STUDY OF CROSSED-FIELD AMPLIFIERS

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The aim of this project is to design and test a shielded-gun, crossed-field amplifier. As part of the project, a design procedure for crossed-field guns is being developed.

Errors in the assembly of the crossed-field electron gun were corrected, and dimensions of the gun were checked against the design. The tube is now on the pump and will be ready for tests in a few days. Minor modifications have been made in the fabrication of the tube. Unlike the old tube, the cathode is not electrically tied to the beam-forming electrodes or the Pierce plates. This enables small assembly errors to be counteracted by slight adjustments of the electrode potentials. The beam collection scheme has been changed. The beam now bends slightly upward in the collector region instead of bending sharply downward as in the old design. This results in a slightly longer rectilinear portion of the beam along which the beam thickness is constant. The effect of using glass side-plates with transparent conducting coating has been evaluated and was found satisfactory. The electrodes
have been extended on the left-hand side of the gun. This reduces the effect of the open left-hand end of the gun on the electric fields near the cathode.

Meanwhile, the computer programs for the design of the electron beam in the shielded-gun crossed-field tube are being checked. It is hoped that some problems with the computer programs will be resolved in a few days. The design of the shielded-gun is expected to be ready during the next report period.

NOISE-Figure Studies on Forward-Wave Crossed-Field Amplifiers

Prof. T. Van Duzer
A. Sasaki

The aim of this work is to understand the noise characteristics of forward-wave crossed-field amplifiers so that appreciable noise-figure reductions can be made. The normal mode approach will be used in the study of noise transducing schemes.

The coupled-mode equations in which space-charge effects are taken into account have been derived previously. These equations were applied to interpret the interaction of the circuit wave with beam waves, from the viewpoint of ac power flow. As a further application of the coupled-mode equations, the noise figures of crossed-field amplifiers were derived in terms of normal mode amplitudes of beam waves at the entrance to an interaction region. We considered four types of crossed-field amplifiers which result from the interaction of the forward-circuit wave with the slow cyclotron wave, the backward-circuit wave with the slow cyclotron wave, the forward-circuit wave with space-charge waves (forward-wave amplifier), and the backward-circuit wave with space-charge waves (backward-wave amplifier). The procedure for finding noise figures is as follows: for example, in a forward-wave amplifier, the space-charge waves synchronize with the forward-circuit wave and
the participation of cyclotron waves in the amplification is neglected. Therefore, the noise output power $N_0$ is derived with only three coupled-mode equations (for one circuit wave and two space-charge waves) and using the boundary conditions at $z = 0$ (entrance to the interaction region).

Normal mode amplitude of forward-circuit wave $A_{o+} = \sqrt{kT}a_{o+}$

- growing space-charge wave $A_g = a_g$
- decaying space-charge wave $A_d = a_d$.

Zero correlation between $a_{o+}$ and $a_g$, $a_d$ is assumed. Here $k$ is Boltzmann's constant, $T$ is room temperature in degrees Kelvin, and $a_g$ and $a_d$ are normal mode amplitudes determined by the noise fluctuation at the cathode. The noise figure can be defined by

$$F = \frac{N_0}{GkT}$$

where $G$ is the gain of the amplifier. Applying this procedure to other types of amplifiers, we obtained the noise figure expressions.

Slow-cyclotron wave amplifiers:

$$F = 1 + \frac{|a_s|^2}{kT} \left( 1 - \frac{1}{G} \right),$$

where $G$ is the gain of the amplifier,

$$G = \cosh \left[ \frac{(\alpha - 1)\beta eDL}{2\sqrt{\alpha}} \right]$$ for forward-circuit wave

$$G = \frac{1}{\cos \left[ \frac{(\alpha - 1)\beta eDL}{2\sqrt{\alpha}} \right]}$$ for backward-circuit wave.
Space-charge wave amplifiers:

\[ F = 1 + \frac{1}{2kT} \left| Pa_g - jQa_d \right|^2 \left( 1 - \frac{1}{G} \right) \]

where

\[ P = \sqrt{1 + S_G^2 + S_G'} \quad Q = \sqrt{1 + S_G^2 - S_G'} \quad S_G^2 = S \left[ \frac{G - 1}{G + 1} \right] \]

and

\[ G = \frac{\cosh \beta_e D \sqrt{S^2 + 1} \ L + S^2}{S^2 + 1} \quad \text{for a forward-wave amplifier} \]

\[ = \frac{S^2 - 1}{S^2 - \cosh \beta_e D \sqrt{S^2 - 1} \ L} \quad \text{for } S > 1 \]

\[ = \frac{1 - S^2}{\cos \beta_e D \sqrt{1 - S^2} \ L - S^2} \quad \text{for } S < 1 \]

where the notations follow those in the last quarterly report. It can be seen in these expressions that the same noise figures are obtained for the forward- and the backward-circuit wave amplifiers, provided the operating conditions, gain \( G \), and space-charge parameter \( S \), are the same. If we could find the conditions which give the minimum \( |a_g| \) and \( |Pa_g - jQa_d|^2 \), the minimum noise figures would be determined. In the process of finding the noise figures, the fulfillment of power conservation was checked. This showed that the input power (the sum of input beam noise and \( kT \)) equals the output power (the sum of output beam noise and \( N_o \)).

In continuation of this work, the criteria for reducing beam noise (the choice of the operating conditions and the location and voltage ratio of the velocity jump) will be studied using the noise behavior in a drift region.
BACKWARD-WAVE NOISE-Figure STUDIES

Prof. T. Van Duzer
N. R. Mantena

Three phases of this project were undertaken during this period:

1. The experimental tube which included the modified long gun is in the shop. It may be recalled that the modified long gun design is such that its diocotron gain is much less than that of the conventional long gun of the Kino type. The experimental results will be used to determine the noise quantities by the matrix inversion scheme proposed in the previous reports. It is expected that this experiment will be completed during the next quarter.

2. Computer calculations on the matrix inversion scheme to determine the noise quantities yielded inconsistent results. For example, even though the computer calculations were cross-checked and were correct, the magnitude square of the surface charge density fluctuations is shown to be negative. Causes such as the possible invalidity of the matrix inversion scheme and algebraic errors in the analysis are being investigated.

3. The noise-figure expression derived by Van Duzer has included a few assumptions and specializations in order to make it tractable for the parametric dependence. The various assumptions were that: a) cold circuit loss is zero, b) the cyclotron waves do not contribute to the total electric field at the circuit, and c) interaction between the circuit wave and the slow cyclotron wave is negligible. A severe restriction on the analysis is that it is applicable only to a centered beam case. By completely removing these restrictions, a very general analysis was made to determine the noise quantities for interaction between the circuit wave and the space charge waves. This can be written as
where "t" refers to the total quantity, and all the quantities are to be evaluated at the circuit plane. $E_{1z}$ is the longitudinal component of the circuit electric field; $\sigma_{1t}$, $v_{1t}$, $v_{1yt}$, and $v_{1z}$ are, respectively, the fluctuations in the surface charge density, the transverse beam position, and the two velocity components at the entrance plane of the interaction region. $E_{i | i=1 \text{ to } 5}$ are the amplitudes of the circuit wave, the two space charge waves, and the cyclotron waves. The coefficients $a_{ij}$ of the matrix are dependent on Gould's parameters $g$, $d$, $D$, $S$ and the incremental propagation constants $\delta_i$. For the circuit wave and space charge waves,

$$\beta_i = \frac{\omega}{u_o} (1 + jD\delta_i), \quad i = 1 \text{ to } 3$$  \hspace{1cm} (2)

and for the cyclotron waves,

$$\beta_4 = \frac{(\omega + \omega_c)}{u_o} (1 + jD\delta_4)$$ \hspace{1cm} (3)

$$\beta_5 = \frac{(\omega + \omega_c)}{u_o} (1 + jD\delta_5).$$ \hspace{1cm} (4)

$\delta_1$, $\delta_2$, and $\delta_3$ are determined from the determinantal equation.

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\[(\delta + j b \pm d)(\delta^2 + j \beta g S \delta - S^2) = \pm \delta \tag{5}\]

where the superscripts and subscripts refer to the forward and backward-wave interactions, respectively; and \(b\) and \(d\) are the velocity slip and the cold circuit loss parameters, respectively. For the particular case of space charge wave-circuit wave interaction considered here, \(\delta_4\) and \(\delta_5\) have been determined to be

\[
\delta_4, 5 = \pm j \frac{S}{\sinh((\beta_e \pm \beta_m)(a + d))} e^{\pm (\beta_e \pm \beta_m)(a - d)} \tag{6}\]

where \(\beta_e = \frac{\omega_m}{u_o}\), \(\beta_m = \frac{\omega_c}{u_o}\), and \(a\), \(d\) are the distances of the beam from the sole and circuit planes, respectively. In deriving these, the usual assumptions were made, such as \(D \ll 1\), \(D\delta \ll 1\), and \(DS \ll 1\).

Using (1), noise-figure expressions for the forward and backward wave interactions are being derived and will be solved to make a detailed theoretical comparison with our experimental results. It is believed that the inclusion of cyclotron waves and the removal of restrictions may account for the 10 dB difference between the theoretical and experimental noise figures previously observed.

**CATHODE-REGION STUDIES ON CROSSED-FIELD TUBES**

Prof. T. Van Duzer

R. Y. C. Ho

The aim of this work is to study the effect of a crossed magnetic field on potential minimum stability. The procedure is to calculate the shot noise smoothing factor for a wide range of values of the crossed magnetic field.

The first part of this work has been completed. It is concluded that the space-charge feedback model based on dc analysis with fixed trajectories, sheet beam flow for each current generator at the cathode,
and parabolic potential distribution near the cathode, shows no possibility of instability caused by space-charge feedback. A technical report covering these studies is being prepared.

It is believed that, although the model we used suggests no possibility of instability, there is hope of making a convincing explanation of the noise phenomena in crossed-field guns after including other factors when calculating coefficients for the space-charge feedback matrices. In continuation of this work, we will study the potential distribution near the cathode where a large crossed magnetic field is present. The transverse discreteness error will be reduced by using more beams in each segment of cathode. Initial velocities will be put into the electron streams.

CHARACTERISTICS OF THE SMOOTH-BORE MAGNETRON

Prof. C. Süskind
K. Mouthaan

The objective of this research is to obtain a theoretical description of the characteristics of the smooth-bore magnetron. The theoretical expressions for the space-charge density, electric field, and anode current were given previously. The theoretical results were compared with available experimental data. A graphical comparison of the theoretical and experimental results is presented in the following.

Figure 1 shows the theoretical space-charge distribution in the interaction space of the smooth-bore magnetron. Figures 2 and 3 present experimental curves obtained by Mathias. The results (not given here) of the more extensive investigation of Nedčerman show that the experimental space-charge density is indeed independent of the strength of the magnetic field.

Figure 4 gives the theoretical distribution of the electric field in the interaction space. Figure 5 is the experimental curve obtained by Reverdin for the case of space-charge limited operation.
Figures 6 and 7 show the comparison of the theoretical anode current and the experimental anode current, obtained by Hartman. Figure 6 is a comparison for the cathode-anode spacing of 11/32 in.; Figure 7 compares the cathode-anode spacing of 15/16 in.

As mentioned previously, the theoretical and experimental results agree considerably better than could be expected in view of the experimental shortcomings. The statistical theory of the motion of the electrons in the interaction space of the smooth-bore magnetron is thus supported by experimental evidence, and it may be concluded that the new theory is a valid description of crossed-field electron motion.

The results obtained for the characteristics of the smooth-bore magnetron may be particularly useful for design purposes. The inter-relation between such quantities as anode voltage, magnetic field, cathode-anode spacing, and beam velocity, beam charge, anode current, and circulating current is explicitly given by the theoretical expressions so that the amount of empirical work that usually goes into the design of magnetrons can be considerably reduced.

References

Fig. 1. Theoretical space-charge distribution.
Fig. 2. Experimental space-charge distribution (Mathias).
Fig. 4. Theoretical distribution of electric field.

Fig. 5. Experimental distribution of electric field (Reverdin\textsuperscript{3}).
Fig. 6. Comparison of theoretical and experimental (Hartmann$^4$) anode current; $d = 11/32$ in.
Fig. 7. Comparison of theoretical and experimental (Hartman) anode current; d = 15/16 in.
A new crossed-field gun combining the characteristics of a Kino gun and Brillouin flow was designed and constructed. An excellent beam shape and more than 98% beam transmission were obtained.

A matrix inversion method was used to calculate cathode noise quantities for several types of crossed-field guns. Restrictive assumptions under which a noise-figure expression had been previously derived were removed and a more generally applicable expression was derived.

Noise-figure expressions are being used to study velocity jumps as noise reducing transducers. Optimums operating conditions for minimum noise are also being investigated.

The first phase of study on crossed-field potential minimum was completed. The assumptions and the space-charge model on which this initial phase was based did not lead to any instability in the crossed-field beam. A more realistic physical model will be assumed and is expected to lead to more useful information on noise phenomena.

A new diffusion theory of electron motion in crossed fields was completed. Expressions for the static characteristics of the magnetron were derived and compared with experimental results. The validity of the theory is supported by the close agreement observed between experiment and theory.
Crossed-Field Amplifier
Crossed-Field Guns
Noise-Figure
Potential-Minimum
Smooth-Bore-Magnetron

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