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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES

L. G. HANSCOM FIELD, BEDFORD, MASSACHUSETTS

An Evaluation of an Important Advance in Network Synthesis Theory

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MICROWAVE PHYSICS LABORATORY PROJECT 5635

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Abstract

A discussion of a recent significant advance in network synthesis theory is presented. This "breakthrough" was accomplished by D. Hazony of the Case Institute of Technology and by D.C. Youla of the Pol technic Institute of Brooklyn, who independently of each other developed methods for unifying the theory of two-port cascade synthesis. Both methods are based on Richards' theorem, and both introduce the gyrator artificially. Different methods of proof are used, however. A valuable "cookbook recipe" was developed by Youla. Hazony managed to extend the method to n-ports. In all, this epoch-making achievement has resulted in an important, simple, and beautiful method of network synthesis.

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Illustrations

An Evaluation of an Important Advance in Network Synthesis Theory

I. INTRODUCTION

During the past five years, extraordinary theoretical advance has occurred in network synthesis theory. In 1959, Dov Hazony, at the Case Institute of Technology, Cleveland, Ohio, in a proposal to the AFCRL Microwave Physics Laboratory outlined a research program for finding a general synthesis method that would include the methods by Brune, Darlington, and Bott and Duffin. This research was promptly sponsored under Contract AF19(604)-3887. Very soon interesting results were obtained by Hazony and his research group. The results were so general and so simple as to merit the term "breakthrough".

It is an interesting coincidence that the first part of the theory was independently developed by Prof. Dante C. Youla at the Polytechnic Institute of Brooklyn, also under an AFCRL Microwave Physics Laboratory Contract AF19(604)-4143.

An attempt will be made in this report to present the basic ideas leading to and continuing the exploitation of this advance. We begin by discussing a positive real function and Richards' theorem.

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2. POSITIVE REAL FUNCTIONS

A complex function $Z(s)$ is a positive real function, prf, if the following conditions are fulfilled:

$$\begin{aligned}
&\text{(a)}\quad Z(s) \text{ is real, if } s \text{ is real;}\\
&\text{(b)}\quad \operatorname{Re} Z(s) \geq 0 \text{ , if } \operatorname{Re} s = 0 \text{ ; and}\\
&\text{(c)}\quad \operatorname{Re} Z(s) > 0 \text{ , if } \operatorname{Re} s > 0 \text{ .}
\end{aligned} \tag{1}$$

Here, $s = \sigma + j\omega$, where σ is a damping constant, $\omega = 2\pi f$, f is the frequency, and s is a complex frequency function.

Alternatively we can say that $Z(s)$ is a prf if:

(a) $Z(s)$ is real, if s is real;
(b) $\operatorname{Re} Z(s) \geq 0$, if $\operatorname{Re} s = 0$; and
(c) $Z(s)$ is analytic in the right half plane (that is, it has a derivative at each point of the right half plane); poles on the ω axis are simple with positive residues. (2)

3. RICHARDS' THEOREM

Richards' theorem,[1] found in 1947, is a form of Schwartz's lemma (known since 1869). It says:

"If Z(s) is a rational positive real function (prf) with the numerator and the denominator of the same degree, then

$$Ri(s) = \frac{kZ(s) - sZ(k)}{kZ(k) - sZ(s)} \tag{3}$$

is prf." (k is a real number.)

Richards used $k = 1$, and later on Bott and Duffin introduced the k. The proof that Richards used is the following:

Let $$w = \frac{s-k}{s+k} \tag{4}$$

and

$$f(w) = \frac{Z(s) - Z(k)}{Z(s) + Z(k)} \quad . \tag{5}$$

Then we can write Richards' formula (3)>

$$\frac{f(w)}{w} = \frac{1 - Ri(s)}{1 + Ri(s)} \quad . \tag{6}$$

Now, Schwartz's lemma says: "Let the analytic function f(w) be analytic inside the unit circle $|w| = 1$, and let $f(0) = 0$. If, in $|w| < 1$, $|f(w)| \leq 1$, then

$\frac{f(w)}{w} < 1, \quad |w| < 1.$"

The theorem can be used directly. Therefore,

$|1 - Ri(s)| < |1 + Ri(s)|,$

so that Re $Ri(s) > 0$, if $s > 0$.

The other conditions valid for a prf are easily checked. Thus, Ri(s) is prf. Q. E. D.
Another proof of Richards' theorem, found by Hazony will be given later on.

4. ANALYSIS AND SYNTHESIS OF NETWORKS

If we apply a unit impulse, defined by $\delta(t) = 0$, $t \neq 0$, and

$$\int_{\infty}^{\infty} \delta(t) dt = 1,$$

to the input of a two-port network having the transfer impedance function F(s), then we obtain a signal f(t) at the output. The connection between F(s) and f(t) is given by the Laplace transformation

$$\left.\begin{aligned} F(s) &= \int_0^{\infty} f(t)\, e^{-st}\, dt \\ f(t) &= \frac{1}{2\pi j} \int_{\gamma_s} F(s)\, e^{st}\, ds \end{aligned}\right\} \quad . \tag{7}$$

Figure 1 shows the analysis and synthesis of two-ports. The synthesis problem can be divided into two parts, the approximation part and the realization part. (See Figure 2.) A Laplace transformation is called LT; a star indicates an approximated function.

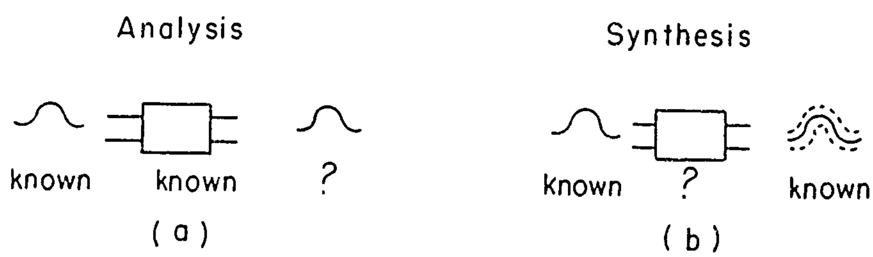

Figure 1. Analysis and Synthesis of Two-Ports

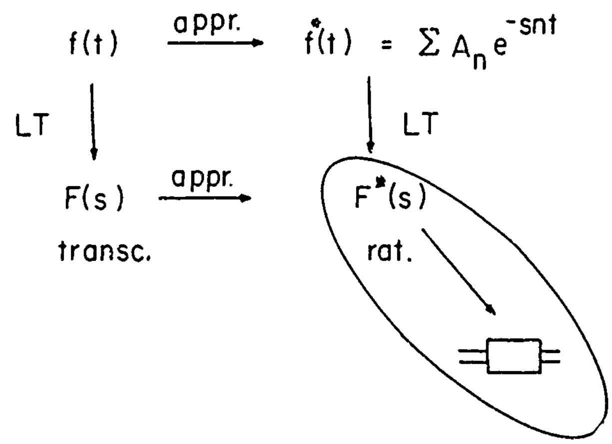

Figure 2. The Synthesis Problem

5. CIRCUIT ELEMENTS

The circuit elements used are:

(a) resistance, r,

(b) inductance, L,

(c) capacitance, C,

(d) unity coupled transformer, T.

See Figure 3 where

$$\begin{cases} L_1 + L_2 = L_p \\ \qquad L_2 = M \quad ; \\ L_2 + L_3 = L_s \end{cases}$$

$M = \sqrt{L_p L_s}$; therefore $L_1 L_2 + L_1 L_3 + L_2 L_3 = 0$.

(e) gyrator (Tellegen, 1948).

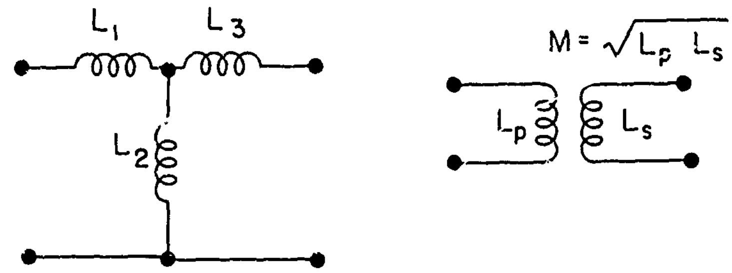

Figure 3. Unity Coupled Transformer. (L_1 or L_3 is negative.)

See Figure 4 where

$$\begin{cases} V_1 = z_{12} I_2 \\ V_2 = -z_{12} I_1 \end{cases} .$$

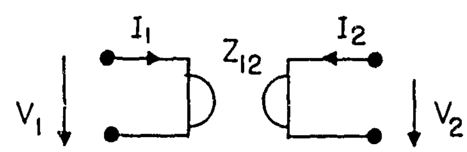

Figure 4. Gyrator

6. FOSTER (1924)

Foster[2] synthesized lossless one-ports by splitting the input impedance function into partial fractions.
Example:

$$Z(s) = \frac{2s^2+1}{s(s^2+1)}$$

$$= \frac{A}{s} + \frac{B}{s^2+1} = \frac{1}{s} + \frac{s}{s^2+1}$$

$$= \frac{1}{s} + \frac{1}{s + \frac{1}{s}} .$$

(See Figure 5.)

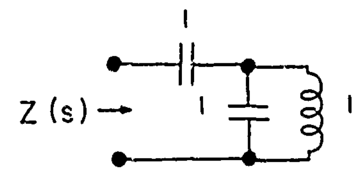

Figure 5. Foster Network

7. CAUER (1926)

Cauer[3] synthesized lossless one-ports by splitting the input impedance function into continued fractions.

Example:

$$Z(s) = \frac{2s^2+1}{s(s^2+1)} = \frac{2s^2+1}{s^3+s}$$

$$= \frac{1}{\frac{s}{2} + \frac{s/2}{2s^2+1}} = \frac{1}{\frac{s}{2} + \frac{1}{4s + \frac{1}{s/2}}} .$$

(See Figure 6.)

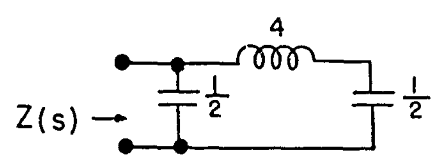

Figure 6. Cauer Network

8. BRUNE (1931)

Brune[4] was the first to synthesize lossy networks. He found that a rational positive real function (sometimes called a "Brune function") could be synthesized by using resistances, inductances, capacitances, and unity coupled transformers.

Procedure (see Figure 7):

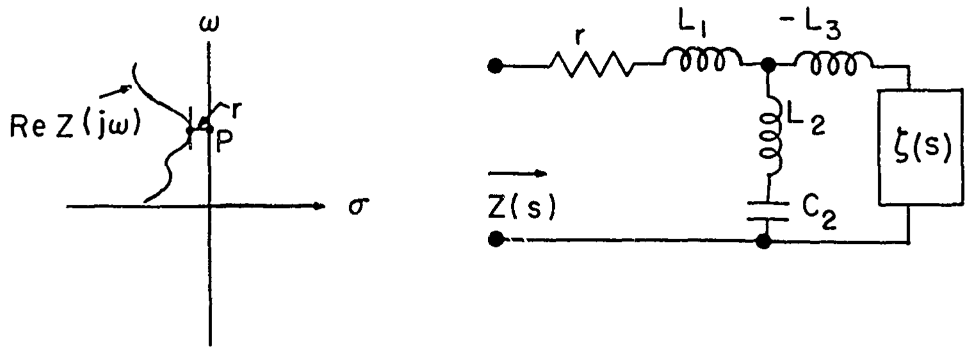

Figure 7. Brune Cycle

(a) $\operatorname{Re} Z(j\omega)_{min} = \Gamma \;;\; Z(s) - \Gamma = Z_1(s)$,

(b) $\because$ $Z_1(j\omega)$ is reactive at P ,

(c) $Z_1(s) - sL_1 = Z_2(s)$,

(d) $Z_2(s)$ has a zero at P ,

(e) $Y_2(s)$ has a pole at P ,

(f) $$Y_2(s) - \frac{s^2 L_2 C_2 + 1}{s L_2} = Y_3(s) = \frac{1}{Z_3(s)} .$$

The function $Z_3(s)$ is not prf.

Excellent idea by Brune: L_1 , L_2 , and L_3 form a unity coupled transformer.

(g) $Z_3(s) + sL_3 = \zeta(s)$

The function $\zeta(s)$ is prf and has the degrees of its numerator and denominator lowered by two compared with Z(s).

9. DARLINGTON (1939)

Darlington[5] also studied lossy networks. He found that any prf Z(s) function could be synthesized by a lossless two-port terminated in a resistance usually selected to be one ohm. See Figure 8, where

$$V_1 = z_{11} I_1 + z_{12} I_2 ,$$
$$V_2 = z_{12} I_1 + z_{22} I_2 , \qquad (8)$$
$$V_2 = -\zeta I_2 .$$

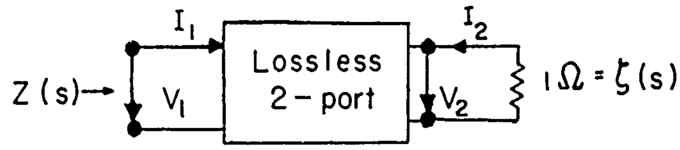

Figure 8. Darlington Synthesis

From Eq. (8) we get

$$Z(s) = \frac{V_1}{I_1} = \frac{z_{11}\zeta + z_{11}z_{22} - z_{12}z_{21}}{\zeta + z_{22}} . \qquad (9)$$

Reciprocal networks have $z_{12} = z_{21}$. Darlington compared Eq. (9) with Z(s) written as

$$Z(s) = \frac{m_1 + n_1}{m_2 + n_2} \qquad (10)$$

where m and n are even and odd parts. Thus, he obtained z_{11}, z_{12}, and z_{22}. From these values four different types of networks could be extracted, called A, B, C, and D networks by Darlington. (See Figure 9.) A difficulty may appear in that z_{12} can be irrational. In such a case Darlington multiplied the numerator and the denominator of Z(s) by a surplus factor.

10. BOTT AND DUFFIN (1949)

Bott and Duffin[6] were the first to synthesize a lossy network without the use of a unity coupled transformer. They based their method on Richards' theorem. A simplified treatment has been given by Hazony.[7, 8] He splits Z(s) as follows:

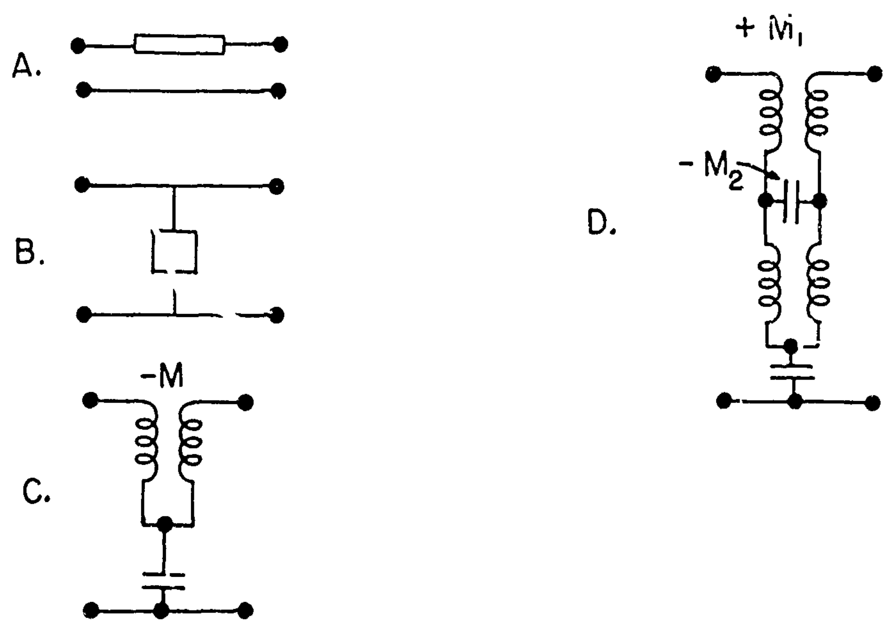

Figure 9. Darlington Networks

$$Z(s) = k\,\frac{kZ(s) - sZ(k)}{k^2 - s^2} + s\,\frac{kZ(k) - sZ(s)}{k^2 - s^2}$$ (11)

$$= Z_1(s) + Z_2(s) \ .$$

The function Z(s) is prf.

(a) $Z_1(s)$ is real, if s is real,

(b) $\operatorname{Re} Z_1(j\omega) = \dfrac{k^2}{k^2 + \omega^2} \operatorname{Re} Z(j\omega)$,

therefore $\operatorname{Re} Z_1(s) \geq 0$, if $s = 0$.

(c) $Z_1(s)$ is analytic in the right half plane.

Thus, $Z_1(s)$ is prf according to Eq. (2). Similarly, $Z_2(s)$ is prf.

When

$s \rightarrow \infty$, $Z_1(s) \rightarrow \dfrac{kZ(k)}{s}$.

Therefore $$Z_1(s) = \frac{1}{A + \frac{s}{kZ(k)}} = \frac{Z(k)}{\frac{kZ(k)-sZ(s)}{kZ(s)-sZ(k)} + \frac{s}{k}} \tag{12}$$

$$Z_1(s) = \frac{Z(k)}{\frac{1}{Ri(s)} + \frac{s}{k}} \quad . \tag{13}$$

Thus, Ri(s) is prf, which proves Richards' theorem. Similarly,

$$Z_2(s) = \frac{Z(k)}{Ri(s) + \frac{k}{s}} \quad . \tag{14}$$

From Eqs. (11), (13), and (14), Z(s) can be synthesized by the network shown in Figure 10. This is the network found by Bott and Duffin. It is a balanced bridge (first described by Reza[9]), because

$$\frac{k \cdot Z(k)}{s} \cdot \frac{sZ(k)}{k} = Z^2(k) \quad ,$$

and

$$Z(k)\, Ri(s) \cdot \frac{Z(k)}{Ri(s)} = Z^2(k) \quad .$$

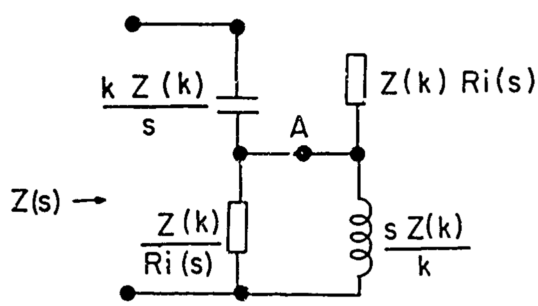

Figure 10. Bott-Duffin Balanced Bridge

The procedure that Bott and Duffin used in order to reduce Ri(s) is the following:

(a) Z(s) was made minimum resistive by subtraction of a resistance r (as in the Brune procedure);

(b) Therefore $Z(j\omega_o) = j\omega_o L$ (at a specific $\omega_o > 0$ for example) ;

(c) let $k = \frac{Z(k)}{L}$;

(d) for this k, $Ri(j\omega_o) = 0$;

(e) therefore, Ri(s) has a zero on the imaginary axis and the degree can be lowered by means of the Foster method. (See Figure 11.)

(f) $\frac{1}{Ri(s)}$ has a pole on the imaginary axis and can be reduced in a similar way.

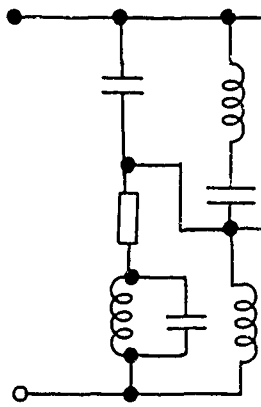

Figure 11. Bott-Duffin Network

The Bott and Duffin method does not make use of unity coupled transformers. The price they paid consisted in the fact that the method required many circuit elements. In the beginning the method was not very well understood. In order to get fewer elements Reza,[9] Pantell,[10] and Fialkow and Gerst[11] studied the Bott and Duffin method thoroughly and managed to find unbalanced bridge circuits that all had one element less than Bott and Duffin's balanced bridge. Reza[9] and Storer[12] showed, however, that the work of deriving these unbalanced bridges could be simplified to a high degree by inserting a specific impedance at A in Figure 10 and then applying a Y - Δ transformation.

11. FIALKOW AND GERST (1955)

Both the Brune and the Bott-Duffin methods are based on the fact that Z(s) first is made minimum resistive. This is usually a rather complicated procedure. It was therefore a big advantage when Fialkow and Gerst[13] developed a method that does not require any minimum resistive Z(s). The reasoning is the following:[8]

Both the numerator and the denominator in the Richards' function

$$Ri(s) = \frac{kZ(s) - sZ(k)}{kZ(k) - sZ(s)} \tag{3}$$

are zero for $s = k$, therefore $s - k$ can be cancelled. If there is any other factor $s - k_o$, which can be cancelled, then

$$kZ(k_o) - k_o Z(k) = 0 ,$$

and

$$kZ(k) - k_o Z(k_o) = 0 ,$$

which means that (with corresponding signs),

$$k_o = \pm k$$

and

$$Z(k_o) = \pm Z(k) .$$

We already know $k_o = +k$, but $k_o = -k$ yields

$$Z(k_o) + Z(-k_o) = 0 ,$$

or

$$\mathrm{Ev}\, Z(k_o) = 0 . \tag{15}$$

This was already known by Richards when he wrote his original article.[1] Thus, instead of the complicated minimization procedure used by Bott and Duffin, Fialkow and Gerst used Eq. (15). Real values of k_o led to balanced bridges. For complex k_o these authors ran into difficulties. The following procedure was developed:

Let $k_o = a + jb$; $Z(k_o) = A + jB$.

When $k = 0$: $\quad Ri(s) = \dfrac{Z(0)}{A + jB} = Z(0)\sqrt{A^2+B^2}\, e^{-j \tan^{-1} B/A}$,

$k = \infty$: $\quad Ri(s) = \dfrac{A + jB}{Z(\infty)} = \dfrac{\sqrt{A^2+B^2}}{Z(\infty)}\, e^{j \tan^{-1} B/A}$.

This means that $Ri(s)$ is real for a specific k . A balanced bridge was obtained in which the degrees of the resulting impedances were unchanged. The procedure was then repeated for $k_o = a + jb$, and another bridge was obtained with reduced degrees of the resulting impedance. The quite complicated method led to a series of iterated bridges. Many circuit elements were required.

12. HAZONY (1959) AND YOULA (1961)

In 1959 Hazony[14, 15] obtained a new and basic insight: Why not use Richards' theorem twice when $\text{Ev}\, Z(s) = 0$ had a complex root: first for $k_1 = a + jb$, and then for $k_2 = a - jb$. This led to a generalization of Richards' theorem. Thus, the following cases were obtained:

(a) k real. Yields a balanced bridge as shown above. The network was simplified in cascade representation by Hazony and Schott.[16] They used a gyrator as shown in Figure 12. If k satisfies $\text{Ev}\, Z(s) = 0$, the degrees of the numerator and the denominator are lowered by one.

(b) k complex. $k_1 = a + jb$; $k_2 = a - jb$

$$R_1(s) = \frac{\left(1 + s^2 \frac{A}{abB}\right) \cdot Z(s) - s\, \frac{Z(a) \cdot Z(b)}{abB}}{\left(1 + s^2 \frac{B}{abA}\right) - sZ(s) \cdot \frac{1}{abA}} \tag{16}$$

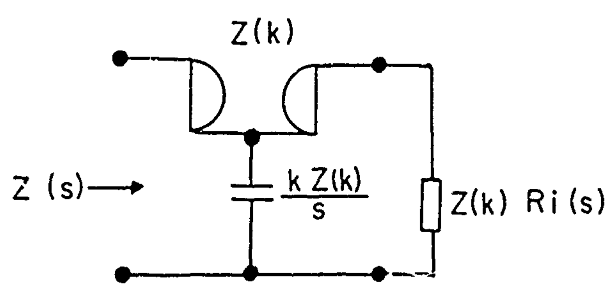

Figure 12. Cascade Representation of Balanced Bridge

$$\text{where} \quad \left.\begin{aligned} A &= \frac{aZ(b) - bZ(a)}{a^2 - b^2} \\ B &= \frac{aZ(a) - bZ(b)}{a^2 - b^2} \end{aligned}\right\} .$$

The network, Figure 13, was obtained by a Darlington type synthesis.[8, 17] If k_1 and k_2 satisfy $\text{Ev}\, Z(s) = 0$, the degrees of the numerator and the denominator are lowered by two.

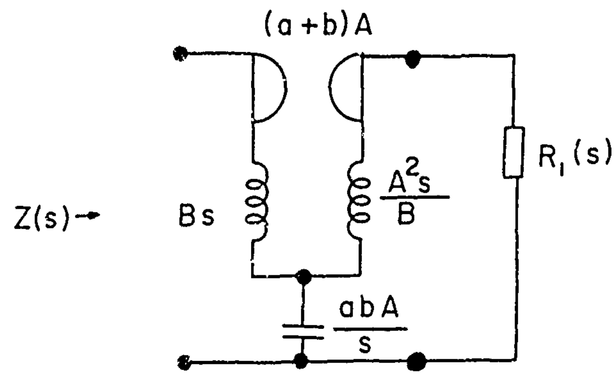

Figure 13. Network Obtained by Applying Richards' Theorem Twice

(c) k imaginary. $k_1 = jb$; $k_2 = -jb$. The gyrator is eliminated. Z(s) is minimum resistive. A Brune network is obtained by using l'Hospital's rule:[8]

$$\left. \begin{aligned} A &= \frac{j\omega_0 Z'(j\omega_0) - Z(j\omega_0)}{2j\omega_0} \\ B &= \frac{j\omega_0 Z'(j\omega_0) + Z(j\omega_0)}{2j\omega_0} \end{aligned} \right\} .$$

(See Figure 14.) If k_1 and k_2 satisfy Ev Z(s) = 0 , the degrees of the numerator and the denominator are lowered by two.

The method of using Richards' theorem and the gyrator artificially as an intermediate step has also been worked out by Youla.[18] Youla adds to the usefulness of the method by giving a "cookbook" for the different cases so that more complicated networks can be calculated by computers.

The gyrator is a highly idealized circuit element and it can be eliminated by repeating the method using the same k-values. If (a) k is real, Darlington's C network is obtained, and if (b) k is complex, Darlington's D network is obtained.[19,20] (See Figure 15.) If k_1 and k_2 are double roots of Ev Z(s) = 0 , the degrees of the numerator and the denominator are lowered by two in the Brune case, by four in the D network case, and by two in the C network case.

13. GENERALIZATION TO N-PORTS

In 1961 Hazony and Nain[21] extended the above results to n-ports by considering positive real matrix functions instead of scalar functions. So, for example, Eq. (11) was generalized to

$$[\,Z(s)\,] = k \cdot \frac{k[\,Z(s)\,] - s[\,Z(k)\,]}{k^2 - s^2} + s \cdot \frac{k[\,Z(k) - s[\,Z(s)\,]}{k^2 - s^2}$$

$$= [\,Z_1(s)\,] + [\,Z_2(s)\,] \quad . \tag{17}$$

A pr matrix is defined in the following way:[8]

(a) $[\,Z(s)\,]$ is an n by n symmetric matrix.

(b) The matrix element Z_{pq} is a rational function of s with real coefficients.

(c) For any choice of real numbers $n_1, n_2, \ldots n$ the associate function $Z(s)$ defined by the follo\ ing equation is prf:

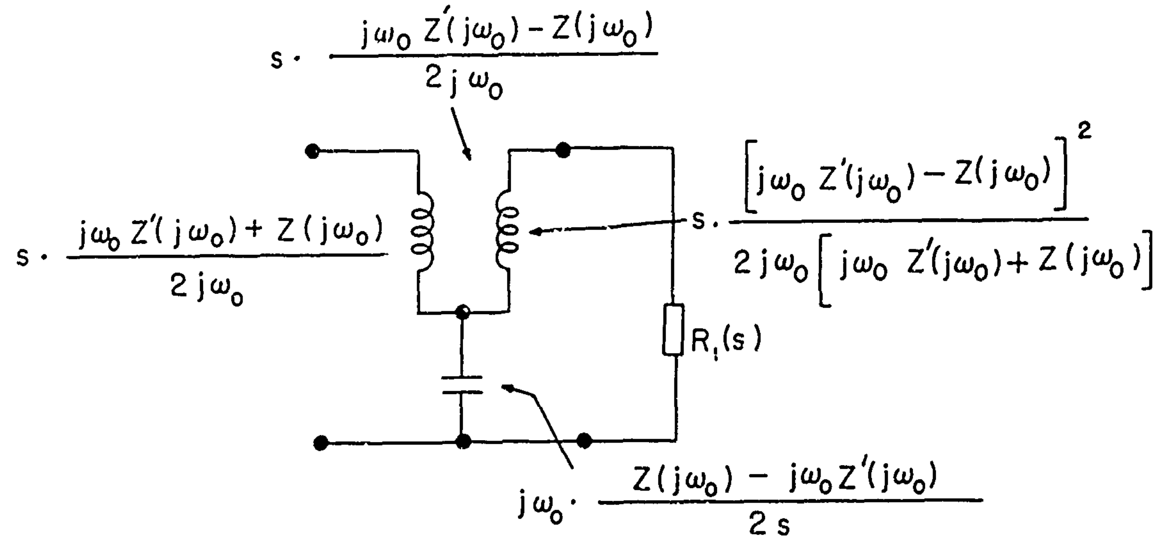

Figure 14. Brune Network

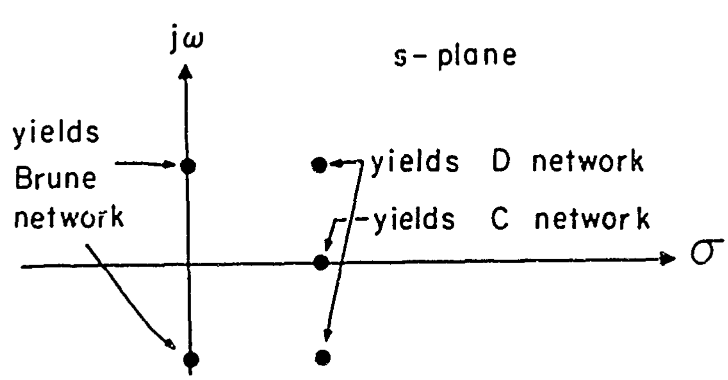

Figure 15. Positions of Roots of Ev Z(s) = 0 in the s-plane

$$Z(s) = \sum_{1}^{n}\sum_{1}^{n} z_{pq}\, n_p\, n_q \equiv [N]_t\, [Z(s)]\, [N]$$

with $[N]$ being a column matrix.

The generalized Richards' theorem is as follows:

If $[Z(s)]$ is a pr matrix, then

$$[Ri(s)] = [Z(k)]\,(k[Z(k)] - s[Z(s)])^{-1}\left(k[Z(s) - s[Z(k)]\right) . \tag{18}$$

is a pr matrix. Synthesis through the matrix Richards' transformation has been performed by the team consisting of E. K. Boyce, R. V. Duffin, H. V. Nain, and D. Hazony. (See Reference 8.) In this work, Ev $Z(s) = 0$ is replaced by det Ev $[Z(s)] = 0$.

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