NOL COPPER-BALL ACCELEROMETERS

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NOL COPPER-BALL ACCELEROMETERS

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ABSTRACT: The report describes a number of ball-crusher, peak-reading accelerometers and their application for recording simple as well as complex shocks. Some concepts on shock spectra, as they apply to these devices, and a new method for determining the peak of a shock recorded with preset copper-ball accelerometers are discussed. Ten accelerometer models cover a shock range from approximately 30 g to 450,000 g; their corresponding natural frequencies range from 244 cps to 14,800 cps.

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U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

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This report supersedes NAVORD Report 6925, which was prepared in July 1960. Since that time, the Laboratory has developed several new designs to extend the shock range and the applications of the accelerometers. This work was carried out on Project SUBROC, Task No. RU35 00 000/W030 A0 01 001, and the Mk 73 VT Fuze Program, Task No. RM24 00 010/212 1/F008 08 01.

The assistance of Mr. L. A. Vagnoni of the Environment Simulation Division in preparing material on shock analysis is acknowledged.

J. A. DARE
Captain, USN
Commander

V. M. KORTY
By direction
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INTRODUCTION

1. Copper-ball accelerometers are used extensively at NOL for field and laboratory investigation of shock where it is impractical to use more sophisticated instrumentation. The accelerometers, designed during World War II, still consist of a simple inertia weight and a copper ball as their principal parts. However, there have been frequent changes in the size and shock range of the accelerometers and several changes in their mechanism of operation. The first major modification was reported by reference (a). Several inertia-actuated discriminator mechanisms were incorporated in the earlier design to extend its utility. The discriminators are used when it is known that several shock pulses occur in close time sequence and when it is desired to measure the maximum acceleration of one pulse or a portion of a pulse while discriminating against shocks incidentally present. Recently, another major modification extended the shock range and natural frequency of the device to record complex, high-g shocks spectrally. Also, a preset accelerometer was developed to record the highest peak of a simple or complex shock.

CONVENTIONAL AND DISCRIMINATING ACCELEROMETERS

GENERAL DESCRIPTION

2. The conventional accelerometers consist of four principal parts: a housing, an inertia weight, a copper ball, and an anvil or base. Figure 1 illustrates the basic design. The back-up wedge prevents the inertia weight from rebounding during shock and the rubber sleeve holds and centers the ball on the anvil. The discriminating accelerometers utilize the same basic design and incorporate two types of discriminator mechanisms to program the action of the inertia weight. Details of the discriminators are discussed in paragraphs 15 and 16.

3. The accelerometers (conventional and discriminator) have a shock range from approximately 30 g to 20,000 g and range in natural frequency from 244 cps to 1890 cps. Shock range and natural frequency are dependent on the inertia weight since all accelerometers use the same copper balls. Table 1 lists the characteristics of each accelerometer.

PRINCIPLE OF OPERATION

4. The principle of operation of the accelerometers is quite simple: When a shock pulse is applied to the base of the device, the inertia weight exerts a force on the copper ball and permanently deforms it. The inertia force, figure 1, is opposite in direction to the applied shock. The deformation is
Fig. 1. Conventional Accelerometers
converted to load from the static calibration curve, figure 2. The peak acceleration (g) indicated by the accelerometer is the static load divided by the inertia weight value in pounds.

5. This simple method for calculating acceleration is accurate to within 10 percent for simple pulses whose rise time is greater than five times the natural period of the accelerometer. Where this condition does not exist, an amplification factor must be used to calculate the actual peak acceleration — more about this later.

Table 1

CHARACTERISTICS OF CONVENTIONAL AND DISCRIMINATING COPPER-BALL ACCELEROMETERS

<table>
<thead>
<tr>
<th>Mod No.</th>
<th>Nominal Shock Range* g</th>
<th>Inertia Weight (W) grams</th>
<th>Natural Frequency (fₚ) cps</th>
<th>Natural Period (Tₚ) ms</th>
<th>Discriminator Actuation Time** ms</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>30 to 300</td>
<td>976.0</td>
<td>244</td>
<td>4.09</td>
<td></td>
</tr>
<tr>
<td>4*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 to 25</td>
</tr>
<tr>
<td>2</td>
<td>190 to 1900</td>
<td>168.0</td>
<td>589</td>
<td>1.70</td>
<td></td>
</tr>
<tr>
<td>5*</td>
<td>400 to 4000</td>
<td>80.4</td>
<td>852</td>
<td>1.17</td>
<td>1 to 15</td>
</tr>
<tr>
<td>7*</td>
<td>600 to 6000</td>
<td>47.8</td>
<td>1104</td>
<td>0.905</td>
<td>.1 to 2</td>
</tr>
<tr>
<td>3</td>
<td>1000 to 10,000</td>
<td>23.3</td>
<td>1582</td>
<td>0.632</td>
<td></td>
</tr>
<tr>
<td>6*</td>
<td>2000 to 20,000</td>
<td>16.3</td>
<td>1891</td>
<td>0.529</td>
<td>2 to 4</td>
</tr>
</tbody>
</table>

* Discriminating accelerometers (fig. 6) use the same inertia weights.

** Approximate time for the discriminator inertia collar (fig. 6, item 1) to complete the discriminating action within the nominal shock range.

6. All NOL accelerometers employ 5/32-inch diameter annealed copper balls. The balls exhibit acceptable linear load-deformation characteristics over a wide range. Under dynamic loading the balls also exhibit near-perfect plastic deformation. The impact velocity change of a mass striking the ball is only four percent higher than the striking
Fig. 2. Sample Static Calibration Curve for 5/32" (NOL Group D-1956 Lot) Annealed Copper Balls Deformed on Two Sides
velocity of the mass. This indicates that the elastic recovery of the ball after impact is negligibly small. Detailed information on the properties of copper balls is presented in reference (b).

7. The response of the accelerometer essentially follows the differential equation of a mass-spring system up to the maximum deflection of the copper ball. The response has a linear dependence with temperature, the deformation at -65°F being approximately 15 percent less than that at 160°F for equivalent loads. Under nonaxial loads up to 60° obliquity, the accelerometer responds to the peak axial component with less than 10 percent error.

BEHAVIOR AS AN ACCELEROMETER

8. When the shock to be measured has a very short rise time or is too short in duration for the accelerometer to respond accurately, an amplification factor must be used to calculate true peak acceleration. These factors are taken from response curves for mass-spring systems subjected to simple shock pulses - the effects of shock on simple elastic structures and the theory for plotting shock response curves is described in reference (c). A typical curve is illustrated in figure 3.
This curve and others were computed on the Environment Simulation Division Analog Computer — computer operations and curves for several other idealized pulses are presented in Appendix A. Plotted in figure 3 is the response of a single-degree-of-freedom mass-spring system in the region where the natural period of the system \( T_n \) is less than five times the pulse duration \( T_0 \). The natural period of the accelerometers is determined from the equation

\[
T_n = 2\pi \sqrt{\frac{W}{Kg}}
\]

where \( K \), figure 2, is the slope* of the curve in pounds per foot and \( W \) is the value in pounds of the inertia weight — the natural period of each accelerometer is listed in Table 1. Briefly, the response curves are used as follows:

a. Assuming that the pulse (figure 3) is too short for the accelerometer to respond accurately, an estimate is made of its duration.

b. From the time ratio calculated, an amplification factor is found.

c. Peak acceleration is calculated using the same method described in paragraph 4, but in this case the value found is divided by the amplification factor.

\[
A_{\text{peak}} = \frac{\text{Static Load}}{W \cdot \text{Amplification Factor}}
\]

**BEHAVIOR AS VELOCITY METER**

9. The principles described above apply strictly to the behavior of the device as an accelerometer. Under certain conditions, the instrument will behave as a velocity meter. It is important for the user to determine whether the shock of interest can be measured in terms of acceleration or must be measured in terms of velocity change. For shocks impulsively applied (shocks whose durations are much shorter than the natural period of the mass-spring system) the instrument behaves more as a seismic device than as an accelerometer.

* The slope drawn favors the initial portion of the curve since \( K \) determined from shock measurements \( (a = \omega^2 \delta) \) is generally higher.
In this range, copper ball deformations indicate the velocity change applied to the instrument.

10. That an accelerometer will behave as a velocity meter is evident from figure 4, which shows the amplification factor of a linear mass-spring system as a function of the duration of an applied rectangular pulse. Note that for pulses of duration much shorter than the natural period \( T_0/T_n < .3 \), the peak response varies linearly with the duration. Consequently, since the velocity change is directly proportional to the duration, it follows that the resulting deformation is directly proportional to the velocity change.

![Amplification Factor vs Time Ratio](image)

**Fig. 4. Response of Mass-Spring System to Rectangular Pulse**

11. As demonstrated earlier, pulse shape influences accelerometer response. However, for impulsively applied shocks, pulse shape has little effect in the region where the mass-spring system behaves as a velocity meter. Figure 5 illustrates how the device responds to several typical pulses—note that in this application the time ratio is inverted. The shorter the impulse time \( T_0 \), the smaller the variation with
Fig. 5. Velocity Meter Response

Ratio of induced to actual velocity change
pulse shape. It is generally recommended that the device be used as a velocity meter only in the range where the time ratio \( \frac{T_n}{T_o} \) is greater than 5. At ratios less than 5, measurements should be interpreted in terms of acceleration using appropriate amplification factors — see paragraph 8.

12. When the instrument is used as a velocity meter (in the range recommended above) velocity change can be determined with better than 5% accuracy from the equation

\[
\Delta V = \frac{\omega n \delta}{12} = \frac{\pi F n \delta}{6} \text{ fps}
\]

where \( \Delta V \) is the velocity change, \( \omega \) is the circular natural frequency in radians per second, and \( \delta \) is the copper ball deformation in inches.

**DISCRIMINATOR MECHANISMS**

13. One serious limitation with earlier model copper-ball accelerometers was their inability to distinguish between the shock pulse under investigation and shocks incidental to the test environment — in many tests, two or three shocks occur in close time sequence. This multiple shock problem was overcome by adding discriminator mechanisms to the accelerometer. The mechanisms program the accelerometer inertia weight to act on the copper ball only during the period of interest.

14. The most useful application for discriminating copper-ball accelerometers has been for measuring shocks encountered by high velocity missiles during their entry into water. These shocks generally consist of a high deceleration (impact) lasting for a fraction of a millisecond, followed by a low deceleration (drag) several orders of magnitude longer in duration than the impact. Frequently field tests are conducted in shallow water to facilitate recovery of the test vehicles. Thus, a third shock occurring during impact with the bottom must be considered. To be useful, then, the copper-ball accelerometer must be capable of discriminating against one or several shocks during a particular test.

15. Figure 6 shows the two principal types of discriminators used with NOL accelerometers. The Mod 5 mechanism was incorporated in the Mods 1, 2, and 3 accelerometer designs, figure 1. With discriminators, the gages are designated as the Mod 4, 5, and 6 Copper-Ball Accelerometers, respectively. Because it has limited use, the Mod 7 discriminator has been incorporated in the Mod 2 accelerometer design only.
Mod 4, LD 542408 and Mod 6, LD 542410 are similar in design.

Inertia Mod 6, LD 542410 are similar in design.

Mod 5 Accelerometer (LD 542409)

Fig. 6. Discriminating Accelerometers
16. Referring to figure 6, the discriminator mechanisms operate as follows. In the Mod 5 accelerometer, shock actuates the inertia collar (item 1), which shears the Teflon screw (item 2). Several milliseconds later, depending on the position of the set screw (item 3) and the intensity of the shock, the collar engages the restraining pin (item 4), driving it inward, causing the pin to release the anvil (item 5). Thus, the ball drops clear of the inertia weight and recording is interrupted. In the Mod 7 accelerometer, the operation is similar except that in this device, recording is delayed for several milliseconds until the restraining cam (item 3) releases the inertia weight to operate on the ball.

17. The Mod 5 accelerometers discriminate against all but the initial portion of shock. For example, during the entry of high velocity missiles into water, the accelerometer will record the peak shock occurring at impact and discriminate against water-entry drag and bottom impact. The recording period of the accelerometer may be varied by adjusting the set screw, item 3, figure 6. The Mod 7 accelerometer is designed to discriminate against the initial portion of shock, as for example the impact phase of water entry. The device permits recording either the peak of water-entry drag or bottom impact, whichever is higher. Construction details for conventional and discriminating accelerometers are given in reference (d). Appendix B discusses the character of two-phase (water-entry) shock and its effect on the response of the Copper-Ball Accelerometer.
18. The high-g accelerometers shown in figure 7 consist of a 9-mass multifrequency unit, Mod 8, a 12-mass preset unit, Mod 9, and a single-mass unit, Mod 10. The Mod 8 and 9 accelerometers are compact devices weighing less than one-half pound; the Mod 10 is a 1/2-ounce accelerometer no larger than most piezoelectric pickups.

19. In operation, the accelerometers are basically similar to the conventional types. They differ only in that no backup wedge - see figure 1 - is used to prevent inertia weight rebound. This has not proven to be a serious problem at high shock levels. Normally, this type of shock is characterized by a single severe impulse in which most of the energy is expended in a matter of microseconds. Except in rare cases, high frequency oscillations following the initial pulse are of such small displacements and account for so little of the energy expended, that the hammering effects are minimal. More will be said about this later. Also, unlike the conventional accelerometers, the high-g inertia weights are less than 10 grams and thus responsive only to very high shock loads. Once the weight has deformed the ball under maximum load, any repeated loading of lower g value or of very high frequency will have negligible effect. This is particularly true in the preset type of accelerometers where each copper ball is deformed before it is subjected to shock. Characteristics of both types of accelerometers are listed in Table 2. The original designs and some of the types of shock measured with the accelerometers are described in reference (e).
### Table 2

**CHARACTERISTICS OF HIGH-\(g\) COPPER--BALL ACCELEROMETERS**

<table>
<thead>
<tr>
<th>Type</th>
<th>Inertia Weight No.</th>
<th>Inertia Weight (W) grams</th>
<th>Natural Frequency ((f_n)) cps</th>
<th>Natural Period ((T_n)) (\mu)sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod 8</td>
<td>1</td>
<td>9.60</td>
<td>3,390</td>
<td>295</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.90</td>
<td>4,010</td>
<td>249</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.64</td>
<td>5,500</td>
<td>182</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.12</td>
<td>7,220</td>
<td>138</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.50</td>
<td>8,560</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.05</td>
<td>10,350</td>
<td>96.5</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.83</td>
<td>11,500</td>
<td>87.0</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.67</td>
<td>12,800</td>
<td>78.0</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.50</td>
<td>14,800</td>
<td>67.5</td>
</tr>
<tr>
<td>Mod 9</td>
<td></td>
<td>5.71*</td>
<td>3,210</td>
<td>312</td>
</tr>
<tr>
<td>Mod 10</td>
<td></td>
<td>0.78</td>
<td>11,900</td>
<td>84.0</td>
</tr>
</tbody>
</table>

* Ball is deformed on two sides in the preset accelerometer; 1/3 the weight of the ball is added to the inertia weight value. Other accelerometers are ball socket types; thus ball weight is not a factor.
MULTIFREQUENCY ACCELEROMETER

20. Details of the Mod 8 accelerometer are shown in figure 8. The device contains nine mass-spring elements ranging in natural frequency from 3390 cps to 14,800 cps. The corresponding shock range is from a nominal 250 g (9.60 gram mass) to 450,000 g (0.500 gram mass). Each mass-spring element responds to the shock in keeping with its own natural frequency. Thus for each shock measured the accelerometer records nine peaks at nine different frequencies to obtain shock spectra directly. The shock spectra concept as it applies to copper-ball accelerometers is discussed in paragraph 24.

21. Each ball in the nine-mass accelerometer rests in a socket. Each socket is precision machined to form fit over about 40 percent of the copper ball. During shock the ball deforms on one side only. This scheme prevents the ball from rotating during shock, eliminates the weight of the ball as a factor in calculating acceleration, and increases the spring constant (K) of the ball to a value nearly twice that of a ball deformed on two sides (figure 2). The higher spring constant, figure 9, plus the sizable reduction in the size of the inertia weights significantly increases the natural frequency range of the accelerometer over that of the conventional type.

22. The Mod 8 (and Mod 10) accelerometers require more precision in their manufacture than do the other models. As seen in figure 8, critical parts are held to very close tolerances. In addition, copper balls are sorted in lots by increments of 0.0001. NOL ball lots vary from 0.1552 to 0.1555. The last operation in the manufacture of the accelerometers consists of fitting all the inertia weights to one lot of copper balls (for example, a 0.1554 lot). Each mass is placed in a fixture to hold it perpendicular to a lapping table and lapped on one side by hand until it is the proper length. Once this has been done, the accelerometer is stamped with the ball size for which it was fitted and only that size ball is used with it.

23. The accelerometers also require greater precision in the calibration and measuring of the balls. As shown in figure 9, the calibration extends from 0.0002 to 0.020 deformation. The 0.020 limit is imposed by the ball socket. Beyond this deformation the ball bulges against the socket edge and becomes nonlinear. Ball deformations greater than 0.001 are measured with conventional micrometers. However, for smaller deformations an optical micrometer is used to measure the diameter of the ball flat (chordal plane) and this value is converted to deformation.
FIG. 8 MOD 8 ACCELEROMETER DETAILS
* Spread between maximum and minimum deformations is 13%

Fig. 9. Sample Static Calibration Curve for 5/32" (NOL Group D-1956 Lot) Annealed Copper Balls Deformed on One Side
SHOCK SPECTRA

24. The following definition, taken from reference (f), best describes the shock spectra concept:

A shock spectrum is the plot of the maximum absolute values of the relative displacements, multiplied by scaling factors if desired, of a set of either damped or undamped single-degree-of-freedom oscillators with negligible mass which have been subjected to shock motion, the values being plotted as a function of natural frequency of the simple oscillators. These graphs can be constructed with units of displacement, velocity, or equivalent static acceleration by the choice of the scaling factors; unity, \( w \), or \( w^2/g \) respectively, where \( w \) is the angular frequency of the oscillator.

Most severe shocks, even those where the initial pulse has the highest peak or contains most of the impact energy, have spurious oscillations which combine to excite simple oscillators in such a way that the "maximum absolute values" can occur after several cycles. While the copper-ball accelerometer is a single-degree-of-freedom, mass-spring system, it does not behave as a simple oscillator. The ball permanently deforms, allowing motion in one direction only. Spectra plotted from its peak values will differ significantly at some frequencies from maximum absolute values of simple oscillators.

25. To demonstrate their response to complex shock, the Mod 8 and two Mod 3 Copper-Ball Accelerometers were tested on the Mk 7 Mod 0 Drop Shock Tester, reference (e), and the WOX-126A Test Set, reference (g). Oscillograms of the shocks produced are shown in figure 10. Also shown are typical analogs to identify the theoretical operating range of the copper-ball accelerometers. Analog computer operations are described in Appendix A.

26. The agreement between the analog and the copper-ball accelerometer spectra, especially for the high initial peak shock (13,200 g), is unusually close. Two factors may account for this: (1) Most of the shock energy for the 13,200 g shock is expended during the initial pulse. This interpretation is supported by the fact that most of the analog runs for this shock show the maximum peak to be in the first cycle — figure 10 is typical of the analog responses. (2) The copper-ball accelerometer is subjected to a shock with much higher frequency content than that shown in the figure 10 transients. The 20 Kcps flat response system used to record these shocks did not have the range to accurately respond to the very high frequencies known to be part of these complex shocks.
SHOCK WITH HIGH INITIAL PEAK

**WOX-126A SHOCK TRANSIENT**

* RESPONSE OF 6400 cps ANALOG (see Fig. 11) TO ABOVE TRANSIENT

* RECORDING RANGE OF COPPER-BALL ACCELEROMETER

SHOCK WITH MULTIPLE HIGH PEAKS

**MK.7 SHOCK TRANSIENT**

* MAXIMUM PEAK RESPONSE OF 6400 cps ANALOG (see Fig. 11)

* RECORDING RANGE OF COPPER-BALL ACCELEROMETER

* TAPE RECORDED WITH 20 Kcps FLAT SYSTEM. INPUT TRANSIENT FED THROUGH 20 Kcps LOW-PASS FILTER.

**FIG. 10** COMPLEX SHOCK TRANSIENTS AND ANALOG RESPONSES TO SHOW THE COPPER-BALL ACCELEROMETER RECORDING RANGE
Fig. 11. Analog and Copper-Ball Accelerometer Spectra of Complex High-g Shock
27. The greater errors in measuring multiple-peak shocks with the copper-ball accelerometer are on the low side. This deficiency is not considered serious where the information is intended to provide guidelines for design. If the limitations of the accelerometer are known, compensation for error should pose no difficulty. In artillery projectile fuze applications, shock spectra from copper-ball accelerometer measurements helped to correlate component damage with shock level, and assisted in identifying the region of maximum resonance in the projectile-fuze system.

Preset Accelerometer

28. The preset accelerometer differs from those previously described only to the extent that the balls are partially deformed before they are used to measure shock. The preset-ball method for measuring shock has been used for many years at NOL, but not in the shock range above 20,000 g. Originally, a group of conventional accelerometers was preset and clustered in one container to make up an instrumentation unit. The Mod 9 accelerometer, figure 12, consists of a simple, compact unit containing 12 preset, mass-spring elements, each weighing six grams—this includes the weight of the copper ball.

29. The length of the weights, numbered 1 through 12, increases progressively by nominal increments of 0.004 inch. When the accelerometer is assembled, the parts are forced together in an arbor press or vise. In the process the balls are deformed in steps of about 0.004 inch from 0.003 inch to 0.047 inch. The balls may be preset separately by using the accelerometer mounting stud as a press. Using the stud to preset 12 balls at once is not recommended. Accelerometer housings may be shortened to increase the preset values for higher shock levels. The Mod 9 body is for a preset range from about 3000 g to 41,000 g.

30. Dimensional precision is not as critical with the preset accelerometer as it is with the Mod 8. Slight variations in inertia weight length or copper ball size do not affect accelerometer accuracy. Preset deformations are measured and recorded before each test and, as will be demonstrated, are used only to establish a base for the shock deformations recorded later.

Preset Theory

31. With the design of the preset accelerometer, a new method for analyzing shock recordings was developed. Briefly, the method consists of plotting shock deformations against static preset accelerations and fitting a curve through the points to intersect the preset acceleration axis. The point
FIG. 12 MOD 9 ACCELEROMETER DETAILS
of intersection is considered to be close to the peak value of the shock. For simple pulses of very short duration, curve fitting or extrapolation can be done mathematically.

32. Preset analysis is an extension of the method described in paragraph 8. Because each ball in the Mod 9 accelerometer is deformed before it is subjected to shock, the response curves for simple mass-spring systems (figure 3 and Appendix A) were replotted to take into account the effect of preset. The curves are plotted for time ratios up to 0.7 — for most high-g shock applications, pulses are fairly short compared to the natural period of even the highest frequency element in the Mod 9 accelerometer.

33. To demonstrate how these response curves theoretically support the "extrapolation" method of analysis, several hypothetical high-g shocks are analyzed. In this case the shock parameters are known and the response of the preset accelerometer is determined — in practice the operation is reversed. The peak of the shock is assumed to be 70,000 g and the duration \( T_0 \) 25 microseconds. The pulses are the shape of those illustrated in figure 13 and others included in Appendix A. From Table 2, the natural period \( T_n \) of the Mod 9 accelerometer is 312 microseconds. The time ratio \( \frac{T_0}{T_n} \) for all mass-spring elements is 0.08. To simplify the demonstration, it is assumed that eleven of the accelerometer balls are preset in increments of 10% from 0% to 100%, as plotted in the response curves, and that the twelfth ball is preset to 110%. From the curves an amplification factor is found for each preset value and a response acceleration is calculated. For example, for the rectangular pulse, figure 13, the 50% preset factor is .69. The response acceleration \( A_R \) is

\[
A_R = (70,000)(.69) = 48,300 \text{ g}
\]

This is equivalent to a static load of 640 pounds or a copper ball deformation of 0.0515 (see static calibration, figure 2). At 50% preset, the ball deformation is 0.0380. The shock deformation for 70,000 g, therefore, is 0.0135. This value is plotted against the corresponding preset acceleration. Figure 14 illustrates how the curves intersect the abscissa at the 70,000 g preset.

34. Of significance in the curves shown in figure 14 is the following:

a. Only in the case of the rectangular pulse, figure 13, is the curve linear. Since pulses of this character are seldom encountered in elastic structures, especially for
Fig. 13. Response of Preset Mass-Spring System to Rectangular Pulse
Fig. 14. Theoretical Response of Preset Copper-Ball Accelerometer to Simple Pulses
very severe shocks, the slope for most curves should follow those of the half-sine and triangular pulses.

b. For short time ratios there is little difference in the response of preset accelerometers to triangular pulses.

c. Following the procedure described in paragraph 33, it is possible by trial and error to fit a theoretical curve which approximates the curve plotted from copper ball measurements. From the theoretical curve, an estimate can be made of the duration of the principal shock (that portion of the shock during which most of the impact energy is expended).

35. Figure 15 shows a typical curve fitted through data plotted from three preset accelerometers. For comparison, the accelerometers were subjected to a shock close to the range of those described in paragraph 33. The last five readings from the 31,000 to 83,000 g accelerometer are typical for any complex shock in this range. In fitting a curve, these "flat" end values are generally ignored.

200,000-g ACCELEROMETER

36. The Mod 10 accelerometer, figure 16, is made up of a single inertia weight and a base with a ball socket similar to those in the Mod 8 accelerometer, figure 8. The compact body of the accelerometer is made of stainless steel. A screw cap clamps the ball and weight together elastically to prevent the ball from turning during shock. The accelerometer requires the same precision in its construction and use as that described in paragraph 23.

37. The inertia weight for the device was especially selected to provide a simple and compact accelerometer to monitor high-g shock tests conducted on the Mk 7 tester, reference (e). Once a machine has been calibrated spectrally, a single mass-spring accelerometer is adequate to insure that the calibrated shock levels are maintained. Usually, selection of a mass-spring element is made to provide as high a frequency response as possible and enough deformation for a particular shock to make ball measurements easy and to minimize errors.

ADVANTAGES AND LIMITATIONS

38. The addition of discriminators and high-g gages to the family of NOL Copper-Ball Accelerometers has helped to reduce some of the more serious limitations encountered in the past. The high-g accelerometers are much simpler mechanically. Also, because they record nine peaks for a single
Fig. 15. Response of Preset Copper-Ball Accelerometer to Mk 7 Mod 0 Drop Shock Tester High-g Shock
shock, they describe complex shock more completely than do the single-mass devices. With increasing emphasis being placed on the spectral method of defining simple and complex shock, this type of accelerometer should find wide application. Because they are compact and light-weight, they are ideal as backup systems, even where it is possible to use continuous recording instruments.

39. There are few known alternatives to using peak reading devices for measuring very severe shocks. Even those accelerometers rated as having natural frequencies in the range of 100,000 cps and maximum sensitivities of 100,000 g have failed mechanically or have malfunctioned electrically to the extent that acceleration-time data has been in doubt. Typical tests at these extreme levels are described in reference (e). Compounding the transducer problem has been the limitation in tape recorders. The most advanced equipment still does not have the response necessary to record shock transients of the order of 50,000 to 100,000 cps. The same is true of equipment used to compute shock spectra electronically from recorded shock transients at these extreme levels.

40. Despite the fact that design improvements have significantly increased the usefulness of copper-hall accelerometers, they by no means solve all the problems of shock-recording instrumentation. These devices indicate only the peak values of acceleration or velocity change. Also, they require that some of the characteristics of the shock under study be known to interpret measurements with any acceptable accuracy - such shock parameters as impact velocity, target penetration, the contact time of colliding objects, etc. Without this knowledge shock measurements can be extremely difficult to interpret.

41. While the accelerometers are simple-mechanical devices, they cannot be used effectively by untrained personnel. The judicious selection of gages for a particular job, the care which must be taken in readying gages for test, and the accurate analysis of shock data require more than a general knowledge of shock environment and of the response of elastic systems to dynamic loads.

42. Admittedly, peak reading devices are a poor substitute for continuous recording instrumentation. However, unless the response of existing instruments can be improved and recording equipment can be shock hardened to survive today's more severe shock environment, peak reading devices will continue to be valuable tools in the measurement and interpretation of shock.
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(a) NAVORD Report 6925, NOL Copper-Ball Accelerometers, Jul 1960
(b) J.R. Townsend, The Development, Annealing, and Calibration of Copper Tarage Balls, Final Report, OSRD 6160, 1948
(c) J.M. Franklin, Effects of Impact on Simple Elastic Structures, David Taylor Model Basin Report 481, Apr 1942
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(e) NOLTR 63-263, Explosive Component Test Adaptor for Drop Shock Tester Mk 7 Mod 0, Dec 1963
(g) NOLTR 61-106, Test Set WOX-125A (Drop Shock Tester)
APPENDIX A

RESPONSE CURVES FOR SIMPLE AND PRESET SINGLE-DEGREE-OF-FREEDOM MASS-SPRING SYSTEMS

A-1. The response curves presented in this report were computed with the Environment Simulation Division's General Purpose Analog Computer. For convenience, the computer is set up to perform mathematical computations and to analyze the response of undamped structures. Analysis of complex shock, where damping is generally a factor, is done on a Special Purpose Analog Computer. Spectra presented in figure 11 were run with two percent damping to account for a slight amount of friction between the copper ball and the inertia weight during the crushing process.

A-2. Figures A-1 through A-4 are response curves for simple mass-spring systems; figures A-5 through A-8 are response curves for preset mass-spring systems. Admittedly, the idealized input pulses used in the computations are seldom duplicated in practice, but they are fairly representative of most simple shock pulses. The pulse illustrated in figure A-4 is one generally identified with free-fall drop testers.

A-3. Response curves were obtained by programming the analog computer to simulate a family of single-degree-of-freedom mass-spring systems. Each system was subjected to the desired idealized shock pulse. The pulses illustrated in the response curves were produced by a function generator. The maximum response for each oscillator was plotted vs the ratio of oscillator period to pulse duration.

A-4. The preset systems were simulated using the same oscillators. The percent of preset was chopped from the input pulse and only the portion of the acceleration pulse above the preset value was applied to the analog accelerometer. The response (output of analog accelerometer) was then summed with a voltage set to represent the preset value. To check the validity of this method, several points for the half-sine input were calculated and compared with the computer solution. Preset values were from 10% to 90%.
Fig. A-1. Response of Mass-Spring System to Half-Sine Pulse

Fig. A-2. Response of Mass-Spring System to Triangular Pulse
Fig. A-3. Response of Mass-Spring System to Sawtooth Pulse

Fig. A-4. Response of Mass-Spring System to Versed-Sine Pulse
Fig. A-5. Response of Preset Mass-Spring System to Sawtooth Pulse
Fig. A-6. Response of Preset Mass-Spring System to Half-Sine Pulse
Fig. A-7. Response of Preset Mass-Spring System to Triangular Pulse
Fig. A-8. Response of Preset Mass-Spring System to Terminal Sawtooth Pulse
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Appendix B

ANALYSIS OF THE RESPONSE OF THE COPPER-BALL ACCELEROMETER TO TWO-PHASE SHOCK

B-1. Nomenclature.

\( \Delta V \) - Velocity change (fps)
\( \omega_n \) - Circular natural frequency (rad/sec)
\( d \) - Copper-ball deformation (inches)
\( f_n \) - Natural frequency (cps)
\( x \) - Instantaneous deflection (ft)
\( a(t) \) - Input acceleration (ft/sec\(^2\))
\( x_{\text{max}} \) - Maximum deflection (ft)
\( V_0 \) - Velocity change of first phase (fps)
\( A \) - Acceleration of second phase (ft/sec\(^2\))
\( p \) - Laplace operator
\( \delta \) - Copper-ball deformation (ft)

B-2. Measurement of the impact velocity change of water-entry shock has long been a subject of prime interest in the design of weapons at NOL. The copper-ball accelerometer has been used almost exclusively to measure these shocks. As indicated in paragraph 11 of the report, the device behaves as a velocity meter when exposed to impacts of duration less than one-fifth the natural period of the accelerometer. For simple impacts, velocity change can be determined from copper ball deformation by using the formula

\[
\Delta V = \omega_n d = \frac{\pi f_n d}{6} \text{ fps}
\]  

However, for shocks where the first phase (the impact phase) is followed without time lag by a second shock (drag phase) of significant amplitude and duration, the motion of the mass no longer follows the simple equation (1) relationship.

B-3. Since in any case the copper-ball accelerometer has very little damping and is a peak indicating, mass-spring system, its response to shock can be determined from the general equation

\[
\dddot{x} + \omega_n^2 x = a(t)
\]
From this equation the copper ball deformation, $x_{\text{max}}$, can be calculated. The type of two-phase pulse under consideration is shown in figure A-1.

Fig. B-1. Idealized Water-Entry Shock Pulse

Writing the Laplace transform of equation (2) with the above shock pulse as the driving function, we have

$$\overline{x} = \frac{A}{p(p^2 + w_n^2)} - \frac{V_o}{p^2 + w_n^2}$$  \hspace{1cm} (3)

The solution of equation (3) is

$$x = \frac{A}{w_n^2} [1 - \cos w_n t] + \frac{V_o}{w_n} \sin w_n t$$  \hspace{1cm} (4)

The copper-ball deformation is $x_{\text{max}}$. Deformation is obtained by differentiating equation (4) with respect to time and setting the resulting expression equal to zero.

$$\dot{x} = \frac{A}{w_n^2} \sin w_n t + V_o \cos w_n t = 0$$  \hspace{1cm} (5)

from which we find that

$$\tan w_n t = - \frac{V_o w_n}{A}$$  \hspace{1cm} (6)

Calculations show that the maximum will occur in the second quadrant.
From this expression we find that

\[ \sin\omega_n t = \frac{V_0\omega_n}{\sqrt{A^2 + V_0^2\omega_n^2}} \]  
(7)

\[ \cos\omega_n t = \frac{-A}{\sqrt{A^2 + V_0^2\omega_n^2}} \]  
(8)

Substituting equations (7) and (8) into equation (4), we find that

\[ x_{\text{max}} = \delta = \frac{A + \sqrt{A^2 + V_0^2\omega_n^2}}{\omega_n^2} \]  
(9)

Solving for \( V_0 \) we have

\[ V_0 = \sqrt{\omega_n^2\delta^2 - 2A\delta} \]  
(10)

B-5. Equation (10) indicates that when the drag acceleration is known, the actual velocity change for the first phase of shock motion can be calculated. Curves for the
Mod 2* (80.4-gram, 852 cps) accelerometer, see Table 1, are presented in figure B-2. It is obvious from the above analysis that very significant errors can be made in the measurement of velocity change if the effect of drag is not considered.

* Because of its range and compactness, the Mod 2 accelerometer has been standardized for monitoring simulated water-entry shock in the laboratory. The same inertia weight is used in the Mod 5 accelerometer to measure water-entry shock in the field.
Fig. B-2. Response of Mod 2 (80.4-gram) Copper Ball Accelerometer to Water-Entry Shock
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Three types of peak-reading copper-ball accelerometers are described. The mechanical devices measure peak shocks from approximately 30 g to 450,000 g; their corresponding natural frequency is from 244 cps to 14,500 cps. Their operation as accelerometers, as well as velocity meters, is discussed. Theoretical dynamic response curves are included to assist in analysis and interpretation of shock measurements, and information on the use of multi-mass accelerometers to determine the peak and spectra of high-g shocks is presented.
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Title: NOL Copper-Ball Accelerometers
Approved by Commander, U.S. NOL

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1. Figure 8, page 15 - insert "Inertia Weight Data" table shown below under NOTES: (upper right corner of figure).

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