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ANALYTIC STUDY OF AIRCRAFT AGILITY
IN THE TURNAROUND MANEUVER

By
Clifton G. Wrestler, Jr.

September 1965

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA
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RESEARCH TECHNICAL MEMORANDUM 38

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ABSTRACT

Calculations of the minimum distance and time required for an aircraft to return to a point without changes in altitude are presented in this report for the initial velocities of 50, 100, 150, and 200 knots. The aircraft was allowed to decelerate at various predetermined values while maintaining a constant normal load factor. When the minimum distance was determined, the time corresponding to minimum distance was examined to see if it was close to the minimum time computed. No allowance was made for pilot or aircraft response time, and the physical limitations of the aircraft were not explored.
CONTENTS

ABSTRACT ........................................ iii

LIST OF ILLUSTRATIONS .................. vi

LIST OF SYMBOLS .......................... vii

INTRODUCTION .................................. 1

DISCUSSION .................................. 2

Turnaround Maneuver .......................... 2
Theory ............................................ 3

RESULTS .................................. 10

REFERENCES .................................. 17

DISTRIBUTION .................................. 18
ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Typical Flight Path Trajectory</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Minimum Turning Distance; Initial Velocity, 50 Knots</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Time to Return to Target; Initial Velocity, 50 Knots</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>Minimum Velocity During Maneuver; Initial Velocity, 50 Knots</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>Minimum Turning Distance; Initial Velocity, 100 Knots</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>Time to Return to Target; Initial Velocity, 100 Knots</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>Minimum Velocity During Maneuver; Initial Velocity, 100 Knots</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>Minimum Turning Distance; Initial Velocity, 150 Knots</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>Time to Return to Target; Initial Velocity, 150 Knots</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>Minimum Velocity During Maneuver; Initial Velocity, 150 Knots</td>
<td>14</td>
</tr>
<tr>
<td>11</td>
<td>Minimum Turning Distance; Initial Velocity, 200 Knots</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>Time to Return to Target; Initial Velocity, 200 Knots</td>
<td>15</td>
</tr>
<tr>
<td>13</td>
<td>Minimum Velocity During Maneuver; Initial Velocity, 200 Knots</td>
<td>16</td>
</tr>
</tbody>
</table>
SYMBOLS

d( ) Derivative
D Deceleration along flight path g units
G Acceleration due to gravity feet per second²
k Acceleration along straight line path g units
m Mass pounds-second² per foot
n Normal load factor (lift/weight) g units
R Radius vector from origin to aircraft feet
s Length along curve
T Time
V Velocity feet per second
X Coordinate
Y Coordinate
Z Coordinate
θ Angle of tangent to the flight path with the X coordinate radians
ρ Radius of curvature feet
(*) Derivative with respect to time

SUBSCRIPTS

A Conditions at point A
B Conditions at point B
o Initial conditions
INTRODUCTION

The increase in airmobility within the Army has precipitated many new functions and roles for Army aircraft. The helicopter has become the workhorse of the Army; it serves as a troop transport and for logistic support, rescue, and even armed escort. The helicopter must be maneuverable and agile to perform these missions. The armed escort maneuverability and agility requirements are more demanding because of the nature of its mission. It is evident that some criteria should be established by which maneuverability and agility can be judged.

The turnaround maneuver is a means by which an aircraft, after passing over a point, turns around in the minimum time and distance and returns to that point. The intentions of the pilot determine the last phase of the maneuver. If he wants to land, he will accelerate towards the landing area, then decelerate to land. The turnaround maneuver described in this report does not include the deceleration to land but continues the acceleration to obtain a minimum time.

The maneuver brings into play the maximum capabilities of the aircraft. The aircraft must simultaneously decelerate and sustain a large normal load factor. It must then sustain a constant velocity turn until headed towards the area of interest and then accelerate in a straight line towards the point. If the distance and time required versus the entry speed are specified, the aircraft's agility and maneuvering capability are dictated in terms of acceleration, deceleration, and normal load factor.

The graphs shown in this report provide (1) a means for measuring the aircraft's agility and (2) a quick means of determining the times and distances to be specified for the various types of aircraft to arrive at the average desired capability over a given speed range.

For example, if an initial velocity of 100 knots, a minimum distance of 670 feet, and a time to return to target of 13.4 seconds are specified, the aircraft must have a deceleration capability of 0.4g, a normal load factor capability of 1.8 over the speed range from 35 to 100 knots, and an acceleration capability of 0.7g. These figures are obtained from a cross plot of Figures 5 and 6 by plotting deceleration versus load factor for the specified time and distance, respectively. The intersection of the two lines gives the desired results. Figure 7 gives the lowest speed during the maneuver.
DISCUSSION

TURNAROUND MANEUVER

The maneuver begins when the aircraft is directly over the target. The aircraft is allowed to decelerate with a given constant force while maintaining a given normal load factor. At some point whose coordinates are $X_A$ and $Y_A$, the aircraft stops decelerating and continues with the same normal load factor in a constant radius turn until the aircraft is aligned with the target. The aircraft proceeds in level flight with a given acceleration.

Figure 1 shows a typical flight path with the notation used in the derivation of the equations of motion. The aircraft is originally approaching the target, which is located at the coordinate system from the minus X axis. Upon passing the target, the aircraft rolls to the left to establish the proper bank angle for the normal load factor specified and starts decelerating. T forces are applied instantaneously, and the aircraft responds with no delay. At point A the aircraft stops decelerating, and at point B the aircraft stops turning and starts accelerating in a straight line towards the target.

From the equations of motion, it was noted that the key point from which to determine the minimum distance was point A. Similarly, the minimum time was a function of point A and of the constant acceleration to the target. The minimum distance point and the minimum time point are not necessarily at the same velocity.
There are three basic portions to the flight path, and the equations of motion are different for each of these parts. In the first part, from the origin to point \( A \), the intrinsic equations of motion (that is, the equations of motion tangent and normal to the flight path) are as follows:

\[ m\dot{V} = -Dmg, \quad (1) \]
\[ \frac{mV^2}{\rho} = mg\sqrt{n^2 - 1}, \quad (2) \]
and
\[ m\dot{\rho} = 0, \quad (3) \]

where \( \rho \) is the radius of curvature. Equation (1) is the tangential equation of motion. Equation (2) is the normal equation of motion. Equation (3) merely states that the maneuver is performed at constant altitude. The radius of curvature is described by

\[ \rho = \frac{ds}{d\theta} = \frac{ds}{dt} \frac{dt}{d\theta} = \frac{V}{d\theta}, \quad (4) \]

where \( \theta \) is the angle that the tangent of flight path makes with the \( X \) axis. Equations (1) and (2) can be written without the mass terms, since the mass is considered to be a constant:

\[ \dot{V} = -Dg \quad (5) \]
and
\[ \frac{V^2}{\rho} = g\sqrt{n^2 - 1}. \quad (6) \]

If equation (4) is used in equation (6), it can be written

\[ \frac{V^2}{V \frac{dt}{d\theta}} = g\sqrt{n^2 - 1} \quad (7) \]
or

\[ V\dot{\theta} = g\sqrt{n^2 - 1}. \quad (7A) \]

Taking the time dependency from equations (5) and (7A) leads to the following differential equation:

\[ \frac{dV}{V} = -\frac{Dd\theta}{\sqrt{n^2 - 1}}, \quad (8) \]
which has the following solution:

\[ \ln(V) = \frac{-\theta D}{\sqrt{n^2 - 1}} + C, \]  \hspace{1cm} (9)

where \( \frac{D}{\sqrt{n^2 - 1}} \) is a constant and \( C \) is a constant of integration that can be determined from the initial conditions. If the initial conditions

\[ V = V_0 \text{ and } \theta = 0 \]

are applied to equation (9),

\[ \ln\left(\frac{V}{V_0}\right) = \frac{-D\theta}{\sqrt{n^2 - 1}} \]  \hspace{1cm} (10)

or

\[ V = V_0 e^{\frac{-D\theta}{\sqrt{n^2 - 1}}}. \]  \hspace{1cm} (10A)

In order to solve for the time required to perform the first portion of the maneuver, either equation (5) or equation (7) must be used. Since equation (10A) can readily be used in equation (7A) to give a differential equation in time and velocity that can be readily solved, equation (7A) becomes

\[ dt = \frac{V_0}{g\sqrt{n^2 - 1}} e^{\frac{-D\theta}{\sqrt{n^2 - 1}}} d\theta, \]  \hspace{1cm} (11)

which can be integrated to yield

\[ t = \frac{-V_0}{gd} \frac{V}{V_0} + C, \]  \hspace{1cm} (12)

where \( C \) is a constant of integration that can be determined by use of the initial conditions

\[ t = 0, \ V = V_0. \]

Thus, equation (12) can be simplified to read

\[ t = V_0 \left(1 - \frac{V}{V_0}\right) \frac{1}{Dg}. \]  \hspace{1cm} (13)
Thus far, the equations have been the intrinsic equations that are independent of the coordinate system used. In order to describe the path and the velocity along the path of the aircraft, an X-Y coordinate system was chosen with the origin at the target and the initial flight path along the X axis. The equations of motion are as follows:

\[ \dot{X} = \frac{dX}{ds} \frac{ds}{dt} \quad (14) \]

and

\[ \dot{Y} = \frac{dY}{ds} \frac{ds}{dt} \quad (15) \]

where

\[ \frac{ds}{dt} = V, \quad (16) \]

\[ \frac{dX}{ds} = \cos \theta, \quad (17) \]

and

\[ \frac{dY}{ds} = \sin \theta. \quad (18) \]

Therefore, equations (14) and (15) can be written as

\[ \dot{X} = V \cos \theta, \quad (19) \]

and

\[ \dot{Y} = V \sin \theta. \quad (20) \]

Equations (19) and (20) are differential equations which are best solved if the time variable is eliminated. If equation (7A) is used, equations (19) and (20) become differential equations in \( \theta \) instead of in time; thus,

\[ dX = \frac{V^2}{g\sqrt{n^2} - 1} \cos \theta \, d\theta, \quad (21) \]

and

\[ dY = \frac{V^2}{g\sqrt{n^2} - 1} \sin \theta \, d\theta. \quad (22) \]
However, $V$ is a function of $\theta$, and it is described by equation (10A). Substantiating equation (10A) in equations (21) and (22) yields the following integral equations:

$$X_A = \int_0^{\theta_A} \frac{V_0^2 e^{-\frac{2D\theta}{\sqrt{n^2 - 1}}}}{g \sqrt{n^2 - 1}} \cos \theta \, d\theta,$$

and

$$Y_A = \int_0^{\theta_A} \frac{V_0^2 e^{-\frac{2D\theta}{\sqrt{n^2 - 1}}}}{g \sqrt{n^2 - 1}} \sin \theta \, d\theta.$$

Equations (23) and (24) can be readily integrated to yield the following solutions:

$$X_A = \frac{V_0 \sqrt{n^2 - 1}}{g (4D^2 + n^2 - 1)} \left[ \left( \frac{V_A}{V_0} \right)^2 \left( \sin \theta_A - \frac{2D}{\sqrt{n^2 - 1}} \cos \theta_A \right) - \frac{2D}{\sqrt{n^2 - 1}} \right],$$

and

$$Y_A = \frac{V_0 \sqrt{n^2 - 1}}{g (4D^2 + n^2 - 1)} \left[ \left( \frac{V_A}{V_0} \right)^2 \left( \frac{-2D}{\sqrt{n^2 - 1}} \sin \theta_A - \cos \theta_A \right) + 1 \right].$$

Equations (25) and (26) describe the position of the aircraft, while equation (13) gives the time for the first portion of the flight path. The flight path from point $A$ to point $B$ is a constant speed maneuver. The intrinsic equations of motion are as follows:

$$m\ddot{V} = 0,$$

$$\frac{mV^2}{\rho} = mg \sqrt{n^2 - 1},$$

and

$$m\ddot{z} = 0.$$
Equation (29) is identical to equation (3), as is expected in a constant altitude maneuver. The time required to reach a heading back to target is given by

$$t = \frac{V_A}{g \sqrt{n^2 - 1}} (\theta_B - \theta_A).$$  \hspace{1cm} (30)

The position in relation to the target is given by

$$X = X_A + \frac{V_A}{g \sqrt{n^2 - 1}} (\sin \theta - \sin \theta_A)$$  \hspace{1cm} (31)

and

$$Y = Y_A + \frac{V_A}{g \sqrt{n^2 - 1}} (\cos \theta_A - \cos \theta).$$  \hspace{1cm} (32)

where $X_A$ and $Y_A$ are the coordinates of the point where the aircraft stops decelerating and $\theta_A$ is the angle of the tangent to the flight path at point A.

The last portion of the flight path is the straight line acceleration from point B to the origin, 0. The aircraft accelerates from the velocity at point A to the final velocity at point 0. The intrinsic equations of motion are as follows:

$$m \ddot{V} = kmg,$$  \hspace{1cm} (33)

$$\frac{mV^2}{\rho} = 0,$$  \hspace{1cm} (34)

and

$$m \ddot{z} = 0.$$  \hspace{1cm} (35)

The item of interest is the time that the aircraft takes to return to target. The distance from point B to the origin is given by

$$R_B = \frac{1}{2} kg t^2 + V_A t.$$  \hspace{1cm} (36)
Thus, the time required to return to the target is

\[ t = \frac{-V_A \pm \sqrt{V_A^2 + 2 \text{kg} R_B}}{\text{kg}}. \]  

(37)

The velocity at the target can be obtained from equations (33) and (37):

\[ V = kgt + V_A \]  

(38)

or

\[ V = \sqrt{V_A^2 + 2 \text{kg} R_B}. \]  

(39)

The total time for the maneuver is the sum of the time for each of the three parts given by equations (13), (30), and (37), and it is expressed as one equation:

\[ t = \frac{V_o(l-V_A)}{Dg} + \frac{V_A}{g \sqrt{n^2-1}} \left( \theta_B - \theta_A \right) + \frac{-V_A + \sqrt{V_A^2 + 2 \text{kg} R_B}}{\text{kg}}. \]  

(40)

In order to minimize the time, equation (40) has to be differentiated with respect to \( V_A \) and set equal to zero; then the roots have to be extracted and tested to see whether a maximum, a minimum, or a point of inflection has been obtained. If the derivative of equation (40) is equal to zero,

\[ \frac{dt}{dV_A} = 0 = \frac{-1}{Dg} + \frac{1}{g \sqrt{n^2-1}} \left( \theta_B - \theta_A \right) + \frac{V_A}{g \sqrt{n^2-1}} \left( \frac{d\theta_B}{dV_A} - \frac{d\theta_A}{dV_A} \right) \]

\[ - \frac{1}{g} \frac{1}{g} \left( V_A^2 + 2 \text{kg} R_B \right) \left( \theta_B - \theta_A \right) + \frac{V_A + gk \frac{dR_B}{dV_A}}{g}. \]  

(41)

However, \( \theta_B \) has not been defined except as the tangent to the flight path at point B; but from geometrical considerations,

\[ \theta_B = \pi + \arctan \frac{Y}{X} + \arctan \frac{\rho_A}{R_B}, \]  

(42)

where \( X \) and \( Y \) are defined by equations (31) and (32). After equation (41) has been set equal to zero and an effort has been made to solve for \( V_A \), it is apparent that a solution for all of its roots is a very difficult task.
A value for $\frac{dt}{dV_A}$ has been generated by the computer, and a change of sign is observed as the value crosses the zero point.

There are two cases that must be considered when the radius vector from the target is considered: (1) when the constant speed turn contributes to the magnitude of the radius vector and (2) when the constant speed turn does not add to the magnitude of the radius vector. In the first case,

$$R = \sqrt{X^2 + Y^2} + \rho,$$

where $X$ and $Y$ are the coordinates of the center of the constant speed turn and are defined by equations (44) and (45):

$$X = X_A - \rho \sin \theta_A,$$  \hspace{1cm} (44)

$$Y = Y_A - \rho \cos \theta_A.$$  \hspace{1cm} (45)

The second case, $R$, is simply given by the radius vector to the flight path at point A. It is expressed by

$$R = \sqrt{X_A^2 + Y_A^2}.$$  \hspace{1cm} (46)

The derivative of $R$ with respect to $V_A$ was taken in the first case and set equal to zero. The resulting equation does not lend itself to an easily obtained closed solution. The value of the derivative was generated with the condition that if the flight path became normal to the radius vector, $R$, the solution for minimum $R$ was obtained.
RESULTS

The curves shown in Figures 2 through 13 represent a summary of the computer runs made to minimize the distance required to perform the maneuver. It can be seen that the minimum time occurred at nearly the same velocity at which the minimum distance occurred; no further effort was devoted to defining the minimum time, since the time for minimum radius was within 1/2 second of the minimum time.

The straight line acceleration back to the target was assumed to be 0.7g. This is an average value for the range of decelerations used. Even though this value of acceleration is on the high side for aircraft, a change from 0.7g will not radically change the overall time for maneuver.

Figures 2, 5, 8, and 11 show the minimum turning distance required to perform the required maneuver for the initial velocities of 50, 100, 150, and 200 knots, respectively. Similarly, Figures 3, 6, 9, and 12 show the time required to perform the minimum turning distance maneuver, and Figures 4, 7, 10, and 13 show the velocity at point A for initial velocities of 50, 100, 150, and 200 knots. The velocity at point A is the most sensitive of the three parameters. During the computer runs, provisions were made to iterate to obtain more accurate results; however, since the radius vector would not change more than a fraction of 1 percent and the time would be within 1 second, it was felt that further iterations would not be necessary.
Figure 3. Time to Return to Target: Initial Velocity, 50 Knots.

Figure 4. Minimum Velocity During Maneuver: Initial Velocity, 50 Knots.
Figure 5. Minimum Turning Distance; Initial Velocity, 100 Knots.

Figure 6. Time to Return to Target; Initial Velocity, 100 Knots.
Figure 7. Minimum Velocity During Maneuver: Initial Velocity, 100 Knots.

Figure 8. Minimum Turning Distance: Initial Velocity, 150 Knots.
Figure 9. Time to Return to Target; Initial Velocity, 150 Knots.

Figure 10. Minimum Velocity During Maneuver; Initial Velocity, 150 Knots.
Figure 11. Minimum Turning Distance: Initial Velocity, 200 Knots.

Figure 12. Time to Return to Target: Initial Velocity, 200 Knots.
Figure 13. Minimum Velocity During Maneuver:
Initial Velocity, 200 Knots.
REFERENCES


### Analytic Study of Aircraft Agility in the Turnaround Maneuver

**Author:** Wrestler, Clifton G., Jr.

**Report Date:** September 1965

**Abstract:**
Calculations of the minimum distance and time required for an aircraft to return to a point without changes in altitude are presented in this report for the initial velocities of 50, 100, 150, and 200 knots. The aircraft was allowed to decelerate at various predetermined values while maintaining a constant normal load factor. When the minimum distance was determined, the time corresponding to minimum distance was examined to see if it was close to the minimum time computed. No allowance was made for pilot or aircraft response time, and the physical limitations of the aircraft were not explored.
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