ANALYSIS AND TEST OF SIGNAL TRANSMISSION IN
A MULTIPHASE FLUID MIXTURE

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SILVER SPRING, MARYLAND

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For
U. S. Army Aviation Materiel Laboratories
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SUMMARY

Analytic studies and experimental measurements of the velocity of sound in a two-phase mixture of gas and liquid were performed to provide a basis for analyzing performance of fluid computing networks with multiphase fluids and to stimulate the application of such conditions to fluid computation. In particular, the work was directed toward the investigation of mixtures with a large percentage of gas, a regime where little experimental data has been available.

The analytic studies include a review of the classical approach to wave propagation and attenuation, which pertains to a homogeneous mixture, and a development of an analytic model consisting of layers of gas and liquid normal to the flow direction for the non-homogeneous case. Calculations based on this model show that the speed of sound and attenuation in a two-wave mixture having high gas content depend on the detailed nature of the mixture. This is borne out by the experiment.
FOREWORD

The advent of fluid computing networks and sensors with no moving parts has stimulated interest in the possible applications and performance of signal transmission in multiphase fluids.

The purpose of this study is to analyze and test fluid signal transmission in multiphase fluids.

The project was performed under the technical direction of Mr. George W. Fosdick, the designated representative of the Contracting Officer, of the U. S. Army Aviation Materiel Laboratories.

The project was organized at Bowles Engineering under the direction of the Chief Engineer, Mr. Edwin M. Dexter. The Project Manager was Mr. Francis Manion. The engineering staff included Mr. Charles Lomas, Mr. Ron Humphrey, and Mr. Vincent Neradka.
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LIST OF SYMBOLS

A  Area, feet$^2$
$c_v$ Specific heat at constant volume, BTU per pound - degrees Rankine
$c$ Speed of sound, feet per second
$F$ Force, pounds
$L$ Length of fluid, feet
$m$ Mass, slugs
$P_o$ Ambient pressure, psi
$P_w$ Wave pressure, psi
$p$ Pressure, psi
$Q$ Volume flow, feet$^3$ per second
$T$ Temperature, degrees Rankine
$t$ Time, second
$v$ Velocity, feet per second
$\gamma$ Ratio of specific heats
$\rho$ Density, slugs per feet$^3$
$\phi$ Mass ratio of gas to liquid

Subscripts

$g$ Refer to gas
$l$ Refer to liquid
$m$ Refer to mixture
INTRODUCTION

In pure fluid systems, which must operate under extreme environmental conditions, the possibilities exist for operation with fluids in a multiphase condition. Also it may be useful to induce multiphase conditions to effect desired performance or to sense performance by the condition of the fluid. In these cases, a knowledge of the transmission characteristics of multiphase fluids is necessary to the adequate analysis of the system.

The transmission characteristics are primarily the speed of propagation of the information signal, the attenuation of the signal, and the noise level introduced during transmission.

The following section is a brief review of classical wave propagation theory to present the fundamental relationships. Subsequent sections give a description of an analytical model to predict the behavior of multiphase fluids and test results. Appendix I gives a more complete analysis of the analytical mode and predicted performance under various conditions. Appendix II describes the experimental equipment.
CONCLUSIONS

In the regime of high gas content, 70 percent or more gas by volume, the liquid tends to form droplets. This is usually the case when the liquid is dispersed by atomization. Under these conditions the propagation velocity is essentially that of the gas alone. Since the droplets appear only as obstructions, there is a continuous path of gas for the sound to follow. The attenuation is due to orifice effects in the restrictions between droplets. This attenuation is expressed as the ratio of the amplitude of the signal wave at the output compared to the amplitude of the input over the section being tested.

In this same regime of high gas-liquid ratio, the propagation velocity can be considerably reduced by isolating the gas volumes with liquid films. This can be effected by reducing the liquid surface tension. Under this condition the sound attenuation is increased markedly because of the transmission of the propagation wave through successive liquid barriers. The conversion of the sound wave from the gas to the liquid medium is very inefficient. For this reason the sound velocity wave, for most practical cases, is completely removed, and the transmission of an information signal is related to the time required to sequentially compress the gas volumes with the liquid motion. This is analogous to the homogeneous mixture case where the speed of propagation is at a minimum for the 50-percent mixture by volume.

In the regime of low gas content, the hydrophone test and the tests with the shock tube both indicate that liquids with large gas bubble flows create noise and greatly attenuate the signal before any significant increase in the propagation velocity occurs.
CLASSICAL ANALYSIS OF WAVE PROPAGATION AND ATTENUATION

The transmission characteristics of fluid lines are primarily the speed of wave propagation, the degree of signal attenuation, and the noise level introduced during transmission. The following analysis shows that the ratio of a change in the pressure in a fluid to the change of density describes the velocity of sound and that the product of the fluid mass density and speed of propagation is its characteristic impedance.

To obtain the characteristic impedance, consider the equation that the applied force is equal to the true rate of change of momentum,

\[ F = \frac{d}{dt} \left( \frac{m}{v} \right) \]

This force can also be written as the product of the mass of fluid moved by the wave per unit time times the velocity at which this mass moves,

\[ F = \rho Av \cdot c v. \]

Pressure is equivalent to a force per area, or

\[ P = \rho cv. \]

The volume flow through the pipe is \( Av \). The ratio of the pressure to the volume flow is the impedance of the fluid:

\[ \frac{P}{Q} = \frac{\rho cv}{Av} = \frac{\rho c}{A} \]  \hspace{1cm} (1)

This impedance when taken per unit area, is defined as the characteristic impedance, \( \rho c \). The similarity of this quantity to electrical impedance follows from the fact that pressure is analogous to voltage and volume flow is analogous to current. The analogy can be carried to the example of power transfer. In electrical battery systems, the maximum power transfer occurs when the battery and load have equal resistance. In multiphase acoustical systems, the power transfer is best when the \( \rho c \) of both phases is the same. This analogy brings out the importance of the use of the term \( \rho c \) as a transmission characteristic.
If the continuity equation is written for fluid on both sides of the wave front and is combined with Equation 1, it is found that the propagation speed is related to the ratio of the change of pressure to the change of density.

\[ c^2 = \frac{dp}{d\rho} \]  \hspace{1cm} (2)

This relationship is true for perfect exchanges of energy, i.e., isentropic (reversible processes). More rigorous analyses can be performed to compute the propagation speed under conditions where the exchanges are not perfect, when an energy loss is present.

S. W. Gouse and G. A. Brown present such a thermodynamic analysis of mixtures which results in the formula

\[ \left( \frac{c_m}{c_g} \right)^2 = \frac{\left( \frac{p_g}{\rho_1} + \phi \right)^2 \left( \frac{C_{v1}}{C_{vg}} + \phi \right)}{\phi \left( 1 + \phi \right) \left( \frac{C_{v1}}{C_{vg}} + \phi \right)} \]  \hspace{1cm} (3)

where \( c_m \) is the sonic velocity of the mixture, \( c_g \) is the sonic velocity of the gas, \( \phi \) is the mass ratio of gas to liquid, \( C_v \) is the specific heat at constant volume, and \( \gamma \) is the ratio of specific heats for gas. The subscripts \( g \) and \( l \) refer to gas and liquid respectively. This analysis is a more exact version of the approximation discussed earlier. The values of speed of sound computed on this base are approximately those shown in Figure 1.

Figure 1. APPROXIMATE EFFECT OF AIR-WATER MASS RATIO ON SPEED OF SOUND IN THE MULTIPHASE FLUID MIXTURE.
This thermodynamic approach somewhat obscures what is actually happening. Further, although the approach is rather precise, it is contingent on the assumption that the mixture is homogeneous in phase composition. In addition, there are other assumptions such as: (1) the gas behaves as a perfect gas, (2) the liquid phase has a constant isentropic compressibility, and (3) the gas and liquid are always at the same temperature. The non-realization of these assumptions affects the resulting formula.

In order to present a clearer understanding of the slow wave propagation, a simple analytic model was developed. This analytic model shows that the slow propagation velocity predominantly results from the time taken to compress the gas by actual liquid motion.

ANALYSIS OF PROPAGATION VELOCITY BASED ON A LIQUID-GAS MODEL

The propagation speed given by Equation 3 is the result of a thermodynamic approach, but the form of the equation somewhat obscures what is actually happening. Further, although it is derived by a rather precise approach, it is contingent upon an assumption that the mixture is homogeneous in phase composition. While such homogeneity is readily obtained in mixtures which are predominantly liquid, it is extremely difficult to obtain in a mixture which is predominantly a gas.

This difficulty is partially responsible for the lack of prior data relative to the speed of sound in mixtures which are less than 30-percent liquid by volume. Further, it is evident that the non-homogeneous case will be the condition most frequently encountered in a pure fluid system. For example, in a fluid system, a passageway predominantly filled with gas but also containing liquid, which may or may not fully block the passageway, is often encountered. Such mixtures are far from homogeneous.

Consequently, an analytic model is developed here for mixtures which can accommodate both a homogeneous condition and the non-homogeneous condition. The development of this analytic model is described in detail in Appendix I. A brief description is as follows: Consider an extreme case wherein all of the liquid is in one portion of the passageway and all of the gas is in the remaining
portion of the passageway as illustrated by Figure 2:

![Diagram: Pressure Wave, Liquid, Gas]

Figure 2. ANALYTIC MODEL.

The transit time of the pressure wave through the mixture is the sum of the propagation time through the liquid and through the gas, plus the time required to displace the liquid-gas interface (i.e., the liquid slug) sufficiently to compress the gas.

$$\text{Total time} = \frac{L_1}{c_1} + \frac{L_g}{c_g} + t \quad (4)$$

where

- \(L_1\) = Length of liquid portion
- \(L_g\) = Length of gas portion
- \(c_1\) = Speed of sound in liquid
- \(c_g\) = Speed of sound in gas
- \(t\) = Time to displace the liquid

Actually, an acoustic level pressure wave is transmitted directly from the liquid to the gas, but because of the gross mismatch of the impedance characteristics the pressure level is severely attenuated. The result is that an acoustic wave amplitude is essentially zero after it has passed through several of these interfaces. However, when the liquid is displaced as a result of the pressure difference across the liquid, the full pressure wave amplitude is transmitted from the liquid to the gas. This latter pressure wave is strong and is the pressure wave of interest. The inertia of the liquid and the amplitude of the pressure wave establish the acceleration which the liquid slug will experience. The
adiabatic thermodynamic equations relate the liquid displacement to the pressure level developed within the gas. The combination of these effects and the above equation for total time of signal transmission provides the following equation for speed of sound in the mixture:

\[ c_m = \frac{1}{L_1/c_1 + L_g/c_g + \left\{ \frac{2 \rho_1 L_g (1 - L_g)}{(P_w - P_o) (P_w/P_o)} \left[ 1 - \left( \frac{P_o}{P_w} \right)^{1/2} \right] \right\}^{1/2}} \]  

(5)

wherein \( L_1 \) and \( L_g \) are equivalent to the volume percentages of the mixture.

The details of this derivation are presented in Appendix I.

Figure 3 is useful in visualizing the behavior predicted by this analytic model. Data were computed for a number of ambient pressures for the mixture. The upper curve at 3205 psia corresponds approximately to the triple point, above which there is no difference between water as a liquid and a gas. This curve has the monotonic characteristic one would expect for a mixture of two compressible fluids, i.e., the slope is of constant sign. The other two curves are at lower ambient pressures (one at 14.7 psia and the other at 0.25 psia) where the effect of compressibility of the two fluids influences the speed of sound. These two curves are not monotonic, and each exhibits a major regime wherein the speed of sound is less than that of either the gas alone or the liquid alone. Figure 4 shows the predicted effect of pressure wave amplitude.

This analytic model is based on the assumption that the liquid will be accelerated by the pressure wave. The motion of the liquid-gas interface acts as a piston, creating a compression wave within the gas downstream of the liquid. This condition exists when the liquid completely blocks the passageway. The complete blockage can occur with a foam mixture or when the liquid slugs extend completely across the passageway. Under these conditions the liquid will respond and compress the gas in accordance with the analytic model. An additional and significantly different condition is experienced in the pure fluid system, wherein the liquid droplets are suspended in the mixture but do not block the passageway. Under these conditions, a gaseous continuum exists throughout the
Effect of Air-Liquid Volume Ratio on Propagation Velocity

Analytical Model

$P_w$  $P_0$

- $P_0 = 3205$ psia
- $P_w = 1$ psia
- $T = 705^\circ F$
- $P_0 = 14.7$ psia
- $T = 60^\circ F$
- $P_0 = 0.25$ psia
- $P_w = 1.25$ psia
- $T = 50^\circ F$
- $P_0 = 22.05$ psia
- $P_w = 44.1$ psia

Figure 3. VOLUME RATIO, AIR TO WATER.
Figure 4. EFFECT OF PRESSURE WAVE AMPLITUDE ON VELOCITY PROPAGATION FOR MIXTURES OF AIR AND WATER.
entire passageway. A gas continuum as low as 10 percent by open area will transmit far more signal power than the 90-percent liquid area due to impedance mismatch. As a result, the propagation velocity is that of the gas alone.

Figure 5 gives the propagation velocity for water-air mixtures at room conditions and includes the additional regime for high percentage gas mixture in which the speed of sound will be critically dependent upon how the liquid is distributed within the mixture. It is interesting that the behavior of a mixture in which all of the liquid is in a single slug, i.e., minimum homogeneity, and the behavior of a homogeneous mixture will be approximately the same, while the behavior for intermediate drop sizes will deviate significantly.

Figure 6 gives the values of impedance, $\rho c$, calculated for the homogeneous case by this model and also for the region where the fluid is not homogeneous.

TEST RESULTS

Two series of tests were conducted, the first series using the shock tube as a means of generating a sharp wave front pneumatic signal and the second series using a hydrophone driver. These test fixtures are described in Appendix II. The shock tube is 25 feet long with a 0.75-inch-square section driven by a solenoid valve. The property of the shock tube is its ability to steepen a wave front during transmission to provide a fast rise input signal to the test section at the far end of the tube. In the test section two transducers are located 6 inches apart. The output signals are presented, with appropriate amplification and delay, on a dual beam oscilloscope.

SHOCK TUBE TESTS

As an extreme condition, tests were conducted with 100-percent air in the test chamber of the shock tube (Figure 7). The pressure wave propagation speed indicated is 1162 feet per second. The signal levels shown and instrumentation indicate that the attenuation of the pressure wave over a 6-inch path in air is approximately 3.67 decibels. For comparison purposes, tests were conducted with the test chamber filled with carbon dioxide. These test results are
Figure 5. PROPAGATION SPEED OF AIR-LIQUID MIXTURES AT AMBIENT CONDITIONS.
Figure 6. CHARACTERISTIC IMPEDANCE PER UNIT VOLUME OF AN AIR-LIQUID MIXTURE AT AMBIENT CONDITIONS.
Figure 7. SHOCK TUBE TEST OF PROPAGATION VELOCITY
(Air at Room Ambient Conditions).
presented in Figure 8 and indicate a propagation speed of 908 feet per second, which compares reasonably well with the anticipated value of 885 feet per second.

The next test points were obtained by spraying water in the test section with an atomizer, creating mists with mass ratios of 4.85, 5.7 and 6.35 pounds of liquid per pound of air. These mass ratios are equivalent to volume ratios of approximately 1/2-percent liquid. This is an interesting range because it is a practicable operating condition in possible applications involving controlled characteristics. The analysis, based on the homogeneous mixture, indicates that the reduction of propagation speed at high gas to liquid ratios is about 13 percent per 1-percent change of ratio. The test results should therefore show approximately a 6-percent change of propagation from the 100-percent air condition to the 99-1/2-percent air condition. The test results show no significant change.

Figure 9 shows the test results with the atomized water and, for comparison, a 100-percent air test. A 6-percent change is equivalent to about 1/2 centimeter. There is no indication that such a speed change occurred.

This effect of water droplets was as predicted by the analytic model in that the liquid was in the form of droplets which were of large diameter yet sufficiently small that they did not block the passageway. Under such conditions the mixture does not act as a homogeneous mixture. The major effect of the liquid spray was to increase attenuation. The liquid provided an additional reduction of pressure wave strengths, over a 6-inch path, of 3.85 decibels. The total attenuation of the air and liquid spray mixture was 7.52 decibels for a 6-inch sweep of the pressure wave.

The next test point, Figure 10, is a mixture of 75-percent air. The liquid was distributed throughout this air in the form of thin films of bubble surfaces which extended in such a fashion as to form a foam which completely filled the passageway. The analytical model predicts a propagation velocity under these conditions of 84 feet per second. From Figure 10 two velocities were established at 83 feet per second and 89 feet per second, which is in excellent agreement with the calculated values of Figure 3 for ambient pressure conditions. The means utilized for establishing mixture in this case were by comparing the initial volume of foamed liquid with the volume after the foam had condensed.
Sweep Rate \( t = 1 \text{ millisecond per centimeter} \)
(The input is delayed 0.1 millisecond).

Figure 8. SHOCK TUBE TEST OF PROPAGATION VELOCITY
(Carbon Dioxide at Room Ambient Conditions).
Figure 9. SHOCK TUBE PROPAGATION VELOCITY TESTS
(Air and High Gas Liquid Ratios at Ambient Conditions).
Figure 10. SHOCK TUBE PROPAGATION VELOCITY TEST
(Air-Liquid Foam Mixture, 75-Percent Air).
Tests of the next group are typified by Figures 11 and 12. In these tests, air was bubbled through the liquid-filled test section. The appearance of the resulting mixture is shown by the sketches in Figure 11. As is apparent from the oscillograph traces, the large noise content of these traces made it extremely difficult to interpret the test results. However, propagation velocities of 1930 feet per second and 833 feet per second were estimated for the mixtures of Figure 11 A and B, respectively, and 2000 feet per second and 2270 feet per second for the mixture of Figure 12, A and B, respectively. No attempt was made to establish precisely the true mixture ratios involved in these tests, as data are available for this regime based on the results of others.

Figure 13 shows the other extreme of this series of tests, wherein the test section was filled with aerated tap water. No attempt was made to establish the percentage of air in this mixture. The propagation speed of 2000 feet per second indicated an effective mixture of 4-percent gas.

HYDROPHONE TEST

The second series of tests was conducted with the hydrophone arrangement which is described in detail in Appendix II. The hydrophone was pulsed, and pressure measurements were made directly above the hydrophone at a distance of 2 inches. Measurements of a non-aerated water sample showed a propagation velocity of 4150 feet per second. Additional tests were conducted wherein air was forced through four 0.08-inch-diameter tubes in the bottom of the test section and, subsequently, through eight 0.005-inch-diameter tubes at the bottom of the test section. The results for each arrangement were the same. There was no change in propagation velocity for air flow rates up to 2 cubic inches per second, where the detector signal had become so noisy that the time measurement could not be made.

Figure 14 shows the effect of aeration provided by an Alka Seltzer tablet. This tablet generated many minute gas bubbles. Figure 14 shows the ringing in the output transducer resulting from the pulsed hydrophone. Immediately after the Alka Seltzer was inserted, the input was completely absorbed (Figure 14 B). The absorption decreased with time. About 15 minutes was required for a major portion of the gas to be released, but attenuation still existed. Although the gas is carbon dioxide, similar results are obtained with air.
Sweep Rate 0.2 Millisecond Per Centimeter Input Delayed 0.1 Millisecond

Figure 11. SHOCK TUBE PROPAGATION VELOCITY TESTS (Mixtures Induced by Bubbling Air in Water).
Figure 12. SHOCK TUBE PROPAGATION VELOCITY TESTS (Mixtures Induced by Bubbling Air in Water).
Figure 13. SHOCK TUBE PROPAGATION VELOCITY TEST (Aerated Tap Water).
Figure 14. PROPAGATION VELOCITY TESTS
(Hydrophone Test of Unaerated Water and Water Aerated With Alka Seltzer).
Figure 15. PROPAGATION VELOCITY TESTS
(Hydrophone Test of Soda Water Showing the Decay of Absorption Characteristic With Time).
Figure 15 shows the same effect in club soda. Figure 15 A is the performance immediately after pouring. The propagation velocity was measured at 3300 feet per second. Figure 15 B shows the response 20 minutes after pouring of the club soda, where the propagation velocity has risen to 3700 feet per second. The same propagation speed was measured 1 hour later. In Figure 15 B the trace amplitude produced greatly increased, showing a reduction in the amount of attenuation.
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ANALYSIS OF PROPAGATION VELOCITY BASED ON A LIQUID-GAS MODEL

In the simplest form, a homogeneous two-phase mixture can be depicted as adjacent liquid and gas volumes. This model also applies to extreme cases of nonhomogeneity where the mixture is a gross separation of gas with slugs of liquid interspersed.

The total transport time of wave through such a model is the sum of the wave propagation time through the liquid and through the gas, plus the time required to displace the liquid sufficiently to compress the gas. This sum can be written as

\[ \text{Transport time} = \frac{L_1}{C_1} + \frac{L_g}{C_g} + t \]  

where
- \( L_1 \) = the length of the liquid portion (percentage)
- \( L_g \) = the length of the gas portion (percentage)
- \( t \) = the time to displace the liquid
- \( C_1 \) = propagation velocity in the liquid
- \( C_g \) = propagation velocity in the gas

Assuming the model is of unit length, the apparent propagation velocity for the mixture is

\[ C_m = \frac{1}{\frac{L_1}{C_1} + \frac{L_g}{C_g} + t} \]  

Actually, an acoustic wave will be transmitted directly from the liquid to the gas, but because of the gross mismatch of characteristic impedances, the pressure level will be reduced 30 decibels for each transformation cycle. As a result, after the acoustic wave has passed through several of these interfaces, the amplitude of the truly acoustic wave will be essentially zero. However, when the liquid displaces as the result of the pressure difference, the full pressure wave amplitude will be transmitted from the liquid to the gas. The result is that the latter wave is far stronger because it passes through half as many interfaces (i.e., only from gas to liquid), and for all useful purposes in a pure fluid system, this is the wave of interest.
The required displacement of the liquid to compress the gas depends on the process of compression. For this analysis, an isentropic process was assumed. This means that the displacement can be found by the equation

\[
\left( \frac{P_W}{P_0} \right)^{1/2} = \frac{\rho_W}{\rho_0} = \left( \frac{L_g}{L_g - \Delta L} \right)
\]  

(8)

where \(P_W\) is the pressure wave amplitude and \(P_0\) is the ambient pressure level.

Solving for \(\Delta L\),

\[
\Delta L = L_g \left[ 1 - \left( \frac{P_0}{P_W} \right)^{1/2} \right]
\]  

(9)

But the displacement \(\Delta L\) is also a function of time and pressure difference across the liquid mass. Neglecting the compliance of gas and assuming \(\Delta L = 1/2\) at \(\Delta L\), the displacement can be written in terms of the pressure difference and time, \(t\). This expression is

\[
\Delta L = \frac{P_W - P_0}{\rho_1 \cdot L_1} \cdot \left( \frac{t^2}{2} \right)
\]  

(10)

Substituting this expression for the displacement into the expression for the compression, \(\Delta L\) is eliminated, and the time, \(t\), is defined as a function of the mixture by volume, the pressure difference, and the pressure ratio.

\[
t = \sqrt{\frac{2 \rho L_g \cdot L_1 L_g}{(P_W - P_0)} \cdot \left( 1 - \frac{P_0}{P_W} \right)^{1/2}}
\]  

(11)

Since the length of the model is unity, it follows that \(L_1 = 1 - L_g\), and the value of \(L_g\) can be taken between zero and one. The expression for \(t\) is then

\[
t = \sqrt{\frac{2 \rho L_g (1-L_g)}{(P_W - P_0)} \left[ 1 - \left( \frac{P_0}{P_W} \right)^{1/2} \right]}
\]  

(12)

The apparent propagation velocity of the mixture is
\[ C_m = \frac{L_1}{C_1} + \frac{L_q}{C_g} + \frac{1}{\left(\frac{P_w - P_o}{P_w}ight)^{1/2}} \sqrt{\frac{2 \rho_1 \cdot L_q (1 - L_q)}{P_w - P_o} \cdot \left(1 - \left(\frac{P_o}{P_w}\right)^{1/2}\right)} \]

or simplifying

\[ C_m = \frac{1}{\frac{L_1}{C_1} + \frac{L_q}{C_g} + \frac{1}{\left(\frac{P_w - P_o}{P_w}\right)^{1/2}} \sqrt{\frac{2 \rho_1 \cdot L_q (1 - L_q)}{P_w - P_o} \cdot \left(1 - \left(\frac{P_o}{P_w}\right)^{1/2}\right)}} \]

This equation yields the result that \( C_m = C_1 \) when \( L_g = 0 \), and \( C_m = C_g \) when \( L_g = 1 \), as it should.

In the denominator the term for \( t \) is dominant because of \( C_1 \), unless extremely high pressures are used. Thus, for a fixed pressure amplitude, the maximum value of \( L_g (1 - L_g) \) determines minimum apparent propagation velocity. \( L_g (1 - L_g) \) has its maximum at \( L_g = 0.50 \), which is a mixture of 50-percent gas and 50-percent liquid by volume. This is encouraging for the analytical model because it yields results that compare with the thermodynamic method. But, in addition, it also provides insight into the actual delay mechanism.

From the above equation for \( C_m \), it is apparent that the propagation speed depends not only on the mixture but also on the pressure difference and the pressure ratio. This means that the ambient pressure level and the amplitude of the pressure wave affect the apparent propagation velocity.

Figure 3 presents the computed "apparent propagation velocity", based on the above formula for pressure wave amplitudes of 0.1, 1.0, 7.35, and 29.4 psig. Although each of these conditions has a minimum velocity at 50-50 ratio mixture, the effect of the pressure wave amplitude is apparent. Figure 4 is a computed plot of the propagation velocity as it depends on the pressure wave amplitude for five mixtures of air and water. All of the data of Figures 3 and 4 were obtained by assuming an isentropic compression.

Figure 3 also shows the effect of ambient pressure level on the apparent propagation speed. The upper band is established by the maximum ambient pressure which will allow the presence of a two-phase
mixture. An increase in pressure above 3205 psi involves a regime in which there is no apparent difference between the liquid and a gaseous phase of water. As a result, a pressure wave of 1 psig was used in the calculations. The other extreme of $P_0$ is limited by the minimum pressure which will still permit the existence of a two-phase mixture. For water at 60°F, this pressure is 0.25 psia. The pressure wave for this case was also 1 psig. These two curves give the envelope of all possible propagation velocities. Note the velocity in dry air for the upper limit curve. The increase in acoustic speed for 100-percent air results from the fact that the temperature of this upper curve is 705°F, whereas the temperature for the other curves is 60°F.

The assumption that $\Delta L$ could be approximated by the expression

$$\Delta L = \frac{P_w - P_0}{\rho_1 \cdot L_1} \cdot \frac{t^2}{2}$$  \hspace{1cm} (14)$$

is somewhat in error. This approximation does not consider the fact that during compression the air pressure is increasing and retards the motion of the liquid mass. For values of $t$ that are less than twice $L_g/C_g$, this equation is correct. The validity of approximation when $tL_g/C_g$ depends on the damping of the mixture. Actually, for low damping the approximation is close. Since there is inertia and compliance in the liquid slug and the air volume, there is a resonant frequency which will show up in output wave as noise. This frequency is

$$f = \frac{1}{2\pi} \cdot \sqrt{\frac{1}{2\mathcal{L}C}}$$ \hspace{1cm} (15)$$

where $\mathcal{L}$ is inertia and $C$ is compliance.

The conditions of validity for this analysis or for the previous thermodynamic analyses at all mixtures are important.

At a 50-50 mixture and increasing the liquid portion, the propagation velocity increases but remains far below that of the liquid alone. There is data of Karplus and others who have shown that this reduced propagation speed from a liquid to a 50-50, two-phase mixture does exist. There was a lack of data on the gaseous side of the two-phase mixture to verify if the reduced propagation speed existed in this type of mixture. The data of this report indicates that for non-homogeneous, highly gaseous mixtures, wherein the liquid is in large droplets, the propagation speed is the same as that of the gas,
although the attenuation will be high. This situation can exist in a pure fluid system with multiphase flow. Referring to the analytical model, if the liquid section is very short, \( L_1 \) is small. Surface tension tends to form the liquid into droplets rather than leave it as a membrane which blocks the passageway. The result of this droplet formation is that there now exists a gaseous continuum through the entire passageway. Because of the gross mismatch between water and air, a 10-percent gaseous continuum will transmit far more acoustical power than the 90-percent liquid center section. As a result, as soon as a gaseous free path is formed, the apparent velocity of sound is that of the gas. As the droplet size is decreased, the mixture approaches homogeneity. Therefore, the plot of velocity of sound as a function of the volume mixture will follow a single curve for low percentages of gas but becomes an area or family of curves dependent upon the character of liquid distribution within the mixture as the percentage of gas is increased above 50 percent. Such a pattern is presented in Figure 5 and is the behavior of the proposed analytic model.

The experimental data obtained seem to confirm this characteristic.
APPENDIX II

EXPERIMENTAL TEST ARRANGEMENTS

Two experimental test arrangements were constructed to measure the velocity of pressure wave propagation through a two-component, two-phase mixture. These arrangements were an air shock tube and a liquid hydrophone arrangement.

SHOCK TUBE

The first arrangement utilized the Bowles Engineering Corporation repetitive cycle shock tube which is 25 feet long and which has a 0.75-inch-square test section. The shock tube serves to steepen any inclined wave that results from the method of inducing pressure wave. The arrangement is shown in Figure 16. At the bottom of the shock tube, four piezoelectric pressure transducers were rubber mounted in the side of a special extension of the shock tube. These transducers were spaced exactly 6 inches apart. Spacing between the last transducer and the end of the test section was also 6 inches.

The first transducer was used as a trigger for the oscilloscope trace. In this way, the time elapsed, as the wave passed from the first transducer to the second, is displayed as a distance on the oscilloscope screen. This distance, in addition to the sweep rate, provides the information necessary to compute the speed of the wave.

In order to expand the time scale, thereby increasing the accuracy of the measurement, the first scope trace was electronically delayed (20 to 100 microseconds), as appropriate to the particular test.

Time measurements between the two transducers were measured for air with no liquid, for the entire test section filled with aerated liquid, for liquid spray into air, for air bubbling into liquid, and for the test section filled with carbon dioxide.

Liquid sprays into air were made with liquid content of 4.8 to 6.3 pounds of liquid per pound of air.

Air bubbled into liquid included mixtures up to 75-percent air.
Figure 16. SHOCK TUBE TEST ARRANGEMENT.
HYDROPHONE TEST ARRANGEMENT

The hydrophone test arrangement was a water shock tube. The arrangement is shown in Figure 17. The tube was 6 inches long with an inside diameter of 2.25 inches. At the bottom of the tube, a driver hydrophone was used to provide the disturbance.* The disturbance was then detected by the detector hydrophone which was placed 2 inches above the driver. By triggering the oscilloscope on the driver, the time difference between driver and detector was measured. A reference time increment was made by placing the detector adjacent to the face of the driver. This permitted the effective delay of the driver to be measured. The test showed a 10-microsecond delay which was subtracted from all readings.

To operate the driver as a pulse generator, a step function was introduced at a low repetition rate, such as 100 cps. The hydrophone responded to the step transient and emitted a high frequency ringing. This ringing damped before the subsequent step input. An oscilloscope photograph showing the response of the detector hydrophone to the step function repetition rate is presented in Figure 18. This photograph was made at a slow sweep speed of the scope so that the decay of the ringing could be seen.

To make a measurement of wave velocity, the scope was triggered by the electrical step input, and the time increment between the electrical step and the detection of the ringing by the detector hydrophone was photographed. The sonic velocity was then obtained by subtracting the 10-microsecond driver time delay and dividing this time increment into the distance between the driver and the detector hydrophone.

Tests were conducted with liquid alone, with air bubbles forced in through four 0.08-inch-diameter holes, and with air bubbles forced through 0.005-inch-diameter holes. In addition, tests were conducted in water with small quantities of Alka Seltzer and in club soda. The former generated many small bubbles of gas. The soda also contained many CO₂ bubbles in solution.

* The driver hydrophone does not respond to low frequency. Its useful range is 50 kilocycles to 100 kilocycles. An initial attempt was made to measure the change in standing wave frequency as air bubbles were forced into the bottom of the tube, but this proved to be an impractical method because the resonance of the tube influenced the results.
Figure 17. HYDROPHONE TEST ARRANGEMENT.
Figure 18. HYDROPHONE TEST ARRANGEMENT
(Response of Hydrophone to Electrical Step Input).
"Analysis and Test of Signal Transmission in a Multiphase Fluid Mixture"

Analytic studies and experimental measurements of the velocity of sound in a two-phase mixture of gas and liquid were performed to provide a basis for analyzing performance of fluid computing networks with multiphase fluids and to stimulate the application of such conditions to fluid computation. In particular the work was directed toward the investigation of mixtures with a large percentage of gas, a regime where little experimental data has been available.

The analytic studies include a review of the classical approach to wave propagation and attenuation, which pertains to a homogeneous mixture, and a development of an analytic model consisting of layers of gas and liquid normal to the flow direction for the non-homogeneous case. Calculations based on this model show that the speed of sound and attenuation in a two-wave mixture having high gas content depend on the detailed nature of the mixture. This is borne out by the experiment.
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