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TECHNICAL MEMORANDUM

DESIGN CRITERIA AND ANALYSES FOR THIN-WALLED PRESSURIZED VESSELS AND INTERSTAGE STRUCTURES

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FOREWORD

This report, based on a review of the various philosophies of design criteria currently used in the missiles and spacecraft industry, presents an approach for preliminary-design analysis of thin-walled pressurized vessels and interstage structures. It is intended primarily as an aid toward more rapid and consistent estimations of structural gages and weights.

Although the report emphasizes preliminary design usage, the material presented is not limited to preliminary design analysis.

Examples are included of use of the equations and graphs presented.

ACKNOWLEDGMENT

The author wishes to express his appreciation to R.L. Hammitt, L.L. Long, and M.L. Vaughn for helpful information which they contributed to this report.

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NOMENCLATURE

a	major elliptical radius, cylindrical radius
b	minor elliptical radius
c	refers to combined stresses
C	critical axial compressive stress coefficient $= \frac{\sigma_{cr}}{\eta E} \frac{r}{t}$
C_p	buckling stress coefficient for cones
C_Δ	buckling stress coefficient for cones

$$= p_{cr}\, r_{eq}\, L_{eq}^2/Et^3 \left(= 0.904 C_p \text{ for } \mu = 0.3\right)$$

d	diameter
d_c	diameter of cylinder = 2a
d_s	diameter of sphere
e	elliptical eccentricity $= \left[1 - \left(1/k^2\right)\right]^{1/2}$
E	parameter $= \left\{2k + 1/\left(k^2 - 1\right)^{1/2} \ln \left[k + \left(k^2 - 1\right)^{1/2}\right] / \left[k - \left(k^2 - 1\right)^{1/2}\right]\right\}$
E	Young's modulus
f	stress
f_c	stress in cylindrical portion
f_h	stress in head
f_s	stress in spherical portion
F_y	yield point allowable of material corresponding to 0.2% offset on stress-strain diagram
k	ellipse ratio = a/b
K	stress factor for a particular ellipse ratio, k
l_c	length of cylindrical portion
Δl_c	length of cylinder corresponding to minor radius
Δl_{in}	length of cylindrical portion of interstage structure

L	overall length (= l_c + 2 x minor radius of head for propellant tank)
L	length of generatrix of cone or of conical frustrum
L_{eq}	length of equivalent cylinder = $(0.75r_1 + 1.45r_2)\ L/2.2r_2$
m	refers to membrane stress only
p	working pressure
P_{cr}	critical load
p_{cr}	critical pressure
r	radius of cylinder
r_{eq}	radius of cone at base perpendicular to generatrix = $r_2/\cos\alpha$
r_1	minor radius of truncated cone
r_2	base radius of cone ($r_2 > r_1$)
R	crown radius = ka
t_c	thickness of cylindrical shell
t_{cr}	thickness of crown
t_e	equivalent thickness = $(t_h + t_{cr})/2$
t_{in}	gage of interstage structure
t_k	thickness of knuckle
t_s	thickness of spherical head
t_{s_m}	spherical head gage determined on basis of membrane stresses only
V	volume
w	density of material
w_{in}	density of interstage structure

W	weight
W_{hc}	weight of nead plus cylindrical shell
W_{s}	weight of spherical tank
ΔW_{G}	weight gain
α	cone half angle
η	plasticity coefficient
μ	Poisson's ratio
σ_{cr}	critical axial compressive stress
$\Delta\sigma_{cr}$	increase in critical axial compressive stress

SUMMARY

This report considers the problem of varied philosophies of design criteria in the missiles and spacecraft industry in relation to preliminary design work. The report offers a set of design ground-rules consistent with safety factors and materials allowable as an effective approach not only for structural preliminary design but also for general application by the structural design analyst.

Specific uses involving design of thin-walled pressurized vessels and interstage structures are presented. Criteria with respect to factors of safety and use of buckling probability curves are included for structures in which a buckling type of failure under axial compression predominates.

Formulae for determining gages for tanks and interstages are given and a set of graphs is provided which permits rapid calculation of the weights of tanks. The report also contains a graph which shows the weight penalty incurred for tanks which deviate from a spherical shape, and there is a graph presenting the volume of a tank as a function of the diameter.

Critical axial compressive stress coefficients for different philosophies of structural design are graphically illustrated and curves for allowable load versus thickness based on these philosophies are presented.

Section 1
INTRODUCTION

A survey of missile-and-spacecraft design practices has demonstrated the need to establish consistent design criteria for thin-walled pressurized vessels and interstage structures. Up to the present, attempts to achieve consistency of results from the various design criteria in use in industry have been somewhat obscured by efforts to correlate experimental results with the various strength theories. The two theories used most widely for establishing the thicknesses of the walls of pressurized structures of isotropic material are as follows:

- Maximum Stress Theory, $f = f_y$ (subscript y refers to yield)
  This theory assumes that failure is not influenced by the presence of one stress acting at right angles to the other.
- Distortion Energy Theory or Shear Energy Theory for Two-Dimensional Stress, $f_1^2 + f_2^2 - f_1 f_2 = f_y^2$
  This theory is based on Hooke's Law and, consequently, requires use of the yield stress. Further, this theory assumes that the material has the same yield point in tension and in compression.

The thickness of the walls based on equations of the maximum stress theory is about 15 percent greater than the thickness obtained from equations of the distortion energy theory. However, the maximum stress theory is more generally used. One reason is that this theory is easier to use. Also, since it is more conservative, it is more appropriate for use for pressure vessels which are subject to premature failures due to mismatched welds, or to failure caused by some other stress concentration.

Another design area where there is a great deal of inconsistency is that of monocoque structures under axial compression. Here, large discrepancies between theory and experimental results are well documented. Recent attempts to deal with these differences have been made using failure-probability theory coupled with various factors of safety.

Design criteria and methods of analyses leading to consistent results in preliminary design applications are presented in the following pages.

Section 2

THIN-WALLED PRESSURIZED STRUCTURES

2.1 CLASSIFICATION

Structures loaded by internal pressurization are classified generally in the missile and spacecraft industry as follows:

2.1.1 Propellant Tanks

Ordinarily, propellant tanks form an integral part of the basic primary structure, constitute a fairly large percentage of the total structural weight, and are moderately to highly pressurized.

2.1.2 Pressure Vessels

Generally, pressure vessels are removable, experience a great many more cycles of pressurization than do propellant tanks, and are highly pressurized.

2.2 SAFETY FACTORS CRITERIA AND ALLOWABLES

For missile and spacecraft applications each company has its own value for a safety factor. This is also true for the military services and for civilian governmental agencies. (See Appendixes A and B which show the results of a recent survey together with present LMSD policy.)

It is seen clearly from the Appendixes that any factor selected will have its adherents and its detractors.

2.2.1 Safety Factors

For thin-walled pressurized structures the following safety factors are recommended for preliminary design work. It should be noted that the ratio of ultimate to yield of the material may in some cases warrant a re-evaluation of these factors.

Propellant tanks

• That are non-hazardous to personnel or vital equipment	1.00
• That provide a special precautionary safety measure * for personnel	1.10
• That are hazardous to personnel or vital equipment	1.33

Pressure vessels

• In remotely launched missiles	1.00
• In aircraft and vehicle-launched missiles	1.33

The pressure considered in the above applications is the working pressure, defined as the maximum pressure to which the component is subjected under steady state conditions or the effect of launch or catapult loads, whichever is the more severe.

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* For example, for the booster for a manned capsule which has an ejection device with an exceptionally high degree of reliability.

2.2.2 Material Allowable

Inseparable from safety factor is the material allowable which is generally based on uniaxial-stress data. Admittedly the use of any material allowable based on uniaxial-test data is an oversimplification for multiaxial-stress systems such as pressurized tanks. The strain hardening characteristic, which is an important factor, the heat-treat cycle, and the shape of the vessel influence the maximum pressure a vessel can withstand. Depending on the value of the strain hardening parameter, for example, the burst stress as a function of the ultimate uniaxial tension stress can vary approximately plus-or-minus 15 percent. Ductile materials in the annealed state have burst-stresses which vary between 90 percent to 105 percent while cold-rolled metals range between 100 and 110 percent of the maximum. These variations in values are based on information contained in Reference 1. (See, also References 2 and 3, two recent papers which include, among other topics, further discussion of the effects of strain hardening.)

A fairly reliable and acceptable allowable for preliminary design is the yield point (0.2 percent offset) of the material for the minimum expected strength ("A" values of ANC-5/MIL-HDBK-5) in combination with whatever special factors are required for welds, or other stress risers. A weld efficiency of 85 to 90 percent is a reasonable value. If considered desirable, built-up lands at the welds can be used with little weight penalty.

Section 3
STRUCTURAL ANALYSIS METHODS

3.1 GENERAL

At a sufficient distance from the juncture of the ends with the cylindrical or conical shell (where interaction does not occur), the maximum stress in the shell due to internal pressure generally is determined on the basis of the simple hoop-membrane stress formula.

For preliminary design purposes the gages and weights of the cylindrical or conical shell will be determined using the simple hoop-membrane formula. The weights of the heads (or ends) will be determined using an equivalent thickness of the knuckle-crown arrived at by accounting for the superposition of membrane, discontinuity, and local bending stresses. Elliptical and spherical heads only are compared. It should be noted that the spherical-head equations are special cases of the elliptical.

Examples of the use of the following equations and graphs are included at the end of this report.

3.2 ELLIPTICAL AND SPHERICAL ENDS

By assuming an equivalent head-thickness which is an average value of the knuckle-and-crown thickness a fairly close weight value is obtainable for elliptical ends. (Reference 4) For completeness, some of the more general equations of the referenced article are included here.

3.2.1 Equivalent Head-Thickness for Elliptical Ends

The equivalent head-thickness is determined as follows:

(a) Knuckle thickness, t_k

$$t_k = Kpa/f \tag{1}$$

where f = stress

K = stress factor for a particular ellipse ratio, $k = a/b$. (An envelope curve for K vs k for combined membrane, discontinuity, and local bending was obtained from Reference 5.)

a = major elliptical radius (or cylindrical radius)

b = minor elliptical radius

p = pressure

(b) Crown thickness, t_{cr}

$$t_{cr} = pR/2f \tag{2}$$

where R = crown radius = ka

(c) Equivalent thickness, t_e

$$t_e = \left(t_k + t_{cr}\right)/2 = pa\left(K + k/2\right)/2f \tag{3}$$

3.2.2 Elliptical (oblate spheroid) Weight and Volume

(a) Surface area of an ellipsoid is:

$$2\,\pi a^2 + \pi b^2/e\,\ln\left[(1 + e)/(1 - e)\right] \tag{4}$$

where e = eccentricity = $\left(1 - 1/k^2\right)^{1/2}$

(b) Ellipsoidal head weight is:

$$W = \left(\pi a^3 wp/2fk\right)\left(K + k/2\right)\left\{2k + 1/\left(k^2 - 1\right)^{1/2} \ln\left[\frac{k + \left(k^2 - 1\right)^{1/2}}{k - \left(k^2 - 1\right)^{1/2}}\right]\right\}$$

$$= \pi a^3 E/2k \left(K + k/2\right) wp/f = \pi a^2 wt_e E/k \tag{5}$$

where w = density of material and

E is the term in the large brackets

$$= 2k + 1/\left(k^2 - 1\right)^{1/2} \ln\left[\frac{k + \left(k^2 - 1\right)^{1/2}}{k - \left(k^2 - 1\right)^{1/2}}\right] \tag{6}$$

E is plotted in curve of Figure 1. (Note: Reference 4 is in error for $k > 2.5$)

For just one head the weight is, of course, just 1/2 W.

(c) Volume of ellipsoid is:

$$V = 4/3\ \pi a^2\ b = 4/3\ \pi a^3/k \tag{7}$$

(d) Weight/Volume

$$W/V = 3E/8\ (K + k/2)\ wp/f = 3Ewt_e/4a \tag{8}$$

This relationship is the basis for Figure 2.

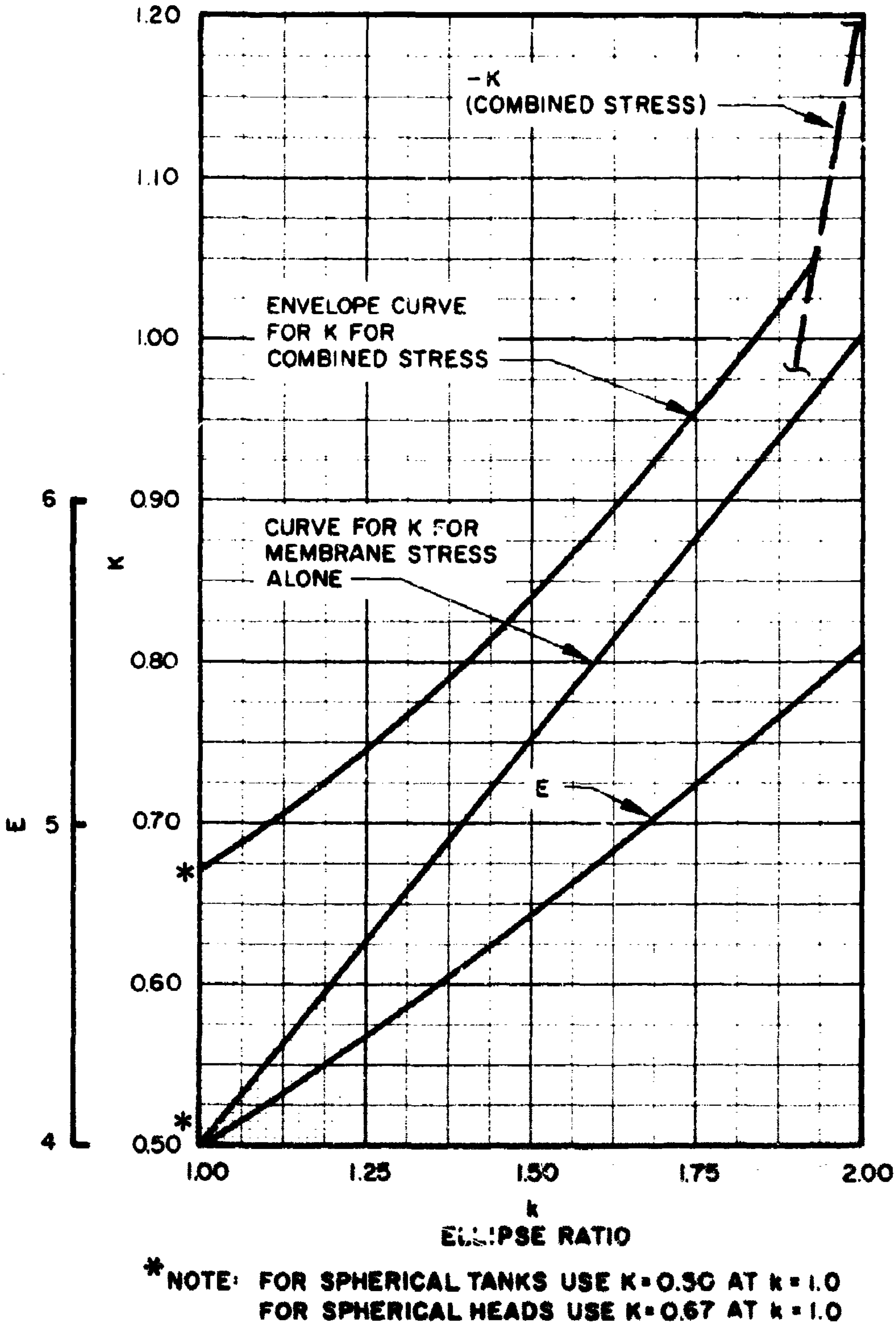


Figure 1 Ellipse Ratio vs Knuckle Stress Factor and the Parameter E

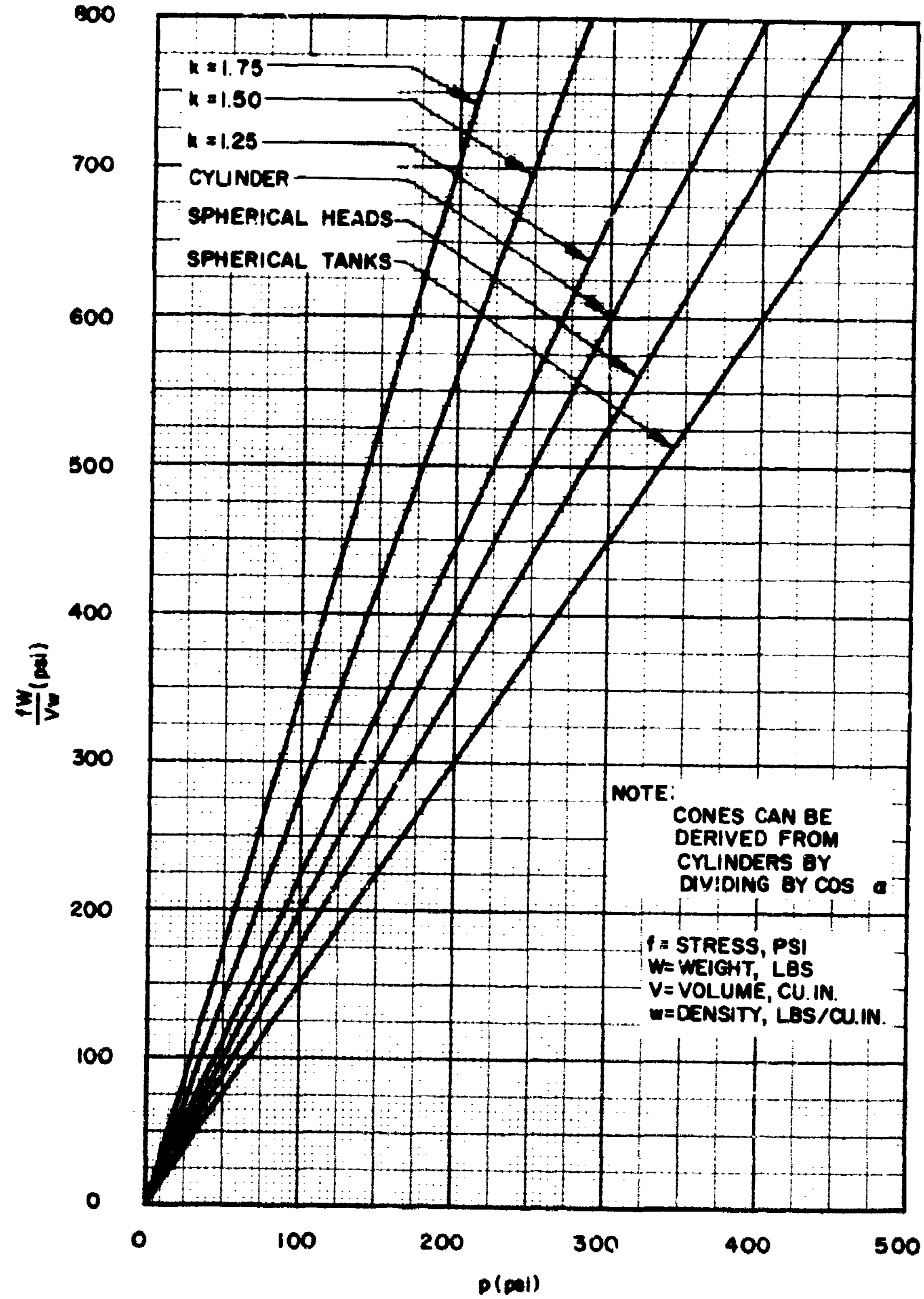


Figure 2 Parameter fW/Vw vs Pressure

3.2.3 Thickness Relationships

(a) Thickness of elliptical head vs thickness of cylindrical shell

$$t_e/t_c = (K + k/2)/2 \tag{9}$$

(b) Thickness of elliptical head vs thickness of spherical head

$$t_e/t_s = \left(2a/d_s\right)\left(K + k/2\right) \tag{10}$$

where subscript e refers to ellipse and s refers to sphere

3.2.4 Spherical Plate External Loading Critical Pressure

For the case of a spherical plate buckling under external pressure with $d^2rt > 100$ and $r/t > 1500$, $f_{cr} = 0.2Et/r$. Setting this equal to $pr(K + k/2)/2t = 0.585\ pr/t$, the buckling pressure p_{cr} is determined to be:

$$p_{cr} = 0.342\ Et^2/r^2 \tag{11}$$

3.3 CYLINDRICAL SHELLS

3.3.1 Thickness

$$t_c = pd_c/2f = pr/f \tag{12}$$

where d_c = diameter of cylinder

3.3.2 Weight

$$W = \pi d_c l_c w t_c = \pi d_c^2 l_c w p/2f \qquad (13)$$

where l_c = length of cylinder

3.3.3 Volume

$$V = \pi d_c^2 l_c/4 \qquad (14)$$

3.3.4 Weight/Volume

$$W/V = 4wt_c/d_c = 2wp/f \qquad (15)$$

This relationship is also plotted in Figure 2.

3.4 CONICAL SHELL

3.4.1 Weight/Volume

This relationship is derived from the cylindrical by dividing by the cosine of the cone half angle:

$$W/V = 2wp/f \cos \alpha \qquad (16)$$

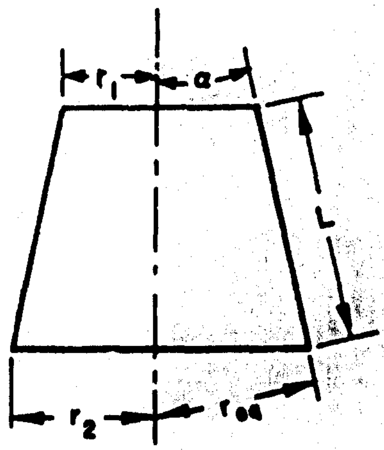
3.4.2 External Loading Critical Pressure

The critical pressure that will buckle a conical section under external loading can be determined from

$$P_{cr} = C_p \frac{\pi^2}{12\left(1 - \mu^2\right)} \frac{Et^3}{r_{eq}\, L_{eq}^2} \tag{17a}$$

$$= C_\Delta E\left(\frac{t}{r_{eq}}\right)^2 \Big/ \left(L_{eq}^2/r_{eq}t\right) \tag{17b}$$

for μ (Poisson's ratio) a constant



where L_{eq} as a function of r's and L is taken as

$$L_{eq} = \frac{0.75\, r_1 + 1.45\, r_2}{2.2 r_2}\, L \tag{18}$$

A 90 percent probability curve of $L_{eq}^2/r_{eq}t$ vs C_Δ for $\mu = 0.3$ is plotted in Figure 3. The information for determining this curve was obtained from Reference 6.

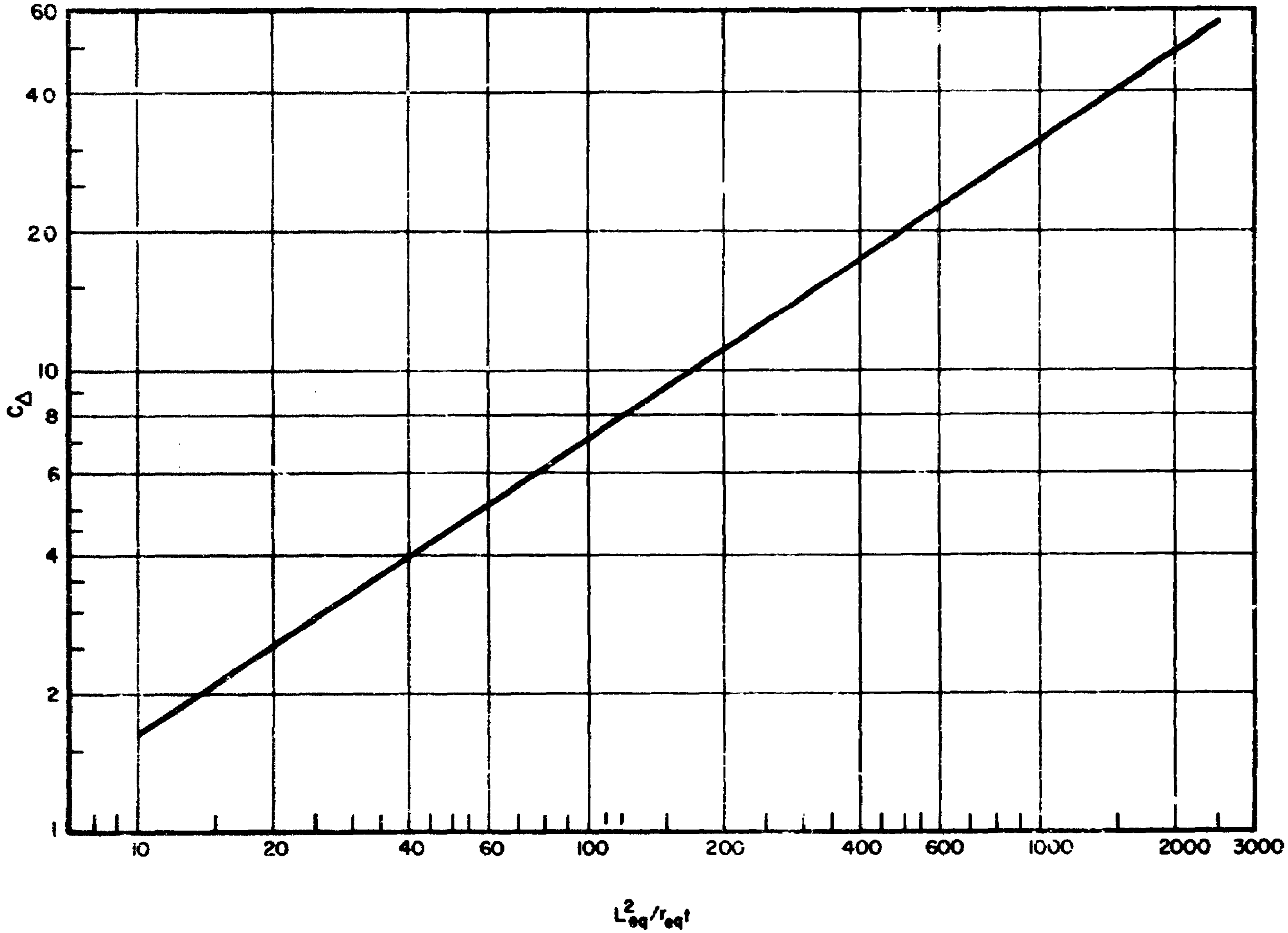


Figure 3 Buckling Stress Coefficient for Cones Under External Pressure vs Length-Radius-Thickness Parameter

3.5 RELATIONSHIP OF VOLUME AND DIAMETER AS A FUNCTION OF FINENESS RATIO, L/d

It can be shown that

$$d = \left[\left(\frac{12}{\pi}\right)\left(\frac{V}{3L/d - 1/k}\right)\right]^{1/3} \tag{19}$$

where L is the overall length. Figure 4 depicts this relationship graphically for spherically ended tanks. Table 1 which accompanies this figure gives volumetric multiplying factors for determining the volume of elliptically ended tanks with the same cylindrical diameter as the spherically ended tanks.

3.6 SPHERICAL TANKS VS CYLINDRICAL TANKS WITH SPHERICAL OR ELLIPTICAL HEADS

Weight relationship as a function of fineness ratio, L/d is determined to be:

$$\frac{W_{hc}}{W_s} = \frac{\left[3E/8 \left(K + k/2\right)wp/f\right]\left[4\pi a^3/3k\right] + \left(2wp/f\right)\left(\pi d_c{}^2 l_c/4\right)}{\left[3/2 \left(K + k/2\right)wp/f\right]\left(\pi d_s{}^3/6\right)}$$

$$= \frac{\left[a^3E/k \left(K + k/2\right)wp/f_h\right] + \left(d_c{}^2 l_c\right)\left(wp/f_c\right)}{d_s{}^3/2 \left(K + k/2\right)\left(wp/f_s\right)} \tag{20}$$

where W_{hc} = weight of head plus cylindrical shell

W_s = weight of spherical tank

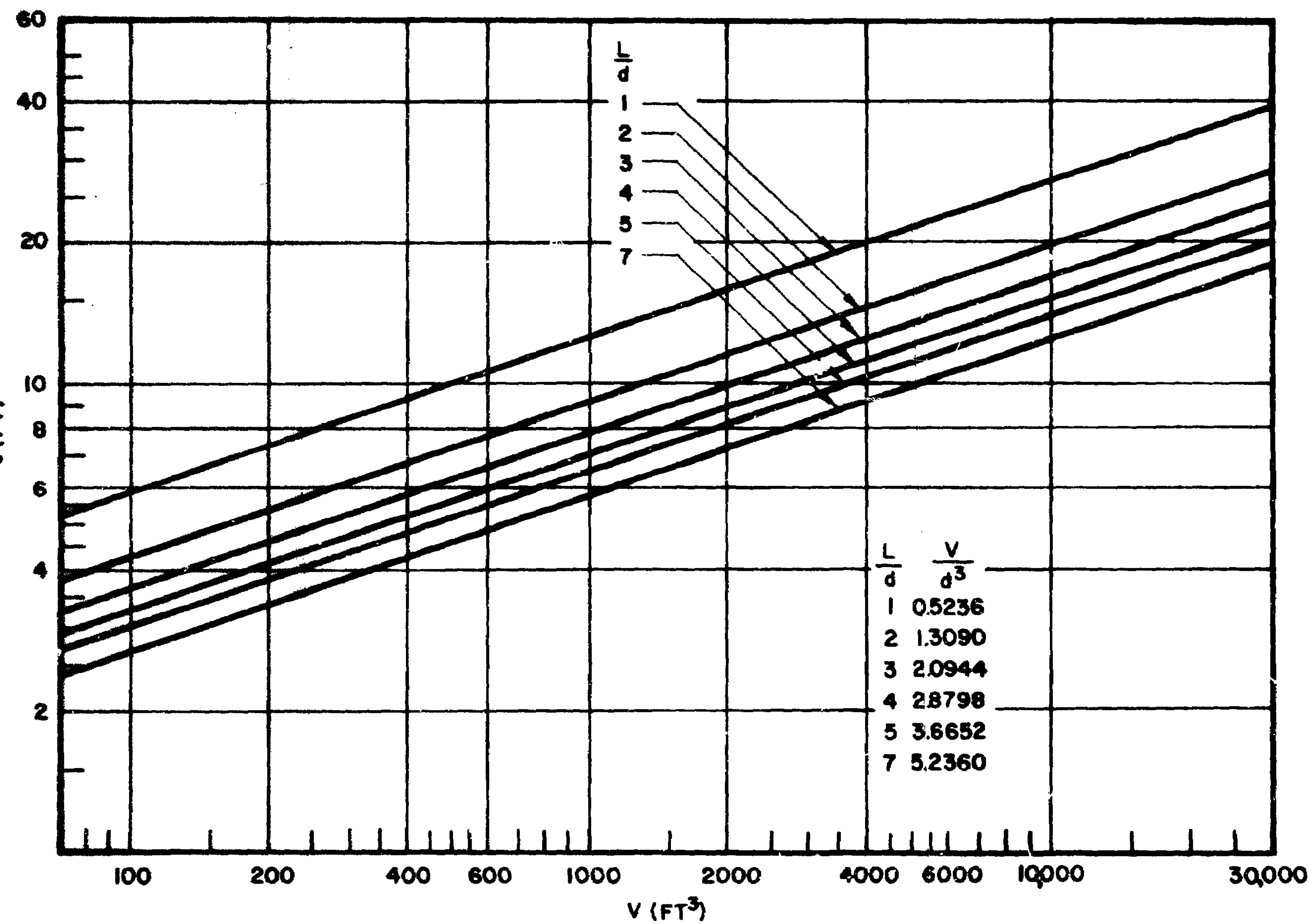


Figure 4 Volume of Spherically Ended Cylindrical Tanks vs Diameter (with Table I)

TABLE I

VOLUMETRIC MULTIPLYING FACTORS FOR ELLIPTICALLY ENDED TANKS WITH SAME CYLINDRICAL DIAMETER AS SPHERICALLY ENDED TANKS

(WITH FIGURE 4)

$\frac{L}{d}$	k	Multiply Spherically Ended Volumes by
2	1.25	1.0400
	1.50	1.0667
	1.75	1.0857
3	1.25	1.0250
	1.50	1.0417
	1.75	1.0536
4	1.25	1.0182
	1.50	1.0303
	1.75	1.0390
5	1.25	1.0143
	1.50	1.0238
	1.75	1.0306
7	1.25	1.0100
	1.50	1.0167
	1.75	1.0214

A simplification is obtained by assuming for the head and cylinder and sphere the same allowable stress, F_y and the same material

$$\frac{W_{hc}}{W_s} = \frac{{d_c}^2 E/2k + 4 d_c l_c/(K + k/2)}{2{d_s}^3/d_c (K + k/2)} \tag{21}$$

Now for equal volumes of sphere vs head plus cylinder

$$\frac{\pi {d_s}^3}{6} = 4/3\ \pi a^3/k + \pi {d_c}^2 l_c/4 \tag{22a}$$

$${d_s}^3 = {d_c}^3/k + 3/2\ {d_c}^2\ l_c = d_c\left({d_c}^2/k + 3/2\ d_c l_c\right) \tag{22b}$$

Finally equation (20) reduces to

$$\frac{W_{hc}}{W_s} = \frac{E{d_c}^2/4k + 2d_c l_c/\left(K + k/2\right)}{\left[\dfrac{{d_c}^2/k + 3/2\ d_c l_c}{K + k/2}\right]}$$

$$= \frac{E/4k + 2l_c/d_c\left(1/K + k/2\right)}{1/k + \left(3/2\ l_c/d_c\right)/\left(K + k/2\right)} \tag{23}$$

Figure 5 based on equation (23) shows the weight penalty due to using other than a spherical tank. The lower curve of each set is for the membrane while the upper is for combined stresses. In Reference 7, Figure 6.2 also shows the curve for the spherical tank where membrane stresses only were considered. The tick mark on each of the curves indicates the L/d value at which the diameter of a tank with a cylindrical body and elliptical heads is the same as the

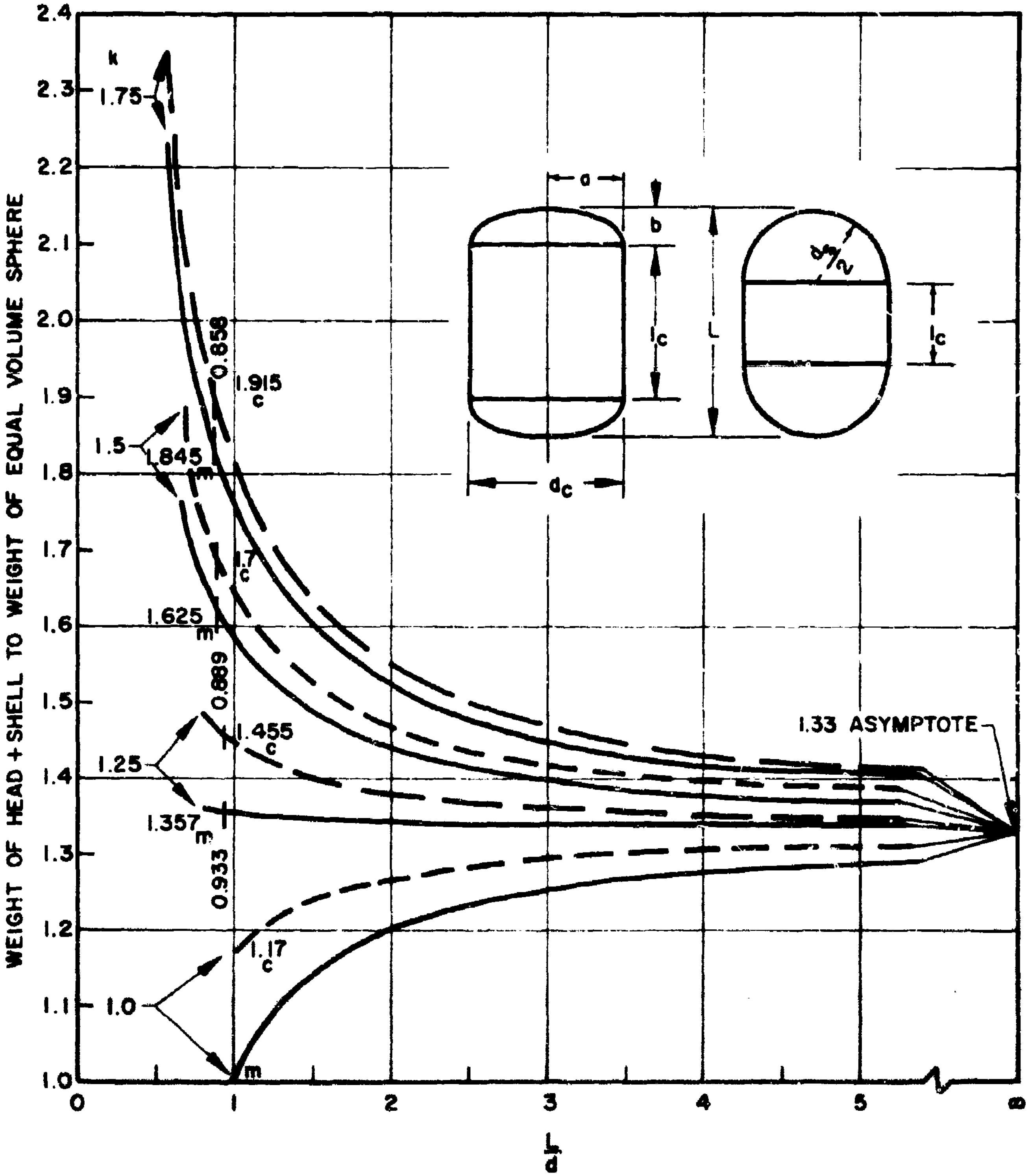


Figure 5 Weight Increase in a Pressurized Structure Due to Deviating from a Spherical Shape

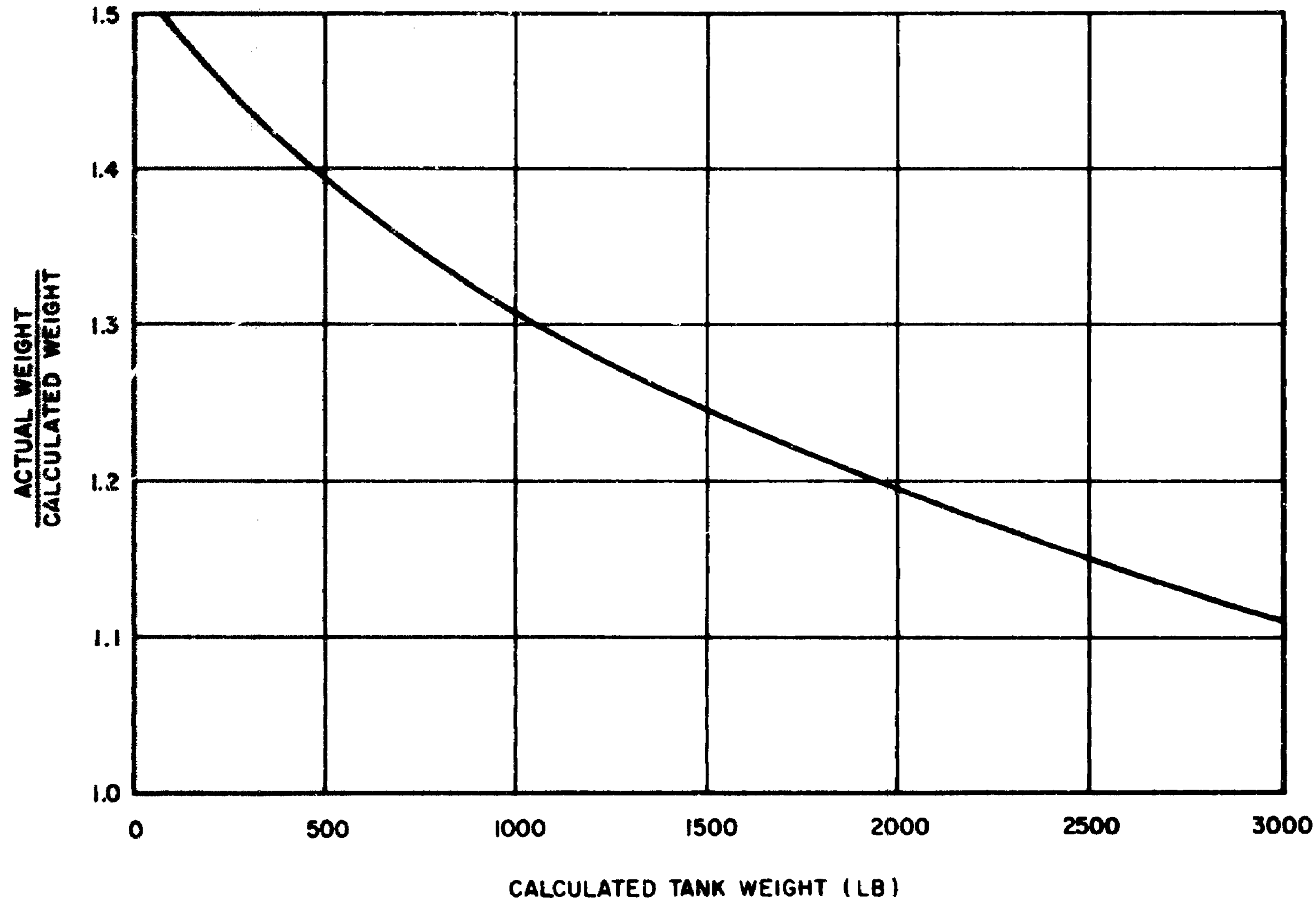


Figure 6 Ratio of Actual to Calculated Weight for Tanks vs Calculated Weight

diameter of a spherical tank of equal volume. For comparative values of cylindrical bodies with spherical heads vs elliptical heads at other L/d values merely add the difference between the chosen value and L/d = 1; e.g., a spherically-headed tank of L/d of 1.5 has the same diameter and volume as an elliptically-ended tank with k = 1.25 when the latter has an L/d of .993 + .5 = 1.433. To the left of the tick marks an equally-volumed elliptical tank has a larger diameter, to the right a smaller.

3.7 THEORETICAL VS ACTUAL WEIGHTS

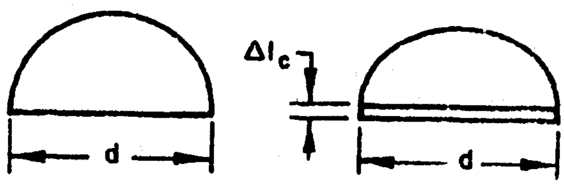
The values obtained from the graphs and equations presented are theoretical. It should be noted therefore that where structural reinforcements are required because of doors, structural-support fittings, or baffles, affecting the stress distribution, the actual weights will be greater. For small tanks this increase may be in the neighborhood of 50 percent whereas for large tanks the actual weight may be about 15 percent greater. Figure 6 shows this relationship. The curve is based on an evaluation of the weights of a few liquid-propellant tanks. However, it is thought to be indicative of what can be expected for all tanks. When more comprehensive data becomes available this figure will be revised.

Section 4
TANKS AND INTERSTAGE STRUCTURAL WEIGHT INTERRELATIONSHIP

From Figures 2 and 5 it is obvious that a spherical tank alone is the lightest design - a well known fact. Also, a cylindrical tank with spherical heads is a lighter design than a cylindrical tank with elliptical heads. However, the weight of an elliptically ended tank when coupled with the shorter length of an interstage structure may be lighter than a comparable combination of a spherically ended tank and an interstage structure.

A break-even thickness may be determined as follows (considering one section of the tank):

For equal diameters from Eq. (22b) the length, Δl_c, of the cylindrical section of the ellipse-cylinder combination for equal volume with a sphere is



$$\Delta l_c = d/3\ (1 - 1/k) = 2a/3\ (1 - 1/k) \qquad (24)$$

From Eqs. (5), (13), and (24) the weight penalty chargeable to the use of an ellipsoidal head is determined as the difference between the weight of the ellipsoidal head and the differential length of the cylindrical portion as compared to the weight of the spherical head, i.e.,

$$\Delta W_p = \pi d^3 E/32k\ (K + k/2) wp/f_e + \pi d^3/6\ (1 - 1/k) wp/f_c - \pi d^3/8\ (K + k/2) wp/f_s \quad (25)$$

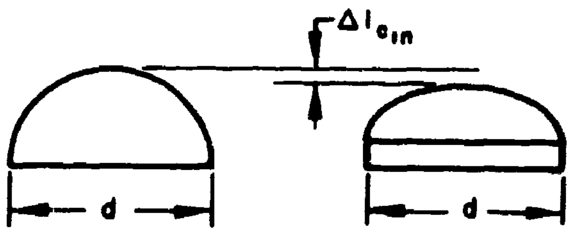
For equal stresses with K for the spherical head = 0.67, therefore,

$$\Delta W_p = wp/f\ \pi d^3/8\ \left[(K + k/2)E/4k + 4/3\ (1 - 1/k) - 1.17\right] \quad (26)$$

where subscript p refers to penalty.

The weight gain due to the shorter interstage on the elliptical head as compared to the spherical head is obtained as follows:

For equal diameters the differential length, $\Delta l_{c_{in}}$ is



$$\Delta l_{c_{in}} = d/2 - d/2k = d/2\ (1 - 1/k) \quad (27)$$

where subscript in refers to interstage

Weight gain = $\Delta W_G = 2\pi w_{in}\, d/2\; t_{in}\,(d/2 - d/2k)$

where subscript G refers to gain

$$\Delta W_G = \pi w_{in}\, d^2/2\; t_{in}\,(1 - 1/k) \tag{28}$$

The interstage gage to equalize weights is determined by equating (26) to (28)

$$\begin{aligned}
t_{in} &= \frac{w}{w_{in}} \frac{d_p}{4f} \frac{1}{1 - 1/k} \left[(K + k/2)\,\frac{E}{4k} + \frac{4}{3}\,(1 - 1/k) - 1.17\right] \\
&= \frac{w}{w_{in}} \frac{t_c}{2} \left[\frac{K + k/2}{k - 1}\left(\frac{E}{4}\right) - \frac{1.33}{(k - 1)} + 0.16\,\frac{k}{(k - 1)}\right] \\
&= \frac{w}{w_{in}}\, t_{s_m} \left[\frac{K + k/2}{(k - 1)}\left(\frac{E}{4}\right) - \frac{1.33}{(k - 1)} + 0.16\,\frac{k}{(k - 1)}\right] \\
&= \frac{w}{w_{in}}\, t_e \left[\frac{E}{4(k - 1)} - \frac{1.33}{(k - 1)(K + k/2)} + 0.16\,\frac{k}{(k - 1)(K + k/2)}\right]
\end{aligned} \tag{29}$$

where t_{in} = interstage thickness

t_{s_m} = spherical head thickness based on membrane stress only

The necessity for using a larger gage than that obtained from Eq. (29) indicates that, with respect to weight, it is more economical to go to an elliptical head.

Section 5
CYLINDRICAL AND CONICAL INTERSTAGES UNDER AXIAL LOADING

Equation (29) determines the trade-off thickness for interstage structure when coupled with the propellant tank requirements.

5.1 CYLINDRICAL SECTIONS

In ballistic missile applications the primary loading condition is axial compression for which the design allowable for thin-skinned monocoque structures is based on buckling criteria.

Unpressurized buckling curves for cylinders with two different values of probability are plotted in Figure 7a, with the Kanemitsu-Nojima curve and Gerard's curve included for comparison. Figure 7b indicates the effect of pressurization. (The dashed line for 99 percent probability is an approximation.) Note that the total critical external load that can be supported is the sum of the P_{cr} for $p = 0$ plus the ΔP_{cr} for $p > 0$ plus the pretension load, i.e., $P_{cr_{total}} = 2\pi E t^2 (C + \Delta C) + \pi r^2 p$.

5.2 CONICAL SECTIONS

For conical interstage structure an approximation of the critical buckling load can be obtained by multiplying the cylindrical buckling load by cos α

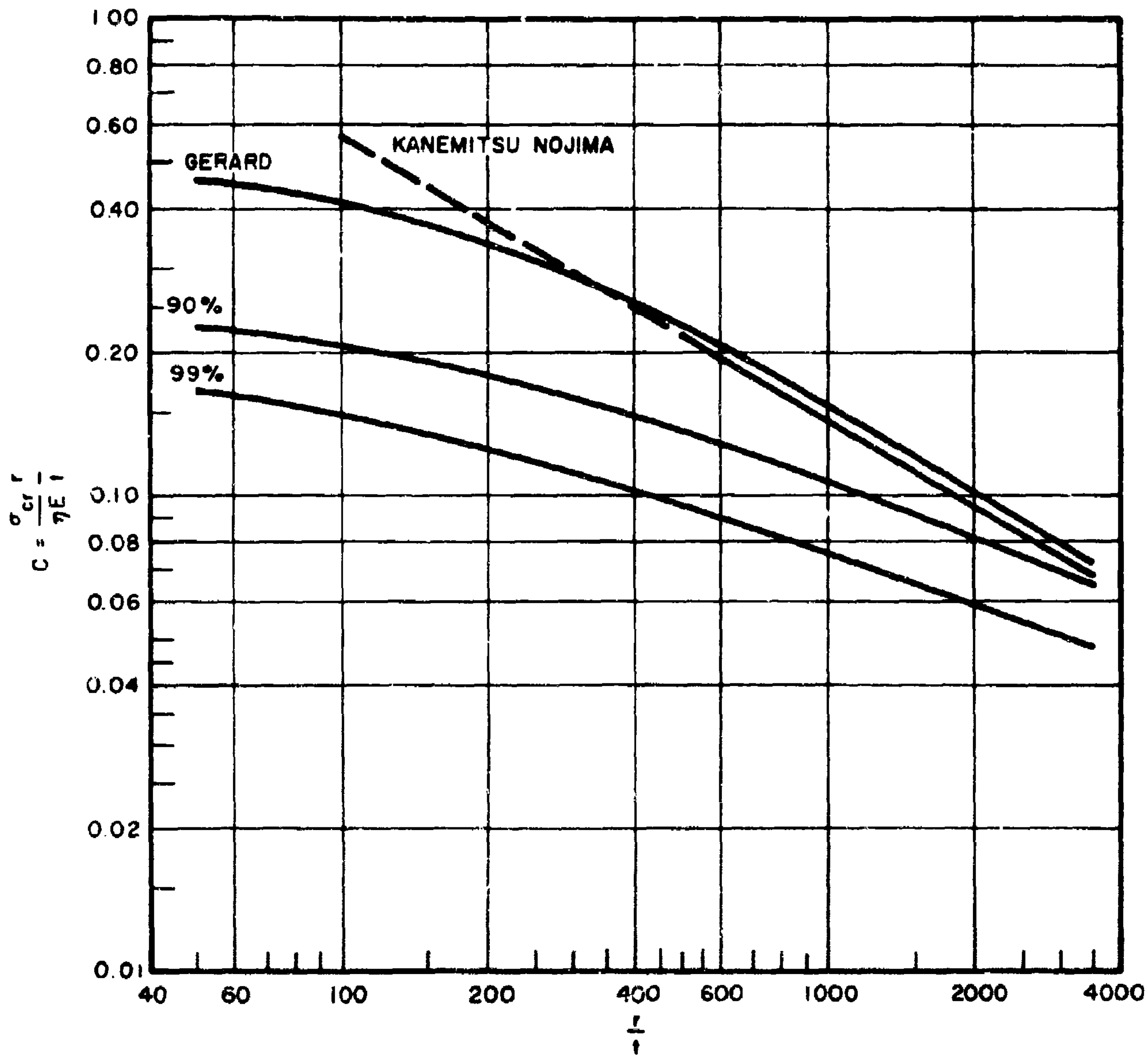


Figure 7a Critical Axial Compressive Stress Coefficient vs r/t

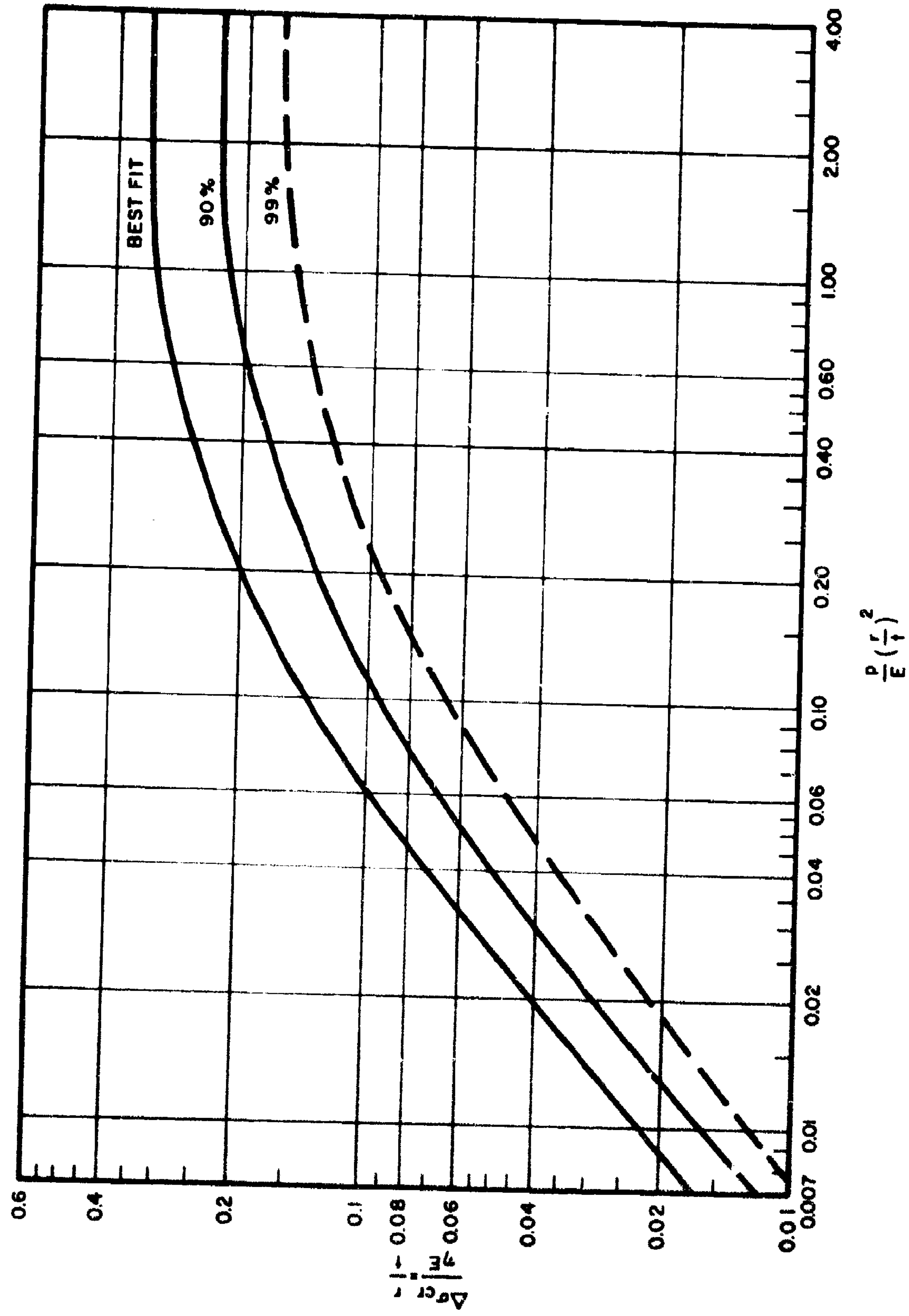


Figure 7b Increase in Compressive Buckling Stress Due to Internal Pressure

5.3 INTERSTAGE SAFETY FACTOR CRITERIA AND BUCKLING ALLOWABLES

A tabulation of safety-factor criteria and allowables used in industry for missile applications is shown as Appendix C.

5.3.1 Kanemitsu-Nojima, and 90 and 99 Percent Failure Probability Curves

A 1.25 factor of safety applied to limit to obtain ultimate has been used with allowables determined from the 90 percent failure probability curves. Another approach has been to use a 1.00 factor of safety in conjunction with a 99 percent failure probability curve. Present practice for manned aircraft is to use a 1.50 factor of safety coupled with the Kanemitsu-Nojima curves.

Figures 8, 9, and 10 show the Kanemitsu-Nojima buckling curve and the 90 and 99 percent failure probability curves.

Table II shows a comparison of the gages obtained from the different design philosophies employed. Arbitrary values were used for loads. The r/t values and the L/r values are generally within the range of interest of present ballistic missile applications. It can be seen from these curves and the tabulation that a factor of safety of 1.00 in conjunction with 90 percent probability is, for the range of interest which concerns us here, as conservative as piloted aircraft philosophy holding to a 1.50 factor of safety.*

5.3.2 Recommended Values for Safety Factor and Probability Allowable

For preliminary design purposes the following values are employed for structures subject to a buckling type of failure:

Application	Factor	Probability
Non-hazardous to personnel or vital equipment	1.00	90%
Special precautionary safety of personnel	1.00	90%
Hazardous to personnel or vital equipment	1.10	90%

* This assumes, of course, a rigorous "loads" correlation.

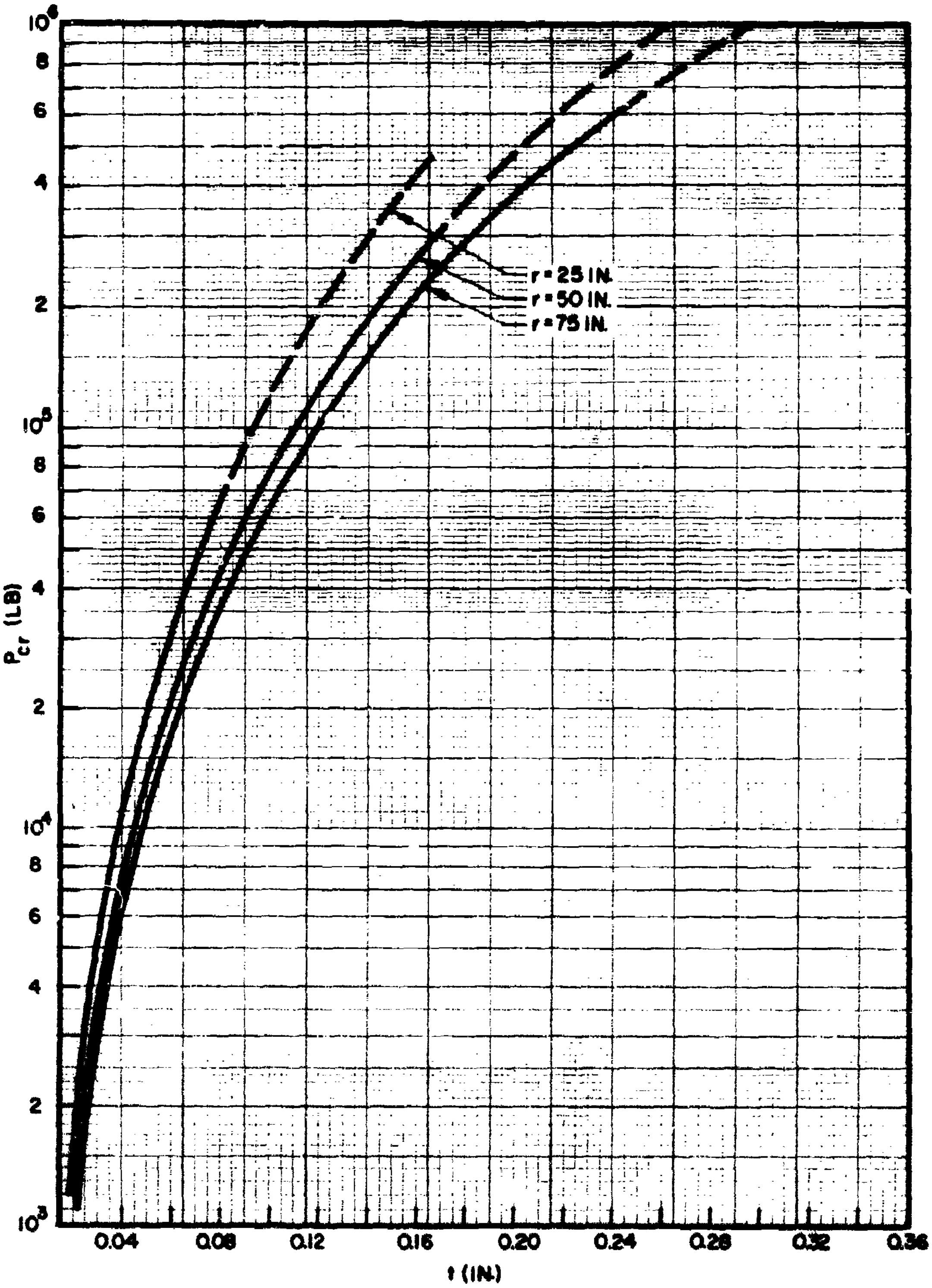


Figure 8 Allowable Load vs Thickness for r = 25, 50 and 75 Inches (Based on Kanemitsu-Nojima Data) E = 6 x 10^6 psi

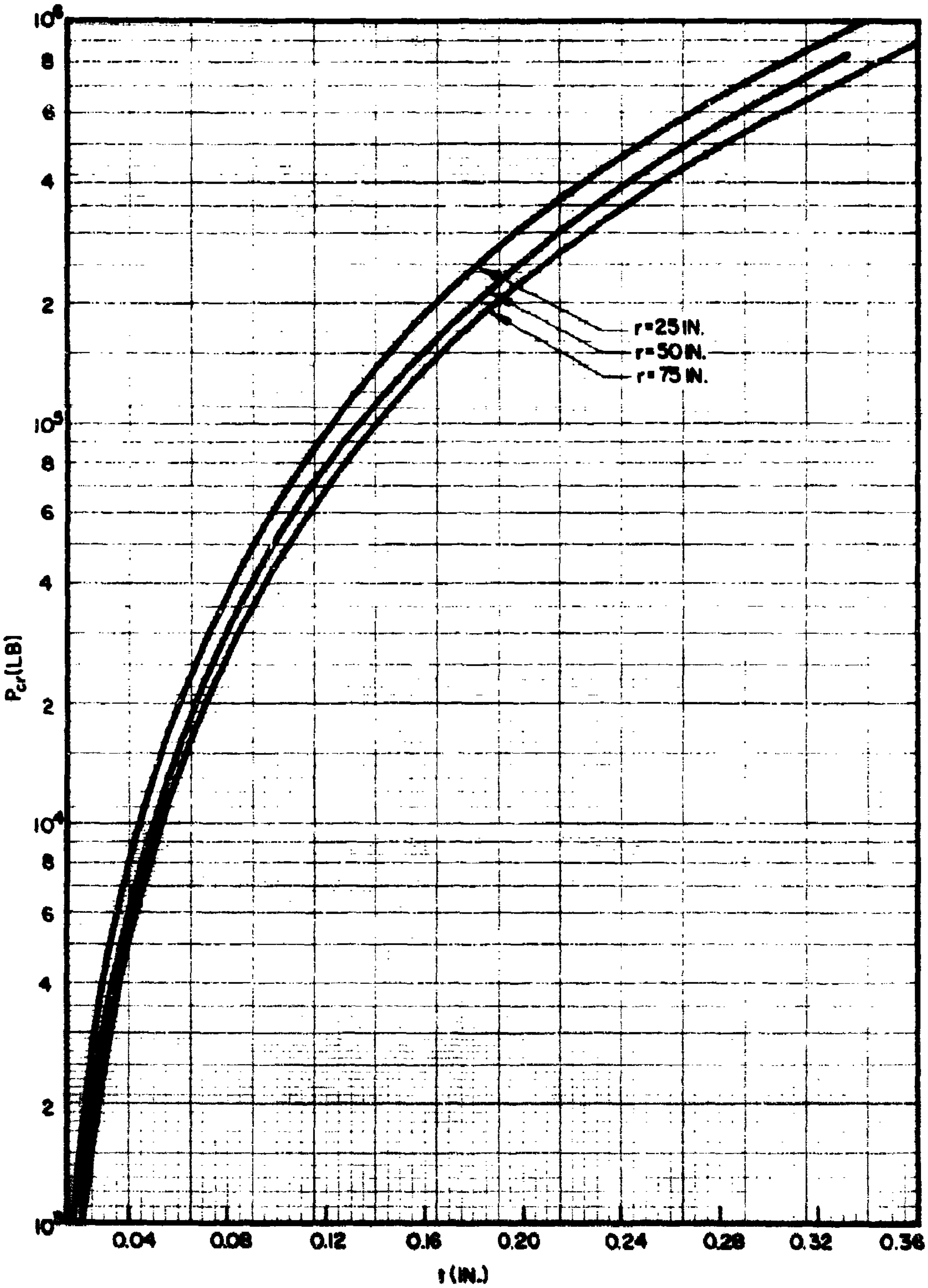


Figure 9 Allowable Load vs Thickness for r = 25, 50 and 75 Inches (Based on a 90% Probability) $E = 6 \times 10^6$ psi

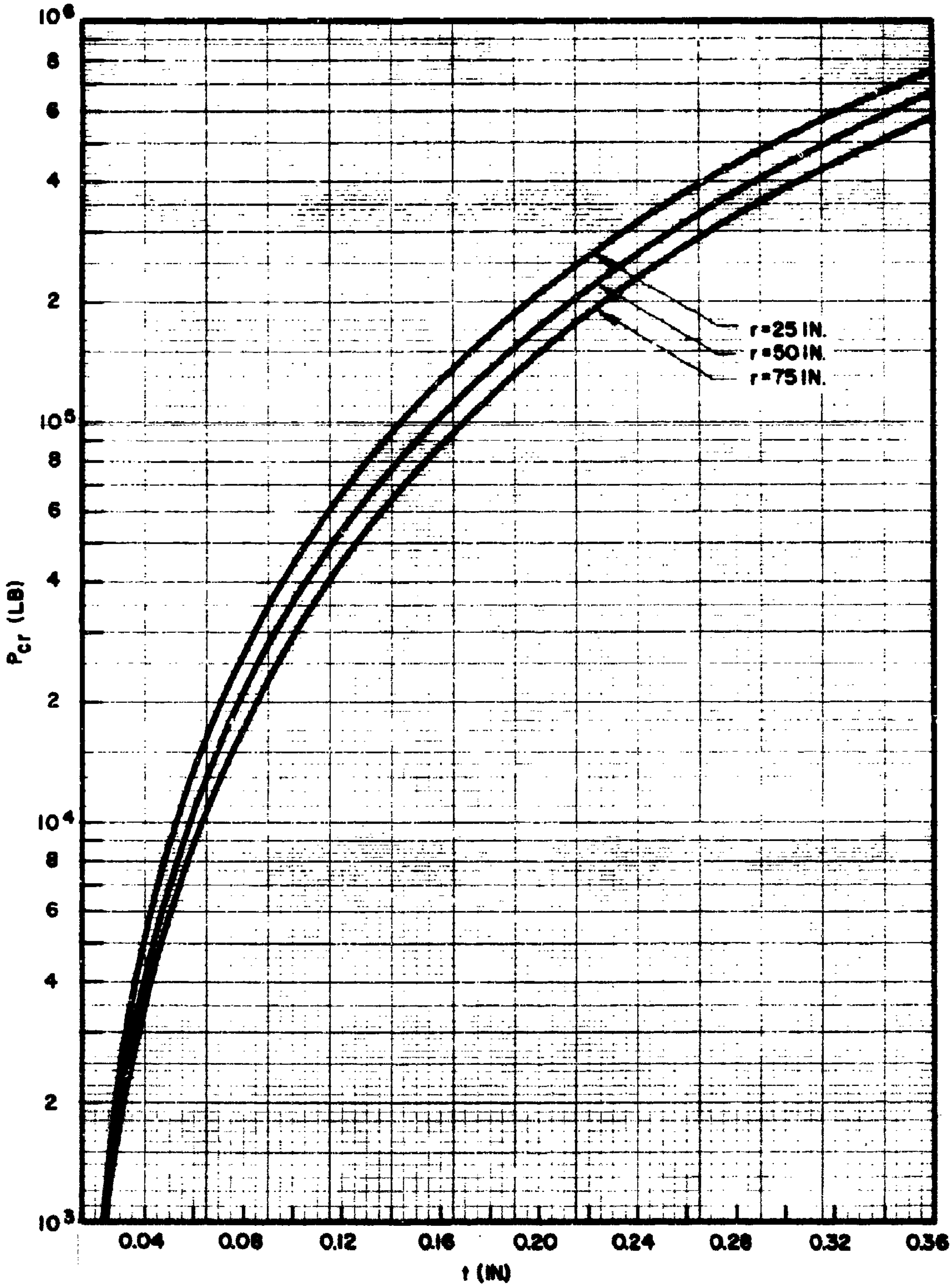


Figure 10 Allowable Load vs Thickness for r = 25, 50 and 75 Inches (Based on a 99% Probability) $E = 6 \times 10^6$ psi

TABLE II

MATERIAL THICKNESSES BASED ON VARIOUS DESIGN PHILOSOPHIES

	①	②	③	④	⑤	⑥	⑦
r (in)	P_{limit} (lb)	$P_{ultimate}$ (lb)		MATERIAL THICKNESS (in) BASED ON			
		1.25 x ①	1.50 x ①	③ &K&N	① &90%	② &90%	① &99%
25"	177,000	221,000	265,200	.134*	.157	.174	.181
	57,600	72,000	86,400	.087	.091	.105	.118
	16,300	20,400	24,500	.055	.055	.061	.065
	2,720	3,400	4,080	.027	.025	.027	.033
50"	706,000	884,000	1,060,800	.266*	.313	.343	.366
	230,000	288,000	345,600	.177	.192	.212	.222
	65,600	82,000	98,400	.109	.110	.123	.130
	10,800	13,500	16,200	.054	.051	.056	.058
75"	1,590,000	1,989,000	2,386,800	.415*	.469	.512	.550
	518,000	647,000	776,400	.268	.289	.317	.339
	147,000	184,000	220,800	.161	.167	.185	.197
	24,300	30,400	36,500	.080	.077	.085	.090

Notes:

1 $E = 6 \times 10^6$ psi. For magnesium this would correspond to a temperature of approximately 350°F.

2 The Kanemitsu-Nojima values are based on $\frac{L}{r} > 4$.

3 *Kanemitsu-Nojima's values have been extrapolated beyond their recommended cutoff $\left(\frac{r}{t} < 500\right)$. They are shown merely for comparative purposes.

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1. N. A. Weil, "Bursting Pressures and Safety Factors for Thin-Walled Vessels", J. Franklin Inst., February 1958, pp. 97-116

2. I. Rattinger and R. H. Gallagher, "A Survey of the Analysis and Design Problems of Integral Propellant Tanks", paper presented at National Summer Meeting of the Institute of Aeronautical Sciences, Los Angeles, California, 16-19 June 1959

3. E. B. Kinnaman, R. E. Jacobsen, and C. F. Tiffany, "An Approach to the Practical Design of Reliable, Light Weight, High Strength Pressure Vessels", paper presented at National Summer Meeting of the Institute of Aeronautical Sciences, Los Angeles, California, 16-19 June 1959

4. R. E. Wong, "Rapid Weight - Strength Analysis of Pressure Vessels" Product Engineering, 14 October 1957, pp. 95-98

5. Bell Aircraft Corporation, Design Charts for the Elastic Stress Distribution in Pressure Vessels with Elliptical Heads, by I. Rattinger, Report No. 02-984-016, Buffalo, N. Y., 22 October 1954 with revisions, 1 December 1955

6. Lockheed Missiles and Space Division, Structural Reliability Methods for Cylinders and Cones, by G. W. Reed and E. J. Pipes, LMSD-4507, Sunnyvale, California, May 1959 (U)

7. Lockheed Aircraft Corporation, California Division, Orbital and Space Vehicles, by Prof. P. E. Sandorff, Report No. 13251, Burbank, California, July - August 1958 (Company Private Data)

EXAMPLES

EXAMPLE 1 - USE OF FIGURE 4

Given:

$$d = 10' = 120''$$

$$L = 30' = 360''$$

$$L/d = 3$$

Determine:

1. Volume of spherically-ended tank
2. Volume of elliptically-ended tank with $k = 1.25$
3. Volume of elliptically-ended tank with $k = 1.75$

Solution:

1. Figure 4 gives a volume of 2094.4 ft^3. As a check using Equations (7) and (14)

   $$\text{Volume} = \frac{\pi}{4}(100)(20) + \frac{4}{3}\pi(125) = 2094.4 \text{ ft}^3$$

2. From Table of Figure 4 for $k = 1.25$

   Volume = 1.0250(2094.4) = 2146.76 ft^3. As a check using Equations (7) and (14)

   $$\text{Volume} = \frac{\pi}{4}(100)(22) + \frac{4}{3}\pi(125)/1.25 = 2146.76 \text{ ft}^3$$

3. From Table of Figure 4 for $k = 1.75$

   Volume = 1.0536 (2094.4) = 2206.6 ft^3

EXAMPLE 2 - USE OF FIGURE 2

Given:

Volume = 2094.4 ft^3

Propellant tank for a non-hazardous application with a working pressure of 50 psi and a material yield stress of 40,000 psi with a weld efficiency of 85 percent.

Allowable F.˙. = .85(40000) = 34000 psi. w = .100 lb/in^3

Determine:

1. Weight of spherical tank
2. Weight of spherically-ended tank of same volume with an L/d = 3
3. Ratio of weight of spherically-ended tank to spherical tank of equal volume

Solution:

1. fW/Vw = 75 for spherical tank corresponding to a p = 50 psi

$$W_1 = \frac{75 \times 2094.4 \times 1728 \times .100}{34000} = 798 \#$$

2. From solution to example 1:

Cylinder Volume = 1570.8 ft^3

Head Volume = 523.6 ft^3

fW/Vw = 100 for cylinder

fW/Vw = 87.5 for spherical head

$$W_2 = \frac{100 \times 1570.8 \times 1728 \times .100}{34000} + \frac{87.5 \times 523.6 \times 1728 \times .100}{34000}$$

$$= 798. + 233. = 1031 \#$$

3. $\dfrac{W_2}{W_1} = \dfrac{1031}{798.} = 1.292$

EXAMPLE 3 - USE OF FIGURE 5

Given:

Volume = 2094.4 ft^3

Propellant tank for a non-hazardous application with a working pressure of 50 psi and a material yield stress of 40,000 psi with a weld efficiency of 85 percent.

Allowable F.S. = .85(40000) = 34000 psi. w = .100 lb/in^3

Spherical tank weight = 798 # from example 2

Determine:

1. Weight of spherically ended tank of same volume with an L/d = 3
2. Weight of elliptically ended tank with k = 1.75 of same volume and with same diameter as spherically ended tank with L/d = 3

Solution:

1. For L/d = 3, weight of head and shell to weight of equal volume sphere = 1.292

W = 1.292 x 798 = 1031 #

2. From curves for combined stresses for k = 1.75, tick mark gives a value of .858 which when added to difference in L/d's = .858 + 2 = 2.858. Reading the ratio value, one obtains 1.479.

W = 1.479 x 798 = 1180 #

As a check use Figure 2:

$$V_{head} = \frac{4}{3}\pi(125)/1.75 = 299.2 \text{ ft}^3$$

$$V_{cyl} = \frac{\pi}{4}(100)(22.86) = 1795.2 \text{ ft}^3$$

For head with a p = 50 psi and k = 1.75

$$fW/Vw = 175$$

$$W_h = \frac{175 \times 299.2 \times 1728 \times .100}{34000} = 266$$

For cylinder

$$fW/Vw = 100$$

$$W_{cyl} = \frac{100 \times 1795.2 \times 1728 \times .100}{34000} = 912$$

$$W_h + W_{cyl} = 1178 \#$$

EXAMPLE 4 - USE OF EQUATION (29)

Given:

A magnesium interstage with w = 0.065 lb/in^3 and an aluminum propellant tank for a non-hazardous application with w = 0.100 lb/in^3. Working pressure is 50 psi, allowable stress = 34000 psi, a = 120 inches.

Determine:

Thickness of interstage where an elliptically ended tank with k = 1.75 and a spherically ended tank have the same weight. Both tanks are of the same volume.

Solution:

$$t_{in} = \left(\frac{.100}{.065}\right)\left(\frac{50 \times 120}{4 \times 34000}\right)\left(\frac{1}{1 - 1/1.75}\right)\left[\frac{4}{3}\left(1 - 1/1.75\right) + \frac{5.11}{4 \times 1.75}\left(.961 + 1.75/2\right) - 1.17\right] = 0.12''$$

Therefore if the interstage gage is greater than 0.12" the elliptically ended tank - interstage combination is lighter.

EXAMPLE 5 - USE OF FIGURE 7a

Given:

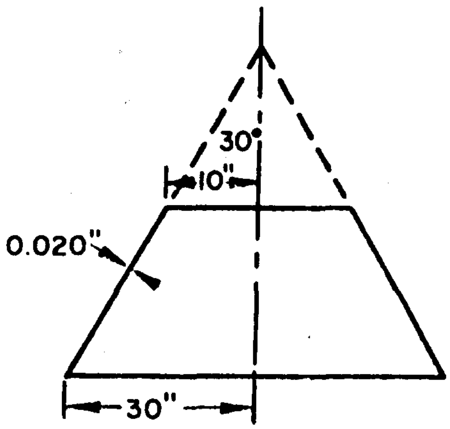
$\alpha = 30°$

$r_1 = 10''$

$r_2 = 30''$

$E = 30 \times 10^6$ psi

$t = .020''$



Determine:

Critical load of cone

Solution:

$$\frac{r_1 + r_2}{2t} = \frac{20}{.02} = 1000$$

C = 0.108 (Based on 90 percent curve, Figure 7a)

$$P_{cr_{cyl}} = \frac{.108}{1000}\left(30 \times 10^6\right)\left(\pi \times 20 \times .02\right) = 8250 \text{ lbs}$$

$P_{cr_{cone}}$ = [illegible] ° = 71[illegible] lbs.

Appendix A

THIN-WALLED PRESSURIZED STRUCTURES -- FACTORS OF SAFETY IN USE BY VARIOUS AGENCIES *

Facility / Strength	Yield (0.2% offset)	Ultimate
Jet Propulsion Laboratory		1.33
Convair (San Diego)	1.0 Proof	1.25 Burst 4/
Convair (Pomona)	2.0	
Northrop		1.50
Hughes	1.15	
Boeing	1.10 Proof 1/	1.25 Burst 1/
	2.00 2/	4.00 2/
Chance Vought	1.50	2.00
Redstone Arsenal	2.00	
Johns Hopkins University	2.00	
Martin		2 to 3
Goodyear		2 to 3
Grumann	1.00	1.50 Liquid
	1.00	2.00 Gas
General Electric	1.15	
Bell	1.33 2/	1.50 2/
Cornell	2.00	
McDonnell	2.00	
North American	1.50 2/	1.25 1/
	4.50 3/	
Lockheed MSD	1.40 1/ (XN)	
	2.00 2/	
	1.20 1/ (XA)	1.60 1,5/ (XA) 1.25 1,6/
	1.50 2/ (XA)	2.00 2/ (XA)

1/ No personnel hazard involved

2/ Hazardous to personnel

3/ State code for testing with personnel present

4/ Used in a special precautionary safety-of-personnel application

5/ Up to '59

6/ '59 on

* Note: Although the particular applicability with respect to personnel hazard was not reported in all cases it can be assumed that the smaller factors are for a non-hazardous application.

Appendix B

INTEGRAL AND NON-INTEGRAL TANKS -- FACTORS OF SAFETY AS RECOMMENDED BY U.S. AIR FORCE

Reference:

Air Force Handbook of Instruction for Airplane Designers, ARDCM 80-1, Volume II, Guided Missiles

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Part B, Airframe Design

Integral Tanks, Section 15.3.1, Safety Factors

A minimum of 1.20 x 1.25 times all critical loads and pressures for conditions considered non-hazardous to personnel or vital equipment

A minimum of 1.33 x 1.50 times all critical loads and pressures for conditions considered hazardous to personnel or vital equipment

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Part D, Propulsion Systems

Tanks, Section 3.1.3.2, Safety Factors

Safety factors for all types of removable tanks are the same.

They are:

For aircraft and vehicle-launched missiles

Proof pressure = 1.5 x working pressure
Burst pressure = 1.33 x proof pressure

For remotely launched missiles

Proof pressure = 1.2 x working pressure
Burst pressure = 1.33 x proof pressure

Nominal working pressure is defined as the maximum working pressure to which a component is subjected under steady state conditions, or the effect of launch or catapult loads, whichever is the more severe.

Appendix C

PRIMARY STRUCTURE -- FACTORS OF SAFETY IN USE BY VARIOUS AGENCIES

Facility / Strength	Condition with respect to personnel				Basis for Allowables 1/	
	Hazardous		Non-hazardous			
	Yield	Ultimate	Yield	Ultimate	Gage	Probability
Convair	1.175 $F_{ty} < 0.85\ F_{tu}$					
Hughes			1.15			
Douglas				1.33	**Nominal**	**B**
Boeing	1.00	1.50	1.00	1.25	**Nominal**	**A**
Chance Vought	1.15	1.50			**Nominal**	**A**
Bureau Ordnance	1.00	--	1.00		**Nominal**	**A, B**
Martin	1.15	--	1.15	1.50	**Nominal**	**A, B**
Grumman	1.15	1.50	1.15			
Bell	--	1.50	--	1.25	**Nominal**	**A, B**
Goodyear	--	1.50		1.25	**Nominal**	**A**
Northrop	1.00	1.50	1.00	1.25	**Nominal**	**B**
North American		1.50	1.10	1.25	**Reduced** 2/ **Nominal**	**A** **B**
Redstone Arsenal	1.50			1.25	**Nominal**	**A**
Lockheed MSD		1.50		1.25	**Nominal**	**A, B**

1/ A and B Probability refer to ANC 5/MIL-HDBK-5 Material Property values where A = **minimum guaranteed** (99%) and B is 90%

2/ Reduced gage due to forming