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On the Radiative Corrections to the Muon Polarization from Pion Decay (*).

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Summary. — It is noted that the polarization of muons (or electrons) from pion (or kaon) decay, $\pi(K) \to \mu(e) + \nu$, is not complete when radiative corrections are taken into account. It is shown that the correction is very small.

It is usually stated that, as a consequence of the two-component neutrino theory, the muons from pion decay,

(1) $$\pi \to \mu + \nu ,$$

are completely polarized. The pion is spinless and in its rest system the decay products travel in opposite directions. From conservation of angular momentum it follows that complete polarization for the neutrino implies complete polarization for the muon. Present experiments [1-3] are in excellent agreement with this conclusion.

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(1) M. BARDON, P. FRANZINI and J. LEE: *Phys. Rev. Lett.*, 7, 23 (1961).
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It is interesting to note, however, that when radiative corrections to (1) are included the polarization of the muons is no longer complete. The radiative corrections are of two types: *a*) vertex and self-energy corrections, which involve only virtual photons and which add coherently to the lowest order amplitude for (1), and *b*) inner bremsstrahlung, involving the emission of real photons, which adds incoherently to (1). Both types are separately infra-red divergent, but in any physical situation, corresponding to a definite muon energy resolution, this divergence cancels in the usual way.

The argument that muon polarization must be complete still holds in the presence of all diagrams of type *a*). The radiative corrections to the muon polarization are thus due only to the emission of real radiation. This means that the infra-red divergent terms, which dominate soft photon emission, must also give completely polarized muons. Consequently, the radiative correction to the polarization of muons from pion decay can be expected to be very small.

We have recently calculated the transition rates for radiative kaon and pion decay in another context ([4]). We wish here to report the explicit results, which may be obtained from such a calculation, for the lowest-order radiative correction to the polarization of the emitted muons (or electrons) ([5]). It will be sufficient to consider the inner bremsstrahlung resulting from the gauge-invariant phenomenological coupling

$$\mathscr{L}_{\text{eff}} \sim -f[(\partial_\alpha - ieA_\alpha)\pi][\bar{\mu}\gamma_\alpha \tfrac{1}{2}(1+i\gamma_5)\nu]\,. \tag{2}$$

An explicit calculation, for example, of the effects of an intermediate vector boson indicate that they are negligible for low-energy photons ([6]).

As we have indicated, the Feynman diagrams which contribute to the radiative correction, up to first order in the fine structure constant α, can be separated into those involving only virtual photons (including the lowest order diagram) and those involving real photons, as in Fig. 1. The transition probability is the incoherent sum of the virtual and real photon parts

$$\Gamma = \Gamma_v + \Gamma_r\,. \tag{3}$$

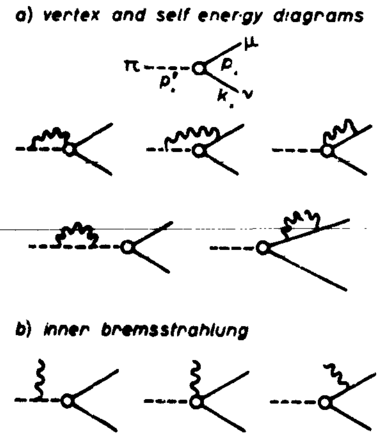

Fig. 1. – Lowest-order Feynman diagrams for radiative corrections to (1).

([4]) E. S. GINSBERG and R. H. PRATT: *Phys. Rev.*, **130**, 2105 (1963).
([5]) E. S. GINSBERG and R. H. PRATT: *Bull. Am. Phys. Soc.*, **7**, 502 (1962).

The muon polarization is defined to be (for definiteness, let us consider π^+-decays)

$$\mathscr{P}=\frac{\Gamma(-)-\Gamma(+)}{\Gamma(-)+\Gamma(+)}, \tag{4}$$

where $\Gamma(\pm)$ is the decay rate for muons with helicity ± 1.

The matrix element for the emission of real photons has been given elsewhere ([4]). The only difference here is that since we are interested in polarized muons we must insert a projection operator,

$$H_-=\tfrac{1}{2}(1-i\gamma_5\, n\cdot\gamma)\,, \tag{5}$$

in front of the muon Dirac spinor, where $p\cdot n=0$ and $n^2=-1$. (In the muon rest frame, H_- projects onto the spin direction antiparallel to n.)

On invariance gronds, the matrix element for the lowest-order diagram plus the vertex corrections is proportional to

$$\delta^4(p'-p-k)\bar{u}_\mu H_-\left(p'_\alpha-\frac{\alpha}{2\pi}I_\alpha\right)\gamma_\alpha\frac{1}{2}(1+i\gamma_5)u_\nu\,, \tag{6}$$

where the four-vector I_α is a sum of (perhaps divergent) integrals over the virtual photon momentum. The only four-vectors available are p'_α, p_α, and k_α, but due to the Dirac equation and energy conservation we may replace I_α by Ip_α, where I is the sum of scalar integrals. Therefore, the matrix element for the diagrams which contain no real photons is proportional to

$$\delta^4(p'-p-k)m_\mu\left(1-\frac{\alpha}{2\pi}I\right)\bar{u}_\mu H_-\frac{1}{2}(1+i\gamma_5)u_\nu\,. \tag{7}$$

(We have used the fact that $p\cdot\gamma$ commutes with H_-.) From this, we readily obtain in the decay rate (in the center-of-mass system)

$$\Gamma_v=\Gamma_0\left(1-\frac{\alpha}{\pi}I\right)\frac{1}{2}(1+\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{n}})\,, \tag{8}$$

where Γ_0 is the decay rate, in lowest order, for unpolarized muons

$$\Gamma_0=f^2m_\mu^2(m_\pi^2-m_\mu^2)^2(16\pi)^{-1}m_\pi^{-3}\,. \tag{9}$$

The unit vector $\hat{\boldsymbol{n}}$ is parallel (antiparallel) to the muon direction of motion $\hat{\boldsymbol{p}}$ for muons with helicity $-1(+1)$. Thus

$$\Gamma_v(+)=0\,,\qquad \Gamma_v(-)=\Gamma_0\left(1-\frac{\alpha}{\pi}I\right). \tag{10}$$

The muon polarization, correct up to first order terms in α, can now be written

$$\mathscr{P}=\frac{\Gamma_e(-)+\Gamma_r(-)-\Gamma_r(+)}{\Gamma_e(-)+\Gamma_r(-)+\Gamma_r(+)}\simeq 1-2\,\frac{\Gamma_r(+)}{\Gamma_e}\,. \tag{11}$$

As expected, the expression for $\Gamma_r(+)$ is not infra-red divergent (although $\Gamma_r(-)$ is) and the integrals in I cancel out of the expression for $\mathscr{P}$. We find that, integrating over all photon variables,

$$\begin{aligned}\mathscr{P}=1-\frac{\alpha}{\pi}\frac{2m_\pi^2}{(m_\pi^2-m_\mu^2)^2}\cdot\\ \cdot\int_{E_{min}}^{E_{max}}\mathrm{d}E\left\{\left[\frac{2E}{p}\ln\left(\frac{E+p}{E-p}\right)-4\right]\left[\frac{2m_\pi m_\mu^2-m_\pi^2(E-p)-m_\mu^2(E+p)}{m_\pi^2-2m_\pi E+m_\mu^2}\right]+\right.\\ \left.+\left(\frac{m_\pi^2-2m_\pi E+m_\mu^2}{m_\pi}\right)\left[1-\left(\frac{m_\pi^2(E-p)-m_\mu^2}{2m_\pi^2 p}\right)\ln\left(\frac{m_\pi(E+p)-m_\mu^2}{m_\pi(E-p)-m_\mu^2}\right)\right]\right\},\end{aligned} \tag{12}$$

where E and p refer to the energy and momentum of the muon. The integral in eq. (12) is to be interpreted as being over the accepted range of muon energies. This will be determined by experimental conditions (*i.e.*, the muon energy resolution) and in any event, is limited by kinematics to the range

$$E_{max}\equiv\frac{m_\pi^2+m_\mu^2}{2m_\pi}>E>E_{min}>m_\mu\,. \tag{13}$$

Suppose that the quantity $\Delta=(E_{max}-E_{min})E_{max}^{-1}$ is small. We may then expand the integrand in eq. (12) and retain only the lowest-order term of the integral in Δ. The result is

$$\mathscr{P}\simeq 1-\frac{\alpha\Delta^2}{4\pi}\left(\frac{m_\pi^2+m_\mu^2}{m_\pi^2-m_\mu^2}\right)^4\left[1-\left(\frac{4m_\pi^2m_\mu^2}{m_\pi^4-m_\mu^4}\right)\ln\left(\frac{m_\pi}{m_\mu}\right)\right]. \tag{14}$$

Substituting for the masses we find

$$\mathscr{P}(\pi\to\mu+\nu)\simeq 1-0.0045\Delta^2\,.$$

Similarly, the coefficient of Δ^2 for the muon polarization from kaon decay (substituting m_K for m_π in eq. (14)) is 0.00072; for the electron modes of both pion and kaon decays the coefficient of Δ^2 is very close to $\alpha/4\pi$ or 0.00058.

The maximum value of Δ, for the case $\pi\to\mu+\nu$, is obtained by setting

$E_{min} = m_\mu$,

$$\Delta \leq \frac{(m_\pi - m_\mu)^2}{m_\pi^2 + m_\mu^2} \simeq 0.041 \; . \tag{15}$$

Therefore, the expansion in powers of Δ is quite good even for an experiment with no energy resolution whatever. On the other hand, if m_π is replaced by m_K in eq. (15) we only have $\Delta < 0.59$, however an energy resolution of only 16% makes $\Delta \simeq 0.1$. For the electron modes of either pion or kaon decay we may neglect m_e compared to either m_π or m_K so that Δ is essentially the fractional energy resolution of the experiment, which should be on the order of a few percent. Thus, as expected, we conclude that the radiative correction to the polarization is small for all decays $\pi(K) \to \mu(e) + \nu$.

RIASSUNTO (*)

Si nota che la polarizzazione dei muoni (o degli elettroni) da decadimento dei pion (o dei caoni), $\pi(K) \to \mu(e) + \nu$, non è completa se si tien conto delle correzioni radiative. Si mostra che la correzione è molto piccola.

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