LARGE BOOSTER WEIGHT COMPARISON,
GLASS FILAMENT AND STEEL

by

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Steel and glass filaments have been the construction materials for solid propellant rocket chambers of a 3 to 6 foot diameter range produced for four missile systems. Chambers as large as 22 foot diameter currently of interest for boosting space vehicles should use the construction material that makes the boosters (1) more reliable and (2) less costly and/or minimum weight. In the absence of data on reliability and cost, weight comparison of the two construction materials is discussed in detail for (a) present size missile chambers and (b) a proposed very large booster. A booster of 260 inch diameter and 100 feet long between end dome tangency planes made from 18 NiCoMo 250 KSI yield strength steel may be expected to weigh more than a glass filament booster unless the winding problems associated with the large size are not readily solved.

INTRODUCTION

From the viewpoint of chamber diameter, solid rocketry spent its "infancy" with JATO bottles, is now completing its "puberty" with four missile systems of one, two and three stages as it spanned the three-to-six foot range, and contemplates "manhood" as space vehicle boosters of ten, thirteen, twenty-two feet and perhaps larger. From the viewpoint of construction material, the "infancy" was spent with steel of less than 150 KSI yield strength and the "puberty" was spent trying to outgrow steel of 190-210 KSI yield strength for weight reduction and learning about fracture toughness because of premature failures while it also experimented with glass filaments in the hope of achieving a weight reduction over its steel chambers.

While glass has been replacing steel in present missiles, chamber diameters, propellant volume, propellant composition, propellant combustion pressure, and chamber operating pressure are essentially unchanged. With equal structural requirements for the glass and steel chambers, a report of weight comparison could be useful to the industry to guide its selection of chamber construction material as it progresses to its next, larger diameter, phase. In the absence of this comparative information, the industry is about to undertake a giant forward step to the ten foot to twenty-two foot diameter phase, carrying forward both construction materials.

A choice would be desirable from the cost viewpoint and from the reliability viewpoint as well as from the standpoint of inert weight. Until more comprehensive, comparative information becomes available, the present authors consider it important to indicate what weight comparison may be expected for very large boosters currently in the national consciousness. While reducing chamber weight for a booster stage is not as effective from the standpoint of mission performance as analogous savings in the remaining flight structures, it is a sound engineering principle to avoid any
unnecessary weight penalty.

Present state-of-the-art and the art expected to be developed for a large booster are separately discussed for steel and for glass filament construction. Weight comparison is specifically made for a 260 inch diameter chamber of 1200-inch cylindrical length between end dome tangency planes.

THE STEEL CHAMBER

PRESENT STATE-OF-THE-ART

The four missile systems which now use steel chambers, or did earlier in their program, are Minuteman, Nike-Zeus, Pershing, and Polaris. Each has produced and tested a sufficiently large number of chambers to lend assurance to the satisfactoriness of the product. The state-of-the-art of producing this satisfactory product is described in terms of the factors affecting their weight, namely, material and wall-thickness, strength level, and burst factor of safety.

Material and Wall Thickness. The four missile programs have used four different steel grades, namely SAE4340, D6, AMS6434 and H-11. All have been used at a wall-thickness less than 0.200 inch and at a uniaxial yield strength level of 190-210 KSI. The strength level is produced by austenization and tempering of a fully assembled chamber.

The acceptable performance of all grades is presumed to reflect a satisfactory fracture toughness quality in each grade at the stated strength level and thickness, even though laboratory measurement of the fracture toughness property shows rather widely different values between the four steel grades.

Proof pressure testing has been at developed hoop stress equal to or less than the uniaxial yield strength. One missile system has had some proof pressure testing performed satisfactorily at developed hoop stress equal to the biaxial yield strength.

Burst Factor of Safety. Associated with the uniaxial yield strength level of 190-210 KSI, these steels possess uniaxial ultimate strength at the 240-260 KSI level. On the basis of the proof pressure equivalent to uniaxial yield strength or less, these chambers have had a burst factor of safety of 24 percent or more. On the basis of the biaxial burst strength (Ref. 1), the burst margin of safety would appear to be 42 percent. It is important to remember that a zero yield margin is indicated by these numbers for the uniaxial and 15 percent for the biaxial basis.

PROPOSED ART FOR LARGE BOOSTER

Steel Selection. Producing the presently used 190-210 KSI yield strength level by austenization and tempering of a fully assembled chamber is not possible for a 260 inch diameter, 1200 inch long chamber unless someone considers making a heat treatment system for such a size work piece. Utilizing any of the presently employed steel grades at the stated strength level as heat treated parts, assembled by welding without any subsequent post-weld heat treatment (other than local stress relief),
would be possible if land regions were provided adequately thickened in proportion to the strength degradation resulting from welding. This practice is strongly not recommended. The proof test pressure of 960 psig currently associated with booster propellant technology would require a cylinder membrane thickness of about 0.65 in. and a weld region land thickness approximating 1.1 inch. Fracture toughness of the currently used steels at a 190-210 KSI yield strength level is believed to be very low in one-inch thick plate. Its use is not recommended unless adequate fracture toughness evaluation indicates acceptability.

Our choice of material is a maraging steel for the large chamber so that only local aging in the weld regions need be performed; thus a furnace large enough to heat treat the entire vessel will not be required. The 18 NiCoMo maraging steels most feasibly applicable to the local aging procedure can produce three orders of yield strength: 190, 250, and 300 KSI.

The 200 KSI yield strength grade has been found to possess exceptionally high fracture toughness at a thickness commensurate with the chamber size of interest. However, it is certain to result in a chamber weight considerably in excess of a glass chamber. Accordingly a weight calculation is not presented. The reader could make his own, following Appendix A. The 300 KSI yield strength grade has not yet been shown to possess a fracture toughness value sufficiently high to expect reliable performance with it. While a minimum weight steel chamber would be predicated with this strength level, its weight is likewise not presented. Only the 250 KSI yield strength grade is discussed further here.

Since strength level is attained by local aging of weld regions, it is assumed that only 90 percent joint strength is attained thereby. While some reports may indicate a higher value, the 90 percent value is believed not to be overly optimistic. For this value of welded joint strength, manufacturing procedure will make necessary procurement of chamber stock 10 percent thicker than required and this excess weight will be removed (mechanically or chemically) from all but two inches of each edge. Weight calculations are made on the basis of using unit pieces 120 inch by 400 inch for cylinder and end dome portions of a chamber.

Burst Factor of Safety Unlike the steels currently used for chambers, the maraging steels have ultimate strengths only slightly higher than their yield strengths. With little margin between yield and ultimate, only 4 percent, burst margin of safety for the large booster is taken to be equivalent to yield margin of safety. This should not be mistaken for a change from present state-of-the-art as it is only a reflection of the yield/ultimate relationship of the maraging steels. A burst margin of safety of 25 percent is assumed for the large booster on the biaxial basis. This is more optimistic than present practice since it is equivalent to an actual burst margin of 29 percent (with the yield/ultimate margin) in comparison with present practice of 42 percent or more. Justification for this position rests in the fact that the 25 percent margin is actually on yield, a much greater margin than in current practice.

Chamber Weight As is shown in Appendix A, the steel chamber weight could be minimized if the wall thickness of the end domes were tapered to
correspond to the stress variation along the meridian. This is not done and, accordingly, the steel chamber weight to be presented could be further reduced. Final design parameters of the large motor case are summarized in Table I. On the basis of Appendix A calculations, the steel chamber described may be expected to weigh 172,200 lbs.

THE GLASS CHAMBER

PRESENT STATE-OF-THE-ART

Because of the resulting effect on chamber weight, glass chambers may be considered to be of two categories: with off-axis openings for multiple nozzles or thrust termination, or without off-axis openings.

Chambers without off-axis openings can more readily be produced at minimum weight - with no more longitudinal windings than theoretically required - than chambers with off-axis openings. The glass chambers produced for missile systems have been mostly off-axis opening chambers. Present state-of-the-art is described in terms of such chambers. A mental note should be carried that a center-opening chamber should not be of poorer quality.

Unlike the steel chambers for which many steel grades and strength levels were available for selection, only "E" glass filaments with the "801" finish were first available for chamber production. For those familiar with the very early state-of-the-art of glass chambers when burst occurred at composite hoop stress values under 80 KSI, progress will be indicated as present state-of-the-art is discussed in terms of the factors which affect chamber weight.

Cylinder Wall Composite Hoop Strength

Rovings in present use are either 20 or 12 end, each end comprising 204 monofilaments of 3.6 x 10^-10 inch diameter collected together in the filament drawing process. Filament finish in most common use now is designated as "HTS" by the filament manufacturer, who applies this finish during the fiber forming process to enhance the bond to be formed between the glass and the epoxy resin system binders used in the final composite structure, and also to protect the glass filament surface from environmental attack. The "S-994" glass in current use differs in composition from the "E" glass formerly used. The newly developed S-994 glass/HTS finish filaments exhibit 30 percent higher strength than E glass/HTS finish filaments. Comparative values are presented in Table II.

Epoxy resin systems differ between chamber manufacturers as to composition and method of application to the glass filaments (pre-impregnation or wet-winding). However, quantity of resin is quite generally 18 percent by weight, 33 percent by volume. Final composite density is generally of the order of 0.073 lb/in^3, with the S-994 glass having a density of 0.086 lb/in^3 and the epoxy resin a density of 0.042 lb/in^3. A glass volume fraction greater than 0.70 is considered to be indication of too much binder depletion for adequate load transfer between filaments.

Design of filament winding patterns for filament-wound pressure vessels depends upon factors related to design allowable ultimate filament strength, filament stress balances in the hoop and longitudinal directions,
### TABLE I

PARAMETERS FOR LARGE STEEL MOTOR CASE OF 260 IN. DIA. AND 115 FOOT LENGTH

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder Diameter, In.</td>
<td>260</td>
</tr>
<tr>
<td>Overall Length, In.</td>
<td>1384</td>
</tr>
<tr>
<td>Cylinder Length, In.</td>
<td>1200</td>
</tr>
<tr>
<td>Length-to-Diameter Ratio</td>
<td>5.3</td>
</tr>
<tr>
<td>Forward Port Opening Diameter, In.</td>
<td>74</td>
</tr>
<tr>
<td>Aft Port Opening Diameter, In.</td>
<td>108</td>
</tr>
<tr>
<td>Internal Volume, V, in³</td>
<td>70 x 10⁵</td>
</tr>
<tr>
<td>Proof Pressure, Pp, psig</td>
<td>960</td>
</tr>
<tr>
<td>Ultimate Design Pressure, Pp, psig</td>
<td>1200</td>
</tr>
<tr>
<td>Proof/Ultimate Pressure Ratio</td>
<td>0.80</td>
</tr>
<tr>
<td>Burst Factor of Safety</td>
<td>1.25</td>
</tr>
<tr>
<td>Cylinder Wall Hoop Stress at Burst, psi</td>
<td>288,100</td>
</tr>
<tr>
<td>Cylinder Wall Hoop Stress at Proof Pressure, psi</td>
<td>231,000</td>
</tr>
<tr>
<td>Cylinder Wall Thickness, In.</td>
<td>0.539</td>
</tr>
<tr>
<td>Dome Wall Thickness, In.</td>
<td>0.420</td>
</tr>
<tr>
<td>Steel Density, lb/in³</td>
<td>0.289</td>
</tr>
<tr>
<td>Total Structural Weight, Lbs</td>
<td>172,200</td>
</tr>
</tbody>
</table>

### TABLE II

GLASS FILAMENT COMPOSITION AND PROPERTIES

<table>
<thead>
<tr>
<th>Glass Composition</th>
<th>Glass S-994</th>
<th>E-Glass HTS</th>
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</thead>
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<tr>
<td>(wt%)</td>
<td>S-994</td>
<td>E-Glass</td>
</tr>
<tr>
<td>SiO₂</td>
<td>65</td>
<td>52-56</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>25</td>
<td>12-14</td>
</tr>
<tr>
<td>CaO</td>
<td>16-25</td>
<td></td>
</tr>
<tr>
<td>MgO</td>
<td>10</td>
<td>0-6</td>
</tr>
<tr>
<td>B₂O₃</td>
<td>8-13</td>
<td></td>
</tr>
<tr>
<td>Na₂O, K₂O</td>
<td>1-4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>S-994 Glass HTS</th>
<th>E-Glass HTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Gravity</td>
<td>2.49</td>
<td>2.64</td>
</tr>
<tr>
<td>Ultimate Tensile</td>
<td>650,000 psi</td>
<td>486,000 psi</td>
</tr>
<tr>
<td>Strength (Single</td>
<td>700,000 psi</td>
<td>500,000 psi</td>
</tr>
<tr>
<td>Virgin Fiber</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young's Modulus (Single Fiber)</td>
<td>12x10⁶ psi</td>
<td>10.5x10⁶ psi</td>
</tr>
<tr>
<td>NOL-Ring Ultimate Filament Strength</td>
<td>450,000 psi</td>
<td>350,000 psi</td>
</tr>
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</table>
type of dome contour selected, and required winding angle for filaments passing over the end domes. All of these factors combine to result in a design allowable cylinder wall composite stress level for the vessel. Interrelation of these parameters of vessel design is found in Appendix B. Chamber weight for fixed geometry and ultimate pressure level is shown to become a function of the longitudinally (or helically) wound-to-hoop wound composite stress balance and the ultimate cylinder wall strength.

Currently, winding patterns are usually designed such that longitudinal filaments over the chamber end domes develop from 75 to 95 percent of the strength exhibited by the hoop filaments. Selection of this stress balance is dependent upon exact vessel configuration, particularly porting requirements. The filament stress balance and meridional winding angle in turn fix the composite stress balance factor (Fig. B-4 of Appendix B) used for determining weight of the vessel.

Large scale polar ported chambers, efficiently balanced in hoop-to-longitudinal filament ratios, have burst at composite cylinder wall hoop stresses of 120 KSI with E glass and 150 KSI with S-994 glass. Corresponding stresses in the hoop filaments are 260-300 KSI for E glass chambers and 325-375 KSI for S-994 glass chambers. Representative data on full-scale chamber experience is presented in Table III. Specific motor chamber configurations, wall thicknesses, winding pattern details, type of construction, and strength levels are summarized. Cylinder wall composite thickness of these missile system chambers has not been greater than 0.36 inches. In general, vessels described in Table III received a proof pressure cycle at 70-80 percent of ultimate load for at least 60 seconds prior to the burst test phase.

Burst Factor of Safety Many programs employing glass filament-wound pressure vessel components use proof pressure testing at high stress levels, compared to the ultimate vessel strength, as the basis for vessel qualification. Present industry practice with glass filament-wound rocket motor chambers is to use a safety factor of 1.25. For high-performance (high operating stress level) vessels of this type, it is important to observe that currently no reliability level concerning design requirements, can be stated from qualification proof testing alone. This conclusion arises from certain strength-degradation phenomena noted in glass filament-wound vessels subjected to very few cycles and/or relatively short durations at high loads, as compared to the single cycle ultimate vessel strength.

Current practice of adopting an ultimate-to-limit safety factor of 1.25 is rather arbitrary and difficult to justify. Nonetheless, it enjoys such a general acceptance that any reduction of the current value and the consequent weight saving will require a thorough analysis of the available experimental evidence of strength distributions, of the factors influencing such distributions, and of the means (quality control) by which the reliability of the product can be improved. Closely related to this problem are those connected with proof testing, such as the question of the value of proof test as evidence of reliability of those articles which pass it, and the relative merits of a single application of proof pressure and of multiple applications.
<table>
<thead>
<tr>
<th>Chamber Description</th>
<th>Contractor</th>
<th>Contract Number</th>
<th>Nominal Chamber Diameter, in.</th>
<th>Approx. Overall Chamber Length, ft.</th>
<th>Cylinder Wall Thickness, in.</th>
<th>Win. Ang. $\alpha$, degree</th>
<th>Type Port Opening</th>
<th>Longitudinal-To-Hoop Filament Stress Balance $\eta$</th>
<th>Type Glass (HTS Finish)</th>
<th>Ultimate Cylinder Wall Hoop Stress, psi</th>
<th>Cylinder Strength to Density, in.-lb/ft</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polaris A3 First-Stage Typical Performance</td>
<td>Aerojet General Corp.</td>
<td>NOw-63-0050C</td>
<td>54</td>
<td>14</td>
<td>0.320</td>
<td>5' Planar</td>
<td>X</td>
<td>0.92</td>
<td>X</td>
<td>100,000</td>
<td>1.32 x 10^6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>54</td>
<td>14</td>
<td>0.320</td>
<td>5' Planar</td>
<td>X</td>
<td>0.92</td>
<td>X</td>
<td>120,000</td>
<td>1.64 x 10^6</td>
<td>2</td>
</tr>
<tr>
<td>Polaris A3 Second-Stage Typical Performance</td>
<td>Hercules Powder Company/Allegany Ballistics Laboratory</td>
<td>NOrd 16640</td>
<td>54</td>
<td>6.5</td>
<td>0.140</td>
<td>12° Helical</td>
<td>X</td>
<td>0.83</td>
<td>X</td>
<td>100,000</td>
<td>1.32 x 10^6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>54</td>
<td>6.5</td>
<td>0.140</td>
<td>12° Helical</td>
<td>X</td>
<td>0.83</td>
<td>X</td>
<td>135,000</td>
<td>1.85 x 10^6</td>
<td>2</td>
</tr>
<tr>
<td>Minuteman First-Stage S/N TU-226A</td>
<td>Thiokol Chemical Corp.</td>
<td>AF33(600) 42511</td>
<td>65</td>
<td>20</td>
<td>0.355</td>
<td>20° Helical</td>
<td>X</td>
<td>0.75</td>
<td>X</td>
<td>106,900</td>
<td>1.53 x 10^6</td>
<td>3</td>
</tr>
<tr>
<td>Minuteman Second-Stage S/N 1</td>
<td>Aerojet General Corp.</td>
<td>AF33(616) 8442</td>
<td>44</td>
<td>11</td>
<td>0.150</td>
<td>5' Planar</td>
<td>X</td>
<td>0.76</td>
<td>X</td>
<td>125,000</td>
<td>1.67 x 10^6</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>44</td>
<td>11</td>
<td>0.136</td>
<td>5' Planar</td>
<td>X</td>
<td>0.78</td>
<td>X</td>
<td>118,000</td>
<td>1.58 x 10^6</td>
<td>4</td>
</tr>
<tr>
<td>S/N 2</td>
<td></td>
<td></td>
<td>44</td>
<td>11</td>
<td>0.103</td>
<td>5' Planar</td>
<td>X</td>
<td>0.80</td>
<td>X</td>
<td>155,000</td>
<td>2.10 x 10^6</td>
<td>5</td>
</tr>
<tr>
<td>S/N 3</td>
<td></td>
<td></td>
<td>44</td>
<td>11</td>
<td>0.207</td>
<td>21° Helical</td>
<td>X</td>
<td>0.89</td>
<td>X</td>
<td>126,700</td>
<td>1.71 x 10^6</td>
<td>3</td>
</tr>
<tr>
<td>Minuteman Second-Stage SN TU-227A</td>
<td>Thiokol Chemical Corp.</td>
<td>AF33(600) 42511</td>
<td>44</td>
<td>11</td>
<td>0.105</td>
<td>21° Helical</td>
<td>X</td>
<td>0.90</td>
<td>X</td>
<td>121,000</td>
<td>1.62 x 10^6</td>
<td>3</td>
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<tr>
<td>S/N 1</td>
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<td></td>
<td>44</td>
<td>11</td>
<td>0.097</td>
<td>21° Helical</td>
<td>X</td>
<td>0.90</td>
<td>X</td>
<td>150,000</td>
<td>2.08 x 10^6</td>
<td>3</td>
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<tr>
<td>S/N 2</td>
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<td></td>
<td>44</td>
<td>11</td>
<td>0.105</td>
<td>21° Helical</td>
<td>X</td>
<td>0.90</td>
<td>X</td>
<td>150,000</td>
<td>2.08 x 10^6</td>
<td>3</td>
</tr>
</tbody>
</table>
Proof test qualification is only meaningful for detection of weak vessels containing some gross manufacturing flaws and not possessing the strength characteristics established for the design. Successful completion of a proof test cycle does alone not ensure that the vessel will sustain a subsequent pressure cycle to a high load level. Assurance of reliability for the vessel comes only from statistical interpretation of performance results and quality control practices.

Once the design is proven, and all variables frozen, satisfactory manufacturing methods must continue to be employed for assurance that subsequent vessels belong to the design configuration family. Throughout any program, rigid standards for materials acceptance and for manufacturing control in fabricating cases to the proven design configuration are required. The program manager must conduct analyses of (1) test results which fix the degradation characteristic of his particular design and (2) quality control procedures to see if there is any advantage to a regular proof test cycle for each vessel fabricated.

The effect of strength degradation on structural reliability (the probability that a structure will function properly in its environment) has been studied (Ref. 6). When the pressure load and motor case strength are considered to be independent normally distributed random variables, reliability becomes a function of mean chamber strength, standard deviation of chamber strength, mean pressure load, standard deviation of pressure load, proof test pressure for the motor case, and degradation of the motor case strength occurring during and after the proof test. The solution of the reliability relationship for propellant and structural parameters of the Polaris A3 first-stage motor case is presented as Fig. 1.

Because proof testing insures that motor-case strength lies above all but the most extreme possible loads, the curve in Fig. 1 for 0 percent degradation shows high reliability for all factors of safety. The most important feature of the curves in Fig. 1 is the sharp drop off in reliability for factors of safety less than 1.2 percent when the strength degradation is greater than 2 percent. If strength degradation of the composite structure occurs during or after proof testing, motor strengths may fall into the region of more probable loads and the reliability level for the vessel decreases rapidly.

End Dome Contour. Dome contours are elliptical or ovaloid shapes designated variously as geodesic ovaloid (geodesic isothensoid), balanced-in-plane, and zero hoop stress. The specific dome contour and winding pattern for a particular vessel depends upon the length of the vessel and/or the ratio of polar opening diameters to cylinder diameter. Highest performance levels are achieved with geodesic ovaloid and balanced-in-plane contours. For chambers with small center bosses or with large length/diameter ratio, good performance levels have been obtained with zero hoop stress contours.

Planar winding patterns are used in conjunction with balanced-in-plane or zero-hoop stress dome profiles. For geodesic ovaloid head contours helical winding patterns are required.
1.00  
0.99  
0.98  
0.97  
0.96  
0.95  
0.94  
0.93  
1.00  
1.05  
1.10  
1.15  
1.20  
1.25  
1.30  
1.33  
0% DEGRADATION  
2% DEGRADATION  
5% DEGRADATION  
FACTOR OF SAFETY  
RELIABILITY  
WHERE
L = PRESSURE LOAD
S = CHAMBER STRENGTH
D = PERCENT DEGRADATION OF MEAN STRENGTH
K = PROOF TEST PRESSURE MINUS $\mu_S D/100$
N = NORMALIZING FACTOR WHICH ACCOUNTS FOR THE MOTOR CASES LOST IN PROOF TEST

$\mu_S = \text{MEAN STRENGTH OF THE MOTOR CASE STRENGTH, SELECTED AS 80 psi}$
$\sigma_S = \text{STANDARD DEVIATION OF THE MOTOR CASE STRENGTH, SELECTED AS 80 psi}$
$\mu_L = \text{MEAN LOAD SELECTED AS 900 psi}$
$\sigma_L = \text{STANDARD DEVIATION OF THE LOAD SELECTED AS 30 psi}$

FIGURE 1. RELIABILITY VS FACTOR OF SAFETY FOR VARIOUS STRENGTH DEGRADATIONS
With in-plane winding patterns, longitudinal filaments are applied as successive layers of an in-plane pattern of roving tapes laid side-by-side in the cylinder section and tangent to forward and aft polar bosses. Upon completion of longitudinal winding, hoop windings are applied in a side-by-side pattern of tapes which are wound back and forth along the length of the cylinder section until required composite thickness is obtained. Winding of the geodesic ovaloid pattern does not allow side-by-side orientation of helically wound roving tapes. A large number of winding mandrel revolutions are required for each two-layer helical pattern wound over the mandrel surface. Current practice with this winding pattern has been to intersperse helical and hoop windings. Over each two-layer helical pattern, a hoop filament layer is applied followed by a subsequent helical pattern. This sequential winding pattern is concluded by application of several remaining hoop layers upon the last helical pattern.

Filaments in the end domes and along the cylinder do not reach the stress levels found in hoop-wound filaments. In current pressure vessels, helically or longitudinally wound filaments develop about 50-95 percent of the strength exhibited by hoop-wound filaments. In optimized designs with minimum weight and strength redundancy, winding patterns are tailored so that filament stress balance in the composite structure is in about this proportion.

Winding Mandrels Large radial and longitudinal pressures are developed on the mandrel during winding. Rigidity and dimensional stability have been determined to be prime considerations in the selection of a satisfactory mandrel system. From the design standpoint, the mandrel material and configuration must be a compromise of the following basic requirements:

- Strength and modulus adequate to maintain winding tensions at ambient and curing temperatures
- Weight distribution and mandrel reinforcement placement optimized to reduce deflections
- Reliability and ease of mandrel removal without damage to the chamber.

In the development of suitable winding mandrel systems for motor cases, many concepts and materials have been explored. For motor cases with diameters less than 6 feet plaster, metal-segmented and collapsible, or plaster/metal segmented combinations have been required to give the winding mandrel strength and rigidity needed for successful manufacture of high performance vessels.

Winding Tension The proper choice and the careful control of the filament tension is an important aspect of the winding operation. Clearly, a certain amount of tension is necessary to avoid slackness in the filaments. In fact, winding tension as high as 5-10 percent of the ultimate glass roving strength has been shown to be an important factor in achieving best glass strength levels in filament-wound composites. On the other hand, tension in filaments deposited on a curved surface generates
compression in the already deposited layers and in the mandrel. The possibility of developing slackness in the inner filament layers due to the tension in successively wound outer layers exists and depends, among others, on the stiffness of the mandrel. Excessive mandrel deflection during winding and resin cure at elevated temperatures may produce complete loss of tension in inner layers of filaments. It is evident that quite rigid winding mandrels, with respect to the pressure exerted upon the mandrel by the filaments, are required. Additionally, programming of filament tension during winding of successive layers is required to obtain essentially constant residual stress in the filaments of the composite wall of finished chambers.

PROPOSED ART FOR LARGE BOOSTER

The large booster should be made with S-994 glass filament with the HTS finish. In the large diameter chamber this filament can be expected to perform as effectively with an epoxy resin binder as it has performed in the present-day smaller diameter chambers. Wall thickness will, of course, far exceed the present-day 0.36 inches and may, in fact, approach 2.5 inches. Efficiency is not expected to be as high in the very large chamber, as is further discussed here. However, the assumption is made that a longitudinal filament stress/hoop filament stress ratio of at least 0.80 will be attained in the large chamber essentially as it is at present in much smaller ones.

Selection of the proper resin system for use in the large filament-wound structure is an important consideration. Resin system properties of strength, elongation, and toughness are known to be important factors influencing performance of glass filaments in the composite. Accurate control of resin content near a 33 volume percent during winding will be required for fabrication of the thick-walled vessel since (1) excessive resin content may adversely effect the programmed tension control during winding; (2) winding times for vessel fabrication will be long; consequent advancement and partial cure of the resin during winding will not allow bleedout of excess resin; and (3) resin-dry regions of the structure will not perform efficiently. It is possible that a room temperature curing resin may become necessary.

Winding Mandrel For the 260-inch diameter, 100 foot cylindrical length chamber, the winding mandrel may be expected to be a major problem. The inward deflection of mandrel surface from filament-winding pressure increases rapidly with increasing mandrel diameter, and the effects of this are much greater for a large diameter, thick-walled chamber than for small diameter, thinner-walled chambers now manufactured; mandrels should be made even more rigid for the large structure. It is strongly recommended that a greater amount of emphasis be placed on analysis of the effect of winding tension on mandrel deflection, the effect of mandrel compressibility on the wound structure, and on proper control of tension during chamber manufacture.

Wide flanged beams welded to a steel tube shaft and covered with a reinforced plaster shell seemed to serve satisfactorily for a somewhat smaller chamber of 156 inch diameter and 25 foot length (Ref. 7). Other development work is expected to adequately solve the mandrel problem.
Winding Tension  One of the main problems expected for internally pressurized thick-walled filament-wound structures arises from stress concentrations which may be developed from unequal tensions on individual filaments of the composite wall. Programming of roving tension during winding will be required to account for inward deflection of the mandrel during winding and effects of resin system cure in order to produce a nearly constant final stress in the filaments located across the composite wall section. Since the wall of the composite is composed of many layers of glass roving, then each layer can be individually treated to induce stress into that particular layer by varying the tension in the glass roving as it is applied. Use of higher tensions on the inner layers and reduction of tension toward the outer plies should produce a uniform stress distribution throughout the laminate cross-section. By handling each layer as a separate unit, a composite wall can be built up which contains a minimum stress concentration. Analytical, fabricational, and testing experience is being gained on current programs (Ref. 8).

End Dome Contour  Data generated for vessels wound with geodesic or in-plane winding contours do not indicate any superiority for either type of winding applied to the large chamber with polar openings less than 40 percent of the chamber diameter. It appears that dome contour selection thus should be based upon winding machine design concepts and manufacturing experience rather than any expected performance advantage. For purposes of weight computations, an in-plane type winding pattern is selected for the vessel. Based on the chamber length-to-diameter ratio of 5.2 and average polar port opening size 37 percent of the chamber diameter, the longitudinal-in-plane winding angle is 4 degrees.

Burst Factor of Safety  As already discussed, current filament-wound motor case designs are based on a burst factor of safety (ultimate-to-proof pressure ratio) of 1.25 or a proof pressure level 80 percent of the design or demonstrated ultimate vessel strength. The high cost of extremely large motor cases does not allow an extensive structural test program to demonstrate with any degree of reliability ultimate vessel strength. Therefore, more conservative approaches, based on current state-of-the-art and engineering judgment, must be taken to establish both a reasonable margin between operating level and ultimate vessel strength and a design ultimate strength.

Data available from the filament-winding industry on the effects of cyclic and static pressure loading, at high stress levels compared to the single pressure cycle strength, for various configuration pressure vessels have been recently summarized and analyzed in Ref. 9. Of the data summarized, much reflected experience obtained with outdated, low-efficiency vessel designs and fabrication techniques for which low strength materials of construction were used. However, using these data, conservative estimates of strength degradation can be made. Based on the information presented, the indication is that no appreciable strength degradation of the composite structure should be anticipated from cyclic or static pressure loading if the ratio of limit operating or proof pressure is 75 percent of the ultimate vessel strength, a burst factor of safety of 1.33.

The effect of designing with this increased expected burst factor of safety on structural reliability can be seen by a re-examination of Fig. 1. This relationship between reliability and burst factor of safety was
developed for typical parameters of solid propellant filament-wound structures operating at 900 psig. The various levels of degradation indicated are assumed to occur during and after the proof pressure test. It is clearly indicated that the increased expected burst factor of safety significantly improves reliability even when large strength degradations are assumed to be caused by proof testing, by handling, and by environmental attack.

Cylinder Wall Composite Hoop Strength and Chamber Weight

Test data for filament-wound vessels larger than 65 inch diameter are not available. Current programs being conducted under Air Force Contract will establish feasibility and performance data for glass filament-wound constructions as large as 260 inch diameter by 60 foot length.

One study program is being conducted to obtain design parameters, winding patterns, fabricational techniques, and methods of stress analysis that will lead to design and fabrication of optimum weight, filament-wound rocket-motor cases. The status of this program was reported by Fred Darms of Aerojet-General in Ref. 4.

An important part of the contract work was determination of scale factors related to such vessel parameters as vessel geometry (polar port to chamber diameter ratio, chamber-length to chamber-diameter ratio), wall thickness, chamber diameter, and off-center ports. Such scale factors have been established and are presented in Ref. 10. Design allowable filament and composite strengths for a vessel are determined by application of these vessel scale factors to the design allowable glass filament roving strand strength (AGC Strand test). The scale factors presented are predicated on using the current degree of manufacturing process control (structural materials, mandrels, winding machines including filament placement and tension control, composite cure, etc.) in conjunction with present methods of structural design analysis.

In the Polaris program average AGC strand strengths of Owens-Corning S-994/HTS finish 20 end roving have been at least 420 KSI with a typical value of 435 KSI. Using the methods of Ref. 10, application of the single nozzle large booster case scale factors to a glass strand strength of 420 KSI results in a design ultimate cylinder wall strength of at least 130 KSI. This result is consistent with the actual performance of motor cases described in Table III.

However, for realistic estimation of expected large motor case strength, it is necessary to account for the other strength influencing factors already discussed above. A fabrication technology introducing new mandrel systems, new winding machines, longer winding times, and problems of resin system cure, mandrel removal, and handling of the completed chamber is required to produce the ultralarge motor cases. It should be anticipated that large chamber performance will be reduced by the adverse influence of some of these factors. The degree to which performance will be effected is not known; the purpose of work now in progress to fabricate and test filament-wound chambers of 156 and 260 inch diameter is to establish the influence of these factors.

It is believed that the influence factor for large scale-up of
vessel dimensions should not reduce the design allowable cylinder wall stress from 130 KSI to below 100,000 psi for S-994 glass/HTS finish construction. This ultimate strength level of 100 KSI in the cylinder, with a composite stress balance of 0.80, produces stresses of only 240 KSI in the hoop filaments and 190 KSI in the longitudinal filaments. For the design burst factor of safety of 1.33, the resulting proof pressure or limit operating pressure cylinder wall stress is only 75 KSI, a conservative value. Based on this cylinder wall strength the final design parameters of the large motor case are summarized in Table IV. For a chamber ultimate cylinder wall strength of 100,000 psi, with the geometry and pressure level requirements stated, a total vessel structural weight of 131,420 pounds is predicted from Appendix B calculations.

**TABLE IV**

PARAMETERS FOR LARGE GLASS FILAMENT-WOUND MOTOR CASE OF 260 INCH DIAMETER AND 113 FOOT LENGTH

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder Diameter, In.</td>
<td>260</td>
</tr>
<tr>
<td>Overall Length, In.</td>
<td>1356</td>
</tr>
<tr>
<td>Cylinder Length, In.</td>
<td>1200</td>
</tr>
<tr>
<td>Length-To-Diameter Ratio</td>
<td>5.2</td>
</tr>
<tr>
<td>Forward Boss Diameter, In.</td>
<td>83</td>
</tr>
<tr>
<td>Aft Boss Diameter, In.</td>
<td>108</td>
</tr>
<tr>
<td>Internal Volume, ( V ), In(^3)</td>
<td>70 x 10(^4)</td>
</tr>
<tr>
<td>Wrap Angle, Longitudinal-in-plane, Degrees</td>
<td>4</td>
</tr>
<tr>
<td>Cylinder Wall Thickness, In.</td>
<td>1.66</td>
</tr>
<tr>
<td>Composite Stress Balance, ( \varepsilon )</td>
<td>0.80</td>
</tr>
<tr>
<td>Filament Stress Balance, ( \gamma )</td>
<td>0.81</td>
</tr>
<tr>
<td>Proof Pressure, ( P_p ), psig</td>
<td>960</td>
</tr>
<tr>
<td>Ultimate Design Pressure, ( P_B ), psig</td>
<td>1280</td>
</tr>
<tr>
<td>Proof/Ultimate Pressure Ratio</td>
<td>0.75</td>
</tr>
<tr>
<td>Burst Factor of Safety</td>
<td>1.33</td>
</tr>
<tr>
<td>Cylinder Wall Composite Hoop Stress at Burst, psi</td>
<td>100,000</td>
</tr>
<tr>
<td>Cylinder Wall Composite Hoop Stress at Proof Pressure, psi</td>
<td>75,000</td>
</tr>
<tr>
<td>Composite Density, lb/in(^3)</td>
<td>0.073</td>
</tr>
<tr>
<td>Glass Fraction in Composite</td>
<td>0.67</td>
</tr>
<tr>
<td>Glass-Resin Composite Weight, lb.</td>
<td>126,500</td>
</tr>
<tr>
<td>Metal Adapter Weight, lb</td>
<td>4,920</td>
</tr>
<tr>
<td>Total Structural Weight, lb</td>
<td>131,420</td>
</tr>
</tbody>
</table>

**DISCUSSION**

It is the authors' opinion that filament winders face more formidable problems stepping up to 260 inch diameters and 100 feet lengths than do metal chamber manufacturers. Weight predictions for glass chambers have, accordingly, been based on a 100,000 psi burst composite hoop stress rather than a more than 130,000 psi value attained to date. If the 100,000 psi value is not attained, it is predicted that this value would have to be reduced to less than 80,000 psi before the glass weight will equal or exceed the steel chamber weight.
REFERENCES


APPENDIX A

STEEL CHAMBERS

Excluding shirts for attachment to other stages, weight of a steel chamber can be expressed in terms of its components: cylinder, end domes (membranes), and bosses to be welded into domes for igniter and nozzle attachment, thus

$$W_T = W_C + W_{D_{1-2}} + W_B$$

where

- $W_T$ = total chamber weight
- $W_C$ = weight of cylinder
- $W_{D_{1-2}}$ = weight of two domes, membranes without welded in bosses
- $W_B$ = weight welded in bosses (adapters)

Cylinder weight is expressed, simply, in terms of the membrane thickness required to support the hoop stress produced by the burst pressure, thus

$$W_C = \pi D_1 t_1 L s$$

where

- $D_1$ = diameter at mean cylinder wall thickness
- $t_1$ = cylinder wall thickness
- $L_s$ = cylinder length between end dome attachment planes
- $s$ = steel density

Cylinder thickness may be expressed in terms of proof test pressure, burst factor of safety, cylinder hoop stress at burst, and biaxial stress ratio,

$$t_1 = \frac{D_1 P_f F_5 \sqrt{1-6 \times t_2^2}}{2 \sigma_b}$$

where

- $t_1$ = cylinder hoop stress at burst pressure
- $P_f$ = burst pressure/proof pressure ratio (burst factor of safety)
- $F_5$ = proof test pressure
- $\sigma_b$ = 0.5 = biaxiality ratio in cylinder wall (Von Mises Criterion)

Similarly, weight of each dome can be expressed in terms of dome membrane thickness, thus

$$W_D = [A_D - A_{I,N}] t_2 \rho$$

where

- $A_D$ = surface area of one dome
Dome thickness may also be expressed in terms of proof test pressure, burst factor of safety, cylinder hoop stress at burst, and biaxiality ratio. A general formula, applicable to any dome shape would be quite complicated. It is suggested that, unless compelling reasons exist for using another shape, the flattest ellipsoidal dome in which no hoop compression stresses are generated by internal pressure loading is the dome contour of choice. Such an ellipse has a minor axis-major axis ratio of 0.707. With such an ellipse, the meridional stress at (igniter or nozzle) boss attachment will be greater than at cylinder attachment. One has the choice of tapering the dome thickness accordingly, for minimum weight, or maintaining the greater polar thickness throughout.

At cylinder attachment, hoop stress is absent by definition, and biaxiality ratio is zero.

\[ t_{2c} = \frac{P_p D_F s}{4 t_0 b} \]

where \[ t_{2c} = t_2 \] at cylinder attachment

At polar boss (igniter or nozzle) attachment, hoop stress can approach meridional stress and biaxiality ratio becomes

\[ b = \left[ 1 - \frac{r_z}{r_i} \right] \]

where

- \( r_z \) = meridional radius at boss attachment
- \( r_i \) = other principal radius at boss attachment

\[ r_z = \frac{4}{0.707 D_i^2} \left[ \left( \frac{D_i}{2} \right)^2 + \left( \frac{D_i}{2} - 0.707 \right)^2 \right] \]

\[ r_i = \frac{1}{0.707} \left[ \left( \frac{D_i}{2} \right)^2 + \left( \frac{D_i}{2} - 0.707 \right)^2 \right] \]

\[ t_{2p} = \frac{P_p r_z F_s \sqrt{1 - b + b^2}}{20 t_0} \]

where

- \( D_{an} \) = diameter of igniter boss opening or nozzle boss opening, each is calculated separately.
- \( t_{2p} = t_2 \) at polar boss attachment
No formula is available for $W_B$. Experience and judgment assign this value for weight calculation purposes.

For the 260 inch diameter chamber of interest with length 1200 inches, igniter boss opening is taken as 74 inches and nozzle boss opening as 100 inches.

For the chamber of interest and the 16 NiCoMo (250) steel grade selected, the following quantities apply:

- $D = 260$ in.
- $L = 1200$ in.
- $D_1 = 74$ in.
- $L_1 = 100$ in.
- $P = 250,000$ psi
- $n_1 = 1.25$
- $p = 0.289$
- $t = 960$ psi

The cylinder weight is determined to be:

From Eq. 3
\[ z_f = 0.539 \text{ in.} \]

From Eq. 2
\[ W_c = 152,100 \text{ lbs} \]

While Eqs. 6 - 9, inclusive, would show a lesser thickness aft dome than the smaller opening forward dome, the aft dome thickness is used for both in the weight calculations to allow for some service loading in the nozzle area not otherwise accounted for. The weight of the two domes is determined to be:

From Eq. 7
\[ v_1 = 173.2 \text{ in.} \]

From Eq. 8
\[ v_2 = 180 \text{ in.} \]

From Eq. 6
\[ z = 0.96 \]

From Eq. 9
\[ w_{0.2} = 0.420 \text{ in.} \]

From Eq. 4
\[ W_{d,2} = 18,000 \text{ lbs.} \]

Total chamber weight is 152,100 plus 18,000 plus 1,100 lbs for $W_B$, the total being multiplied by 1.004 to allow for a 2 in. wide land around all edges of the planned 120 in. by 400 in. plate at 10 percent greater thickness than membrane calculations to allow for 90 percent weld joint efficiency. Final chamber weight is 172,200 lbs.
Excluding skirts and windings or other devices for attachment to the body, chamber weight is the weight of the several components: cylinder and end dome glass filaments and resin binder, reinforcements around openings, and metal adapters for attaching nozzles, igniter, etc.

\[ W_T = W_C + W_D + W_R + W_M \]  \hspace{1cm} (1)

where

- \( W_T \) = total weight
- \( W_C \) = weight of cylinder, glass filaments and resin
- \( W_D \) = weight of forward and aft dome glass filaments and resin
- \( W_R \) = weight of reinforcements separately added around openings in domes
- \( W_M \) = weight of metal adapters.

Weight of cylinder can be expressed in terms of composite cylinder wall thickness,

\[ W_C = \pi D_C L_C t_1 \sigma \]  \hspace{1cm} (2)

or, in terms of cylinder wall composite hoop stress resulting from an internal pressure,

\[ W_C = \frac{\pi D_C^2 L_C \sigma P}{20_1} \]  \hspace{1cm} (3)

where

- \( D_C \) = mean diameter of complete vessel
- \( t_1 \) = cylinder wall composite thickness
- \( \sigma \) = hoop stress in cylinder wall composite at pressure \( P \)
- \( \sigma \) = composite density
- \( L_C \) = cylinder length between dome tangencies

Similarly, weight of end domes can be expressed in terms of dome thickness at the dome-to-cylinder tangency plane (equivalent here to longitudinal composite wall thickness in cylinder),

\[ W_D = (2\pi D_2 s - \Sigma A) t_2 \sigma \]  \hspace{1cm} (4)

or in terms of the longitudinal composite stress at the tangency plane resulting from an internal pressure,

\[ W_D = \frac{\pi D_2^2 \sigma P}{4\sigma_2} \left[ 0.662(2D_2 - \Sigma D_o) \right] \]  \hspace{1cm} (5)

where

- \( D_2 \) = mean diameter of longitudinal composite
- \( t_2 \) = composite thickness of longitudinally or helically wound filaments
\[ S = \text{developed length of a dome, selected as } 1.324D_2 \text{ (zero hoop stress dome profile)} \]

\[ \Sigma A = \text{total area of openings in both domes} \]

\[ \Sigma D_0 = \text{sum of diameters of openings in both domes} \]

\[ \sigma_z = \text{stress in longitudinal composite at pressure } P \]

End dome weight can also be expressed in terms of composite hoop stress through the relationship

\[ \sigma_z = \frac{z_1}{z} \sigma_1 \]

(6)

where

\[ \zeta = \text{ratio of stress in longitudinally wound composite to stress in hoop wound composite.} \]

Thus,

\[ W_0 = \frac{\pi D_2^2 P}{2(2\zeta+1)\sigma_1} \left[ 0.662 \left( \frac{D_2^2}{\Sigma D_0} \right) \right] \]

(7)

Weight of cylinder and domes could be combined, from Eqs. 3 and 7 into

\[ W_c + W_0 = \frac{\pi D_2^2 P}{2\sigma_1} \left[ \frac{0.662}{2\zeta+1} \left( \frac{D_2^2}{\Sigma D_0} \right) + L_c \right] \]

(8)

For the 260 inch diameter chamber of 1200 inch length between domes, polar porting requirements in the composite dome are estimated to be 0.32D for the igniter attachment adapter in the forward dome and 0.42D for the nozzle attachment adapter in the aft dome; this geometry requires a 4-degree longitudinal-in-plane winding angle. Separate reinforcements about the ports, expressed as the weight \( W_R \), will not be required for this polar port configuration.

The winding pattern is further defined by ratio of stress in the longitudinally wound composite-to-stress in the hoop wound composite, conservatively selected as \( \zeta = 0.80 \). Glass-resin composite density for S-994 glass filament construction (67 vol. %) will be 0.073 lb/in³. Solution of Eq. 8 for this large chamber, to be proof tested at 960 psig with a 1.33 burst factor of safety and developing 100,000 psi cylinder wall composite hoop stress at burst, indicates that the total glass-resin composite weight of the vessel (\( W_c + W_0 \)) will be 126,500 lbs. For the forward and aft dome metal polar adapters, based on scale-up of current design practice in 54 inch diameter chambers, a total weight \( W_R \) of 4,920 lbs is predicted for the 260 inch diameter chamber. Total structural weight \( W_c \) for the large vessel, the sum of glass-resin composite and metal adapter component weights, is 131,420 lbs.

Total structural weights for the 260 inch diameter chamber were computed for other ultimate cylinder wall strengths ranging from a more conservative 60,000 psi to an optimistic 140,000 psi. Resulting weights, wall thicknesses, and filament stresses in the hoop and longitudinal direction are summarized in Fig. B-1.
For handy reference Figs. B-2, B-3, B-4, and B-5, are presented for determining the pressure vessel weight parameters $\varepsilon$ and $\eta$ to be used in Eq. 8 for any specific chamber geometry and filament winding pattern. Figure B-2 depicts the general configuration of a polar port design filament-wound pressure vessel and the in-plane or helical winding patterns used for applying filaments along the cylinder and over the end domes. Circumferential windings are usually applied to the cylinder to provide most of the required hoop strength.

If overall chamber length is $L$, then the cylinder length $L_c$ may be assumed to be given by

$$L_c \approx L - 0.6D$$

The relationship of vessel geometry to winding angle for planar and helical winding patterns is shown in Fig. B-3. From vessel length-to-diameter ratio ($L/D$) and polar port opening size the required winding angle, $\alpha$, can be determined for the particular winding pattern and dome contour selected. For planar winding patterns use the average polar port opening size to select the correct winding angle for the vessel $L/D$ ratio; for helical winding patterns, the polar port opening sizes should be identical in each end dome.

The filament winding angle $\alpha$ relates relative stresses in the longitudinally or helically wound filaments and hoop wound filaments ($\zeta$) to the relative stresses developed in the longitudinal and hoop wound composites ($\epsilon$). Figure B-4 shows the relationship between these factors. A filament stress balance factor $\zeta$ should be assumed and the composite stress balance factor $\epsilon$ selected from Fig. B-4 for the particular winding angle $\alpha$.

The relationship between design allowable hoop-wound filament stress, composite stress balance factor $\epsilon$, and the resulting design allowable cylinder wall hoop stress is shown in Fig. B-5. A design allowable ultimate hoop filament stress is assumed and the resulting cylinder wall stress selected from Fig. B-5 for the value of $\epsilon$ already determined, or the allowable cylinder wall stress selected directly.

With the necessary factors $\epsilon$ and $\eta$ thus established, vessel composite weight can be computed from Eq. 8.
**FIGURE B-1. STRUCTURAL AND DIMENSIONAL REQUIREMENTS FOR LARGE GLASS FILAMENT-WOUND MOTOR CASE OF 260-IN.-DIA AND 113 FT LENGTH**

- **CONFIGURATION**

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CYLINDER DIAMETER, D</td>
<td>260 IN.</td>
</tr>
<tr>
<td>OVERALL LENGTH, L</td>
<td>1356 IN.</td>
</tr>
<tr>
<td>CYLINDER LENGTH, Lc</td>
<td>1200 IN</td>
</tr>
<tr>
<td>L/D</td>
<td>5.2</td>
</tr>
<tr>
<td>FORWARD BOSS DIA.</td>
<td>0.32D = 83 IN.</td>
</tr>
<tr>
<td>AFT BOSS DIA.</td>
<td>0.42D = 109 IN.</td>
</tr>
<tr>
<td>WRAP ANGLE, LONGITUDINAL-IN-PLANE COMPOSITE STRESS BALANCE,</td>
<td>0.80</td>
</tr>
<tr>
<td>FILAMENT STRESS BALANCE,</td>
<td>0.81</td>
</tr>
<tr>
<td>PROOF PRESSURE, ( P_p )</td>
<td>960 psig</td>
</tr>
<tr>
<td>ULTIMATE PRESSURE, ( P_B )</td>
<td>1280 psig</td>
</tr>
<tr>
<td>PROOF/ULTIMATE PRESSURE RATIO</td>
<td>0.75</td>
</tr>
<tr>
<td>BURST FACTOR OF SAFETY COMPOSITE DENSITY,</td>
<td>1.33</td>
</tr>
<tr>
<td>GLASS FRACTION IN COMPOSITE</td>
<td>0.67</td>
</tr>
<tr>
<td>INTERNAL VOLUME, ( \text{in}^3 )</td>
<td>69.75 ( \times 10^6 )</td>
</tr>
</tbody>
</table>
GENERAL CONFIGURATION

VIEW A

TANGENCY PLANE

WINDING PATTERNS

LONGITUDINAL IN-PLANE

HELICAL

FIGURE B-2. FILAMENT-WOUND PRESSURE VESSEL POLAR PORT DESIGN
PLANAR WINDING PATTERNS
(BALANCED-IN-PLANE HEAD
CONTOURS)

HEXAGONAL WINDING PATTERNS
(GEODESIC OVALOID HEAD
CONTOURS)

L/D = VESSEL LENGTH-TO-DIAMETER
RATIO

D_b = AVERAGE POLAR PORT DIAMETER

FIGURE B-3. RELATIONSHIP OF VESSEL GEOMETRY
TO WINDING ANGLE FOR PLANAR AND
HEXAGONAL WINDING PATTERNS
\[ \eta = \frac{\sigma_{Lf}}{\sigma_{Hf}} = \text{DESIGN FILAMENT STRESS BALANCE} \]
\[ \epsilon = \frac{\sigma_{Lc}}{\sigma_{Hc}} = \text{COMPOSITE STRESS BALANCE} \]
\[ \eta = \epsilon \left[ \cos^2 \alpha \left( 1 - \tan^2 \frac{\alpha}{2} \right) \right] \]

WHERE:
\[ \sigma_{Lf} = \text{LONGITUDINALLY OR HELICALLY WOUND FILAMENT STRESS} \]
\[ \sigma_{Hf} = \text{HOOP-WOUND FILAMENT STRESS} \]
\[ \sigma_{Lc} = \text{HOOP-WOUND COMPOSITE STRESS} \]
\[ \sigma_{Hc} = \text{LONGITUDINAL COMPOSITE STRESS (} \sigma_2 \text{)} \]
\[ \alpha = \text{ANGLE BETWEEN LONGITUDINALLY OR HELICALLY WOUND FILAMENTS IN THE CYLINDER AND MERIDIONAL DIRECTION} \]

FIGURE B-4. RELATIONSHIP BETWEEN THE FILAMENT STRESS BALANCE AND RESULTING COMPOSITE STRESS BALANCE FOR DIFFERENT WRAP ANGLES
\[ \sigma_{Hf} = 150,000 \text{ to } 500,000 \]

**Figure B-5. Ultimate Cylinder Wall Stresses for Different Allowable Filament Stresses and Composite Stress Balances**

- **\( \sigma_{Hf} \)** – Design allowable ultimate hoop-wound filament stress
- **\( \epsilon \)** – Composite stress balance
- **\( \sigma_{Cc} \)** – Ultimate cylinder composite wall hoop stress
- **\( \alpha_v \)** – Glass content in composite, volume fraction, selected as 0.675

**Equation:**

\[ \epsilon = \frac{\sigma_{Cc}}{\sigma_{Hf}} \]

**Where:**

\[ \sigma_{Hf} = 2 \epsilon + 1 \]

**Graphical representation:**

- **Y-axis (Composite Stress Balance, \( \epsilon \))**
- **X-axis (Ultimate Cylinder Composite Wall Hoop Stress, \( \sigma_{Cc} \) (psi \times 10^3))**

Values for \( \sigma_{Hf} \) are marked at intervals of 50,000 psi from 150,000 to 500,000 psi on the Y-axis. Corresponding values for \( \sigma_{Cc} \) are plotted on the X-axis for various \( \epsilon \) values, showing the relationship between composite stress balance and ultimate hoop stress for different composite wall configurations.