
Federal Scientific and Technical Information

PREFACE

This Memorandum is part of a continuing research effort by the RAND Cost Analysis Department to improve its capability for estimating costs of future weapon systems. One unavoidable problem in the preparation of such estimates is the uncertainty about something which will only come into being at a future time. We are often uncertain as to the exact nature of the system, of its methods of operation, and also uncertain about the military and political universe in which it will function. None of the methods currently available for dealing with this problem are entirely satisfactory. The work described in this Memorandum is not presented as the final solution either; but it is, rather, offered as a meaningful and useful technique for handling a portion of the total problem.

SUMMARY

Suggested in this Memorandum is a technique for expressing cost estimates of future systems as probability distributions to reflect the uncertainty of the estimate. The impact of this information is shown to be relevant to the decision-making process.

For the purpose of this study, the relationship between the sources of uncertainty and system cost estimates is depicted as an input-output model. Within this framework, a procedure was developed to estimate probability distributions for each of the input uncertainties. From the input distributions, a Monte Carlo procedure is used to generate a series of system cost estimates. A frequency distribution and common statistical measures are then prepared from the set of output estimates to ascertain the nature and magnitude of the system cost uncertainty.

To illustrate the proposed technique, a case study involving the cost estimate of a hypothetical aircraft system with air-to-surface missiles is presented.

CONTENTS

PREFACE .....	iii
SUMMARY .....	v
LIST OF FIGURES .....	ix
 Section	
I. INTRODUCTION .....	1
Sources of Uncertainty .....	1
Relevance to Decision-Making .....	2
Expressing Cost Uncertainty .....	4
II. MONTE CARLO TECHNIQUES FOR ESTIMATING UNCERTAINTY .....	6
Simulating Probability Distributions .....	7
Estimating System Cost Uncertainty .....	8
III. CASE STUDY: INDIVIDUAL WEAPON SYSTEM COST ANALYSIS .....	12
Input Probability Distributions .....	12
Monte Carlo Simulation of System Cost Uncertainty .....	16
Case I: Cost-Estimating Uncertainty .....	16
Case II: Total Uncertainty .....	18
Conclusions .....	24
IV. THE GREATER PROBLEM: UNCERTAINTY IN COST/EFFECTIVENESS STUDIES .....	26
 Appendix	
A. WEAPON SYSTEM COST MODEL INPUTS .....	29
B. WEAPON SYSTEM COST MODEL OUTPUTS .....	39
C. FORTRAN SUBROUTINES FOR ESTIMATING UNCERTAINTY .....	43
BIBLIOGRAPHY .....	49

FIGURES

1. Impact of Cost Uncertainty on Decision Making .....	3
2. Relation of System Cost Uncertainty to Source Uncertainty ..	6
3. Monte Carlo Sampling .....	7
4. Simulated Input Distribution .....	8
5. Input Uncertainty Distributions .....	9
6. Total Training Cost .....	10
7. Input Uncertainty Probability Distributions .....	14
8. Estimating System Cost Uncertainty .....	17
9. Initial Investment Cost (millions of dollars) .....	19
10. Annual Operating Cost (millions of dollars) .....	20
11. Research and Development Costs (millions of dollars) .....	21
12. Five-Year System Cost (millions of dollars) .....	22
13. Total Five-Year System Cost (millions of dollars) .....	23
14. Confidence Limits for Sensitivity Analysis .....	24
15. Estimating Cost/Effectiveness Uncertainty .....	27

## I. INTRODUCTION

A primary function of the long-range planning process is to examine, in a systematic manner, future courses of action in order to identify alternatives which seem preferable to others. The analytic techniques used for such studies are known as cost/effectiveness analysis or systems analysis. As part of the total analytic process, cost analysis deals with the problem of determining the resource impacts of the alternative proposals.

It is an inescapable fact that estimates of resource requirements for future systems are beset with uncertainty. The question is not whether uncertainty exists, but rather in determining the magnitude and nature of the uncertainty. Before addressing this problem, we should take time to examine the sources of resource (cost) uncertainty.

### SOURCES OF UNCERTAINTY

Uncertainties in estimates of resource requirements for future systems arise from many sources. For our purpose it is convenient to consider two categories of uncertainty: requirements uncertainty and cost-estimating uncertainty.\*

Requirements uncertainty refers to variations in cost estimates stemming from changes in the configuration of the system being costed. Here system configuration change means deviations from original specifications or assumptions regarding hardware characteristics and/or system operational concepts. Although each of these sources of uncertainty is of a type over which decision-makers have control, studies have shown that uncertainty about requirements comprises 0 to 80 per cent of the total estimate uncertainty.

Cost-estimating uncertainty refers to variations in cost estimates of a system or force when the configuration of the system or force remains constant. It differs from requirements uncertainty in that decision-makers cannot affect the magnitude of the variations.

---

\* G. H. Fisher, A Discussion of Uncertainty in Cost Analysis, The RAND Corporation, RM-3071-PR, April 1962.

Cost-estimating uncertainty arises from numerous sources: differences between individual cost analysts, errors in the data base used in cost analysis, errors in cost estimating relationships, extrapolation errors, and so forth. Although we shall treat both categories of uncertainty in this Memorandum, cost-estimating uncertainty is more amenable to the discussion and techniques which are presented.

#### RELEVANCE TO DECISION-MAKING

It is under conditions of uncertainty that decision-makers must evaluate and select among alternative proposals for future courses of action. With explicit information describing the uncertainty of each system cost, decision-makers will be aided in two ways. First, the extent and probability that the ultimate system cost could differ from its expected, single-valued estimate can be anticipated and evaluated. With a quantitative measure for the precision of cost estimates, decision-makers should be better able to judge--according to their preferences and attitudes toward risk--the desirability of each system alternative. The second benefit is that decision-makers with this information will be more apt to choose the preferable alternative, especially in cases where the expected costs for competing systems are nearly coincident but have differing uncertainties. To illustrate this last point, consider the following example.

Suppose two systems, A and B, are to be compared using cost (resource requirements) as the criterion for evaluation. Figure 1 shows four cases in which cost estimates are expressed as probability distributions to reflect the actual, though perhaps unmeasurable, uncertainty surrounding each estimate. In Case I, the decision-maker is faced with no problem since all possible costs for System A are lower than System B. Using single-valued estimates (the mean or expected value) would not affect the decision. The situation in Case II is slightly different in that there is some probability that the actual cost of System A will be higher than System B. If this probability is not large, the decision-maker would still select System A. However, when the overlap is significant, the single-valued estimates would no

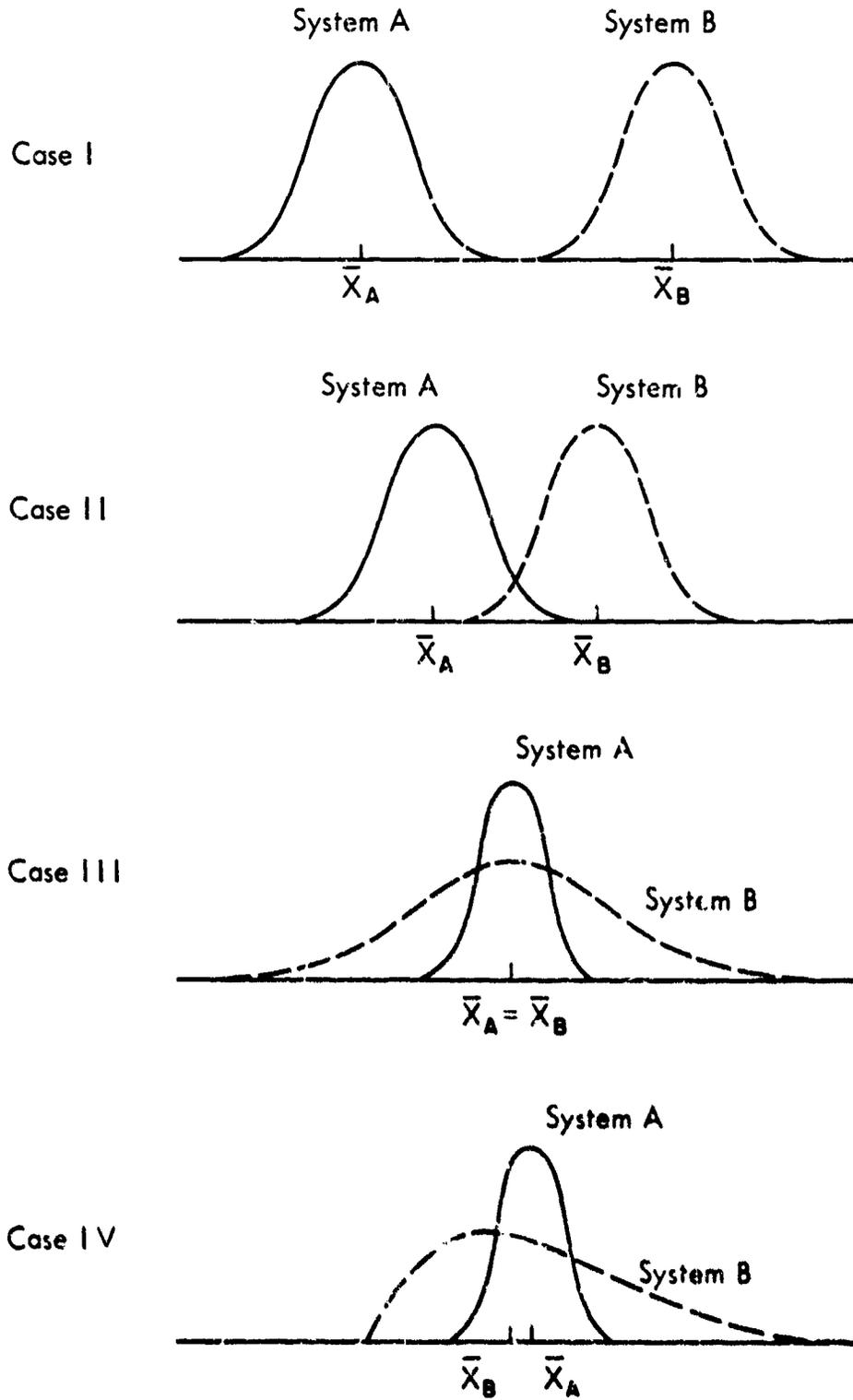


Fig. 1.—Impact of cost uncertainty on decision making

longer provide a valid criterion for system selection. In the third case, both single-valued estimates are the same, but the cost distribution for System B has a larger range. Here the decision-maker's disposition toward uncertainty and risk must dominate the selection process. If he prefers to minimize risk, he will select System A. However, if the decision-maker is willing to risk possible high costs for the chance to obtain a low-cost system, he might prefer System B. Case IV illustrates a more complicated situation where the expected cost of System B is lower but much less certain than System A. If the decision-maker uses only single-valued estimates in this case, he would most likely choose the less desirable alternative.

We should conclude from the previous example that cost uncertainty can be relevant to the decision-making process and an estimate of its nature and magnitude would be a valuable addition to cost/effectiveness studies. Given the context of the uncertainty problem, let us consider some alternative approaches which might be used for expressing cost uncertainty.

#### EXPRESSING COST UNCERTAINTY

A common procedure for describing uncertainty of system cost is to test its sensitivity against system cost inputs when they are varied over their relevant ranges. Sensitivity analyses of this type help to identify major sources of uncertainty and provide valuable information to the system designer. Such a procedure does little, however, to reveal the extent to which the estimated system cost might differ from the actual cost.

Another approach which has been used to express cost uncertainty requires the analyst to specify the lowest and highest possible values for each system cost input in addition to its most-likely value. From the three sets of input values are derived separate system cost estimates. The most-likely values establish the central tendency of the system cost, while the other two are used to determine a range for the cost estimate. Although this approach has considerable merit, it tends to greatly exaggerate the uncertainty of system cost estimates,

since it is extremely unlikely that all system cost inputs will be at the lowest (or highest) values at the same time.

The technique presented in this Memorandum suggests a scheme whereby resource estimates can be expressed as a probability distribution around a mean value. This permits the cost analyst to express in a quantitative manner the uncertainty of his estimate. In the sections to follow, we shall examine the proposed technique for estimating cost uncertainty, present a case study to illustrate the salient features of the technique, and finally discuss possible application of these ideas to the bigger problem of treating uncertainty in cost/effectiveness studies.

## II. MONTE CARLO TECHNIQUES FOR ESTIMATING UNCERTAINTY

In the previous section, it was noted that cost estimate uncertainty results from two primary sources--requirements and cost-estimating uncertainty. The relationship between the system cost uncertainty and its sources can be depicted using a simple input-output model.

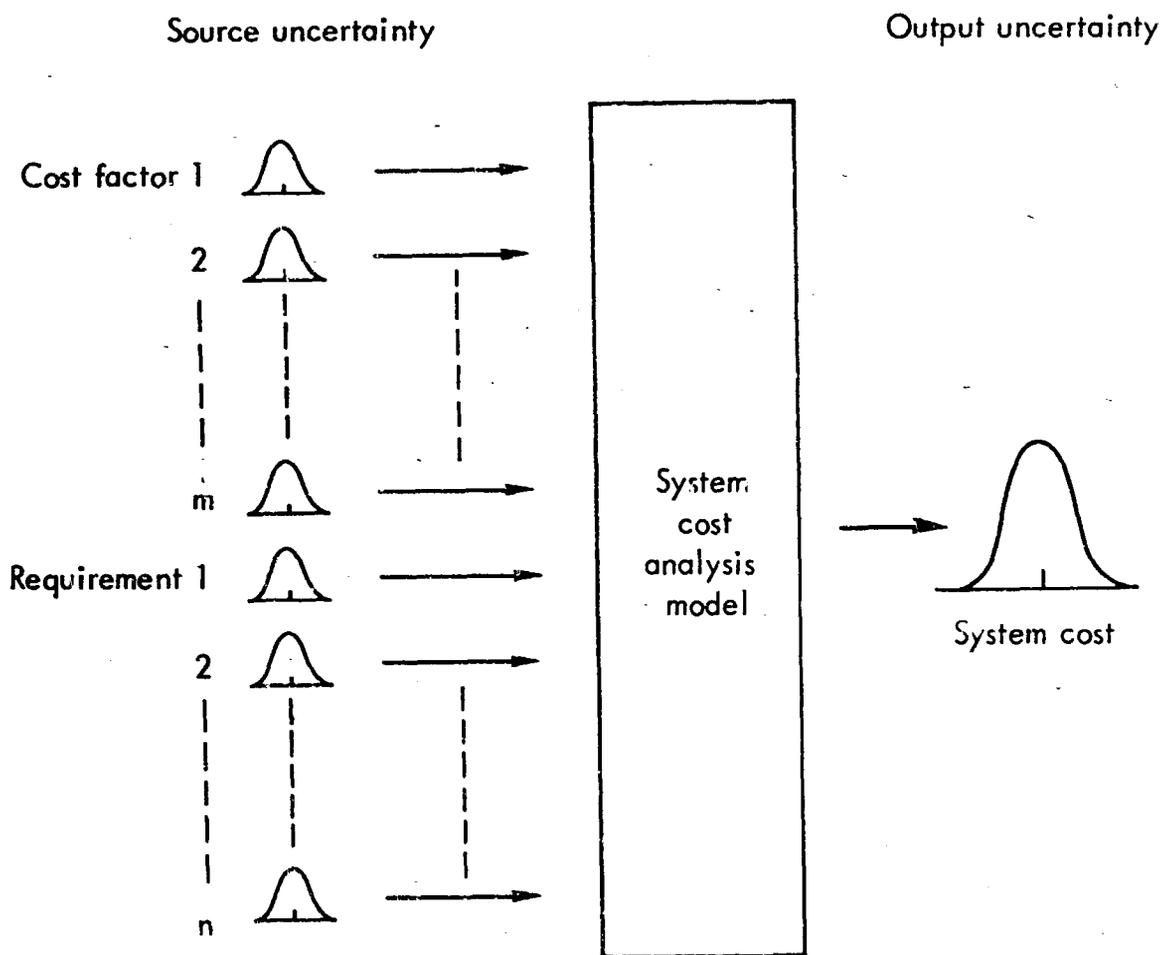


Fig.2— Relation of system cost uncertainty to source uncertainty

The cost analysis model portrayed in the diagram embodies all cost estimating procedures and methodology necessary for estimating the system cost. For our purpose, the cost factors, constants, and estimating relationship coefficients relevant to the system being costed are treated as a set of inputs to the cost model. Associated with each cost input is a probability distribution to reflect its uncertainty. In Section III, it will be shown that these distributions

can be described either statistically or from subjective probabilities. The second input set defines the specifications and requirements, along with accompanying uncertainties, of the system being costed.

As it now stands, the problem statement is to estimate the uncertainty of the total system cost when all input uncertainties, subject to the complex interactions of the cost analysis model, are considered simultaneously. The proposed solution described on the following pages uses a simulation technique to generate the input parameters, then prepares a series of system cost estimates. From the set of output estimates, common statistical measures (mean, standard deviation, range), and a frequency distribution are calculated. Let us first examine the techniques for simulating input uncertainty.

#### SIMULATING PROBABILITY DISTRIBUTIONS

At this point, assume that a cost analyst or system planner can describe each input parameter with a probability distribution. This distribution is then treated as a theoretical population from which random samples are obtained. The methods of taking such samples, as well as problems which rely on these sampling techniques, are often referred to as Monte Carlo Methods.

To illustrate the Monte Carlo procedure for simulating cost input uncertainty, consider the example depicted in Fig. 3.

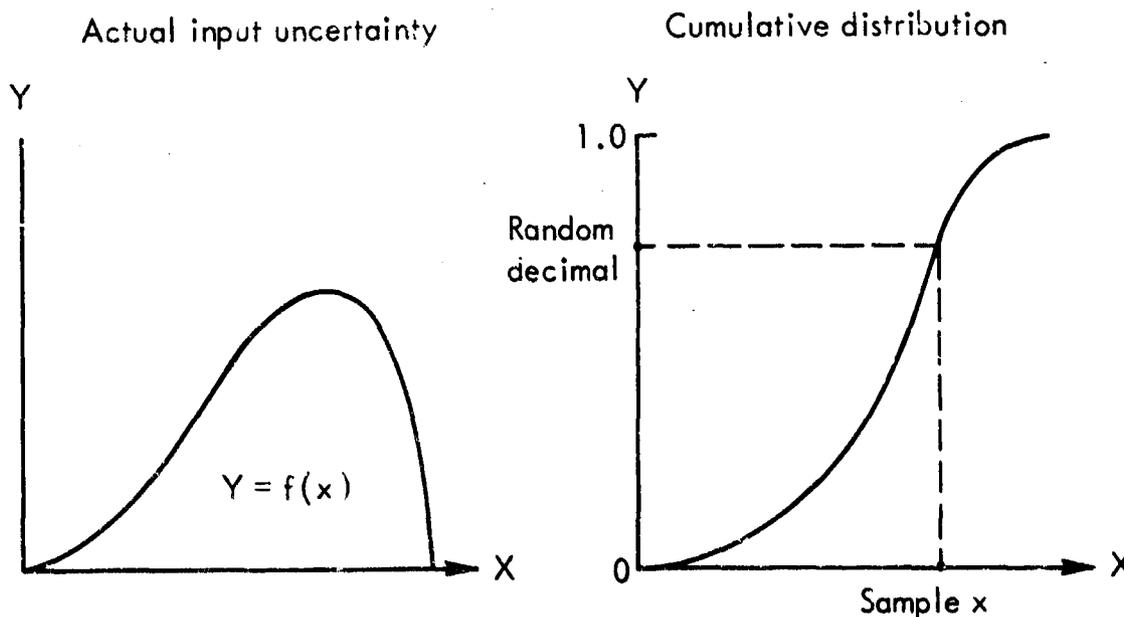


Fig.3-- Monte Carlo sampling

From the probability density,  $Y = f(x)$ , describing the actual (or estimated) input uncertainty, a cumulative distribution is plotted. Next, a random decimal between zero and one is selected from a table of random digits. By projecting horizontally from the point on the Y-axis corresponding to the random decimal to the cumulative curve, we find the value of  $x$  corresponding to the point of intersection. This value is taken as a sample value of  $x$ .

The result, if this procedure is repeated numerous times, is a sample of input values that approximates the required input uncertainty. As seen in Fig. 4, the more repetitions, the better the simulated input distribution.

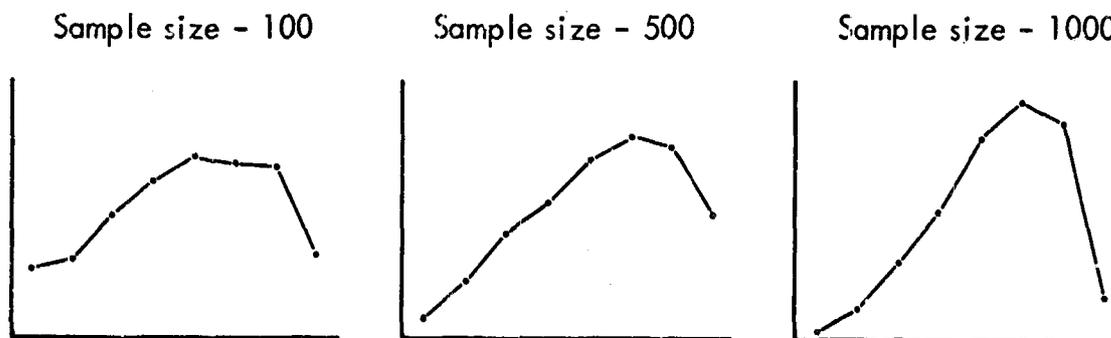
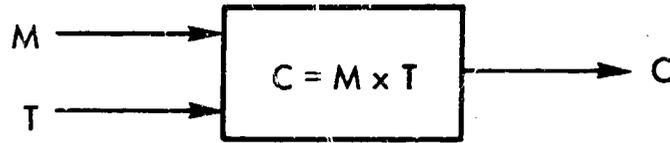


Fig. 4— Simulated input distribution

In the next section where the problem of estimating system cost (output) uncertainty is treated, each of the input parameters must be generated using the Monte Carlo technique just described.

#### ESTIMATING SYSTEM COST UNCERTAINTY

The procedure for estimating system cost uncertainty follows readily once simulated input values have been made. To illustrate the methodology, consider the following simple costing model:



where C = Total training cost (dollars)

M = Manpower requirements (number personnel)

T = Initial training cost (dollars/man)

Suppose the actual uncertainty of the input parameters can be represented with probability distributions as shown below, with L, M, and H denoting the lowest possible, most-likely, and highest possible values, respectively.

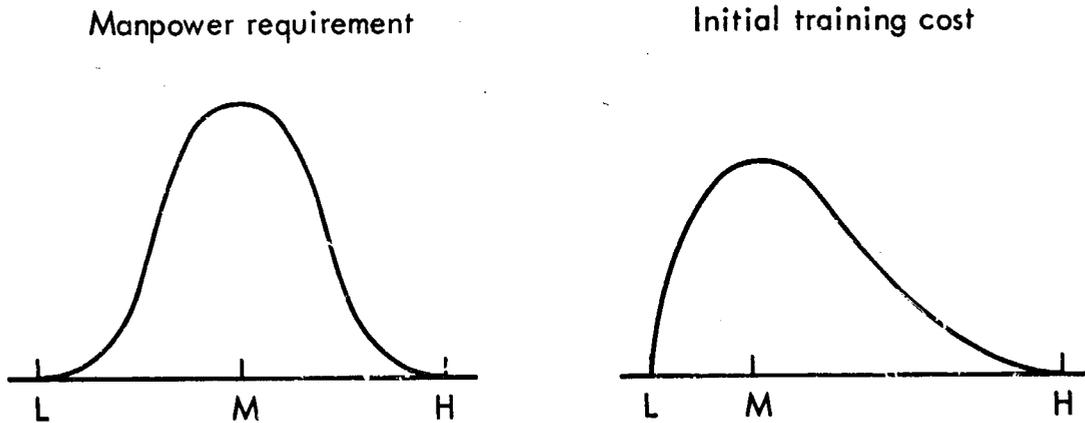


Fig.5— Input uncertainty distributions

Furthermore, assume that these values are as follows:

	<u>L</u>	<u>M</u>	<u>H</u>
Manpower requirement	75	100	125
Initial training cost	3000	4000	7000

From the input distributions, a sample value for both the manpower requirement and the initial training cost is generated by means of the Monte Carlo technique. Using these two sample values, a total training cost is calculated. The procedure is repeated again and again until the nature of the output uncertainty has been established. Table 1 summarizes the procedure for 1000 iterations.

Table 1

MONTE CARLO SIMULATION OF COST UNCERTAINTY

Iteration	M	x	T	=	C
1	83		4,052		336,316
2	108		6,326		683,208
3	103		3,741		385,323
4	101		4,520		456,520
5	92		3,874		356,408
⋮	⋮		⋮		⋮
⋮	⋮		⋮		⋮
1,000	⋮		⋮		⋮
Mean Values	100		4,498		450,000*

\* Rounded to nearest thousand.

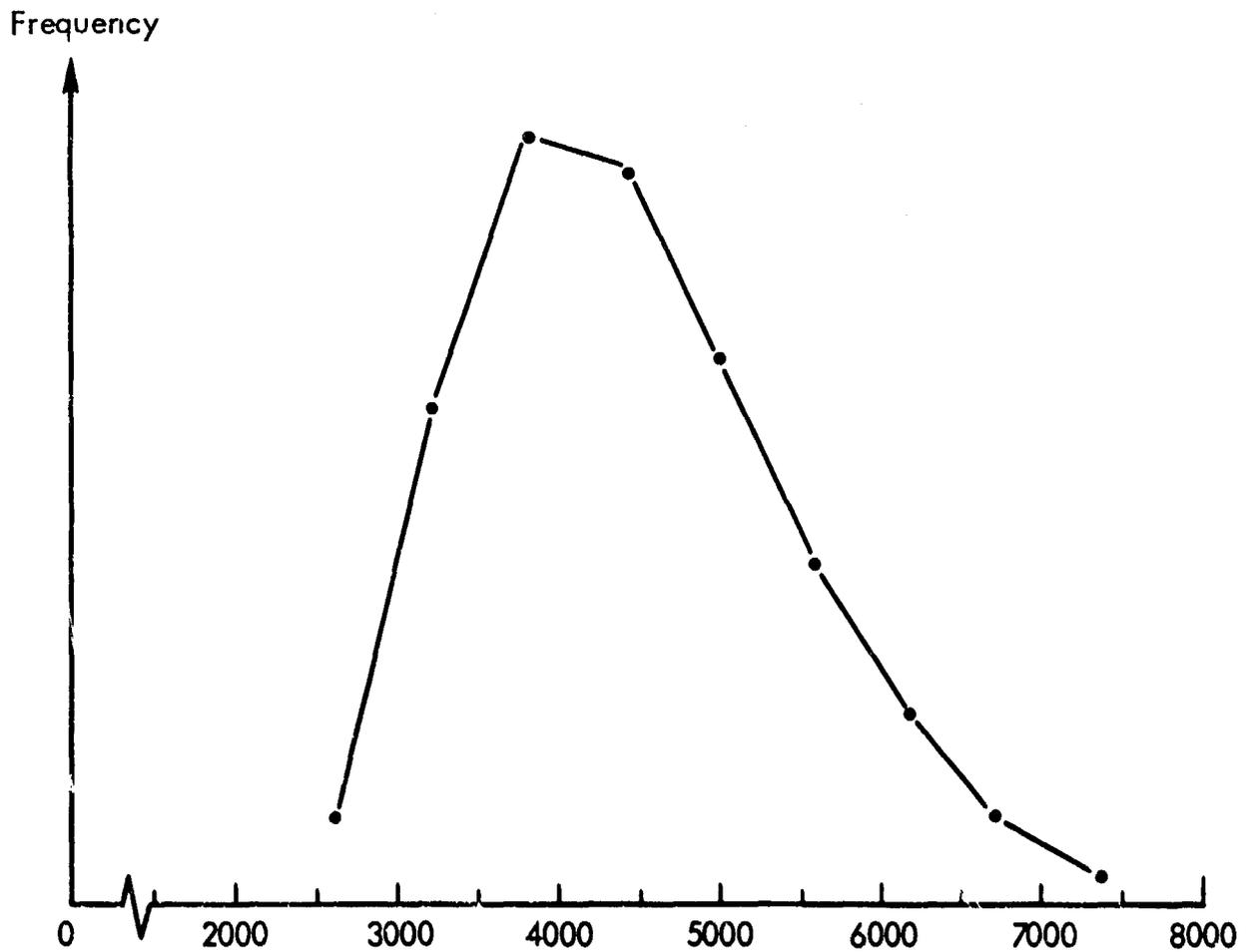


Fig. 6—Total training cost

From the set of cost estimates a frequency distribution as shown in Fig. 6 can be prepared to portray the cost uncertainty. It is interesting to note that the mean value of the total training cost is higher than the traditional, single-value cost estimate (\$400,000) -- the product of the most-likely values for each input factor. The difference between the two estimates occurs because the initial training cost uncertainty is skewed to the right. If the uncertainty distributions of both input factors were symmetric, the two cost estimates would be identical. Of course, the single-value estimate would not describe the nature or magnitude of the output cost uncertainty.

Although this example depicts a very simple costing problem, the techniques which were used to estimate cost uncertainty are applicable to more realistic situations. However, when the scope of the cost analysis problem is expanded -- as the case study presented in Section III -- it is expedient that the costing model be programmed for a computer.

It must be noted that using the Monte Carlo technique to estimate cost uncertainty in this manner requires that all input parameters be mutually independent. For instance, in the example described above it was assumed that the manpower requirement would not be affected by the cost of training. Although there are times when these assumptions might not be true, they should be valid for situations where the inputs have less than order-of-magnitude uncertainty. With cost factor inputs, we can probably conclude that the assumption of independence is true. However, with system requirements we must be more careful. In cases where a functional relationship does exist between two or more inputs, we can often circumvent the interdependence problem by incorporating the relationship within the cost model; or if the problem demands, one could explore more sophisticated techniques for sampling from joint frequency distributions.\*

---

\* D. J. Finney, "Frequency Distribution of Deviation from Means and Regression Lines in Samples from a Multi-variate Normal Population," The Annals of Mathematical Statistics, Vol. 17, 1946.

### III. CASE STUDY: INDIVIDUAL WEAPON SYSTEM COST ANALYSIS

For the purpose of illustrating the concepts and techniques presented in the previous section, a computerized weapon system costing model used by the RAND Cost Analysis Department was found to be quite suitable. The model is used to determine resource requirements for individual weapon systems consisting of either aircraft, aircraft with air-to-surface missiles (ASMs), or strategic missiles (for personnel requirements only). A complete description of the operation and design of the model is available elsewhere.\*

The case study presented in this section is a hypothetical aircraft system with ASMs. To estimate the cost of this system, the computer model requires 195 inputs (27 system requirements and 168 cost factors).\*\* For all practical purposes, the inputs are mutually independent. The uncertainty of the system cost estimate was examined for two cases: one treating only cost-estimating uncertainty, the other the total uncertainty.

Up to this point, it was assumed that all input uncertainty could be described with probability distributions. Before proceeding further with the case study, we must deal with this assumption.

#### INPUT PROBABILITY DISTRIBUTIONS

Distributions expressing input uncertainty can be derived from either statistical parameters or subjective probabilities. When input factors are based on historical data, the first approach is appropriate. Here the standard error of estimate for the factor or estimating relationship defines the magnitude of the input uncertainty. For example, 95 per cent confidence limits can be used to estimate the extreme values. Decisions regarding the symmetry of the uncertainty should reflect the analytic procedure used in developing the factor and the nature of the historical data. When a skewed distribution is suspected,

---

\* H. E. Boren, Jr., Individual Weapon System Computer Cost Model, The RAND Corporation, RM-4165-PR, July 1964.

\*\* See Appendix A for input list.

the approach discussed below should be used to describe the input uncertainty.

In cases where input factors are not derived from historical data (e.g., system requirements) or when the back-up data no longer are available, the cost analyst must utilize subjective probabilities to describe input uncertainty. To simplify this problem the following procedure was developed.

First, the analyst specifies three values for each input factor: the lowest possible, most-likely, and highest possible value. Since the most-likely value is the point estimate normally required, only two additional estimates are needed. The cost analyst should be able to estimate, or at least approximate, the highest and lowest possible values for each input parameter--he might, in fact, welcome the opportunity to qualify his point estimates in this manner. Next, the analyst chooses from the nine probability distributions depicted in Fig. 7 the type which best describes the nature of each factor (or requirement) uncertainty. The selection must be based on whether the analyst considers the uncertainty is skewed left, symmetric, or skewed right and whether the variance (degree of uncertainty) is low, medium, or high. For example, many cost factors have a realistic minimum but no obvious maximum suggesting a distribution that is skewed right. On the other hand, some inputs, e.g., system performance characteristics, often tend to have uncertainties that are skewed left. Although the nine input types describe only a few of numerous possible distributions, the selection should suffice since it is unlikely that an analyst could accurately distinguish between more variations anyway.

All input uncertainties were assumed to be beta-distributed, with the form:

$$f(x) = K(x - L)^{\alpha}(H - x)^{\beta},$$

where  $f(x)$  = probability of value  $x$ ,

$L$  = lowest possible value,

$H$  = highest possible value,

$\alpha, \beta$  = beta parameters.

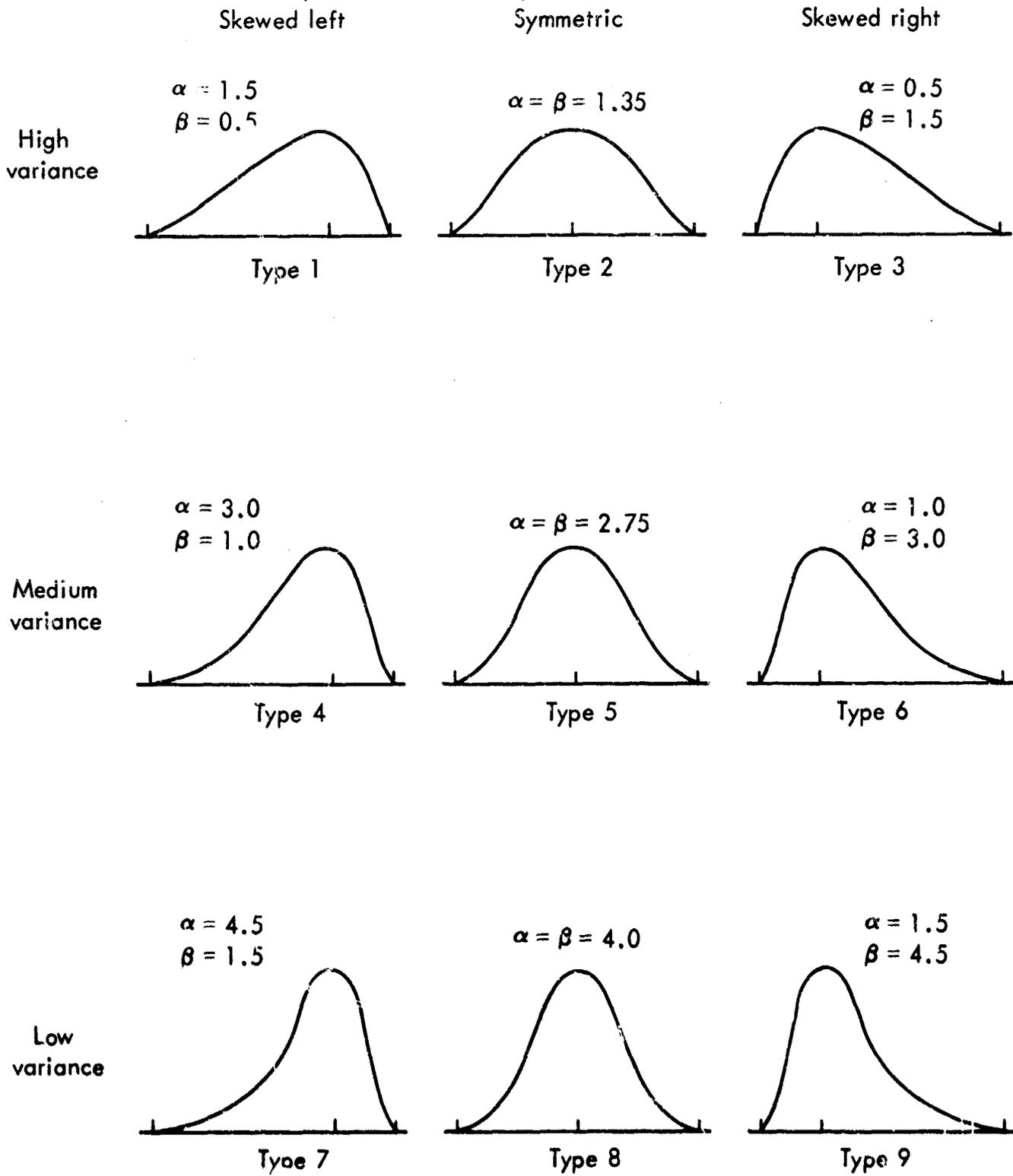


Fig. 7— Input uncertainty probability distributions

The mode (m) of the beta function (the most-likely value) is defined by the equation:

$$m = \frac{\alpha}{\alpha + \beta}$$

For the purpose of describing input uncertainties, beta distributions have many characteristics which would be expected in the actual parameter--finite range, continuity, and unimodality. Precedence in the use of beta functions for this purpose can be found in PERT assumptions regarding the uncertainty distribution of activity durations.\* For these reasons, beta distributions were utilized to describe the subjective probabilities of inputs to the cost model for the case study.

While achieving simplicity and ease of use, one problem exists in the procedure just outlined that needs mentioning. From the equation of the beta function, we note that four parameters,  $\alpha$  and  $\beta$  (implied by a selection of a distribution type) and the high-low values, specify a unique beta distribution. Any estimate of the most-likely input value is, therefore, overdetermined. However, the modal value, being more accurate than either estimated extreme, should be incorporated in the description of the uncertainty distribution. For this reason, it was necessary to define the distribution using  $\alpha$ ,  $\beta$ , the modal value, and range. Because of the types of distributions selected (see Fig. 7), the modal value of the simulated inputs will always be at the first quarter, midpoint, or third quarter of the range depending on whether the distribution is skewed left, symmetric, or skewed right. Furthermore, the calculated high and low values will usually differ slightly from the values specified by the analyst to estimate the input range. However, the discrepancy between suggested and resultant extreme values is not critical.

A list of distribution types (coded 1 through 9) and suggested low, modal, and high values for some of the 195 inputs to the case study cost model are tabulated in Appendix A.

---

\* K. R. MacCrimmon, An Analytic Study of the PERT Assumptions, The RAND Corporation, RM-3408-PR, December 1962.

## MONTE CARLO SIMULATION OF SYSTEM COST UNCERTAINTY

The mechanics of proceeding from input uncertainty expressions to estimates of system cost uncertainty are relatively simple. The flow diagram in Fig. 8 summarizes the necessary steps. A listing of the FORTRAN subroutines for Steps 1, 3, and 6 (called BTABLE, SAMPLE, and HISTO, respectively) is included in Appendix C.

Step 1. In the first step, cumulative beta tables for the nine functional forms are generated. By using 128 increments for each table, a maximum of only 6 binary search steps is required to "look up" beta values for randomly generated decimals.

Step 2. Next, the inputs to the costing model are read and stored. Each input data card contains the low, modal, and high value and functional form for two system input parameters.

Step 3. Using a random number generator, subroutine SAMPLE develops a Monte Carlo sample value for each input parameter based on its mode, range and form. A different random number is used for each Monte Carlo simulation.

Step 4. After a complete set of sample inputs has been generated, the existing cost model is used to estimate the system cost. The results are stored in an output table.

Step 5. Here the computer program tests whether the specified number of system cost estimates have been calculated. If not, a new set of inputs is generated, and the procedure is repeated. When the last iteration is made (1000 repetitions were used in this study), the program proceeds to the last step.

Step 6. From the tabulation of estimated system costs, subroutine HISTO calculates the mean value, standard deviation, and a frequency distribution (for 11 class intervals). These parameters provide the description of cost estimate uncertainty, thus completing the objective of our study.

### CASE I: COST-ESTIMATING UNCERTAINTY

In this analysis, system cost uncertainty was estimated for cost-estimating uncertainty only; system requirements were limited to

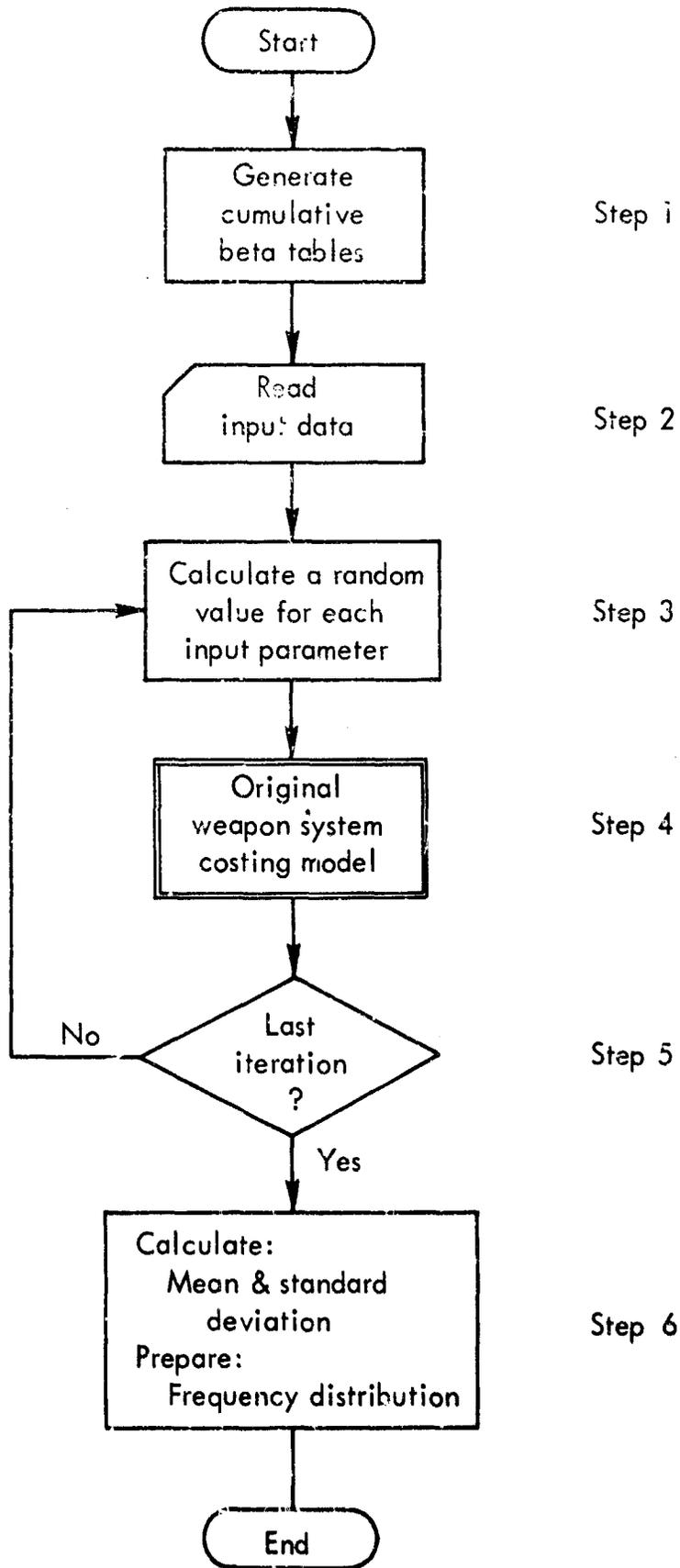


Fig.8—Estimating system cost uncertainty

single-valued inputs. The uncertainty estimates were prepared for major cost categories: initial investment, annual operating, research and development, and five-year system cost; the results of which are presented in Figs. 9 through 12. Included for comparative purposes with each cost distribution is the point estimate obtained by limiting all input parameters to their most-likely values. These estimates are identical to the values obtained when using the costing model in its original form. Appendix B has a listing of the original computer output for the same hypothetical aircraft system. The differences between the mean values of the cost distributions and the point estimates are attributable to the asymmetric form of the uncertainties ascribed to the cost factor inputs and the extent and kind of interactions that take place among them within the costing model. Although only major cost categories were examined, uncertainty estimates for other categories (e.g., detailed cost elements) or major resource requirements (e.g., personnel) could have been prepared with no more effort.

#### CASE II: TOTAL UNCERTAINTY

Here, the effects of both sources of uncertainty, cost-estimating and requirements, were analyzed. Although arbitrary estimates were made to reflect requirement input uncertainty (e.g., low, modal, and high estimates of aircraft procurement level at 40, 100, and 120 units, respectively, with a type 4 distribution), they are in line with previous studies in this field.\* Therefore, the estimate of total system cost uncertainty should be reasonably accurate. As a further check, cost-estimating uncertainty (Case I) was found to be approximately 25 per cent of the total uncertainty (see Fig. 13) which confirms past experience.\*\*

---

\* Robert Summers, Cost Estimates as Predictors of Actual Weapon System Costs: A Study of Major Hardware Articles (U), The RAND Corporation, RM-3061-PR, April 1962 (Secret).

\*\* A. A. Alchian, Reliability of Cost Estimates--Some Evidence, The RAND Corporation, RM-481, October 30, 1950.

Frequency

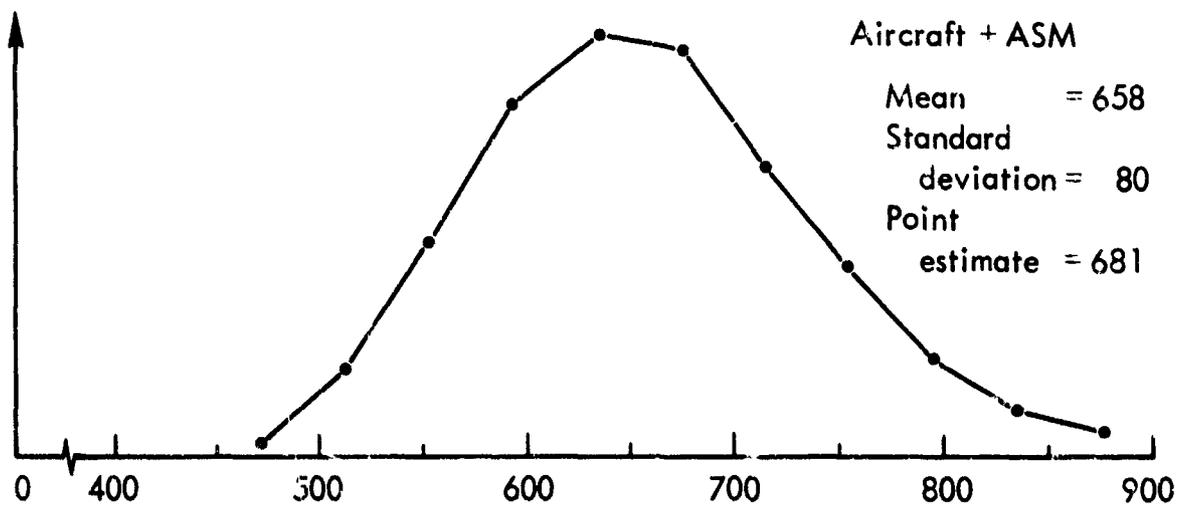
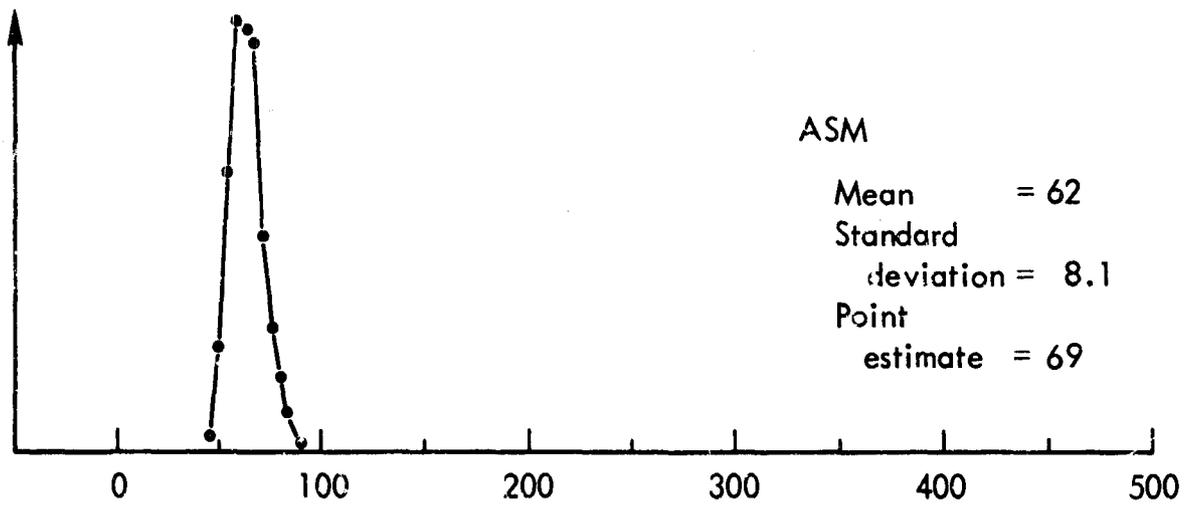
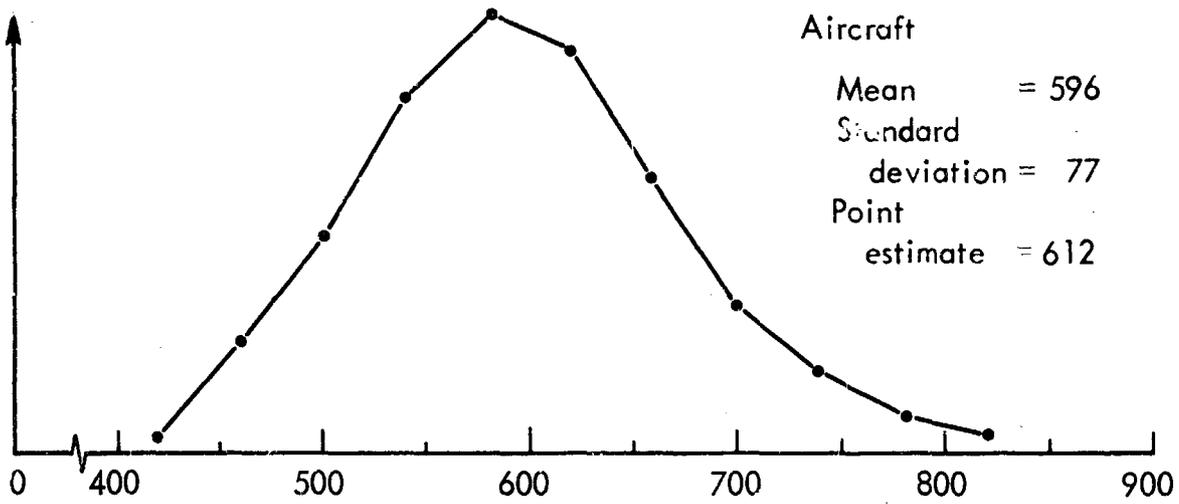


Fig.9—Initial investment cost (millions of dollars)

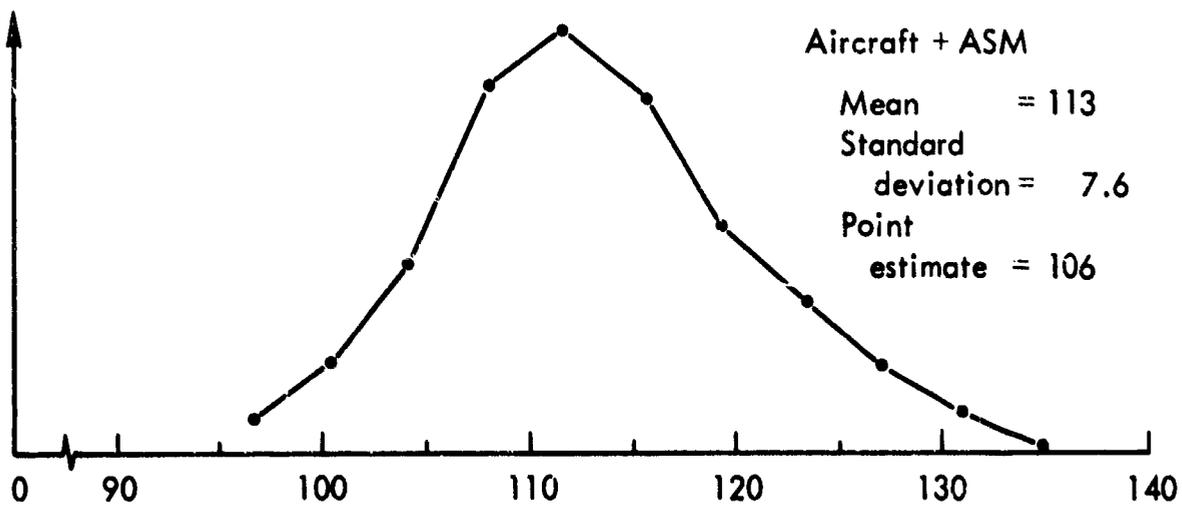
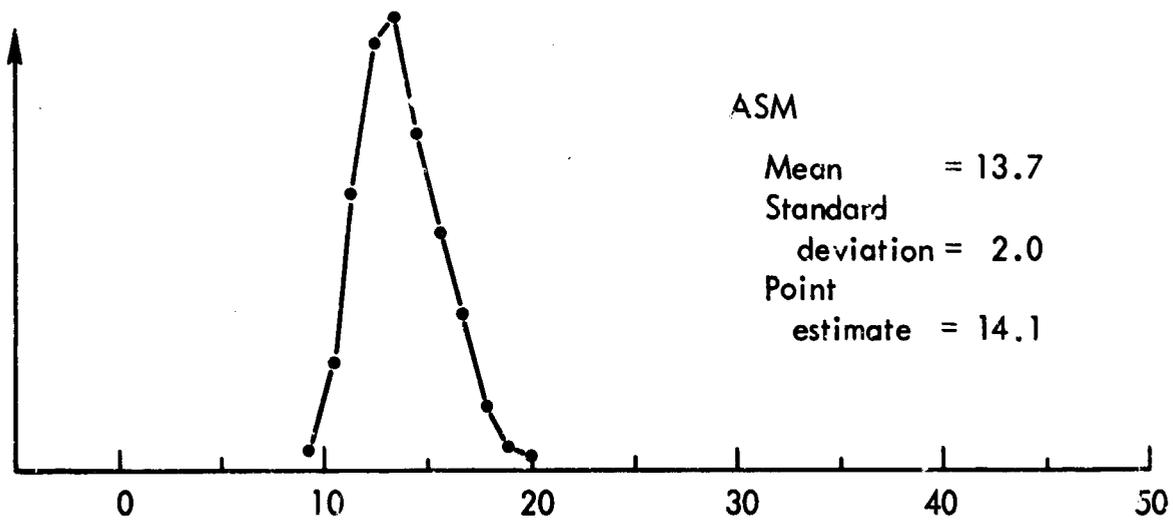
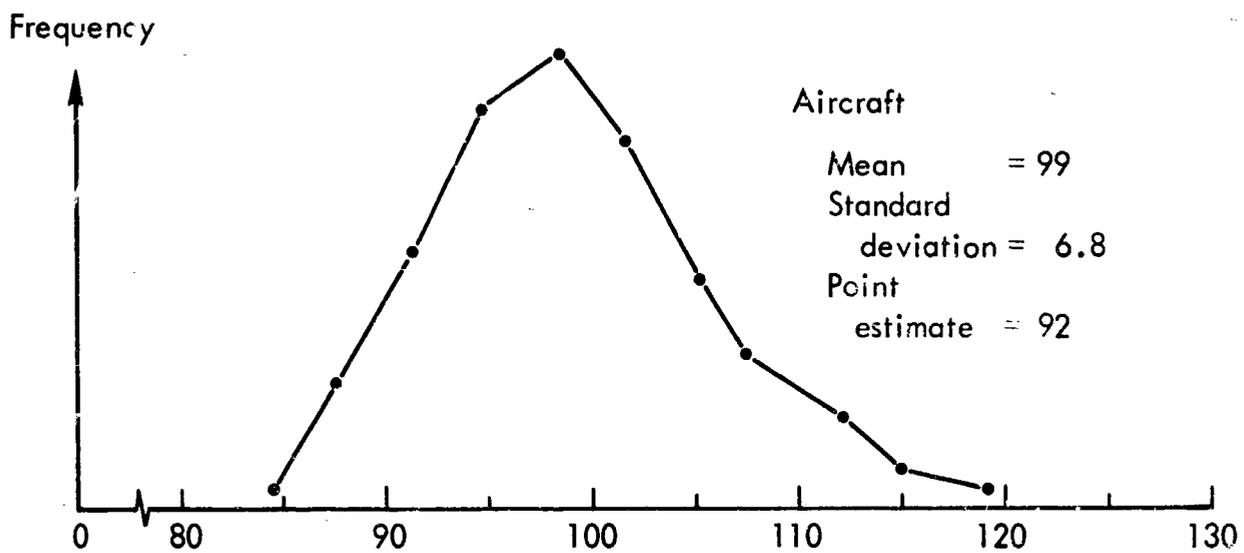


Fig.10—Annual operating cost (millions of dollars)

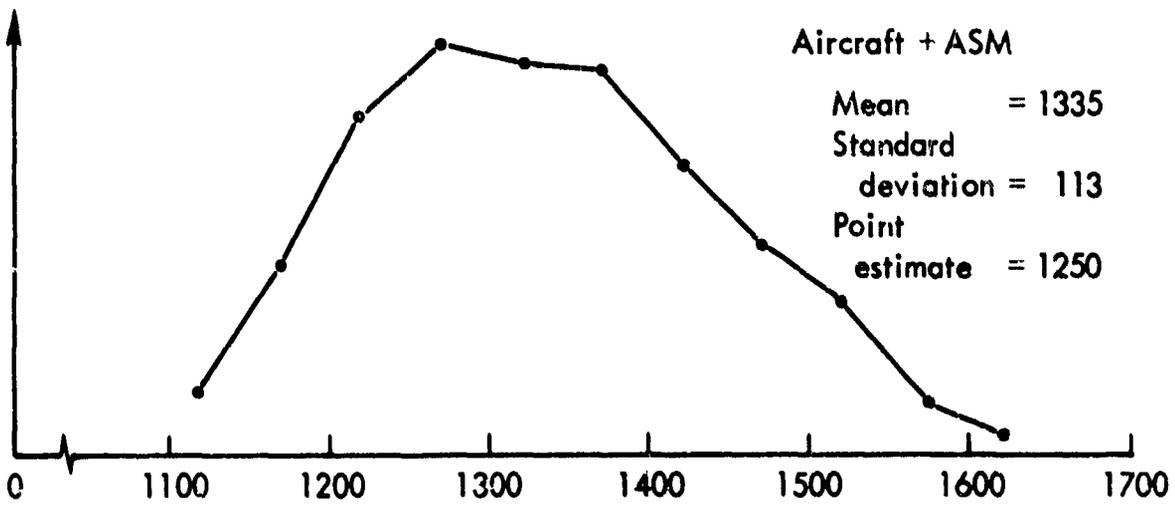
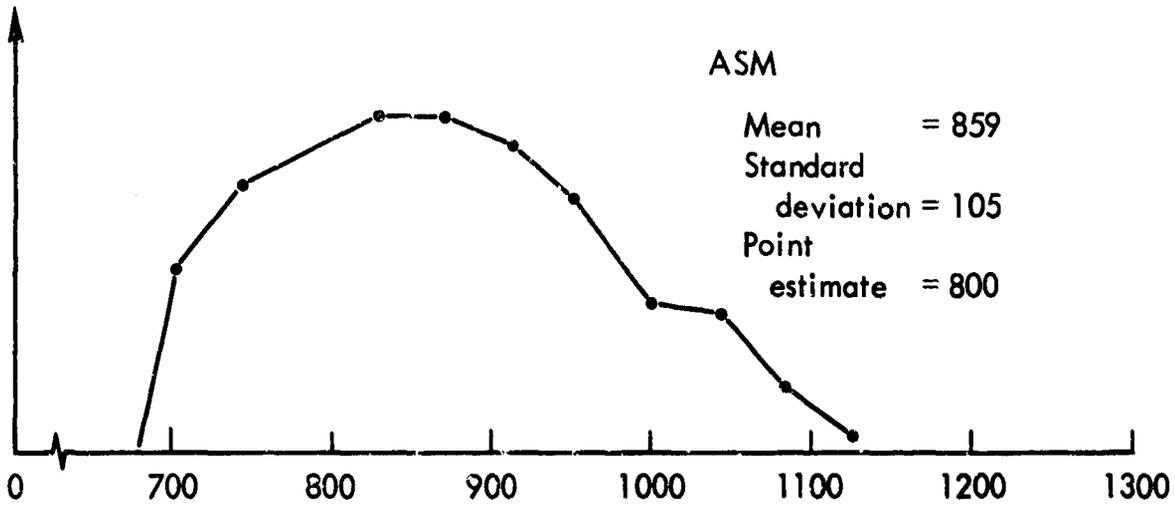
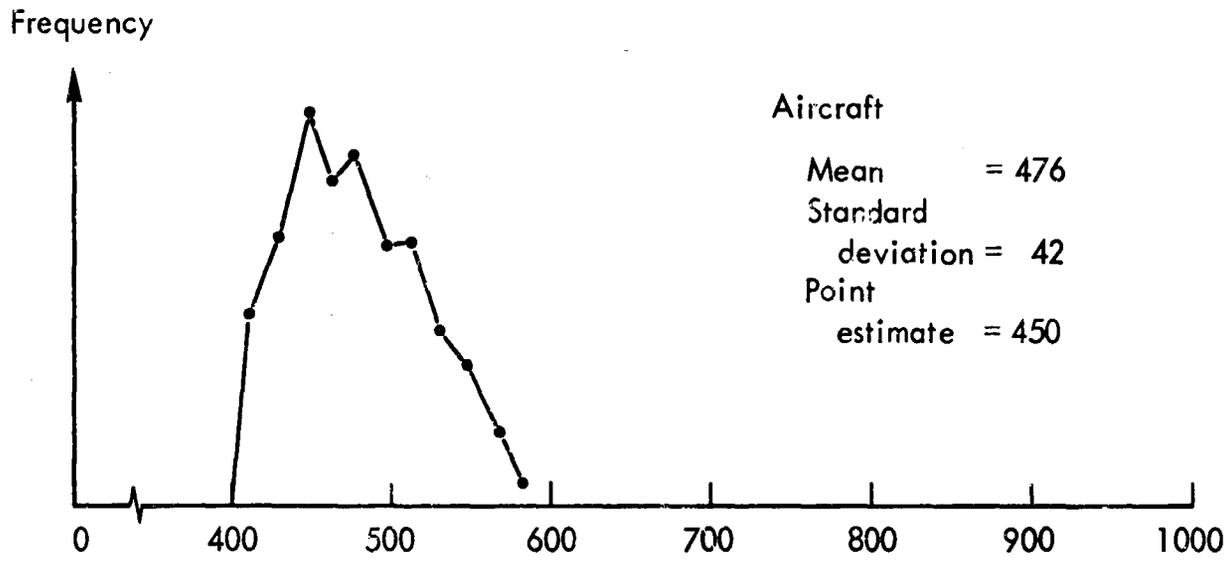


Fig. 11—Research and development costs (millions of dollars)

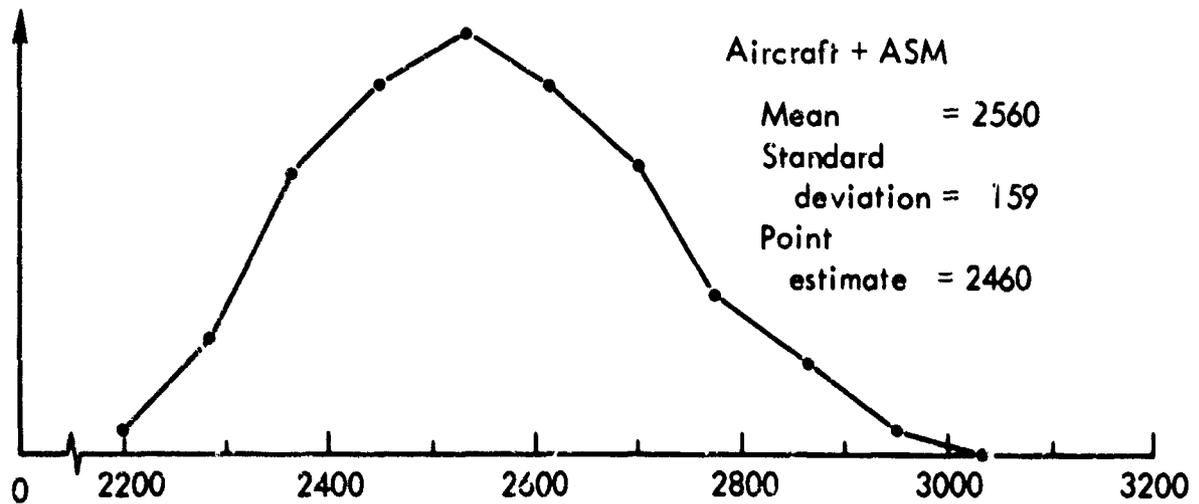
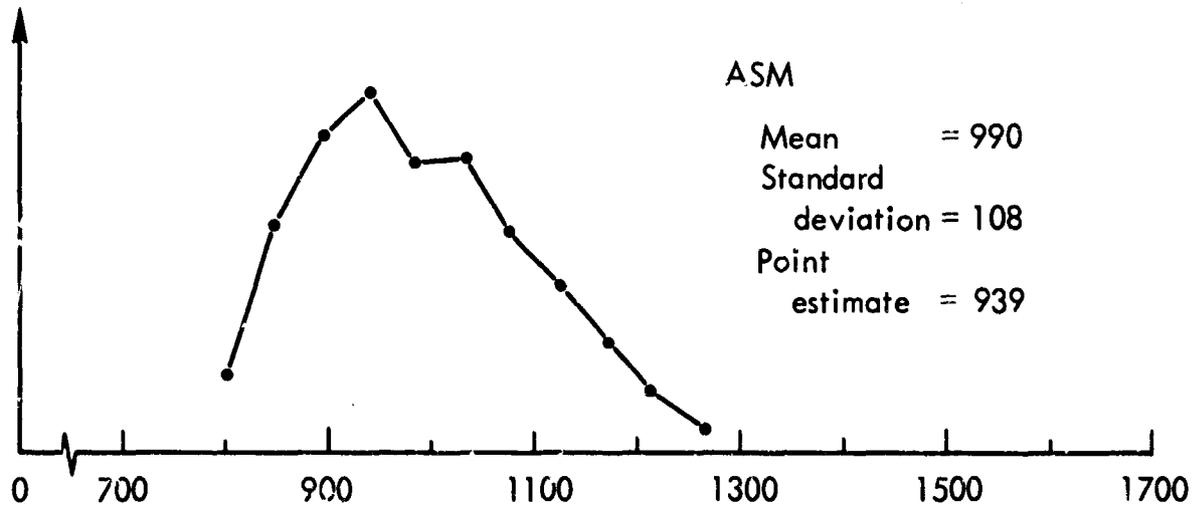
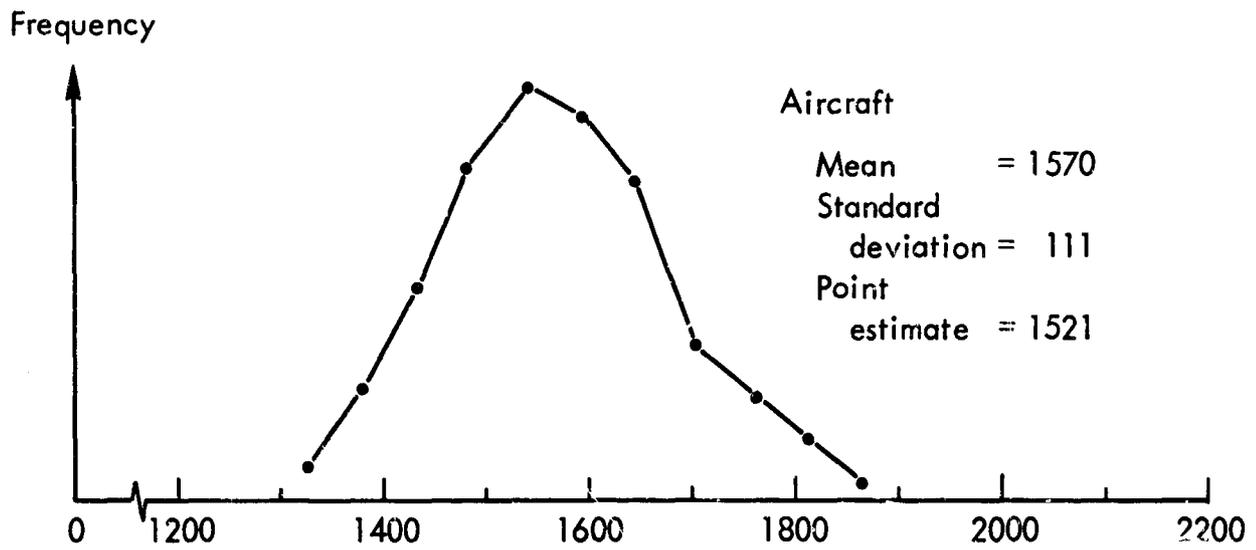


Fig.12—Five-year system cost (millions of dollars)

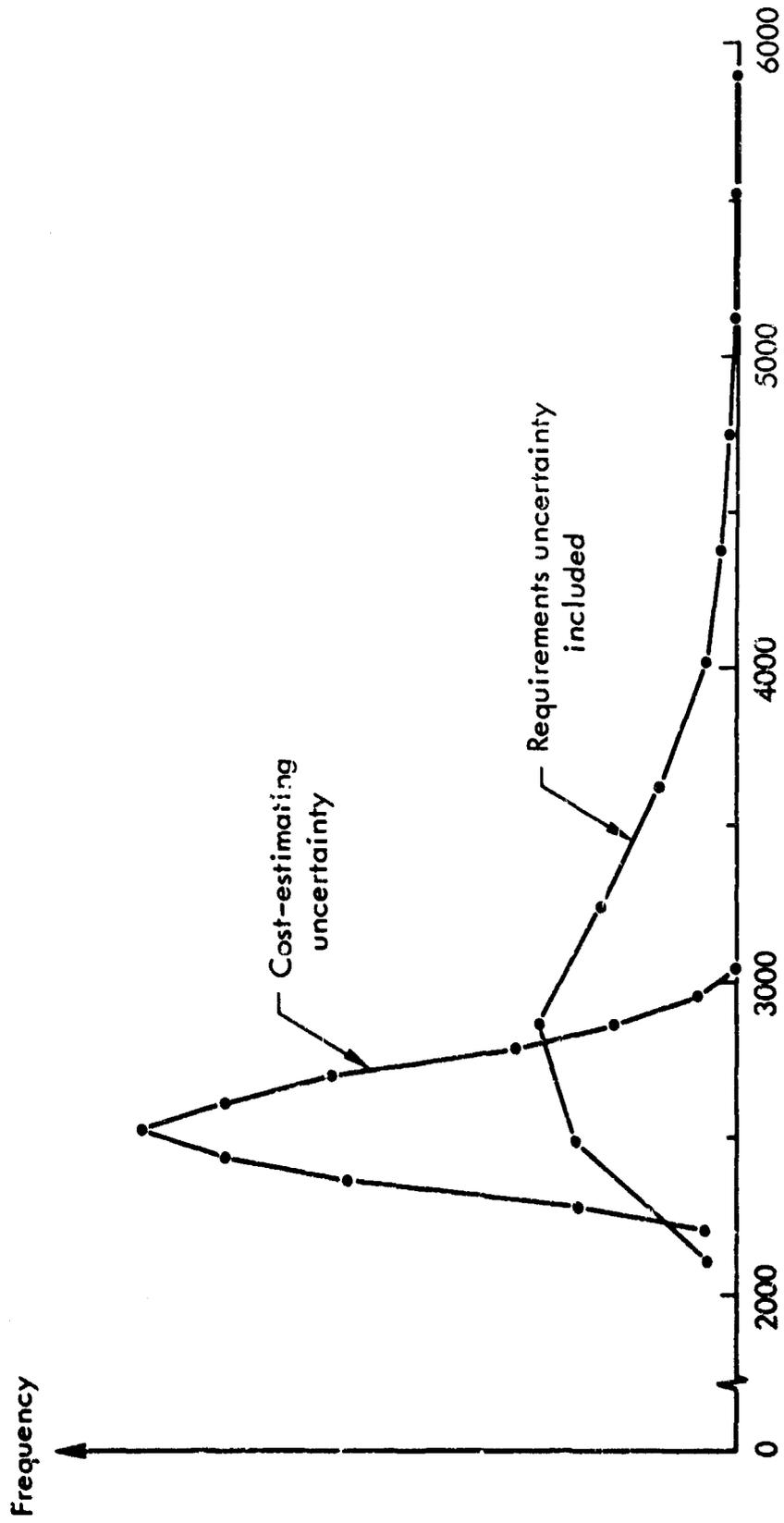


Fig. 13—Total five-year system cost (millions of dollars)

CONCLUSIONS

Although total cost uncertainty can be estimated using Monte Carlo techniques, treating cost-estimating uncertainty alone seems preferable in the context of most cost analysis studies. This stems from the fact that the system planners, who have control over requirement inputs, affect this type of uncertainty. Sensitivity analyses would probably provide the planner with better information regarding the nature and influence of each requirement uncertainty. However, the results of cost-estimating uncertainty can be used to supplement sensitivity analysis of requirements uncertainties. To illustrate this, Fig. 14 depicts a typical analysis where cost category A is related to system requirement X. Confidence limits for the sensitivity curve can be derived from the cost-estimating uncertainty of category A costs--95 per cent limits being two standard deviations (plus and minus) from the expected values.

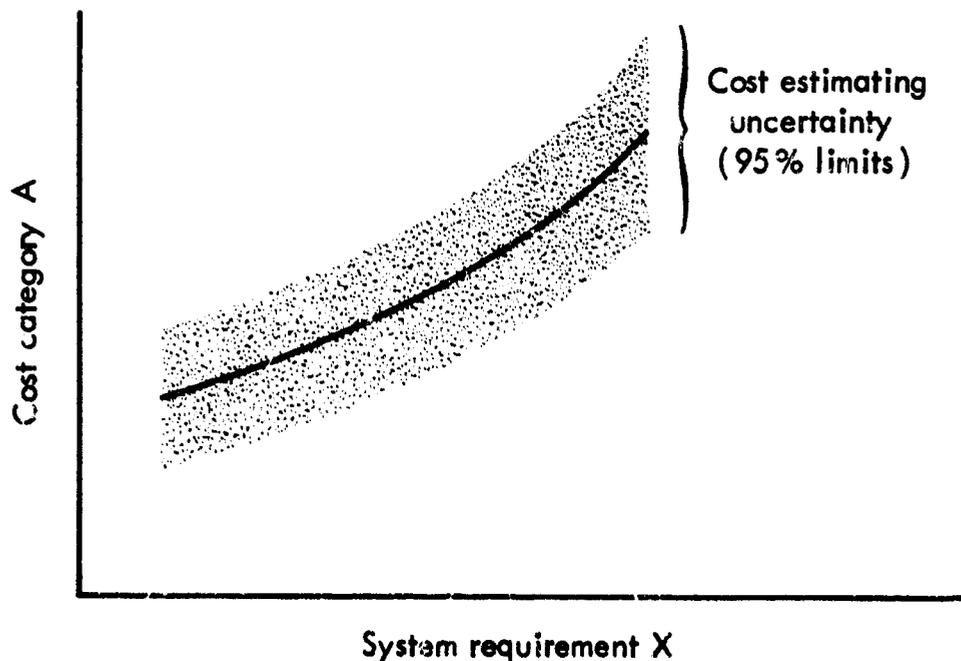


Fig.14—Confidence limits for sensitivity analysis

If the estimated uncertainties for each cost factor input are reasonable, conclusions can be made from Case I regarding the magnitude of cost-estimating uncertainty for cost analyses of this type. For example, the coefficient of variation (standard deviation divided by mean value) tabulated below for the major cost categories, indicates the degree of uncertainty which might be expected.

Table 2

COST-ESTIMATING UNCERTAINTY--PERCENT COEFFICIENT OF VARIATION

Cost Category	Aircraft	ASM
Initial Investment	13	13
Annual Operating	7	14
Research and Development	9	13
Total 5-year Cost	7	11

#### IV. THE GREATER PROBLEM: UNCERTAINTY IN COST/EFFECTIVENESS STUDIES

Up to this point, all discussion regarding the problem of uncertainty was related to cost analysis. Now we shall consider the applicability of the same techniques to the broader context of cost/effectiveness studies. Conceptually, system analysis studies may take either of two forms:\*

- (1) For a specified level of effectiveness, an attempt is made to determine the alternative or combination of alternatives with the minimum resource requirement.
- (2) For a given resource level, an attempt is made to determine the alternative that will achieve maximum effectiveness.

Assume for the purpose of this discussion that both forms of analysis can be stated in terms of comparing cost/effectiveness (C/E) ratios for alternative system proposals. There are, of course, inherent problems in using C/E ratios that must always be accounted for-- e.g., the relative magnitudes of system costs and utility scales. In the first form of analysis, the C/E ratio describes "dollars per unit utility"; with the other form, the ratio is "utility per unit dollar." In each case, both utility and cost have associated uncertainties-- even the given or so-called constant parameter. Depending upon the magnitudes and nature of these uncertainties, their effect can be quite significant to the decision-making process.

It is suggested that this problem can be treated as an extension of the previous cost analysis model as depicted in Fig. 15. As before, a series of system cost estimates is prepared using Monte Carlo techniques to simulate the uncertainties (cost-estimating and/or requirements) of the input parameters. In the same way, Monte Carlo samples can be developed for the system effectiveness inputs to reflect their uncertainty. By combining each system cost estimate with sample sets of effectiveness inputs, a series of cost/effectiveness ratios can be prepared. From the set of C/E ratios, the uncertainty of this parameter can be determined using the procedures previously outlined.

\* G. H. Fisher, Military Systems Cost Analysis (A Summary Lecture for the AFSC Cost Analysis Course), The RAND Corporation, RM-2975-PR, January 1962.

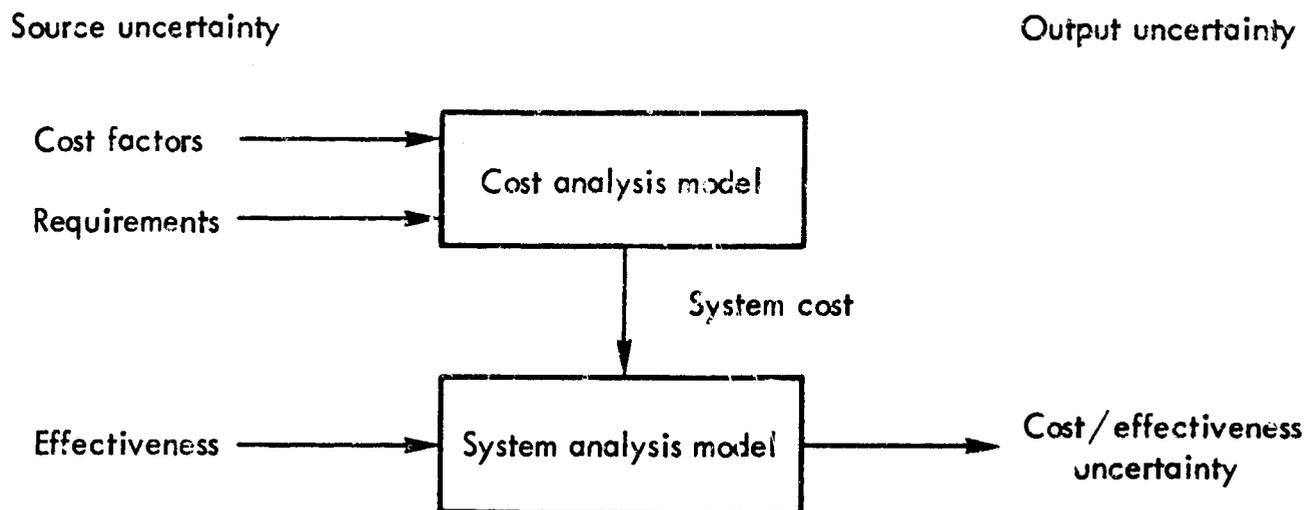


Fig. 15—Estimating cost/effectiveness uncertainty

If cost/effectiveness ratios are prepared for alternative system proposals using the Monte Carlo approach, the impact of the uncertainty on the decision-making process will be analogous to that previously presented graphically in Fig. 1.

Appendix A

WEAPON SYSTEM COST MODEL INPUTS

SYSTEM REQUIREMENT INPUTS\*

	<u>LOW</u>	<u>MODE</u>	<u>HIGH</u>	<u>TYPE</u>
WEAPON SYSTEM DESIGNATOR (STRATEGIC BOMBER = 1.0, STRATEGIC TANKER = 2.0, SM-68B OR SM-80 = 3.0, OTHER STRATEGIC MISSILES = 4.0, DEFENSE WING = 5.0, DEFENSE GROUP = 6.0, TACTICAL (ALL BASES) = 7.0, MATS (ALL BASES) = 8.0)		1.		
WEAPON SYSTEM HOST DESIGNATOR (STRATEGIC BOMBER = 1.0, STRATEGIC TANKER = 2.0, SM-68B OR SM-80 = 3.0, OTHER STRATEGIC MISSILES (SM-65)= 4.0, DEFENSE WING = 5.0, DEFENSE GROUP=6.0, TACTICAL (ALL BASES) = 7.0, MATS (ALL BASES) = 8.0, HOST (NOT A TENANT)=9.0)		9.		
GROUND OR AIRBORNE ALERT DESIGNATOR (CA = 1.0, AA = 2.0)		2.		
ASM PRESENCE DESIGNATOR (ASM'S PRESENT = 1.0, NO ASM'S PRESENT = 2.0)		1.		
NUMBER OF YEARS FOR WHICH WEAPON SYSTEM IS BEING PRICED		5.		
LOCATION (Z/I = 1, ARCTIC O/S = 2, NON-ARCTIC O/S = 3)		1.		
NUMBER OF BASES PER SYSTEM	1.0	1.1	2.0	9
NUMBER OF STATIONS PER SYSTEM	2	3.	6.	9
NUMBER OF AIRCRAFT ON STATION	1.0	1.1	2.0	9
MAXIMUM AVAILABLE FLYING HOURS PER FLIGHT (ENDURANCE)	8	10	12	5
RESERVE FLYING HOURS PER FLIGHT	.8	1.0	1.6	3.
FLYING HOURS FROM BASE TO STATION	1.33	2.33	3.33	8
MONTHLY FLYING HOURS ALLOWED PER CREW	60	120	140	7
PROCUREMENT LEVEL	40	100	120	4
FLYING HOURS PER PERIODIC INSPECTION	400	600	800	5
LENGTH OF PERIODIC IN HOURS	24	32	40	2
FLYING HOURS PER POST-FLIGHT INSPECTION	125	160	175	2

\*Numeric values describe hypothetical aircraft system.

	<u>LOW</u>	<u>MODE</u>	<u>HIGH</u>	<u>TYPE</u>
LENGTH OF POST-FLIGHT IN HOURS	8	10	13	3
NUMBER OF SHIFTS PER DAY	1	2	3	8
LENGTH OF SHIFT IN HOURS	7.0	7.75	8.0	7
AIRCRAFT POL CONSUMPTION RATE IN POUNDS PER FLYING HOUR		13,000		
CREW RATIO		0		
NUMBER OF OFFICERS PER CREW (AIRCRAFT)	4	5	6	8
NUMBER OF AIRMEN PER CREW (AIRCRAFT)	6	7	8	8
WEIGHT OF EACH ASM	4,500	5,000	5,500	5
NUMBER OF ASM'S PER AIRCRAFT	2	4	6	8
NUMBER OF PERSONNEL REQUIRED PER ASM	1	3	5	8
ASM PROCUREMENT LEVEL	80	100	160	6

COST FACTOR INPUTS\*

COMMAND SUPPORT

INITIAL SPARES

RATED NON-CREW TRAINING COST, INITIAL

NON-RATED OFFICER TRAINING COST, INITIAL

NON-CREW AIRMAN TRAINING COST, INITIAL

RATED NON-CREW TRAINING COST, ANNUAL

NON-RATED OFFICER TRAINING COST, ANNUAL

NON-CREW AIRMAN TRAINING COST, ANNUAL

CIVILIAN PAY AND ALLOWANCES Z/I

CIVILIAN PAY AND ALLOWANCES O/S

RATED OFFICER PAY AND ALLOWANCES Z/I

RATED OFFICER PAY AND ALLOWANCES O/S

NON-RATED OFFICER PAY AND ALLOWANCES Z/I

NON-RATED OFFICER PAY AND ALLOWANCES O/S

CREW AIRMAN PAY AND ALLOWANCES Z/I

CREW AIRMAN PAY AND ALLOWANCES O/S

NON-CREW AIRMAN PAY AND ALLOWANCES Z/I

NON-CREW AIRMAN PAY AND ALLOWANCES O/S

INITIAL TRAVEL \$/MAN

ANNUAL TRAVEL \$/MAN Z/I

ANNUAL TRAVEL \$/MAN O/S

ORGANIZATIONAL EQUIPMENT \$/MAN, INITIAL

---

\* The low, most-likely, and high values and distribution type for each cost factor input are specified in the same manner as the system requirement inputs.

ORGANIZATIONAL EQUIPMENT \$/MAN, ANNUAL  
INSTALLATIONS MAINTENANCE \$/MAN, Z/I, TENANT  
INSTALLATIONS MAINTENANCE \$/MAN, Z/I, NON-TENANT  
INSTALLATIONS MAINTENANCE \$/MAN, O/S, NON-ARCTIC, TENANT  
INSTALLATIONS MAINTENANCE \$/MAN, O/S, NON-ARCTIC, NON-TENANT  
INSTALLATIONS MAINTENANCE \$/MAN, O/S ARCTIC, TENANT  
INSTALLATIONS MAINTENANCE \$/MAN, O/S, ARCTIC, NON-TENANT  
INSTALLATIONS REPLACEMENT \$/MAN, Z/I, TENANT  
INSTALLATIONS REPLACEMENT \$/MAN, Z/I, NON-TENANT  
INITIAL TRANSPORTATION \$/MAN  
ANNUAL TRANSPORTATION \$/MAN, Z/I  
ANNUAL TRANSPORTATION \$/MAN, O/S  
INITIAL TRANSPORTATION PERCENT OF SPARES  
ANNUAL TRANSPORTATION PERCENT OF MAINTENANCE COST, Z/I  
ANNUAL TRANSPORTATION PERCENT OF MAINTENANCE COST, O/S  
CONTRACTUAL SERVICES \$/MAN, MATS, Z/I  
CONTRACTUAL SERVICES \$/MAN, OTHER Z/I  
CONTRACTUAL SERVICES \$/MAN, MATS, O/S  
CONTRACTUAL SERVICES \$/MAN, OTHER, O/S  
OTHER SUPPLIES \$/MAN, Z/I  
OTHER SUPPLIES \$/MAN, O/S, ARCTIC  
OTHER SUPPLIES \$/MAN, O/S, NON-ARCTIC  
ANNUAL AMMUNITION COST \$/MAN  
POL STOCKS PERCENT, Z/I  
POL STOCKS PERCENT, O/S  
INSTALLATIONS MAINTENANCE STOCKS PERCENT

STOCKS, ANNUAL SUPPLIES, \$/MAN, Z/I  
STOCKS, ANNUAL SUPPLIES, \$/MAN, O/S, ARCTIC  
STOCKS, ANNUAL SUPPLIES, \$/MAN, O/S, NON-ARCTIC  
INITIAL FOOD STOCKS, \$/MAN, Z/I  
INITIAL FOOD STOCKS, \$/MAN, O/S  
INITIAL CLOTHING STOCKS, \$/MAN, Z/I  
INITIAL CLOTHING STOCKS, \$/MAN, O/S  
INITIAL AMMUNITION STOCKS, \$/MAN  
POL CONSUMPTION VARIANT FACTOR  
POL LBS. PER GALLON  
POL COST PER GALLON (DOLLARS)  
RATED NON-CREW OFFICERS AS PERCENTAGE OF CREW OFFICERS  
OFFICERS AS PERCENTAGE OF WING HQ. PERSONNEL  
SQUADRON HEADQUARTERS PERSONNEL AS PERCENTAGE OF CREW PERSONNEL  
OFFICERS AS PERCENTAGE OF SQUADRON HQ. PERSONNEL  
MAINTENANCE PERSONNEL PER BASE  
MAINTENANCE PERSONNEL PER AIRCRAFT  
OFFICERS AS PERCENTAGE OF MAINTENANCE PERSONNEL  
OFFICERS AS PERCENTAGE OF SUPPORT PERSONNEL  
STRATEGIC WEAPON SYSTEM ADMINISTRATIVE PERSONNEL PER BASE (PURE  
MANNED AIRCRAFT BASE)  
STRATEGIC WEAPON SYSTEM ADMINISTRATIVE PERSONNEL AS PERCENTAGE OF  
DIRECT PERSONNEL (PURE MANNED AIRCRAFT BASE)  
STRATEGIC WEAPON SYSTEM ADMINISTRATIVE PERSONNEL PER BASE (PURE  
MISSILE AND MIXED BASES)  
STRATEGIC WEAPON SYSTEM ADMINISTRATIVE PERSONNEL AS PERCENTAGE OF  
DIRECT PERSONNEL (PURE MISSILE AND MIXED BASES)  
AIR DEFENSE WEAPON SYSTEM ADMINISTRATIVE PERSONNEL PER BASE (WING BASE)

AIR DEFENSE WEAPON SYSTEM ADMINISTRATIVE PERSONNEL AS PERCENTAGE OF DIRECT PERSONNEL OR DIRECT AND SUPPORT PERSONNEL WHEN ON AN AIR DEFENSE BASE (WING BASE)

AIR DEFENSE WEAPON SYSTEM ADMINISTRATIVE PERSONNEL PER BASE (GROUP BASE)

AIR DEFENSE WEAPON SYSTEM ADMINISTRATIVE PERSONNEL AS PERCENTAGE OF DIRECT PERSONNEL OR DIRECT AND SUPPORT PERSONNEL WHEN ON AN AIR DEFENSE BASE (GROUP BASE)

TACTICAL WEAPON SYSTEM ADMINISTRATIVE PERSONNEL PER BASE (ALL BASES)

TACTICAL WEAPON SYSTEM ADMINISTRATIVE PERSONNEL AS PERCENTAGE OF DIRECT PERSONNEL (ALL BASES)

TRANSPORT (MATS) WEAPON SYSTEM ADMINISTRATIVE PERSONNEL PER BASE (ALL BASES)

TRANSPORT (MATS) WEAPON SYSTEM ADMINISTRATIVE PERSONNEL AS PERCENTAGE OF DIRECT PERSONNEL (ALL BASES)

STRATEGIC WEAPON SYSTEM SUPPORT PERSONNEL PER BASE (PURE MANNED AIRCRAFT BASE)

STRATEGIC WEAPON SYSTEM SUPPORT PERSONNEL AS PERCENTAGE OF DIRECT AND ADMINISTRATIVE PERSONNEL (PURE MANNED BASE)

STRATEGIC WEAPON SYSTEM SUPPORT PERSONNEL PER BASE (PURE MISSILE AND MIXED BASES)

STRATEGIC WEAPON SYSTEM SUPPORT PERSONNEL AS PERCENTAGE OF DIRECT AND ADMINISTRATIVE PERSONNEL (PURE MISSILE AND MIXED BASES)

AIR DEFENSE WEAPON SYSTEM SUPPORT PERSONNEL PER BASE (WING BASE)

AIR DEFENSE WEAPON SYSTEM SUPPORT PERSONNEL AS PERCENTAGE OF DIRECT PERSONNEL (WING BASE)

AIR DEFENSE WEAPON SYSTEM SUPPORT PERSONNEL PER BASE (GROUP BASE)

AIR DEFENSE WEAPON SYSTEM SUPPORT PERSONNEL AS PERCENTAGE OF DIRECT PERSONNEL (GROUP BASE)

TACTICAL WEAPON SYSTEM SUPPORT PERSONNEL PER BASE (ALL BASES)

TACTICAL WEAPON SYSTEM SUPPORT PERSONNEL AS PERCENTAGE OF DIRECT AND ADMINISTRATIVE PERSONNEL (ALL BASES)

TRANSPORT (MATS) WEAPON SYSTEM SUPPORT PERSONNEL PER BASE (ALL BASES)

TRANSPORT (MATS) WEAPON SYSTEM SUPPORT PERSONNEL AS PERCENTAGE OF DIRECT AND ADMINISTRATIVE PERSONNEL (ALL BASES)

ADMINISTRATIVE CIVILIANS PER BASE

ADMINISTRATIVE CIVILIANS AS PERCENTAGE OF WING HQ. PERSONNEL

ASM CIVILIANS AS A PERCENTAGE OF ASM MILITARY PERSONNEL

MAINTENANCE CIVILIANS AS PERCENTAGE OF MILITARY MAINTENANCE PERSONNEL

SUPPORT CIVILIANS AS A PERCENTAGE OF MILITARY SUPPORT PERSONNEL

COST OF AIRCRAFT NUMBER 1 (IN MILLIONS OF DOLLARS)

CUMULATIVE AVERAGE COST CURVE SLOPE (0. TO 1.)

SERVICING, PREFLIGHT AND THROUGH-FLIGHT, AND OFF-AND-ON LOADING OF MISSILES

EXTRA DOWN-TIME PER FLIGHT IN HOURS

UNSCHEDULED MAINTENANCE PERCENT (SORTIES)

UNSCHEDULED MAINTENANCE PERCENT (FLYING HOURS)

DEPOT MAINTENANCE COST PER FLYING HOUR

BASE MATERIALS COST PER FLYING HOUR

DEPOT MAINTENANCE COST PER SORTIE

BASE MATERIALS COST PER SORTIE

ATTRITION RATE PER SORTIE

ATTRITION RATE PER FLYING HOUR

INITIAL TRAINING COST PER CREW (AIRCRAFT)

ANNUAL TRAINING COST PER CREW (AIRCRAFT)

FACILITIES COST PER AIRCRAFT

FACILITIES COST PER MAN (AIRCRAFT)

FACILITIES COST PER BASE (AIRCRAFT)

FACILITIES PERCENT FOR UTILITIES (AIRCRAFT)

OVERSEAS FACILITIES COST RATIO (AIRCRAFT)

AIRBORNE ELECTRONICS EQUIPMENT COST PER AIRCRAFT (IN MILLIONS OF DOLLARS)

AIRCRAFT RDT&E (IN MILLIONS OF DOLLARS)

AGE COST FACTOR (AIRCRAFT)

AGE SPARES COST FACTOR (AIRCRAFT)

AGE MAINTENANCE COST FACTOR (AIRCRAFT)

AEE SPARES COST FACTOR (AIRCRAFT)

AEE MAINTENANCE COST FACTOR (AIRCRAFT)

COST OF ASM NUMBER 1 (IN MILLIONS OF DOLLARS)

SLOPE OF ASM CUMULATIVE AVERAGE COST CURVE (0. TO 1.)

ASM INSTALLATIONS COST PER MILITARY MAN

ASM INSTALLATIONS COST PER ASM

ASM INSTALLATIONS COST PER BASE

AIRBORNE ELECTRONICS EQUIPMENT COST PER ASM (IN MILLIONS OF DOLLARS)

ASM RDT&E (IN MILLIONS OF DOLLARS)

ASM PIPELINE FACTOR

ASM OFFICER REQUIREMENT FACTOR

ASM AIRMAN REQUIREMENT FACTOR

ASM OFFICER TURNOVER RATE

ASM AIRMAN TURNOVER RATE

ASM SPARES COST FACTOR

AGE COST FACTOR (ASM)

AGE SPARES COST FACTOR (ASM)

AEE SPARES COST FACTOR (ASM)

ORGANIZATIONAL EQUIPMENT COST PER MAN (ASM)

INITIAL INSTALLATIONS MAINTENANCE MATERIALS COST FACTOR (ASM)

INITIAL TRANSPORTATION COST FACTOR (ASM)

INITIAL TRAINING COST PER OFFICER (ASM)  
INITIAL TRAINING COST PER AIRMAN (ASM)  
INITIAL TRAVEL COST PER OFFICER  
INITIAL TRAVEL COST PER AIRMAN (ASM)  
INITIAL FOOD COST PER MAN (ASM)  
INITIAL CLOTHING COST PER MAN (ASM)  
INITIAL COST OF MISCELLANEOUS SUPPLIES PER MAN (ASM)  
INITIAL AMMUNITION COST PER MAN (ASM)  
ASM MAINTENANCE COST FACTOR  
ASM REPLACEMENT COST FACTOR  
ASM CONSUMPTION COST FACTOR  
AGE MAINTENANCE COST AND REPLACEMENT COST FACTOR (ASM)  
AEE MAINTENANCE COST FACTOR (ASM)  
ORGANIZATIONAL EQUIPMENT MAINTENANCE AND REPLACEMENT COST PER MAN (ASM)  
ASM INSTALLATIONS MAINTENANCE COST FACTOR  
ASM INSTALLATIONS REPLACEMENT COST FACTOR  
ANNUAL TRANSPORTATION COST PER MAN (ASM)  
PAY AND ALLOWANCE COST PER OFFICER (ASM)  
PAY AND ALLOWANCE COST PER AIRMAN (ASM)  
ANNUAL TRAINING COST PER OFFICER (ASM)  
ANNUAL TRAINING COST PER AIRMAN (ASM)  
ANNUAL TRAVEL COST PER OFFICER (ASM)  
ANNUAL TRAVEL COST PER AIRMAN (ASM)  
ANNUAL CONTRACTUAL SERVICES AND OTHER COST PER MAN (ASM)  
ANNUAL AMMUNITION COST PER MAN (ASM)

Appendix B

WEAPON SYSTEM COST MODEL OUTPUTS

AIRBORNE ALERT BASIC CASE -- STRATEGIC BOMBER WEAPON SYSTEM -- HOST

AIRBORNE ALERT CASE NUMBER 1001.

INITIAL INVESTMENT COSTS

	AIRCRAFT	MISSILE	TOTAL
TOTAL PROCUREMENT	359.0	44.9	404.0
PRIMARY MISSION EQUIPMENT	299.2	39.1	338.3
PRIMARY MISSION EQUIPMENT SPARES	59.8	5.9	65.7
TOTAL AGE + AEE	208.4	19.3	227.7
AEROSPACE GROUND EQUIP.	59.8	7.8	67.7
AEROSPACE GROUND EQUIP. SPARES	6.0	2.0	7.9
AIRBORNE ELECTRONICS EQUIP.	105.6	7.0	112.6
AIRBORNE ELECTRONICS EQUIP. SPARES	37.0	2.5	39.4
TOTAL FACILITIES	21.9	1.7	23.7
TOTAL TRAINING AND TRAVEL	15.2	1.1	16.3
TRAINING	14.0	1.1	15.1
TRAVEL	1.2	0.0	1.3
TOTAL OTHER INVESTMENT	7.6	1.4	9.0
INITIAL STOCKS	2.0	0.1	2.2
ORGANIZATIONAL EQUIPMENT	4.5	0.3	4.8
TRANSPORTATION	1.1	1.0	2.0
TOTAL INITIAL INVESTMENT	612.2	68.5	680.7

ANNUAL OPERATING COSTS

	AIRCRAFT	MISSILE	TOTAL
TOTAL OPERATIONS	35.3	9.0	44.3
MAINTENANCE	14.9	5.9	20.8
POL	11.6		11.6
ATTRITION	8.8		8.8
REPLACEMENT		1.2	1.2
CONSUMPTION		2.0	2.0
 TOTAL AGE AND AEE MAINTENANCE	 32.4	 3.7	 36.1
AEROSPACE GROUND EQUIP. MAINT.	6.0	2.0	7.9
AIRBORNE ELECTRONICS EQUIP. MAINT.	26.4	1.8	28.2
 TOTAL FACILITIES MAINT. + REPLACE.	 2.9	 0.2	 3.0
FACILITY MAINTENANCE	0.6	0.1	0.7
FACILITY REPLACEMENT	2.3	0.1	2.3
 TOTAL PERSONNEL	 18.0	 1.1	 19.1
PAY AND ALLOWANCES	14.6	0.9	15.5
TRAINING	2.6	0.2	2.8
TRAVEL	0.8	0.0	0.8
 TOTAL OTHER OPERATING	 3.1	 0.1	 3.2
REPLACEMENT OF ORGANIZATIONAL EQUIP.	0.5	0.0	0.5
TRANSPORTATION	0.8	0.0	0.9
MISCELLANEOUS	1.8	0.1	1.9
 TOTAL ANNUAL OPERATING	 91.7	 14.1	 105.8
  RESEARCH AND DEVELOPMENT	  450.0	  800.0	  1250.0
  TOTAL 5-YEAR SYSTEM COST	  1520.9	  938.8	  2459.7

MISCELLANEOUS DATA

CASE NUMBER	1001.0
NUMBER OF BASES PER SYSTEM	1.0
NUMBER OF STATIONS PER SYSTEM	3.0
ENDURANCE HOURS	10.0
RESERVE FLYING HOURS PER FLIGHT	1.0
FLYING HOURS FROM BASE TO STATION	2.3
OPERATIONAL AIRCRAFT PER SYSTEM	16.0
COST OF AIRCRAFT NUMBER 1	17.0
CUMULATIVE COST CURVE SLOPE (AIRCRAFT)	1.0
PROCUREMENT LEVEL FOR AIRCRAFT	100.0
MONTHLY FLYING HOURS PER CREW	120.0
HOURS IN MAINT. AND SERV. PER MISSION	7.3
LAPSED HOURS PER MISSION INCL. MAINT.	22.0
EFFECTIVE UTILIZATION RATE (PERCENT)	18.7
EFFECTIVE TIME ON STATION (HOURS)	4.3
OPERATIONAL AIRCRAFT PER BASE	16.0
TOTAL PERSONNEL PER BASE	3565.0
NUMBER OF SHIFTS PER DAY	2.0
NUMBER OF MISSILES PER AIRCRAFT	4.0
WEIGHT OF EACH MISSILE	5000.0
COST OF MISSILE NUMBER 1	0.6
CUMULATIVE COST CURVE SLOPE (MISSILE)	1.0
PROCUREMENT LEVEL FOR MISSILES	100.0
OPERATIONAL MISSILES REQ. PER SYSTEM	64.0

PERSONNEL REQUIREMENTS

	OFFICERS	AIRMEN	MILITARY	CIVILIANS	TOTAL
ADMINISTRATION	88.	264.	353.	10.	363.
SQUADRON HQ.	11.	34.	45.		45.
FLIGHT CREW	187.	261.	448.		448.
MISSILE	32.	180.	211.	0.	211.
MAINTENANCE	20.	638.	658.	0.	650.
SUPPORT	113.	1502.	1615.	226.	1841.
TOTAL	450.	2878.	3329.	236.	3565.
RATED NON-CREW	19.				

Appendix C

FORTRAM SUBROUTINES FOR ESTIMATING UNCERTAINTY

```
1 SUBROUTINE BTABLE
  COMMON /BTAB/ NTABLE, CUMB(9,128), XTABLE(128)

  GENERATES CUMULATIVE BETA TABLES FOR NINE BETA EQUATIONS

  DO 10 M = 1,9
    NTABLE = M
    DELTA = 0.0078125
    A = 0.0
    B = DELTA
    DO 10 J = 1,128
      PKLEQ--A LIBRARY ROUTINE-- IS USED TO INTEGRATE BETA EQUATIONS
      CUMB(M,J) = PKLEQ(A,B)
      XTABLE(J) = B
      B = B + DELTA
10 CONTINUE
  RETURN
  END
```

```
1 FUNCTION SSSSS(X)
COMMON /BTAB/ NTABLE, CUMB(9,128), XTABLE(128)
```

THIS SUBROUTINE--USED BY PKLEQ-- DEFINES BETA EQUATION PARAMETERS

```
IF (NTABLE .EQ. 1) GO TO 1
IF (NTABLE .EQ. 2) GO TO 2
IF (NTABLE .EQ. 3) GO TO 3
IF (NTABLE .EQ. 4) GO TO 4
IF (NTABLE .EQ. 5) GO TO 5
IF (NTABLE .EQ. 6) GO TO 6
IF (NTABLE .EQ. 7) GO TO 7
IF (NTABLE .EQ. 8) GO TO 8
IF (NTABLE .EQ. 9) GO TO 9
1 ALPHA = 1.5
  BETA = 0.5
  CONST = 5.1
  GO TO 10
2 ALPHA = 1.35
  BETA = 1.35
  CONST = 10.66
  GO TO 10
3 ALPHA = 0.5
  BETA = 1.5
  CONST = 5.1
  GO TO 10
4 ALPHA = 3.0
  BETA = 1.0
  CONST = 20.0
  GO TO 10
5 ALPHA = 2.75
  BETA = 2.75
  CONST = 95.5
  GO TO 10
6 ALPHA = 1.0
  BETA = 3.0
  CONST = 20.0
  GO TO 10
7 ALPHA = 4.5
  BETA = 1.5
  CONST = 72.5
  GO TO 10
8 ALPHA = 4.0
  BETA = 4.0
  CONST = 630.
  GO TO 10
9 ALPHA = 1.5
  BETA = 4.5
  CONST = 72.5
10 SSSSS = (CONST) * (X**ALPHA) * ((1.0 - X)**BETA)
RETURN
END
```

SUBROUTINE SAMPLE

GENERATES A MONTE CARLO VALUE FOR EACH INPUT PARAMETER

```
COMMON /NUM/ NUM1, NUM2, NUM3, ITER, NI
COMMON /RTAB/ NTABLE, CUMB(9,128), XTABLE(128)
COMMON /DATA/ IDATA(420), GLOW(420), GMODE(420), GHIGH(420),
C ITYPE(420), FLOW(420), FMODE(420), FHIGH(420), NTYPE(420),
C OUT(1000,13)
COMMON AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ, AK, AL,
C AM, AN, AO, AP, AQ, AR, AS, AT, AU, AV, AW, AX, AY,
C AZ, BA, BB, BC, BD, BE, BF, BG, BH, BI, BJ, BK, BL,
C BM, BN, BO, BP, BQ, BR, BS, BT, BU, BV, BW, BX, BY,
C BZ, CA, CB, CC, CD, CE, CF, CG, CH, CI, CJ, CK, CL,
C CM, CN, CO, CP, CQ, CR, CS, CT, CU, CV, CW, CX, CY, CZ,
C DA, DB, DC, DD, DE, DF, DG, DH, DI, DJ, DK, DL, DM, DN, DO, DP,
C DR, DS, DT, DU, DV, DW, DX, DY, DZ, EA, EB, EC, ED, EE, EF, EG,
C EH, EI, EJ, EK, EL, EM, EN, EO, EP, EQ, ER, ES, ET, EU, EV,
C EW, EX, EY, EZ, FA, FB, FC, FD, FE, FF, FG, FH, FI, FJ, FK,
C FL, FM, FN, FO, FP, FQ, FR, FS, FT, FU, FV, FW, FX, FY, FZ,
C GA, GB, GC, GD, GE, GF, GG, GH, GI, GJ, GK, GL, GM, GN, GO,
C GP, GQ, GR, GS, GT, GU, GV, GW, GX, GY, GZ, HA, HB, HC, HD,
C HE, HF, HG, HH, HI, HJ, HK, HL, HM, HN, HO, HP, HQ, HR, HS,
C HT, HU, HV, HW, HX, HY, HZ, IA, IB, IC, ID, IE, IF, IG, IH,
C II, IJ, IK, IL, IM, IN, IO, IP, IQ, IR, IS, IT, IU, IV, IW,
C IX, IY, IZ, JA, JB, JC, JD, JE, JF, JG, JH, JI, JJ, JK,
C JL, JM, JN, JO, JP, JQ, JR, JS, JT, JU, JV, JW, JX, JY, JZ,
C KA, KB, KC, KD, KE, KF, KG, KH, KI, KJ, KK, KL, KM, KN, KO,
C KP, KQ, KR, KS, KT, KU, KV, KW, KX, KY, KZ, LA, LB, LC, LD,
C LE, LF, LG, LH, LI, LJ, LK, LL, LM, LN, LO, LP, LQ, LR, LS,
C LT, LU, LV, LW, LX, LY, LZ, MA, MB, MC, MD, ME, MF, MG, MH,
C MI, MJ, MK, ML, MM, MN, MO, MP, MQ, MR, MS, MT, MU, MV, MW,
C MX, MY, MZ, NA, NB, NC, ND, NE, NF, NG, NH, NI, NJ, NK,
C NL, NM, NO, NP, NQ, NR, NS, NT, NU, NV, NW, NX, NY, NZ,
C OA, OB, OC, OD, OE, OF, OG, OH, OI, OJ, OK, OL, OM, ON,
C OO, OP, OQ, OR, OS, OT, OU, OV, OW, OX, OY, OZ, PA, PB, PC,
C PD, PE, PF, PG, PH, PI, PJ, PK, PL, PM, PN, PO, PP, PQ,
C PR, PS, PT, PU, PV, PW, PX, PY, PZ, QA, QB, QC, QD, QE,
C QF, QG, QH, QI, QJ, QK, QL, QM, QN, QO, QP, QQ, QR, QS,
C QT, QU, QV, QW, QX, QY, QZ, RA, RB, RC, RD, RE, RF, RG,
C RH, RI, RJ, RK, RL, RM, RN, RO, RP, RQ, RR, RS, RT, RU,
C RV, RW, RX, RY, RZ, SA, SB, SC, SD, SE, SF, SG, SH, SI,
C SJ, SK, SL, SM, SN, SO, SP, SQ, SR, SS, ST, SU, SV, SW,
C SX, SY, SZ, TA, TB, TC, TD, TE, TF, TG, TH, TI, TJ, TK,
C TL, TM, TN, TO, TP, TQ, TR, TS, TT, TU, TV, TW, TX, TY,
C TZ, UA, UB, UC, UD, UE, UF, UG, UH, UI, UJ, UK, UL,
C UM, UN, UO, UP, UQ, UR, US, UT, UY, UZ, VA, VB, VC,
C VD, VE, VF, VG, VH, VI, VJ, VK, VL, VM, VN, VO, VP,
C VQ, VR, VS, VT, VU, VV, VW, VX, VY, VZ, WA, WB, WC,
C WD, WE, WF, WG, WH, WI, WJ, WK, WL, WM, WN, WO,
C WP, WQ, WR, WS, WT, WU, WV, WW, WX, WY, WZ, XA, XB,
C XC, XD, XE, XF, XG, XH, XI, XJ, XK, XL, XM, XN,
C XO, XP, XQ, XR, XS, XT, XU, XV, XW, XX, XY, XZ,
C YA, YB, YC, YD, YE, YF, YG, YH, YI, YJ, YK,
C YL, YM, YN, YO, YP, YQ, YR, YS, YT, YU, YV,
C YW, YX, YY, YZ, ZA, ZB, ZC, ZD, ZE, ZF, ZG,
C ZH, ZI, ZJ, ZK, ZL, ZM, ZN, ZO, ZP, ZQ,
C ZR, ZS, ZT, ZU, ZV, ZW, ZX, ZY, ZZ,
COMMON X, PMA, PMO, PMT, XXX, ZA, ZB, ZC, ZCA, ZD, ZDA, ZE,
C ZF, ZFA, ZG, ZGA, ZH, ZHA, ZI, ZIA, ZJ, ZL, ZT, OFA, AMA,
C OFB, AMB, OFC, AMC, OFD, AMD, CVA, CVB, CVC, CVD, TOA, TOB, TOC,
C TOD, XA, XB, XC, XD, XE, XF, XG, XH, XI, XJ, XK, XL,
C XM, XN, XO, XP, XQ, XR, YA, YR, YC, YD, YE, YF, YG,
C YH, YI, YJ, YK, YL, YM, YN, YO1, YO2, YP, YQ, AGE, AGS,
C AGM, AEE, AES, AEM, TAE, TCS, TMC, TSC, TTC, TTT, P1, P2, P3, P4,
COMMON P5, P6, P7, P8, P9, P10, P11, P12, P13, P14, P15, P16,
C P17, P18, P19, P20, P21, P22, P23, P24, P25, P26, P27, P28, P29,
C P30, P31, P32, P33, P34, P35, P36, P37, P38, P39, P40, P41, P42,
C P43, P44, P45, P46, P47, P48, P49, P50, P51, P52, P53, P54, P55,
C P56, P57, P58, P59, P60, P61, P62, P63, P64, P65, P66, P67, P68,
C P69, P70, P71, P72, P73, P74, P75, P76, P77, P78, P79, P80, P81,
C P82, P83, P84, P85, P86, P87, P88, P89, P90, P91, P92, P93, P94,
C P95, P96, P97, P98, P99, P100, P101, P102, P103, P104, P105, P106, P107,
C P108, P109, P110, P111, P112, TITLE(12)
DIMENSION FDATA(1)
EQUIVALENCE (AA, FDATA(1))

DO 99 N=1,196
IF (NTYPE(N) .EQ. 1) GO TO 1
IF (NTYPE(N) .EQ. 2) GO TO 2
IF (NTYPE(N) .EQ. 3) GO TO 3
IF (NTYPE(N) .EQ. 4) GO TO 4
IF (NTYPE(N) .EQ. 5) GO TO 5
IF (NTYPE(N) .EQ. 6) GO TO 6
IF (NTYPE(N) .EQ. 7) GO TO 7
IF (NTYPE(N) .EQ. 8) GO TO 8
IF (NTYPE(N) .EQ. 9) GO TO 9
```

```
SINGLE VALUED INPUT
FDATA(N) = FMODE(N)
GO TO 99
DISTRIBUTION SKFWED LEFT
1 SMODE = 0.75
  M = 1
  GO TO 10
4 SMODE = 0.75
  M = 4
  GO TO 10
7 SMODE = 0.75
  M = 7
  GO TO 10
DISTRIBUTION SYMETRIC
2 SMODE = 0.50
  M = 2
  GO TO 10
5 SMODE = 0.50
  M = 5
  GO TO 10
8 SMODE = 0.50
  M = 8
  GO TO 10
DISTRIBUTION SKEWED RIGHT
3 SMODE = 0.25
  M = 3
  GO TO 10
6 SMODE = 0.25
  M = 6
  GO TO 10
9 SMODE = 0.25
  M = 9
10 CALL RANDOM(R)
    BINARY SEARCH FOR CUM BETA
    J = 64
    DO 13 K = 1,6
    L = 6 - K
    IF (CUMB(M,J) .LT. R) GO TO 11
    IF (CUMB(M,J) .GT. R) GO TO 12
    GO TO 14
11 J = J + 2**L
    GO TO 13
12 J = J - 2**L
13 CONTINUE
14 SAMPLX = XTABLE(J)
    FDATA(N) = FMODE(N) + (FHIGH(N) - FLOW(N)) * (SAMPLX - SMODE)
99 CONTINUE
100 RETURN
    END
```

```

SUBROUTINE HISTO
COMMON /NUM/ NUM1, NUM2, NUM3, ITER, NI
COMMON /DATA/ IDATA(420), GLOW(420), GMODE(420), GHIGH(420),
C  ITYPE(420), FLOW(420), FMODE(420), FHIGH(420), NTYPE(420),
C  OUT(1000,13)
DIMENSION XVAL(25,13), NFREQ(25,13)

CALCULATES MEAN, STANDARD DEVIATION, AND PREPARES HISTOGRAM

DO 99 J = 1,13
CALCULATE INTERVAL RANGE (RINT) FOR NI INTERVALS
XMIN = OUT(1,J)
XMAX = OUT(1,J)
DO 1 K=1,ITER
IF (OUT(K,J) .LT. XMIN) XMIN = OUT(K,J)
IF (OUT(K,J) .GT. XMAX) XMAX = OUT(K,J)
1 CONTINUE
RINT = (XMAX - XMIN) / FLOAT(NI)
CALCULATE MEAN VALUE (XVAL) FOR EACH INTERVAL
XVAL(1,J) = XMIN + RINT/2.
DO 10 N= 2,NI
10 XVAL(N,J) = XVAL(N-1,J) + RINT
DETERMINE DATA FREQUENCY FOR EACH INTERVAL
DO 20 N= 1,NI
20 NFREQ(N,J) = 0
DO 22 K= 1,ITER
DO 21 N= 1,NI
XLIM = XVAL(N,J) + RINT/2.
IF (XLIM .GE. OUT(K,J)) GO TO 22
21 CONTINUE
N = NI
22 NFREQ(N,J) = NFREQ(N,J) + 1
CALCULATE MEAN VALUE (XMEAN)
SUMX = 0.
DO 30 K = 1,ITER
30 SUMX = SUMX + OUT(K,J)
XMEAN = SUMX / FLOAT(ITER)
CALCULATE STANDARD DEVIATION (XSDEV)
SUMSQ = 0.
DO 40 K = 1,ITER
40 SUMSQ = SUMSQ + (OUT(K,J) - XMEAN)**2
XVAR = SUMSQ / (FLOAT(ITER) - 1.0)
XSDEV = SQRT(XVAR)
PRINT OUTPUT
PRINT 50, (XMIN, XMAX, XMEAN, XSDEV)
50 FORMAT (16H1MINIMUM VALUE =, F10.1 / 16H0MAXIMUM VALUE =, F10.1 /
C 16H0 MEAN VALUE =, F10.1 / 16H0STANDARD DEV. =, F10.1 //)
DO 51 N = 1,NI
51 PRINT 52, (XVAL(N,J), NFREQ(N,J))
52 FORMAT (14X,F10.1,I10)
99 CONTINUE
100 RETURN
END
```

BIBLIOGRAPHY

- Alchian, A. A., Reliability of Cost Estimates--Some Evidence, The RAND Corporation, RM-481, October 1950.
- Boren, H. E., Individual Weapon System Computer Cost Model, The RAND Corporation, RM-4105-PR, July 1964.
- Finney, D. J., "Frequency Distribution of Deviates from Means and Regression Lines in Samples from a Multivariate Normal Population," The Annals of Mathematical Statistics, Vol. 17, 1946.
- Fisher, G. H., A Discussion of Uncertainty in Cost Analysis, The RAND Corporation, RM-3071-PR, April 1962.
- , Military Systems Cost Analysis (A Summary Lecture for the AFSC Cost Analysis Course), The RAND Corporation, RM-2975-PR, January 1962.
- Kahn, Herman, Applications of Monte Carlo, The RAND Corporation, RM-1237-AEC, April 1954.
- MacCrimmon, K. R., and C. A. Ryavec, An Analytical Study of the PERT Assumptions, The RAND Corporation, RM-3403-PR, December 1962.
- Quade, E. S. (ed.), Analysis for Military Decisions, The RAND Corporation, R-387-PR, November 1964.
- Sasieni, Maurice, Arthur Yaspan, and Lawrence Friedman, Operations Research--Methods and Problems, John Wiley and Sons, Inc., New York, 1959.
- Schlaiffer, Robert, Probability and Statistics for Business Decisions, McGraw-Hill Book Company, Inc., New York, 1959.
- Sobel, S. A., A Computerized Technique to Express Uncertainty in Advanced System Cost Estimates, The MITRE Corporation, TM-3728, September 1963.
- Summers, Robert, Cost Estimates as Predictors of Actual Weapon System Costs: A Study of Major Hardware Articles (U), The RAND Corporation, RM-3061-PR, April 1962 (Secret).

## DOCUMENT CONTROL DATA

1. ORIGINATING ACTIVITY THE RAND CORPORATION		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED
		2b. GROUP
3. REPORT TITLE ESTIMATING COST UNCERTAINTY USING MONTE CARLO TECHNIQUES		
4. AUTHOR(S) (Last name, first name, initial) Dienemann, Paul F.		
5. REPORT DATE January 1966	6a. TOTAL NO. OF PAGES 58	6b. NO. OF REFS. --
7. CONTRACT or GRANT NO. AF 49(638)-1700	8. ORIGINATOR'S REPORT NO. RM-4854-PR	
9a. AVAILABILITY/LIMITATION NOTICES DDC 1		9b. SPONSORING AGENCY United States Air Force Project RAND
10. ABSTRACT <p>Part of a study to improve techniques for estimating costs of future weapon systems. The Memorandum presents a technique for expressing such cost estimates as probability distributions that reflect the uncertainty of the estimate. This information is shown to be relevant to the decision-making process. The study depicts the relationship between the sources of uncertainty and system cost estimates as an input-output model and, within this framework, a procedure is developed to estimate probability distributions for each of the input uncertainties. From the input distributions, a Monte Carlo procedure is used to generate a series of cost estimates. A frequency distribution and common statistical measures are then prepared from the output estimates to determine the nature and magnitude of the system cost uncertainty. A case study involving the cost estimate of a hypothetical aircraft system with air-to-surface missiles is presented.</p>		11. KEY WORDS Cost estimates Monte Carlo Uncertainty Weapon systems Decision making Cost analysis