

TECHNICAL NOTE NO. 1510  
JULY 1963

LETHAL AREA DESCRIPTION

K. A. Myers

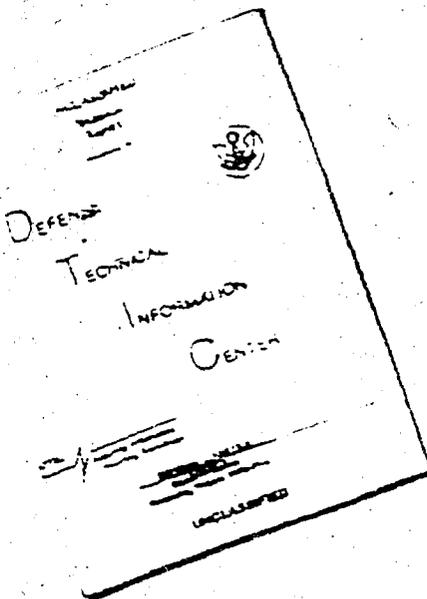
RDT & E Project No. 1M023201A098  
BALLISTIC RESEARCH LABORATORIES

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K. A. Myers

Weapon Systems Laboratory

RDT & E Project No. 1M023201A098

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KAMyers/jdk  
Aberdeen Proving Ground, Md.  
July 1963

LETHAL AREA DESCRIPTION

ABSTRACT

A detailed description of the lethal area problem is given, including a discussion of the procedure presently used at BRL in the computations of lethal areas.

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## INTRODUCTION

This technical note was originally prepared in 1957 as an internal working paper. The intent was to present a detailed description of the lethal area concept at BRL and the method used to compute lethal areas on high speed digital computers. In view of the wide interest in this area over the years and as a consequence of several recent requests for copies, this technical note has been prepared.

The initial portion of this paper consists of an up-dating of the lethal area concepts discussed in BRL Report No. 800<sup>(1)\*</sup> which was necessitated by the introduction of more refined casualty criteria and presented area functions.

The remainder of the paper is devoted to a discussion and description of the techniques and procedures presently used in lethal area evaluations. Although, the possible extensions of the problem are not covered specifically, it is felt that sufficient background is given to allow these special cases to be handled by the reader as necessary.

### LETHAL AREA CONCEPT AND INPUT REQUIRED

Lethal area can be expressed mathematically as follows: Define  $\sigma(x,y)$  as the density of targets in an element of area about the point  $(x,y)$  and denote the probability that a target in that element of area will be incapacitated as  $P_K(x,y)$ . We may then write the equation for the expected number of casualties,  $E_C$ , as follows:

$$E_C = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma(x,y) P_K(x,y) dx dy.$$

Now if we assume the targets are uniformly distributed over the ground plane,  $\sigma(x,y)$  can be represented simply as a constant,  $\sigma$ , and we can write:

$$\frac{E_C}{\sigma} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_K(x,y) dx dy.$$

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\* The parenthetical superscripts refer to reference numbers.

The quantity  $\frac{E_c}{\sigma}$  has the dimensions of area and is called the lethal area. Some operations analysts often refer to this quantity as the mean area of effectiveness (M.A.E.). It is obvious from an examination of this expression, that the lethal area when multiplied by the density of targets, will yield the expected number of casualties. A further examination of the above expression indicates that lethal area is really just a weighted area; the weight for each element of area being determined by the probability of incapacitation function  $P_K(x,y)$ .

Let us now look at some of the geometry associated with the problem. (See Figure 1).

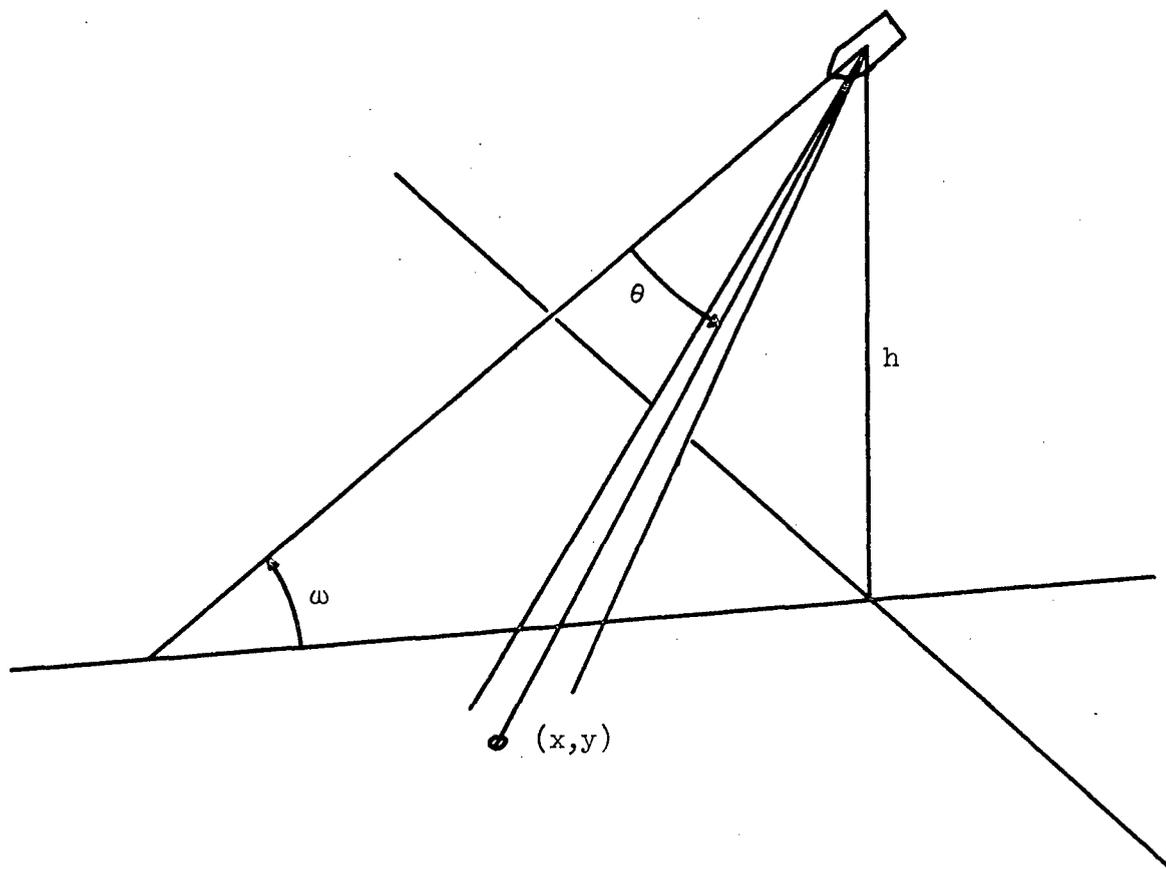


Figure 1

Assume that the shell is approaching the ground at an angle of fall  $\omega$  and velocity  $V_h$ . The shell bursts at height  $h$ . The problem is to find the probability that a target at the point  $(x,y)$  is "killed" by the shell. We see that the target will be located in a small fragment spray leaving the shell about an angle  $\theta$  measured from the nose of the shell. In order to determine the probability that the target will be a casualty, a considerable amount of information will be required. Specifically, the type of information required may be listed as follows:

- (1) Velocity fall off law.
- (2) Casualty criterion.
- (3) Target presented area.
- (4) Fragment mass distribution.
- (5) Fragment density.
- (6) Initial fragment velocity.

The manner in which the velocity falls off with range,  $r$ , is usually approximated in the following manner:

$$V(r) = V_0 e^{-\frac{C_d \rho A_f}{m} r},$$

where  $V_0$  is the initial fragment velocity,  $C_d$  is the drag coefficient,  $\rho$  is the air density,  $A_f$  is the average presented area of the fragment, and  $m$  is the weight of the fragment. In this connection,  $C_d$  and  $A_f$  are generally determined experimentally, unless of course the shape of the fragments are such that  $A_f$  may be estimated theoretically. In any event, once these constants have been determined, we are in a position to determine the velocity of that specific fragment as a function of distance traveled.

During the past few years a concentrated effort has been directed towards obtaining a realistic casualty criterion. To this end, the Biophysics Laboratory at Edgewood Arsenal has developed information pertaining to the wounding power of various size fragments striking the body at various velocities. These data were analyzed by personnel of the Terminal Ballistics Laboratory here at BRL. The resulting casualty criteria originally were reported in BRL Report No. 996.<sup>(2)</sup>

A subsequent medical reassessment of the basic wound data resulted in a modification of these criteria as described in BRL Technical Note No. 1297.<sup>(3)</sup> Essentially, these casualty criteria are a series of functions for various tactical situations which describe the probability that a fragment hitting a target will create a casualty.

With respect to the target presented area  $A_t$ , various "cover functions", or more properly, presented area functions, have been in existence for some time which describe the presented area of the target as a function of the position of the target relative to the burst. Some of the more prominent cover functions which have been used are: (1) a variety of cut-off angles which were intended to simulate foxhole cover; (2) the ORO cover functions which express the presented areas as a function of the aspect angle; and (3) most recently, some new BRL cover functions which have been developed to consider typical terrain features. The latter are described in BRL Memorandum Report No. 1203.<sup>(4)</sup>

The fragmentation characteristics of the shell can adequately be described in terms of initial fragment velocity, fragment density and fragment mass distribution. In most cases, these data will vary as a function of  $\theta$ , the angle measured from the nose of the shell. Typical plots of initial fragment velocity  $V_o$ , and fragment density are shown in Figure 2.

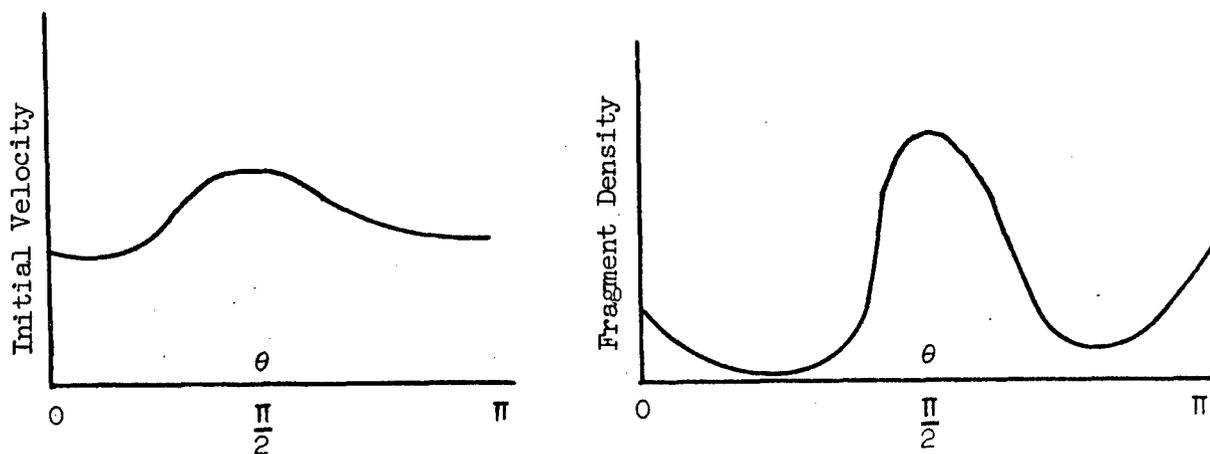


Figure 2

Fragment mass distributions, as the name implies, are used to describe the distribution of fragment sizes within a spray. These mass distributions are also a function of  $\theta$ . In general, these fragmentation characteristics are determined from static detonation tests where fragments are collected in recovery boxes filled with celotex sheets. The fragments are then extracted from the celotex, counted and weighed. Fragment velocity measurements are made with the aid of high speed photography. A description of the techniques employed in the fragmentation tests conducted for the BRL by Development & Proof Services is given in Reference 5.

How then are these basic data combined mathematically to yield  $P_K(x,y)$ ? To illustrate, let us suppose that a small spray about the angle  $\theta$  is comprised of only three fragment sizes; namely  $m_1$  grains,  $m_2$  grains and  $m_3$  grains, and further, that  $q_1$  is the fraction of the fragments in the spray weighing  $m_1$  grains,  $q_2$  is the fraction of the fragments in the spray weighing  $m_2$  grains and  $q_3$  is the fraction of the fragments in the spray weighing  $m_3$  grains. Since we have assumed that the fragments in this spray all leave the shell with the same velocity, we can employ the velocity fall-off law described previously and thus associated with our three fragment sizes we will have three striking velocities;  $V_1$ ,  $V_2$  and  $V_3$ . Using the casualty criteria we can then determine the probability that the target will be incapacitated if hit by a fragment weighing  $m_1$  grains and moving at a velocity  $V_1$ . We denote this probability by  $P_{hk_1}$ ; similarly, we have  $P_{hk_2}$  and  $P_{hk_3}$ . If we then describe the density of all the fragments of the spray to be  $\eta$  fragments per steradian, the density of  $m_1$  fragments will be  $q_1\eta$ ;  $m_2$  fragments  $q_2\eta$  and  $m_3$  fragments  $q_3\eta$ . Now, employing  $A_t$ , the presented area of the target, we should expect  $\frac{q_1\eta A_t}{r^2}$  of the  $m_1$  grain fragments;  $\frac{q_2\eta A_t}{r^2}$  of the  $m_2$  fragments and  $\frac{q_3\eta A_t}{r^2}$  of the  $m_3$  fragments to hit the target. The probability, therefore, that the target will not be "killed" by the  $m_1$  fragments may be represented as follows:

$$(1 - P_{hk_1}) \cdot \frac{q_1\eta A_t}{r^2}$$

Similarly, the probability that the target would not be "killed" by the  $m_2$  fragments would be as follows:

$$(1 - P_{hk_2})^{\frac{q_2 \eta A_t}{r^2}},$$

and for the  $m_3$  fragments as follows:

$$(1 - P_{hk_3})^{\frac{q_3 \eta A_t}{r^2}}.$$

Then the probability that the target will not become a casualty by either of the  $m_1$ ,  $m_2$  or  $m_3$  fragments can be represented by

$$(1 - P_{hk_1})^{\frac{q_1 \eta A_t}{r^2}} \times (1 - P_{hk_2})^{\frac{q_2 \eta A_t}{r^2}} \times (1 - P_{hk_3})^{\frac{q_3 \eta A_t}{r^2}}.$$

For most practical problems, with natural fragmenting shell, there will be many more mass groups than I have discussed here, and it would therefore be advantageous to use the Poisson approximation to the binomial distribution just discussed, i.e.:

$$(1 - P_{hk_i})^{\frac{q_i \eta A_t}{r^2}} \approx e^{-\frac{P_{hk_i} q_i \eta A_t}{r^2}}.$$

Thus, the expression for the probability that none of the fragments in "n" mass groups would create a casualty can be written:

$$e^{-\frac{\eta A_t}{r^2} \sum_{i=1}^n q_i P_{hk_i}}$$

We are now in a position to write the expression for  $P_k$ , the probability that the target is killed, as follows:

$$P_k = 1 - e^{-\frac{\eta A_t}{r^2} \sum_{i=1}^n q_i P_{hk_i}}$$

There has from time to time been considerable discussion regarding this "approximation" of the probability of incapacitation, inferring that the Binomial approach represents a "true" solution. Actually a more correct application of the binomial distribution could lead to an expression of the form:

$$P_k = 1 - \sum_{i=1}^n \pi \left(1 - \frac{A_t P_{hk}}{4 r^2}\right)^{q_i N}$$

where  $N$  represents the total number of fragments in the spray and  $\frac{N}{\Omega} = \eta$ ,  $\Omega$  being the number of steradians subtended by the spray.

This expression for the probability of incapacitation can again be approximated by:

$$P_k = 1 - e^{-\frac{\eta A_t}{r^2} \sum_{i=1}^n q_i P_{hk_i}}$$

Even so, the Binomial itself must be considered only as an approximation since fragments projected from the same shell cannot really be considered as

"independent events". Further, if one considers that  $\frac{\eta A_t}{r^2} \sum_{i=1}^n q_i P_{hk_i}$

is the number of incapacitating fragments expected to strike the target, an estimate of the probability that the target is hit by at least one such fragment, i.e., the probability that the target is incapacitated, can be

estimated directly by the Poisson distribution as:

$$P_k = 1 - \left[ \frac{\eta A_t}{r^2} \sum_{i=1}^n q_i P_{hk_i} \right]^0 e^{-\frac{\eta A_t}{r^2} \sum_{i=1}^n q_i P_{hk_i}}$$

$$= 1 - e^{-\frac{\eta A_t}{r^2} \sum_{i=1}^n q_i P_{hk_i}}$$

To the author's knowledge, no extensive comparisons of lethal areas computed using the Binomial and Poisson forms for the probability of incapacitation have been made. However, in some isolated cases where such comparisons have been made, the difference did not appear significant, particularly in light of other assumptions inherent in the lethal area concept.

#### LETHAL AREA COMPUTATION

Having obtained the input data described in the preceding sections, one is now in a position to proceed to the actual computation of a lethal area. The following sections describe the method presently being used at BRL.

By a transformation to the  $(r, \phi)$  coordinate system the lethal area integral becomes:

$$A_L = \int_r \int_\phi r P_k(r, \phi) dr d\phi ,$$

where  $r$  is the distance from the burst to a point on the ground and  $\phi$  is an angle in the ground plane measured from the projection of the shell trajectory into that plane to the line joining the origin to the point in the ground plane.

The minimum distance to the ground plane is  $h$ , the height of burst, and although theoretically the upper limit on  $r$  would be infinite in practice we shall use the value  $r_{m2}$  such that for all  $r > r_{m2}$  the value of the integrand

would be zero. Taking advantage of the symmetry in  $\phi$  the integral becomes:

$$A_L = 2 \int_h^{r_{m2}} \int_0^\pi r P_k(r, \phi) dr d\phi .$$

Or

$$A_L = 2 \left[ \int_h^{h+1} \int_0^\pi r P_k(r, \phi) dr d\phi + \int_{h+1}^{r_{m1}} \int_0^\pi r P_k(r, \phi) dr d\phi + \int_{r_{m1}}^{r_{m2}} \int_0^\pi r P_k(r, \phi) dr d\phi \right] .$$

If in the second integral we make the substitution  $d(\text{Ln } r) = \frac{1}{r} dr$  we have:

$$\int_{\text{Ln}(h+1)}^{\text{Ln } r_{m1}} \int_0^\pi r^2 P_k(r, \phi) d\phi d(\text{Ln } r) .$$

It will be noted that if one integrates numerically with respect to  $\text{Ln } r$ , taking equal step sizes in  $\text{Ln } r$ , the effect will be to concentrate the evaluations of the integrand in close to the point below the burst where one might expect the integrand to change rapidly, and to spread out the interval in  $r$  between successive evaluations of the integrand at the longer distances. It is also apparent why the integration from  $h$  to  $r_{m1}$  was separated into two parts. Without the separation the lower limit of the integral for  $h = 0$ , would have become  $\text{Ln}(0)$ . It should also be noted that the separation of the integration at  $h+1$  is purely arbitrary. In practice, the first of the three integrals is approximated by:

$$(h+1/2) \int_0^\pi P_k(h+1, \phi) d\phi .$$

In evaluating the third integral, the integration in  $r$  is accomplished in equal steps of  $r$ .

The lethal area integral as evaluated by the machine is then:

$$A_L = 2 \left\{ (h+1/2) \int_0^\pi P_k(h+1, \phi) d\phi + \frac{\ln(r_{m2})}{\ln(h+1)} \int_{r_{m1}}^{r_{m2}} r^2 \left[ \int_0^\pi P_k(r, \phi) d\phi \right] d(\ln r) \right. \\ \left. + \int_{r_{m1}}^{r_{m2}} r \left[ \int_0^\pi P_k(r, \phi) d\phi \right] dr \right\} ,$$

where as indicated the integration over  $\phi$  is considered as part of the integrand with respect to  $r$ .

Generally the fragmentation data are obtained for shell detonated statically. It is necessary therefore, to adjust the data to simulate the fragmentation characteristics of a shell bursting in flight.

The fragmentation data obtained from tests consist of an average initial fragment velocity, average fragment density and a mass distribution for each of several intervals of the angle  $\theta$  between  $0^\circ$  and  $180^\circ$ . Continuous functions of initial fragment velocity and fragment density vs.  $\theta$  are obtained by plotting these quantities at the midpoints of their respective intervals and connecting the points by straight line segments. Mass distributions are considered to be constant over the interval which they represent.

Figure 3 indicates the scheme for adjusting the static fragmentation data to dynamic data.

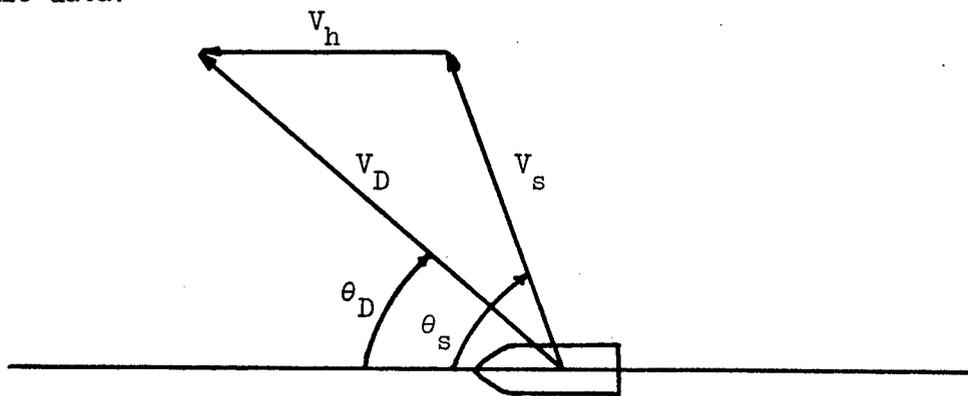


Figure 3

Here a fragment, which would have left the shell at a velocity  $V_s$  and an angle  $\theta_s$  had the shell been at rest, would be thrown forward by the shell's remaining velocity  $V_h$  to an angle  $\theta_D$ , the resultant initial velocity being  $V_D$ . The following relations can be used to determine  $V_D$  and  $\theta_D$ .

$$\begin{aligned} 1) \quad & V_s \sin \theta_s = V_D \sin \theta_D \\ 2) \quad & V_s \cos \theta_s + V_h = V_D \cos \theta_D \\ 3) \quad & V_D^2 = V_s^2 + V_h^2 + 2 V_s V_h \cos \theta_s \end{aligned}$$

In addition to the change in fragment direction and initial velocity it will be noted that if a small fragment spray of width  $2\Delta\theta_s$ , centered at  $\theta_s$  is thrown forward to a spray of width  $2\Delta\theta_D$  centered at  $\theta_D$ , the density of fragments in the spray will change from  $\sigma_s$  to  $\sigma_D$ . Since the number of fragments contained in the spray,  $N$ , will remain constant we have the relations:

$$\sigma_s = \frac{N}{4\pi \sin \theta_s \sin \Delta \theta_s} \qquad \sigma_D = \frac{N}{4\pi \sin \theta_D \sin \Delta \theta_D}$$

or

$$\sigma_D = \sigma_s \frac{\sin \theta_s \sin \Delta \theta_s}{\sin \theta_D \sin \Delta \theta_D}$$

then

$$\lim_{\Delta \theta_s \rightarrow 0} \left( \frac{\sigma_s \sin \theta_s |\sin \Delta \theta_s|}{\sin \theta_D |\sin \Delta \theta_D|} \right) = \sigma_s \frac{\sin \theta_s}{\sin \theta_D} \left| \frac{d\theta_s}{d\theta_D} \right|$$

In order to compute  $\frac{d\theta_s}{d\theta_D}$  we consider the equation:

$$V_s \cos \theta_s \sin \theta_D + V_h \sin \theta_D = V_s \sin \theta_s \cos \theta_D$$

Now since our function of  $V_s$  vs.  $\theta$  is made up of a number of straight line segments we may express  $V_s$  by:

$$V_s(\theta_s) = A_i \theta_s + B_i, \quad \theta_s \text{ in radians}$$

where the constants  $A_i$  and  $B_i$  depend upon the interval in which  $\theta_s$  is contained.

Making this substitution for  $V_s$  and carrying out the differentiation we find:

$$\frac{d\theta_s}{d\theta_D} = \frac{V_D(\theta_D)}{A_i \sin(\theta_s - \theta_D) + V_s \cos(\theta_s - \theta_D)}$$

To obtain the dynamic fragmentation characteristics needed for the lethal area code a supplementary code has been devised which takes as input, a table of initial fragment velocity, density and a count number tabulated for each  $5^\circ$  from  $\theta_s = 0$  to  $\theta_s = 180^\circ$ . The count number is merely a device used to identify the mass distribution associated with a particular interval in  $\theta_s$ . This is accomplished by choosing count numbers so that all count numbers corresponding to angles of  $\theta_s$  from a particular interval, when rounded off to the nearest integer give the integer corresponding to the identification number of the mass distribution associated with that particular interval.

The purpose of the code is to produce a table of dynamic initial fragment velocity, dynamic density and count number tabulated for each  $5^\circ$  of  $\theta_D$  from  $0^\circ$  to  $180^\circ$ .

Essentially what the code does is to determine what fragment angle,  $\theta_s$  in the static test would be vectored forward to each angle  $\theta_D$  as the result of a shell remaining velocity  $V_h$ . Having found the angle  $\theta_s$  for each  $\theta_D$  the machine interpolates in the input table to determine the corresponding initial fragment velocity, density and count number. The interpolated count number then becomes the count number corresponding to the angle  $\theta_D$ . In this way the mass distribution is shifted forward with the  $\theta$  interval to which it corresponds. Then the interpolated values of  $\theta_s$ ,  $V_s$  and  $\sigma_s$  are used to compute  $V_D$  and  $\sigma_D$ . The resulting table is used as input for the lethal area code.

The mass distributions, each identified by an integer, contain a series of fragment weights,  $m_i$ , representative of the  $i^{\text{th}}$  fragment weight group, and a series of normalized percentages,  $q_i$ , representing the fraction of the total number of fragments in each of the fragment weight intervals.

In addition to the fragmentation characteristics, a table relating target presented area and aspect angle,  $\psi$ , to the distance  $r$  is needed for each height at which lethal area is to be computed. The relation between, aspect angle,  $h$ , and  $r$  is shown in Figure 4.

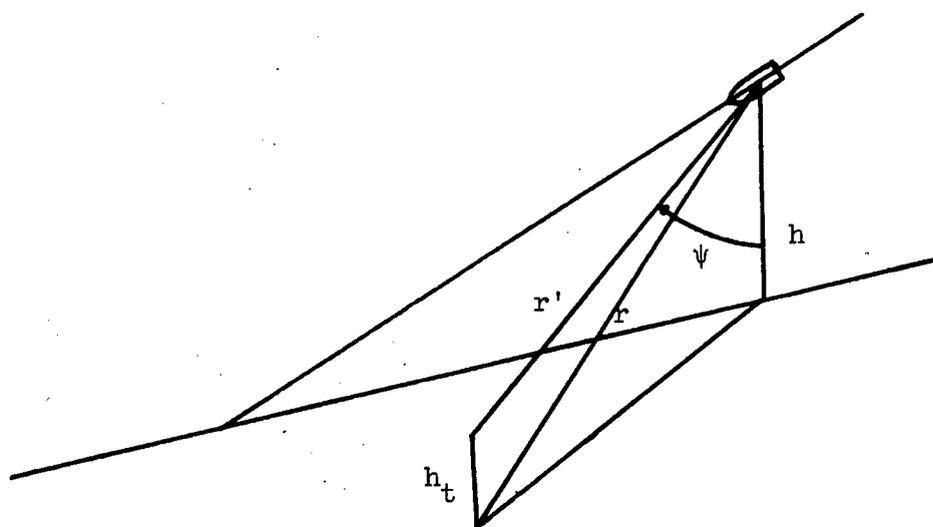


Figure 4

Also shown in Figure 4 are the quantities  $h_t$  and  $r'$ , where  $h_t$  is the height above the ground at which it is desired to compute the probability of incapacitation and  $r'$  is the distance which a fragment would have to travel in order to hit that point. It is apparent that by specifying the  $r$ ,  $h$ , and  $\psi$ ,  $r'$  and  $h_t$  are fixed. This technique is used when it is desired to consider targets at different ranges as being located at different heights above the mean ground plane or where, as might be the case with a standing target, it is desired to base the probability of incapacitation upon the fragment spray striking the center of the man.



1.  $h$  initial height at which lethal area is to be computed
- $\Delta h$  increment by which burst height is to be increased if lethal areas are to be run for several heights
- $h_{\max}$  maximum burst height for which lethal area is to be computed
- $R_{m1}$   $r_{m1}$  (see section describing break down of  $A_L$  integral)
- $R_{m2}$   $r_{m2}$  (see section describing break down of  $A_L$  integral)
- $J$  number of intervals to be used in integration from  $\ln(h+1)$  to  $\ln r_{m1}$  MUST BE EVEN INTEGER.
2.  $-K$  from velocity falloff law  $V_r = V_o \cdot e^{-\frac{Kr}{m^{1/3}}}$
- $M$  number of intervals to be used in integration from  $r_{m1}$  to  $r_{m2}$  MUST BE EVEN INTEGER.
- $\sin \omega$   $\omega =$  shell angle of fall
- $\cos \omega$   $\omega =$  shell angle of fall
- $\omega$   $\omega =$  shell angle of fall MUST BE IN RADIANS.
- $-a$  casualty criterion constant
3.  $b$  casualty criterion constant
- $n$  casualty criterion constant
- $\Delta\theta$  angular interval at which fragment density and initial velocity and "count" numbers are stored in machine
4.  $\Delta\phi$  step size to be used in integration over  $\phi$   
NOTE: NUMBER OF INTERVALS MUST BE EVEN.
- $V_h$  shell remaining velocity
- Run ID number to be used for identification purposes

5.  $C_{\max}$       number of mass distributions + 0.5 .  
Cut off velocity    it is generally assumed that there is some minimum velocity which a fragment must have, regardless of its size, in order that it has any chance of producing casualty.
6. Vector Run    identification number for table of  $\theta_s$ ,  $V_s$ ,  $\sigma_s$ , and count number.
7. q and m        identification number for whole group of mass distributions. Not to be confused with identifying integers for the individual mass distributions.
8. Cover         where tables of presented area and aspect angle vs. r for the burst heights of interest have previously been given to the Computing Laboratory. An identifying symbol is used here.

At each point  $(r_i, \phi_i)$  of the integration grid the probability of incapacitation is computed as follows:

The angle  $\psi$  and the presented area  $A_t$  are determined for the value of  $r_i$  from the table corresponding to the burst height being considered. The angle  $\theta_D$  at which a fragment must leave the shell in order to hit the target can then be computed from the following formula:

$$\cos \theta_D = \sin \omega \cos \psi + \cos \omega \sin \psi \cos \phi$$

When the angle  $\theta_D$  has been determined an interpolation in the fragmentation characteristic table is carried out to obtain  $\sigma_D$ ,  $V_D$ , and the count number corresponding to the angle  $\theta_D$ . The count number is rounded off to the nearest integer and the associated mass distribution consisting of pairs of numbers  $m_i$  and  $q_i$  ( $i=1,2,w$ ) ( $m_i$  = representative mass of  $i^{\text{th}}$  the weight group  $w$  = number of weight groups,  $q_i$  = fraction of fragments in  $i^{\text{th}}$  weight group) is selected. Next the value  $r'$  is computed from the formula:

$$r' = \sqrt{\frac{r^2 - h^2}{\sin \psi}},$$

then the summation:

$$\sum_{i=1}^w q_i P_{hk_i}$$

is evaluated.

Then using the value of  $A_t$  and  $\sigma_D$  previously determined the probability of incapacitation is computed as:

$$P_k = 1 - e^{-\frac{\sigma_D A_t}{(r')^2} \sum_{i=1}^w q_i P_{hk_i}}$$

The numerical integration is accomplished as indicated below:

$$A_L = 2 \left[ (h+1/2)I_1(\phi) + \frac{\Delta \text{Lnr}}{3} \sum_{i=1}^{J+1} \lambda_i r_i^2 I_i(\phi) + \frac{\Delta r}{3} \sum_{i=J+1}^{J+M+1} \lambda_i r_i I_i(\phi) \right],$$

$$\text{where } \Delta \text{Lnr} = \frac{\text{Lnr}_{m1} - \text{Ln}(h+1)}{J} \quad \Delta r = \frac{r_{m2} - r_{m1}}{M}$$

$$\lambda_i = 4 \quad i \text{ even}$$

$$\lambda_i = 2 \quad i \text{ odd and } \neq 1, J+1, \text{ or } J+M+1$$

$$\lambda_i = 1 \quad i = 1, J+1, J+M+1,$$

$$I_i(\phi) = \frac{\Delta \phi}{3} \sum_{j=1}^{\frac{\pi}{\Delta \phi} + 1} v_j P_k \left\{ r_i, (j-1)\Delta \phi \right\}$$

$$v_j = 4 \quad j \text{ even}$$

$$v_j = 2 \quad j \text{ odd and } \neq 1 \text{ or } \frac{\pi}{\Delta \phi} + 1$$

$$v_j = 1 \quad j=1, \frac{\pi}{\Delta \phi} + 1$$

$$r_1 = h + 1$$

$$r_{J+1} = r_{m1}$$

$$r_{J+M+1} = r_{m2}$$

## CONCLUSION

It is hoped that the fairly detailed description of the lethal area problem presented herein will provide the reader with adequate background not only for the computation of lethal areas but also to enable him to consider allied concepts and consider special cases of the lethal area problem. Since virtually all of the evaluation models used by the Weapon Systems analyst are based upon a casualty concept, i.e., expected fraction casualties, number of rounds or weight of ammunition required to produce a given casualty level, etc., an understanding of the probability of incapacitation concept discussed herein is mandatory. Further many evaluation models utilized today are approximations which utilize lethal area as such, giving some consideration to weapon accuracy and target size.

*K. A. Myers*  
K. A. MYERS

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