Quarterly Progress Report
May 15, 1964 - August 15, 1964

Research on the Interactions of Plasma Flow
and Magnetic Fields

Army Missile Command

Contract Number DA-23-072-AMC-168(Z)

Best Available Copy
INTRODUCTION

This quarterly progress report covers the third quarter of a one-year research contract on Interactions of Plasma Flows and Magnetic Fields. The study includes both theoretical and experimental work in magnetohydrodynamics and superconductivity.

The third period has been spent operating the programs initiated and developed in the first and second periods. This report covers these activities.

Denys O. Akhurst
Project Director
PROJECT PERSONNEL

Faculty

Denys O. Akhurst  Project Director
Melvin K. Anderson  (Summer Leave)
William J. Buche  Section A, E, F
Frank W. Lewis  Section A
Edgar C. Tacker  Section E

Graduate Students

Edward L. Burgess  Section B
Martin McCutcheon  Section B, C
Alexander D. Poularikas  Section B, C, D, F, G

Undergraduate Students

Charles R. Duke  Section D, F
Stanley G. Giannopoulos  Section C, D
Charles M. Green  Section A, E
Frank E. Williams  Section A, E

Electronic Technician

James C. Fine  Section A

Consultants

Dr. W. F. Gauster, Oak Ridge National Laboratory
Dr. D. C. Hopkins, Assistant Professor of Physics, University of Missouri, Rolla
Dr. S. M. Day, Assistant Professor of Physics, University of Arkansas, Fayetteville
A. PRESENT STATUS OF EQUIPMENT

To facilitate measurements on the plasma jet, two water flow meters have been added in order to measure the quantity of water cooling the movable and fixed electrodes. Several thermistors have also been added at various points in the cooling system for heat transfer measurements.

Construction of a solenoidal electromagnet for further plasma-magnetic field interaction studies is now in progress. The electromagnet will be mounted inside the first section of the dump tank in order to allow the plasma stream to pass coaxially through the field region.

B. THEORETICAL STUDIES OF PLASMA INTERACTION WITH A MAGNET FIELD

In the induction MW generator, Figure 1, slugs of moving plasma interact with the magnetic field of a solenoid to produce electrical energy. For this type of geometry, there have been few investigations of either an experimental or theoretical nature. However, by making certain assumptions and approximations, an expression for the generated voltage can be determined.

The approach used is to divide the plasma slug into several small, finite rings. Using the mutual inductance between two conducting loops an expression is then found for the induced voltage produced by the interaction of a plasma ring with a current carrying conductor. The results can be generalized to the case of a solenoid and a conducting slug by numerical integration or by graphical means.

Assuming the plasma ring moves coaxially with a constant velocity \( U \) as shown in Figure 2, then the instantaneous value of the mutual inductance between the current carrying conductor and the plasma ring can be written:

\[
M = \frac{2\mu (ab)^{\frac{1}{2}}}{k^2} \left[ (1 - k^2)K(k) - E(k) \right]
\]  \hspace{1cm} (1)

where:

\[
k^2 = \frac{4ab}{(a + b)^{2} + z^2}
\]  \hspace{1cm} (2)

and \( z \) is given by \( z = Ut \). Expanding the elliptic integrals \( K(k) \) and \( E(k) \) in infinite series in terms of the modulus \( k \) yields:

\[
M = \pi \mu (ab)^{\frac{1}{2}} \sum_{i=1}^{24} C_i k^{2i-1}
\]  \hspace{1cm} (3)

where \( C_i \) is a coefficient resulting from the combination of the coefficients.
Simplified MHD Induction Generator

Cylindrical Non-Conducting Channel

Slug of Conducting Material

Coil

FIGURE 1

DRAWN BY JDL

DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF ARKANSAS
of the series expansions of the elliptic integrals. By finding the flux linking the two loops and differentiating accordingly, the induced voltage in the carrying conductor is found to be:

\[ e = \frac{\sigma AU^2 I_1}{2\pi b} \left( M \frac{dM}{dz} + \left( \frac{dM}{dz} \right)^2 \right) \]  

(4)

Where \( \sigma \) is the plasma conductivity, \( A \) the cross-sectional area of the plasma, \( I_1 \) the current in the stationary loop, and \( M \) is from equation (3).

Equation (4) is presented in Figure 3 in terms of the normalized parameters \( D = Z/b, R = a/b, \) and \( E = \frac{8be}{\pi \sigma U I_1 A} \). The values of \( R \) chosen correspond to values which will be used in the numerical integration. An apparatus has been constructed to verify the preceding calculations, and a computer program has been written for the numerical integration. Details of the development of equation (4) are given in Report UAPL No. 18.

REFERENCES

C. PLASMA ELECTRICAL CONDUCTIVITY
1. Theoretical

The electrical conductivity of an ionized gas, or plasma, is dependent upon several variables, i.e., temperature, pressure, cross-section, etc., which are not all independent. Because of this complexity it has been necessary to develop theoretical expressions which are applicable only to certain ranges of ionization. To provide numerical values of the conductivity for comparison with experimental results, these conductivity expressions have been taken from the literature and programmed on an IBM 650 computer. Tabulated values and graphs which show the relationship between the various theories are presented for a temperature range 3,000 - 30,000 °K. Results for five different gases at various pressures are given in Report UAPL No. 18.
2. Experimental Measurement

Because of the difficulties encountered with using direct probe and microwave techniques for plasma conductivity measurement, an R. F. probe using a crystal controlled oscillator has been constructed. The oscillator is coupled to an inductance or search coil which is mounted on a ceramic tube, and when plasma is present, R. F. power is dissipated and appears as an additional resistive load to the oscillator. By proper calibration of the instrument absolute conductivities can be measured.

Calibration was accomplished using KCl and NaCl solutions at different normalities with the zero conductivity condition determined by the use of distilled water. The oscillator was also checked for drifting and a drift of only 1.5% was observed over a 24-hour period of time.

Figure 4 indicates a typical conductivity curve for the plasma jet at low input power as a function of the discharge current in the jet nozzle. The voltage drop across the electrodes was constant at 39 volts, and the gas (nitrogen) flow rate was 0.6 grams per sec. A more detailed description of the apparatus and measurements is given in Report UAPL No. 17.

D. JET VELOCITY MEASUREMENTS

Measurement of the jet velocity was accomplished by discharging a bank of capacitors across the electrodes and measuring the velocity of the luminous ionized gas created by the discharge. The previously described three-gap triggering mechanism was used for a total capacitance of 10 μf and 12,000 volts.

To find the velocity, two photocells (Figure 5) were placed a known distance apart and the time of travel of the luminous gas between the photocells was measured with an oscilloscope. Several tests were performed under two different types of measurements: those of steady state conditions (i.e. those conditions of flow and dump tank pressure which could be held constant for a sufficient length of time) and those of transient conditions (i.e. those conditions of high flow rate and low dump tank pressure which could be held constant only for a short period of time due to the efficiency of the vacuum pump).

Figure 6 shows a typical run for a nozzle with 3/4" input diameter and 1-11/16" output diameter. These measurements were of the transient type.
Mach 2

Mach 1

FIGURE 6

PLASMA VELOCITY

Flow OASE 1 cc
Flow 2 3 cc
Flow 2 05 cc

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ARKANSAS
The vacuum pump was allowed to pull the dump tank pressure down to a specific value and the flow was then set at various levels.

Steady state measurements were made by setting the flow at a specific level and then allowing the pump to stabilize. Measurements were made at various levels and typical values are given below in Table I.

<table>
<thead>
<tr>
<th>Pressure mm Hg</th>
<th>Flow gr/sec</th>
<th>Velocity m/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0.590</td>
<td>410</td>
</tr>
<tr>
<td>15</td>
<td>0.890</td>
<td>380</td>
</tr>
<tr>
<td>17</td>
<td>1.190</td>
<td>350</td>
</tr>
<tr>
<td>23</td>
<td>1.780</td>
<td>320</td>
</tr>
</tbody>
</table>

E. HEAT TRANSFER MEASUREMENTS

Figure 7 shows the distribution of water supply and the location of the thermistors used for heat transfer measurements. The thermistors were calibrated with a standard thermometer so that absolute values of temperature could be measured.

Several tests were conducted to find the amount of heat exchange at the electrodes, but, because of burnout difficulties with the electrodes, the tests were conducted only at low power inputs. Data obtained during a test is given below:

- Input Power = \( P = 39 \times 135 = 5.25 \text{ kw} \)
- Duration = 10 minutes
- Mass flow rate of gas = 1.4 gr/sec
- Dump tank pressure = 15 mm Hg
- Heat exchange
  - (a) Fixed electrode = 135 BTU/min = 2.37 kw
  - (b) Moveable electrode = 27 BTU/min = 0.475 kw

Water flow in moveable electrode = 1.35 Gal/min
Water flow in fixed electrode = 4 Gal/min
V. WEAR OUT OF ELECTRODES

Several tests have been conducted at different input power levels for the purpose of finding the conditions for stable operation. It was found that input power levels of 5 to 7 kW give steady operation up to 10 minutes and the electrode wear is reasonable. Figure 8 shows two anode tungsten electrodes. Before they had been used both the electrodes had flat heads and their length was one inch. The electrode (a) (anode) was used at 5.5 kW power input (discharge voltage x discharge current) for a period of 15 minutes. As shown in the picture, the anode was partly melted close to the end, and only a very small amount of material was dissipated. During this period the cathode showed no effects. When higher input power was applied to the electrodes it was found that the operation was unstable. Another run was conducted at 14.5 kW for a period of 2 minutes, during which time the anode was shortened by half an inch (Figure 8b). The cathode also began to show signs of dissipation.

It is apparent that for higher than 10 kW input power a different configuration of electrodes must be used. At the present time, different experiments are in progress to investigate the influence of current density on the wear of electrodes. By having a better distribution of current density, it is anticipated that the life of the electrodes will be increased and the arc operation will be more stable.

G. EVALUATION OF AC MHD GENERATOR FOR RE-ENTRY

The replacement of metallic conductors by gaseous conductors has already led to the generation of electrical power. This type of generator, called a dc magnetohydrodynamic generator, utilizes the interaction of an ionized gas flow through a transverse magnetic field. A component of current is produced transversely to the stream and to the applied magnetic field. Thus net dc electric power can be produced. This type of generator has some inherent problems in the design and operation. For this reason it has been proposed to investigate the feasibility of adapting the theory of ac induction generators to magnetohydrodynamic power generation. Bernstein et al. have solved the problem for a channel with a rectangular cross section. Multiphase coils are arranged on the upper and lower sides of the channel to provide a magnetic field of the form \( B \exp(j (kx - \omega t)) \) which advances downstream with a phase velocity \( \omega/k \). The gas flows in the same direction with a velocity
greater than the phase velocity, such that an interaction between the conducting gas and the magnetic field takes place. This type of interaction produces ac power. For efficient operation, the conductivity of the gas must be above 100 mhos/meter. During re-entry of a space vehicle, the conductivity of a shock-wave in air at 184,000 feet and Mach No. 20 is 400 mhos/meter and rises rapidly to 1200 mhos/meter at Mach No. 24. This indicates that an induction magnetohydrodynamic generator could efficiently provide power during the re-entry period. In order to investigate this possibility a theoretical study has been initiated and the results will be published soon in a later report.

REFERENCES