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# GENERATION OF WAVES BY WIND

## STATE OF THE ART

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GENERATION OF WAVES BY WIND  
STATE OF THE ART

by

Charles L. Bretschneider

Presented at

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## PREFACE

This report was prepared originally as a series of lectures given at the International Summer Course on "Some Aspects of Shallow Water Oceanology" held at Lunteren, the Netherlands. The subject of this phase of the lectures, "Generation of Waves by Wind," included both deep and shallow water conditions. The decay of swell in both deep and shallow water was also discussed. In addition to the original prepared manuscript, this report includes some material resulting from the discussions during and following the lectures.

# GENERATION OF WAVES BY WIND

## STATE OF THE ART

### I. INTRODUCTION

When air flows over a water surface waves are formed. This is an observable phenomenon. Just why waves form when air flows over the water is a question about nature which has not yet been answered completely or satisfactorily by theoretical means. Furthermore, why do waves have the heights and periods that are observed? All theories begin either with the assumption that waves do form when wind blows over the water surface, or else that waves must already exist by the time the wind begins to blow, or else the theory is immobile.

The brothers Ernst Heinrich and Wilhelm Weber (1825) were the first known to report experiments on waves, and A. Paris (1871) made actual wave observations on the state of the sea. These observations were made aboard the DUPLIEX and the MINIERVA. Although Airy (1848) did theory on waves and tides, wind forces were not included. Other early contributions on wave observations at sea included Abercromby (1888), Schott (1893), and Gassenmayr (1896). However, the best early documentation on wave observations was perhaps that prepared by Cornish (1904, 1910 and 1934). Cornish also attempted to relate wave conditions to meteorological and geographical conditions.

Rather than thinking of the scientist in the role of answering the question "Why do waves form," one might rather think of him as a practical engineer who knows that the phenomenon does occur and who can then recommend what should be done about it. Progress is made only by engineering application of scientific theory. Theory offers no progress, except when implemented; otherwise it is dormant.

Since a little theory has never hurt a practical engineer, an oceanographer, or an applied scientist, it seems quite appropriate to mention various theories which have been proposed through various stages in the advancement of the state of the art.

Although Stevenson (1864) established the first known empirical formula for wave generation, the classical work on wind wave theory was due to Lord Kelvin (1887) and Helmholtz (1888). The Kelvin-Helmholtz theory, which can be found in Hydrodynamics, by Lamb (1945), pertains to the study of the oscillations set up at the interface of two fluid media of different densities -- water and air, for example.

Helmholtz considered the media of mass densities  $\rho_1$  and  $\rho_2$  flowing with velocities  $U_1$  and  $U_2$  with respect to each other as illustrated in Figure 1.

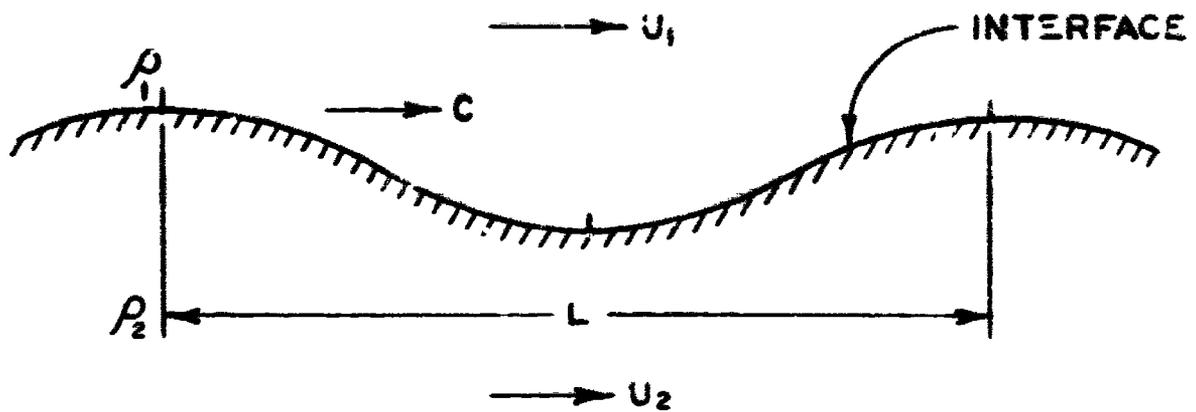


Figure 1

The interface is a wave surface,  $\rho_1$  and  $U_1$  the mass density and velocity of the upper fluid, and  $\rho_2$  and  $U_2$  the mass density and velocity of the lower fluid, respectively. The propagational velocity  $C$  is the speed at which the interface travels in a forward direction, and  $L$  is the distance between two successive peaks of the interface.

Helmholtz (1888) showed that the induced oscillation, if small compared with the distance  $L$ , took the form of a wave train at the interface traveling at the velocity  $C$  such that

$$\rho_1 (U_1 - C)^2 + \rho_2 (U_2 - C)^2 = \left(\frac{g}{k}\right) (\rho_2 - \rho_1) \quad (1)$$

where  $g$  is the acceleration of gravity and  $k$  is the wave number given as  $k = 2\pi/L$ .

Kelvin (1887) derived the same result in a different manner and made some very interesting conclusions. For example, when  $U_1 = U_2 = 0$  it can be shown that

$$C^2 = \frac{g}{k} \left( \frac{\rho_2 - \rho_1}{\rho_1 + \rho_2} \right) = \frac{g}{k} \left( \frac{1 - \frac{\rho_1}{\rho_2}}{1 + \frac{\rho_1}{\rho_2}} \right) \quad (2)$$

If one considers the upper fluid to be air and the lower fluid water, for which  $\rho_1/\rho_2$  is equal to about  $1.29 \times 10^{-3}$ , then Eq. (2) reduces very nearly to the simple form

$$C^2 = \frac{g}{k} \quad (3)$$

which is the classical equation for wave celerity obtained from linear wave theory.

By use of the quadratic formula, Eq. (1) can be solved for  $C$  and one obtains:

$$C = \frac{\rho_2 U_2 + \rho_1 U_1}{\rho_1 + \rho_2} \pm \frac{g}{k} \left[ \left( \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \right) - \frac{\rho_1 \rho_2}{(\rho_1 + \rho_2)^2} (U_2 - U_1)^2 \right]^{1/2} \quad (4)$$

Now one can write Eq. (4) as follows:

$$C = \bar{U} \pm \bar{C} \quad (5)$$

where

$$\bar{U} = \frac{\rho_1 U_1 + \rho_2 U_2}{\rho_1 + \rho_2} \quad (6)$$

and

$$\bar{C}^2 = C_o^2 - \frac{\rho_1 \rho_2}{(\rho_1 + \rho_2)^2} (U_2 - U_1)^2 \quad (7)$$

In the above  $\bar{U}$  and  $\bar{C}$  represent an average of the corresponding values of  $U$  and  $C$ , and  $C_o^2$  is the expression identical to Eq. (2).

If  $C_o^2$  is less than the term involving  $U_2 - U_1$  in Eq. (7), it will be found that  $\bar{C}$  becomes imaginary, which implies a condition of instability in the development of the waves, and this leads to a progressive increase in amplitude. Under these conditions the wind is traveling faster than the waves and there will be a continuous transfer of energy to the waves, which in turn goes into the form of increase in wave height and increase in wave celerity. The term  $U_2 - U_1$  represents the wind velocity relative to the water and is usually expressed simply by  $U = U_2 - U_1$ . The condition of instability is defined when

$$U > \frac{1 + \rho_1 / \rho_2}{\sqrt{\rho_1 / \rho_2}} \quad (8)$$

Since  $\rho_1 / \rho_2 = 1.29 \times 10^{-3}$ , one may obtain

$$U > 28 C_o \quad (9)$$

$$\frac{C_o}{U} < \frac{1}{28}$$

The ratio  $\frac{C_o}{U}$  has been defined as the wave age, and waves are unstable when their wave age is less than 1/28; this instability manifests itself as a progressive increase in wave amplitude.

**From other considerations it can be shown that the smallest velocity that a capillary wave (such as a ripple) can have is 23.2 cm/sec, corresponding to a wave length of 1.7 cm. and a period of .073 seconds. Work by Crapper (1957), Schooley (1960) and Pierson (1961) shows that because of nonlinear effects, the 1.7 cm. is somewhat low. According to Kelvin (1887) the waves will always be unstable if  $U > 28 \times 23.2$  cm/sec. That is, if  $U > 6.5$  m/sec (12.5 knots), then the waves are unstable.  $U = 6.5$  m/sec is also called the critical wind speed required for gravity wave generation. According to Munk (1947) there is a critical wind speed below which waves do not form. There have been numerous articles on the subject of critical wind speed, some supporting and others objecting to the existence of a critical wind speed. Reference is made to the work of Cox and Munk (1956). Later Munk (1957) appears to be dubious as to whether or not a critical wind speed exists, citing the work of Mendelbaum (1956) and Lawford and Veley (1956). If a critical wind speed exists, it appears from all literature sources that it exists approximately between 2 and 6 meters per second.**

The concept of critical wind speed is indeed a controversial subject at present. Nevertheless, there is still belief that there is a critical wind speed somewhere between 4 and 6 m/sec., and that below the critical wind speed the fluid flow is hydrodynamically smooth or laminar and above the critical wind speed turbulence develops and the interface becomes hydrodynamically rough, for which wave amplitudes increase with time and distance.

The difficulty with this theory is that there is a density difference between air and water even when the wind does not blow, and yet no waves are formed. Thus density difference alone is insufficient to start wave generation; the waves must already have existed by some other means, then they can propagate as free gravity waves. If there is a critical wind speed the wind is already blowing, but there are no waves. With an increase in wind speeds, waves do form. Why?

It was not until 1925 that Jeffreys introduced the theory of "shear stress hypothesis," based on the concept of a hydrodynamically rough sea required for wave generation. In his paper "On the Formation of Waves by Wind," Jeffreys (1925) proposed that eddies on the leeward side of the waves resulted in a reduction of normal pressure as compared with the windward face and in a consequent transfer of energy from wind to waves. His results suggested that the wind could add energy to waves only so long as the wind speed was equal to or greater than the wave celerity, and that when the wave celerity became equal to the wind speed the waves reached maximum height and the sea was one of steady state. Also, the lowest wind speed required for wave generation was on the order of two knots, or about one meter per second. It appears that the value of two knots is the best accepted value for the critical wind speed.

During the ten years following the work of Jeffreys, somewhat more detailed observations were made of the sea surface conditions.

For example, Schumacher (1928) reported the first known study of "Stereophotography of Waves" from the German Atlantic Expedition, and Weinblum and Block (1936) also reported results on stereophotogrammetric wave records. This latter contribution gave results of measurements carried out on board the motor ship SAN FRANCISCO, under observation of V. Cornish. Other data on waves during this period included that of Williams (1934), who reported on sea and swell observations, including early methods of obtaining data, and Whitemarsh (1935) who reviewed data on unusual sea conditions as reported by mariners, and discussed the cause of high waves at sea and the effect of these waves on shipping.

After the founding of the Beach Erosion Board in the War Department in the early 1930's, serious research began on theory and formation of gravity waves. The first important report was completed in 1941 and published in 1948, "A Study of Progressive Oscillatory Waves in Water," by Martin A. Mason (1948). This report updated the state of the art to about 1940.

The next great advance in the theory of wave generation in deep water was that by Sverdrup and Munk (1947), although Suthons (1945) had already prepared forecasting methods for sea and swell waves. Whereas Jeffreys (1925) took into account only the transfer of energy by normal stresses, Sverdrup and Munk considered both normal and tangential stresses. (See Figure 2.)

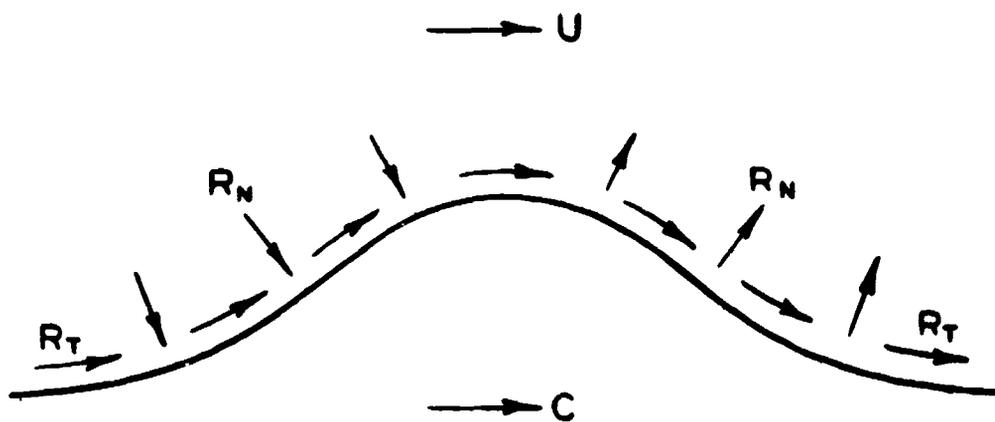


Figure 2

The average rate at which energy is transferred to a wave by normal pressure is equal to

$$R_N = \frac{1}{L} \int_0^L p_z w_0 dx \quad (10)$$

where  $w_0 = -kAC \cos k(x - Ct)$  is the vertical component of the particle velocity at the surface, and  $p_z$  is the normal pressure acting on the sea surface.  $L$  is the wave length.

The average rate at which energy is transmitted to the waves by tangential stress is equal to

$$R_T = \frac{1}{L} \int_0^L \tau u_0 dx \quad (11)$$

$u_0$  denotes the horizontal component of particle velocity at the sea surface, and  $\tau$  is the wind stress.

$$u_0 = kAC \sin k(x - Ct) \quad (12)$$

$$\tau = \gamma^2 \rho' U^2 \quad (13)$$

where  $\gamma^2$  is the resistance coefficient. Various experiments and observations have been made leading to controversial values of  $\gamma^2$  as a function of wind speed. However, a number of different authorities appear to have advocated a value of  $\gamma^2$  close to  $2.6 \times 10^{-3}$ , and it is this value utilized by Sverdrup and Munk (1947).

According to the above arguments, the energy of waves can increase only if  $(R_N + R_T)$ , the rate at which energy is added by both normal and tangential stresses of the wind, exceeds  $R_U$ , the rate at which energy is dissipated by viscosity. The energy added by the wind goes into building the wave height and increasing the wave speed. That is,  $R_H + R_C = R_T + R_N$ , where  $R_H$  is that portion of energy transformed into wave heights and  $R_C$  is that portion of energy transformed into wave speed.

During the early stages of wave development most of the energy is transmitted by normal stresses, but when  $C/U > 0.37$  the transmission by tangential stress is dominant. The effect of the normal stresses dominates for a short time only. During the time that the waves are growing, the effect of the tangential stress is most important. When  $C/U = 1.0$ , energy is added by tangential stress, but there is a small amount lost due to normal pressure, and, for this reason, the relation is written with  $+ R_N$ . When  $R_U = R_T + R_N$ , the waves are said to have reached maximum height and celerity for a particular wind speed and are independent of fetch length and wind duration. This condition is sometimes called the fully developed sea.

According to the work of Sverdrup and Munk (1947), the solution of the hydrodynamic equation describing wave generation entailed a knowledge of certain coefficients or constants resulting from mathematical integration which, of course, could not be determined by theory alone. The appropriate constants were determined by use of empirical data.

Hence, there was no way of knowing whether or not the theory was correct. In order to evaluate these constants it was necessary to resort to empirical wind and wave data, which at that time was very limited. Wind speeds, fetch lengths and wind durations were estimated from meteorological situations, the data of which were also based on very meager coverage. The waves were estimated by visible means. Out of this theoretical investigation grew the concept of the significant wave. The significant wave height was estimated as the average wave height of the waves in the higher group of waves, which later became identified very closely as the average of the highest one-third of the waves in a record of about 20 minutes duration. The significant wave period was the corresponding average period of these waves.

According to the theory as evaluated with "ancient" data for the significant wave, the fully developed sea resulted in the following relations:

$$gH/U^2 = 0.26$$

and

$$\frac{gT}{2\pi} = C/U = 1.37$$

where H and T are the significant wave height and period respectively, and U is the wind speed. It then became quite apparent for any situation, either wind waves or swell, that a whole spectrum of waves was present, including a probability distribution of wave heights and a probability distribution of wave periods. Much of the above work was performed during the days of World War II. Otherwise earlier publications would have appeared in the literature. In fact, as early as 1935, the Imperial Japanese Navy encountered a typhoon in the Pacific Ocean and many observations were taken but were not published until much later by Arakawa and Suda (1953).

The time had then arrived when no further advance in wave generation theory could be made without reliable recorded data and an advance in statistical theory and data reduction and analysis. There is no necessity to discuss wave recording here since this subject is well covered by Tucker (1964).

Barber and Ursell (1948) were perhaps the next to present a very important paper. The results of their investigation proved the existence of a spectrum of waves. A completely new field of theory and research had been initiated, but it should be noted that oceanographers were slow to take advantage of this concept. This research had laid the foundation upon which many advances have been made in the "state of the art," and it is because of this research that the Sverdrup-Munk (1947) works are considered "ancient."

Although much research was carried out during the next few years, no great advances in the state of the art were published until Thijsse and Schijf (1949) presented wave relationships for both deep and shallow water

based on wave data and some considerations of the Sverdrup-Munk theory. Experiments on a paraffin model of wind-generated waves by Thijsse and Schijf show a high negative pressure at the crest of the wave, which is in conflict with the Sverdrup-Munk concept of a constant wind along the free surface, but which is in accordance with Bernoulli's equation for an increase in wind speed at the crest and a decrease at the trough. This experiment was certainly a great contribution.

Johnson (1950) applied the Pi-theorem concept for dimensional analysis and presented wave relationships for deep water based on a collection of numerous data from Abbots Lagoon, California. At the same time the U. S. Army Corps of Engineers (1950) had presented wave data generated under hurricane wind conditions for shallow Lake Okechobee, Florida. The Corps of Engineers also presented wind and wave data for inland reservoirs, Fort Peck, Montana (1951), and later for Lake Texoma, Texas (1953). Bretschneider (1951) presented revised wave forecasting relationships of Sverdrup and Munk (1947), based on the field data of Johnson (1950), laboratory data of Bretschneider and Rice (1951), and numerous other data collected by various authors. Bracelin (1952) presented an unpublished report on observing, forecasting, and reporting ocean waves and surf. An excellent summary of wave recordings was presented by Wiegel (1962).

It seems that the year 1952 witnessed the first acceleration in the "state of the art." Longuet-Higgins (1952) presented the Rayleigh distribution for wave height variability based upon a narrow spectrum. Putz (1952) presented a Gamma-type distribution for wave height and wave period variability based upon analysis of 25 twenty-minute ocean wave records. Darbyshire (1952) and Neumann (1952) each presented wave spectra concepts and relations for wave generation based on collection of wave data. The method of derivation used by Neumann (1952) is controversial; it lead to the introduction of a dimensional constant, and for high frequency, the energy was found to be proportional to  $f^{-6}$ , where  $f = \frac{1}{T}$  wave frequency. Watters (1953) derived the Rayleigh distribution of wave height variability in a less sophisticated manner than Longuet-Higgins, and the data of Darlington (1954) supported the Rayleigh distribution. In fact, the Gamma-type distribution for wave heights of Putz (1952) was represented very closely by the Rayleigh distribution. Ichiye (1953) studied the effects of water temperature on wave generation. Homada, Mitsuyasu and Hase (1953) performed laboratory tests on wind and water.

The first acceleration of the "state of the art" did not mean much in regard to the development of the theory of wave generation, since this was purely empirical, including both field and laboratory data collection, except for the work of Longuet-Higgins (1952) and Watters (1953).

Ursell (1956) had surveyed the problem of wind wave generation and opened with the statement that "wind blowing over a water surface

generates waves in the water by a physical process which cannot be regarded as known;" he concluded that "the present state of our knowledge is profoundly unsatisfactory. "

Bretschneider and Reid (1954) presented a theoretical development for the "Change in Wave Height due to Bottom Friction, Percolation, and Refraction;" Bretschneider (1954) combined these relationships with the wave generation relationships given by Sverdrup and Munk (1947), as revised by Bretschneider (1951), to obtain shallow water wave generation relationships for wave height and wave period as a function of wind speed, fetch length and water depth. Sibul (1955) investigated in the laboratory the generation of wind waves in shallow water. Aside from the above references and the contributions of Thijsse and Schijf (1949) and the U. S. Army Corps of Engineers, Jacksonville District (1950), there is still another contribution on wind-generated waves in shallow water. In 1953 and 1954 Keulegan performed experiments at the National Bureau of Standards; but as far as is known, this information has not been published. However, it has been ascertained that the data of Keulegan is in agreement with that obtained for Lake Okeechobee and is also in agreement with the relationships presented by Bretschneider (1954). Saville (1954) published a useful report on the effect of fetch width on wave generation.

Data collection, although limited in quantity and quality, also persisted during the period from 1950 to about 1955. For example, Unoki and Nankano (1955) in Japan published wave data for Hachijo Island; Titov (1955) presented works in Russian; and Bretschneider (1954) did work on shallow water of the Gulf of Mexico.

There seems to be a transition period during the years 1955 to 1960. His ideas based on theory and verified with data, Krylov (1956 and 1958) presented the Rayleigh distribution for wave height variability and postulated also that the Rayleigh distribution applied to the wave length variability, which could be transformed into a period distribution function. Bretschneider (1957 and 1959) also verified the Rayleigh distribution for wave height and wave length variability. He developed a distribution function for wave period variability which was in agreement with that postulated by Krylov (1958) and was in very close agreement with the Gamma-type distribution function for wave period presented by Putz (1952). Roll and Fischer (1956) made a revision of the spectrum by Neumann (1952), eliminating the dimensional constant, and found that the energy at high frequency was proportional to  $f^{-5}$  instead of  $f^{-6}$  according to the spectrum of Neumann (1952).

It then became apparent that for a narrow spectrum it was safe to assume that the Rayleigh distribution applied equally well for both wave height and wave length variability, the latter readily transformed into a wave period distribution function.

Statistical representation of the sea by wave spectra concepts through the work of Tukey (1959) and Blackman and Tukey (1958) was

greatly promoted by the staff at New York University, particularly Pierson and Marks (1952).

Under the assumption that the joint probability distribution of wave height and period was uncorrelated, Bretschneider (1957, 1958, 1959) proposed a development of a wave spectrum concept. There seemed to be some similarity between the Bretschneider spectrum and that proposed by Neumann (1952), and finally the form of Bretschneider's spectrum resolved as the proposed spectrum of Pierson (1964) based on the similarity theory of Kitaigorodski (1961).

At this time also Bretschneider (1958) again revised the wave forecasting relationships for both deep and shallow water. The practical graphs for wave forecasting are given in the revised version of Beach Erosion Board Technical Report No. 4 (1961). These relationships presently are undergoing a further revision which should increase the accuracy of wave forecasts.

However, during the ten years preceding about 1955, most of the effort was devoted to analytical expressions and little to theory of wave generation. During the days of Jeffreys (1925), Sverdrup and Munk (1947) among others, the concept of wave spectra was not promoted. However, because of the wave spectra enlightenments of Barber and Ursell (1948), Seiwel (1948), Neumann (1952), Darbyshire (1952), Bretschneider (1957), Burling (1959), and Pierson (1964), among others, a new channel was opened for wave generation theory.

At this point mention should be made of the great contributions on wave theory, wave probability distribution functions, and wave spectra proposed by Miche (1954). In particular, Miche proposed the Rayleigh distribution to wave steepness, a theory not previously proposed. This distribution function should have a wide application for engineering studies.

It was not until Phillips (1957) and Miles (1957) that additional theoretical concepts were developed. Sverdrup and Munk (1947) considered that the wind was constant in velocity in order to develop their theory, but this was proven otherwise by Thijsse and Schijf (1949). However, Phillips (1957) considered the fact that the wind was rapidly fluctuating about some mean value. It is very true that winds blowing over water do not consist of streams of air in steady and uniform motion but, rather, of an irregular series of "puffs" and "lulls" carrying eddies and swirls distributed in a disordered manner. The atmospheric eddies, or random velocity fluctuations in the air, are associated with random stress fluctuations on the surface, both pressures (i. e. normal stresses) and tangential stresses. The eddies are borne forward by the mean velocity of the wind and, at the same time, they develop, interact, and decay, so that the associated stress distribution moves across the surface with a certain convection velocity dependent upon the velocity of the wind and also evolves in time as it moves along.

It is these pressure fluctuations upon the water surface that are responsible for the early generation of waves. The tangential stress is not considered, but Phillips (1957) states in some cases that the shear stress action might not be negligible. The theory is in agreement with wave observations during the early stages of generation, but as  $C/U$  approaches unity there are other wave generating processes to take into account, such as sheltering and the effects of variation in shear stresses. Although this theory tends to an under-estimation of wave heights for  $C/U$  close to unity, it may be considered as a great advance in wave generation theory insofar as the initial birth and growth of waves are concerned. A very important aspect results from Phillips (1957) based on dimensional considerations; i. e., for high frequency components the energy varies as  $f^{-5}$ .

Miles' theoretical model for the generation of water waves is based on the instability of the interface between the air flow and the water. The theory of Phillips predicts a rate of growth of the sea proportional to time, whereas after the instability mechanism of Miles takes over, the rate of growth becomes exponential. The Phillips model is an uncoupled model in the sense that excitation (air flow) is assumed to be independent of response (sea motion). The Miles theory represents a coupled model in which the coupling can lead to instability and consequent rapid growth. There can be little doubt that both mechanisms occur in any practical situation. At some frequencies in the spectrum the uncoupled model will govern and at others the instability model will govern. The work of Miles (1960) is a recognized contribution on wave generation theory.

Ijima (1957) presented an excellent paper on the properties of ocean waves for the Japanese area of interest. This study included valuable information on wave spectra obtained under typhoon conditions. Also a decided difference existed between wave spectra obtained on the open Pacific Coast and that obtained for the coast of the Sea of Japan.

Another important effort for obtaining wave spectra was that conducted by members of the New York University: Chase, Cote, Marks, Mehr, Pierson, Ronne, Stephenson, Vetter and Walden (1957). All embarked upon a great task of obtaining the first directional spectrum of a wind-generated sea by stereophotographic techniques, although Weinblum and Block (1936), about 20 years earlier, carried out measurements entailing stereophotogrammetric reproduction of ocean waves on board the motor ship SAN FRANCISCO.

Sulkeikin (1959) presented the Russian methods of forecasting wind waves over water, which followed from his earlier works on the theory of sea waves (1956). Burling (1959) presented a spectrum of waves at short fetches and found a range in values of  $m$  in the high frequency  $f^{-m}$  of the wave spectrum. Korvin-Kroukovsky (1961), in his Theory of Seakeeping, summarizes much of the early work on wind wave generation.

In particular, this book includes a very comprehensive bibliography on waves and wave theory.

In 1961 an International Conference on Ocean Wave Spectra was held at Easton, Maryland, the proceedings of which were published in 1963.\* This conference included about 30 presentations, plus discussions, and had an attendance of less than 100 participants, representing a very large percentage of the scientists and engineers in the world who have been contributing to the advancement of the science of ocean wave spectra. It can be said that this conference brought the "state of the art" up to date. Known and unknown properties of the frequency spectrum of a wind-generated sea, by Pierson and Neumann (1961, 1963) was the most logical paper to lead off the program. Unless one understands the concept of a fully developed sea, the work of Walden (1961, 1963) might be misinterpreted since his data were for very short effective fetches. However, this difficulty was clarified in the discussion of Bretschneider (1961, 1963). Numerous discussions followed and the program continued well on its way throughout the four-day period. The conference produced two sources of directional spectrum: Longuet-Higgins, Cartwright and Smith (1961, 1963) and Munk (1961, 1963). On the last day, with heads still spinning, it became an accepted fact that the one-dimensional linear concepts were not always sufficient to describe the state of the sea.

However, the conference was not intended to bring forth new theory on how waves form when wind blows over the water, except for discussion of the work of Phillips (1957) and Miles (1957), and an introduction "On the Nonlinear Energy Transfer in a Wave Spectrum" by Hasselmann (1961, 1963). Otherwise the theory was limited to that required for data collection, data reduction and analysis, and data presentation and applications. The work of Hasselmann (1961, 1963) is indeed a classical contribution, but it still does not tell us why waves are formed according to pre-described elevations and frequencies.

During the final stages of development of a wind-generated sea, two nonlinear processes could become significant. There is a dissipation of wave energy due to breaking (whitecaps), and a transfer of energy flux between frequency bands may take place. The theory of Hasselmann concerns the latter process. The theory shows that by the fifth order interactions energy can be transferred between frequency bands in the spectrum. In fact, Phillips (1960 and 1961, 1963) has shown that this theoretical energy transfer can occur at certain third order resonant interactions. So far spectral energy transfer is a purely theoretical conjecture and has not been verified by observation or experiment.

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\* In the following material references to the above conference and proceedings are shown by (1961, 1963).

Cartwright (1961) and Pierson (1961) gave brief reports on the papers presented at the 1961 conference, prior to the publication of the proceedings.

After the conference was over, the participants went home to work again, hoping to advance the state of the art. More data were to be collected, and this required further development of instrumentation, and an advancement of statistical theory and computation procedures.

After the conference several important papers appeared, although they had probably been worked on for several years. These included a paper by Korneva (1961) on wave variability which tended to verify the previously mentioned probability distribution functions; a joint paper on "Data for High Wave Conditions Observed by the OWS 'Weather Reporter' in December 1959" by Bretschneider, Crutcher, Darbyshire, Neumann, Pierson, Walden and Wilson (1962); and a paper by Schellenberger (1962) on "untersuchungen uber Windwellen auf linem Binnensee." Pierson and Moskowitz (1963) contributed a new form of the one-dimensional wave spectrum based on the similarity theory of Kitaigorodski (1961) and found out that this spectrum fell somewhere among the other past proposed spectra, considering the inherent errors arising from difficulties in determining and defining wind speeds.

Walden and Pierson (1961) presented data and analysis on wave spectra obtained near the Mellum Plate Lighthouse, located in the somewhat sheltered water off the North Sea coast of Germany.

A good summary of wave theory and wave generation is presented in Volume I of The Sea, edited by Hill (1962), particularly Chapter 19, "Wind Waves," by Barber and Tucker (1962).

Numerous data reports on wave spectra are now becoming available. For example, Moskowitz, Pierson and Mehr (1962, 1963) prepared reports on wave spectra estimated from wave records obtained by the OWS "Weather Reporter I, II and III" and the OWS "Weather Explorer," and Pickett (1962) presented wave spectra for the Argus Island tower off Bermuda.

Bretschneider (1962) presented a concept on modification of wave spectra over the continental shelf, and Ijima (1962) presented an interesting development of the correlation between wave heights and wave periods for shallow water. Kitaigorodski and Strekalov (1962, 1963) presented contributions to an analysis of the spectra of wind wave motion based on experimental data. This work was a continuation of the work of Kitaigorodski (1961).

Goodknight and Russell (1964) presented the first data on large wind waves in shallow water of the Gulf of Mexico. These waves were generated under hurricane wind conditions. The statistical analysis of the data showed that, for all practical purposes, the Rayleigh distribution

was quite satisfactory for representing wave height variability for large hurricane waves in shallow water. The distribution of wave periods did not follow the distribution function of Bretschneider (1957) or that of Putz (1952), but fell somewhere between these distribution functions and the Rayleigh distribution. The data seem to consist of long period waves arriving from deep water combined with local wind waves generated at a large angle to the swells.

Hamada (1963, 1964) presented two very interesting reports based on laboratory experiments of wind wave generation. The  $f^{-5}$  law originally proposed by Phillips (1957) was stated to be applicable to the limiting boundary of the instability. For very short fetches and high wind speeds, Hamada (1964) finds the high frequency relations of  $f^{-7.3}$  and  $f^{-8.94}$ .

According to the work of Bretschneider (1959), the high frequency energy varies with  $f^{-m}$  where  $m = 9$  for very low  $gF/U^2$ , and decreases in magnitude to  $m = 5$  for very large  $gF/U^2$ , corresponding to fully developed seas. Thus there is agreement at initial wave generation for  $f^{-m}$  between Hamada (1963) and Bretschneider (1959), and also agreement at fully developed wave generation  $f^{-5}$  among Phillips (1957), Bretschneider (1959), Pierson (1963), and Hamada (1964). There are still efforts required for the exponential part of the spectral equation, i. e.  $e^{-Bf^{-n}}$  where  $B$  is a constant. According to Bretschneider (1959) and Pierson (1963),  $n = 4$ , but the factor  $B$  is still in some disagreement. The work of Burling (1959) indicates that  $n$  might be larger than  $n = 4$ . Bretschneider (1961, 1963) states that  $n$  might vary between 4 and 8 or 9.

To date, all theories are useful in attempting to understand the mechanisms involved in the generation of waves. None of the theories tells us why waves are formed, let alone why the wave heights and periods are as observed or why there is a spectrum of waves. However, there is enough information to formulate various empirical wave forecasting relationships for certain practical applications. The accuracy of such wave forecasting relationships depends on the accuracy of the wind and wave data collected and used for the empirical relationships. The accuracy of the wave forecasts then also depends upon the accuracy of the wind forecasts.

At this point of the discussion it appears in order to introduce an equation which describes the sea state wave spectrum, including the variability of wave direction. The equation can be written as follows:

$$E(\omega, \phi) = \frac{k^2 \omega t}{2 (g \rho_w)^2} \frac{e^{mt} - 1}{mt} \int_0^\infty \Pi(\underline{k}, \omega) \cos \phi \left[ \left( \frac{\omega U \cos \phi}{g} - 1 \right) \omega \tau \right] d\tau \quad (14)$$

where

$$m = \frac{\rho_a}{\rho_w} \left( \frac{\omega u_* \cos \phi}{g} \right)^2 \omega \beta$$

In Eq. (14)  $\Pi(\underline{k}, \tau)$  is the three-dimensional pressure spectrum as a function of the vector wave number  $\underline{k}$  and time  $\tau$ ;  $U$  is the convection velocity of the pressure systems, and  $u_*$  is the friction velocity of the shear flow.  $\beta$  is the coefficient calculated by Miles (1960), and  $\rho_w$  and  $\rho_a$  are water and air densities respectively.

There is little need to extend the above review any further since the directional spectrum has been discussed quite adequately by Tucker (1964). It is hoped that most of the important contributions on wave generation have been mentioned. Eq. (14) represents the present state of the art on wave spectrum generation theory, but additional empirical data are required.

In regard to practical methods for wave hindcasting, Bretschneider (1964) presented a paper which takes into account the complete problem of deep water waves, storm surge and waves over the continental shelf, the breaking wave zone, the wave run-up on the beach and dunes for the March 5-8, 1962, East Coast Storm. This paper shows the results based on present methods of wave hindcasting and also emphasizes the areas of need for further research. A very important consideration of wind wave generation over the shallow water of the continental shelf is that of the total water depth. The total water depth includes the combined effect of ordinary tide and storm surge. The various problems of wind set-up and storm surge have been discussed by Bretschneider (1958). No further discussion on tides and storm surge is given here since the subjects were well discussed by various lecturers at Lunteren, e.g. Drs. J. R. Rossiter, W. Hansen, P. Groen, J. Th. Thijsse, and J. C. Shonfeld. Additional work on storm surge problems is planned for subsequent reports.

Although the work of Sverdrup and Munk (1947) has been called "ancient," the revised wave generation parameters are still considered the most practical, and the application of these relationships still results in more accurate forecasts than the other underdeveloped techniques. It may not be too long in the future before the wave spectra techniques become more accurate.

## II. PRACTICAL APPLICATIONS -- DEEP WATER

### A. SIGNIFICANT WAVE CONCEPT

The significant wave method of wave forecasting was that originally introduced by Sverdrup and Munk (1947) and is sometimes considered the "ancient" method. The wave forecasting parameters presented by them evolved from theoretical considerations, but the actual relationships required certain basic data for the determination of various constants and coefficients. Hence, this method can be called semi-theoretical or semi-empirical.

The significant wave method entails certain definitions. The significant wave height is the mean or average wave height of the highest 1/3 of all the waves present in a given wave train. The significant wave period represents the mean period of the significant wave height. It was found from the analysis of wave records that the significant height is nearly equal to that height reported from visual observations, and for this reason there was sometimes a certain amount of agreement between various empirical formulas used prior to the development of the theory.

It might be mentioned that the significant wave period represents a period around which is concentrated the maximum wave energy. From the work of Putz (1952), Longuet-Higgins (1952), and Bretschneider (1959), the distribution of the various wave heights can be determined by use of the significant wave height.

The wave parameters obtained from the theoretical work of Sverdrup and Munk (1947) can also be obtained from dimensional considerations. This has been done by Johnson (1950), among others, utilizing the Buckingham Pi-theorem (1914).

The factors on which the wind wave parameters for deep water depend are the wind velocity  $U$ , the fetch length  $F$ , and the wind duration  $t$ . Of the wave parameters only wave height and wave period need to be considered since, in deep water, the wave length  $L = (g/2\pi)T^2$  and the wave celerity  $C = (g/2\pi)T$ . The waves will surely be subject to the influence of gravity and then it may be supposed that:

$$C = f_1(U, F, t, g) \quad (15)$$

and

$$H = f_2(U, F, t, g) \quad (16)$$

Eqs. (15) and (16) state that C and H respectively are functions  $f_1$  and  $f_2$  of U, F, t, g, but apply for deep water only.\*

From the symbols appearing in Eqs. (15) and (16) one may write the dimensions for deep water as follows:

<u>Symbol</u>	<u>Dimensions</u>
C	$LT^{-1}$
H	L
U	$LT^{-1}$
F	L
t	T
g	$LT^{-2}$

Additional relationships can be written if the water depth is taken into account.

For each of the above equations there are five variables and two dimensional units, whence from the Buckingham Pi-theorem the solutions will each be functions of  $5 - 2 = 3$  dimensionless products, with  $2 + 1 = 3$  variables to each product. In respect to Eq. (15) one can write

$$0 = F_1 (CU^a g^b), (FU^c g^d), tU^e g^f) \quad (17)$$

With respect to equation (17) one obtains the dimensions

$$\left[\frac{L}{T}\right]^1 \left[\frac{L}{T}\right]^a \left[\frac{L}{T^2}\right]^b, \left[L\right]^1 \left[\frac{L}{T}\right]^c \left[\frac{L}{T^2}\right]^d$$

and

$$\left[T\right]^1 \left[\frac{L}{T}\right]^e \left[\frac{L}{T^2}\right]^f$$

\* If one considers wave generation in shallow water, Eqs. (15) and (16) become

$$C = f_1 (U, F, t, d, g) \quad (15a)$$

and

$$H = f_2 (U, F, t, d, g) \quad (16a)$$

where d is the water depth.

Equating to unity the sum of the exponents for the corresponding dimensions, one obtains the following equations:

$$\begin{aligned}
 1 + a + b &= 0 \\
 -1 - a - 2b &= 0 \\
 1 + c + d &= 0 \\
 -c - 2d &= 0 \\
 e + f &= 0 \\
 1 - 2 - 2f &= 0
 \end{aligned}$$

The simultaneous solution of the above results in

$$\begin{aligned}
 a &= -1 & d &= 1 \\
 b &= 0 & e &= -1 \\
 c &= -2 & f &= 1
 \end{aligned}$$

Using values of the above exponents, Eq. (17) becomes

$$0 = F_1 \left[ \left( \frac{C}{U} \right), \left( \frac{gF}{U^2} \right), \left( \frac{gt}{U} \right) \right]$$

or

(18)

$$\frac{C}{U} = \psi_1 \left[ \frac{gF}{U^2}, \frac{gt}{U} \right]$$

In a similar manner the corresponding expression from Eq. (16) becomes

$$\frac{gH}{U^2} = \psi_2 \left[ \frac{gF}{U^2}, \frac{gt}{U} \right] \quad (19)$$

It might be mentioned that the Pi-theorem is a most powerful tool if properly used. It is extremely important to realize that the expressions for physical fact must be dimensionally homogeneous; otherwise there are some scientific factors missing.

Equations (18) and (19) represent the wave generation parameters for deep water, based on dimensional considerations.  $\psi_1$  and  $\psi_2$  are functional relations that must be determined by use of wave data.  $gH/U^2$ ,  $C/U$ ,  $gF/U^2$  and  $gt/U$  are defined respectively as the wave height parameter, the wave speed parameter, the fetch parameter, and the wind duration parameter. The wave speed parameter can also be written  $\frac{gt}{2\pi U}$ , which is a better form because the wave period is more easily measured than the wave speed.

Using the above parameters Bretschneider (1951) revised the original forecasting relations of Sverdrup and Munk (1947), utilizing much additional wave data. Before 1951, however, Arthur (1947) also revised the same relations, but did not have the data that were available in 1951. Further revisions of these relationships were made again by Bretschneider (1958). These forecasting relationships have acquired the name S-M-B method for Sverdrup, Munk and Bretschneider.

The final form of the dimensionless wave generation parameters appears in figure 3.

The curve  $tU/F$  is the relationship between wind speed, minimum duration and fetch length, and was determined by numerical integration of the following relationships:

$$\frac{gt}{U} = \int \frac{U}{C_g} \delta \left( \frac{gF}{U^2} \right) ; \quad \frac{C_g}{U} = \frac{1}{2} \frac{C_o}{U} \quad (20)$$

$$\frac{tU}{F} = \frac{gt}{U} \div \frac{gF}{U^2}$$

Wave forecasting relationships given in Figure 4 are based on the dimensionless parameters of figure 3.

For long narrow bodies of water such as man-made reservoirs, rivers, canals, or narrow inlets, corrections need to be made for the fetch length. Figure 5, based on the work of Saville (1954), can be used to calculate an effective fetch length, based on actual fetch length and fetch width. The effective fetch length  $F_e \cong F_{min}$  should be used with Figure 4 to determine significant waves for long narrow bodies of water.

## B. COMPLEX NATURE OF SEA SURFACE

The significant wave description is a simple and practical means of dealing with problems in wave forecasting. However, it is important to recognize that the sea is very complex, made up of many variable heights and periods. Figure 6 shows a schematic interpretation of a typical wave record which might be obtained from a wave recorder. The significant wave height is the average of the highest 1/3 of the waves in a given wave train, of at least 100 consecutive waves, and is therefore a statistical parameter. The significant wave period is the average period of the highest 1/3 of the wave heights, and is common only to the S-M-B forecasting method. The Pierson-Neumann-James (P-N-J) forecasting method considers a mean apparent wave period and ranges in wave period. Both S-M-B and P-N-J consider the probability distribution of wave heights and in both methods the Rayleigh distribution is used. The S-M-B



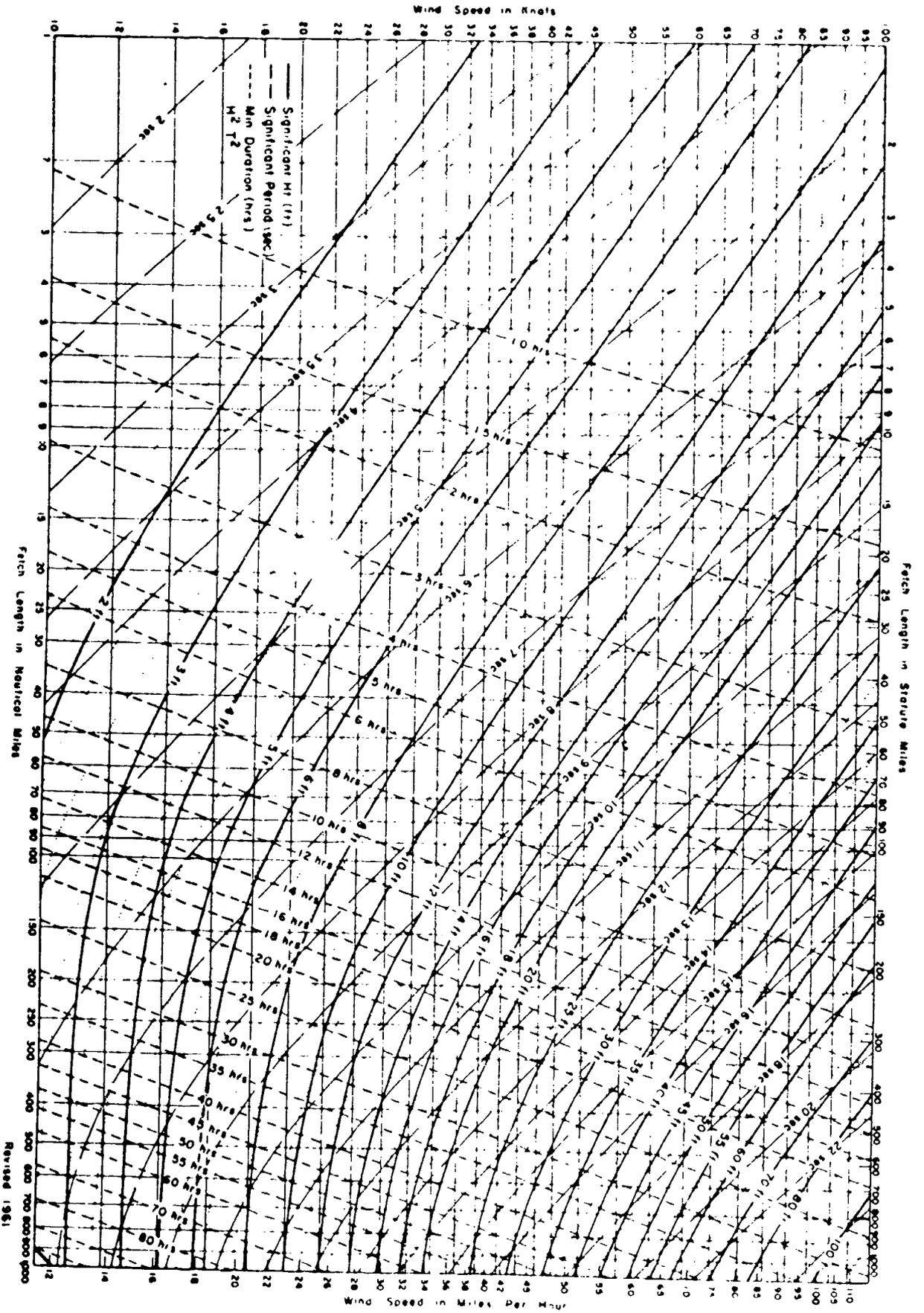


FIGURE 4 DEEP WATER WAVE FORECASTING CURVES  
AS A FUNCTION OF WIND SPEED,  
FETCH LENGTH, AND WIND DURATION  
(FOR FETCHES 1 TO 1,000 MILES)

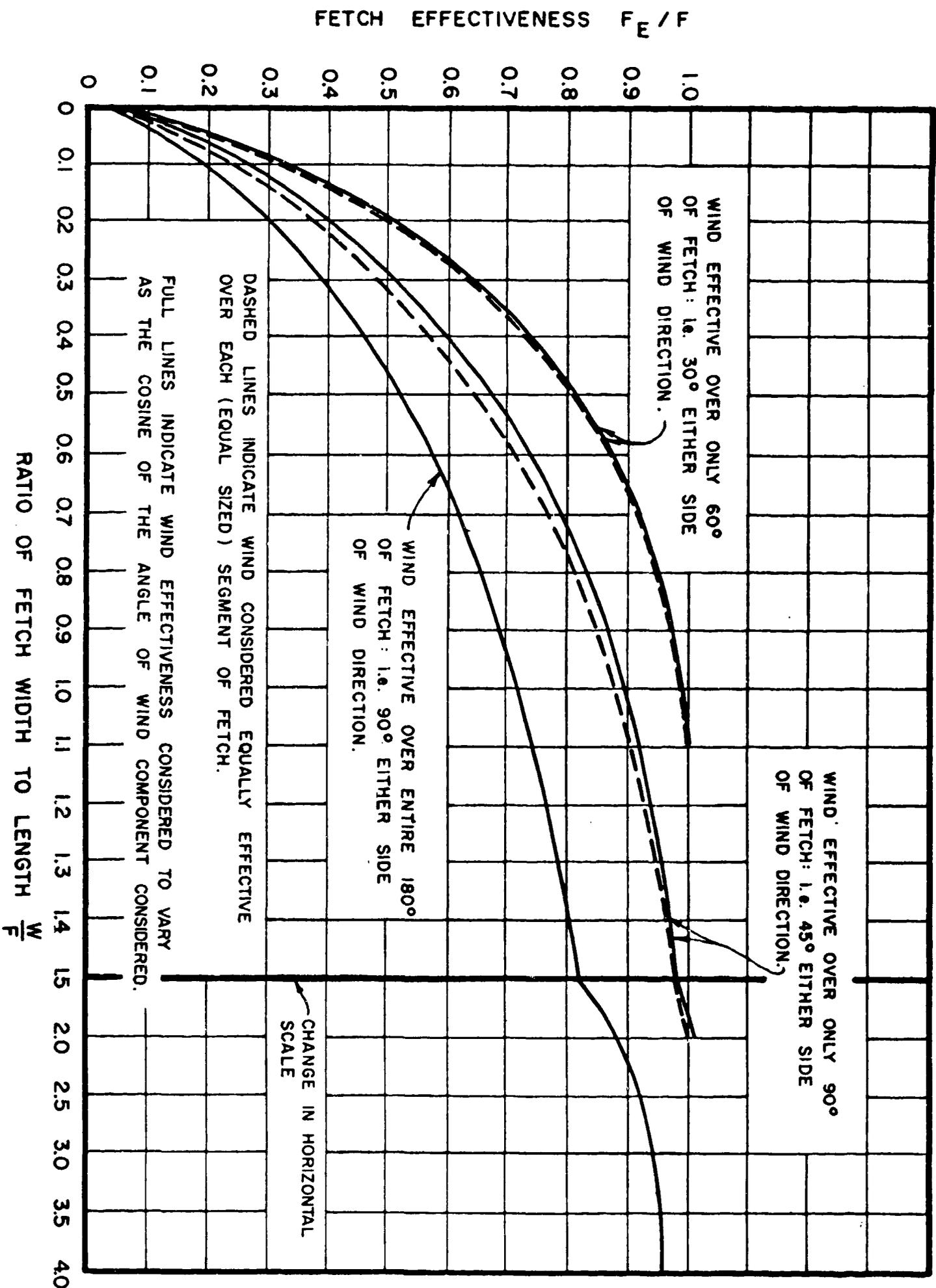
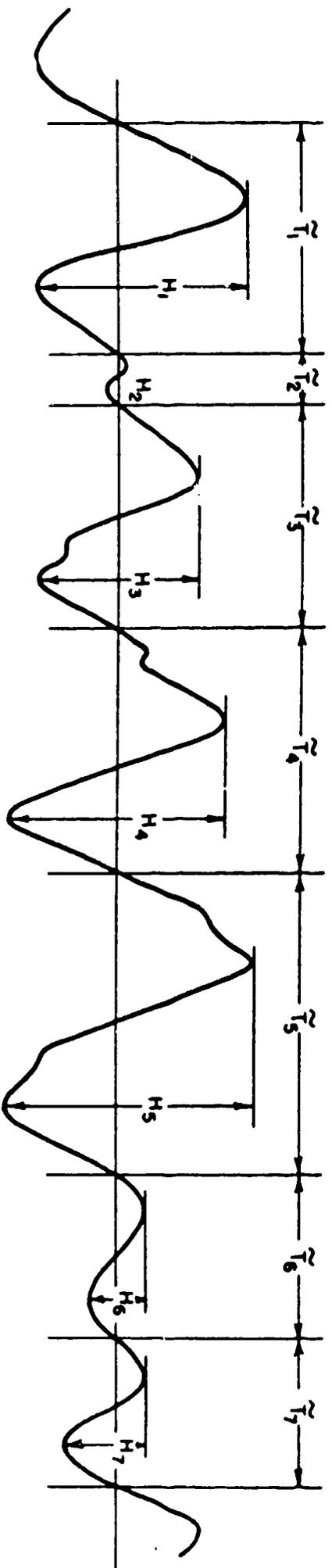


FIGURE 5 RELATION OF EFFECTIVE FETCH TO WIDTH—LENGTH RATIO FOR RECTANGULAR FETCHES



UPPER FIGURE IS THE ZERO-UPCROSSING METHOD  
 LOWER FIGURE IS CREST-TO-TROUGH OR TROUGH-TO-CREST METHOD

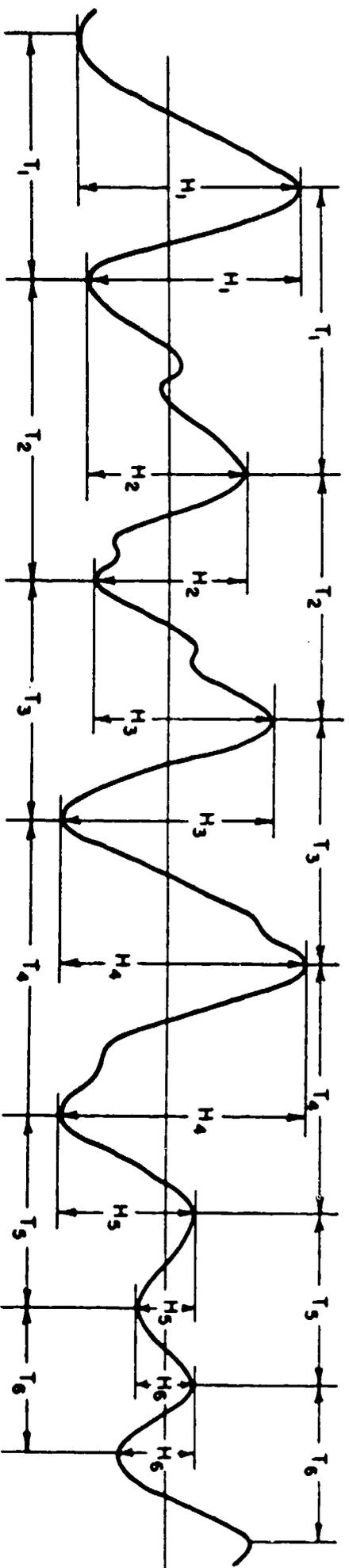


FIGURE 6 METHODS OF WAVE RECORD ANALYSIS

method uses a probability distribution also for wave period. Wilson (1955) introduced the space-time concept for forecasting waves in moving fetches and in 1963 extended the work for use on a high speed computer. Figure 7 is reproduced from Wilson (1955).

Draper and Darbyshire (1963) presented relationships for forecasting maximum wave heights. Figures 8, 9, 10 and 11 are reproduced from Draper and Darbyshire (1963) where it should be kept in mind that the maximum wave is equal to 1.6 times the significant wave height. Figures 8 and 10 are for deep water and Figures 9 and 11 are for coastal waters.

## 1. Wave Variability

a. Significant Wave Height. The significant wave height, as mentioned above, is a term common to both the S-M-B and P-N-J methods of wave forecasting. Just how the significant wave height is related to the probability distribution and also the wave spectrum can best be illustrated by figures 12 and 13. Figure 12 shows the distribution of wave heights as visualized by a histogram, i. e. the number N (or percent P of the total number) of waves in each wave height range. Figure 13 is a schematic diagram of the wave spectrum and in this form is called the period spectrum.

b. Distribution of Wave Heights. The significant wave height is a statistical parameter, and about 16 or 17 percent of the waves will be higher than the significant wave height. It is shown by Longuet-Higgins (1952), Watters (1953), Krylov (1956 and 1958), and Darlington (1954) and verified by Bretschneider (1959) that the distribution of wave heights for a narrow spectrum is given by the Rayleigh distribution. The probability density for wave heights is given by:

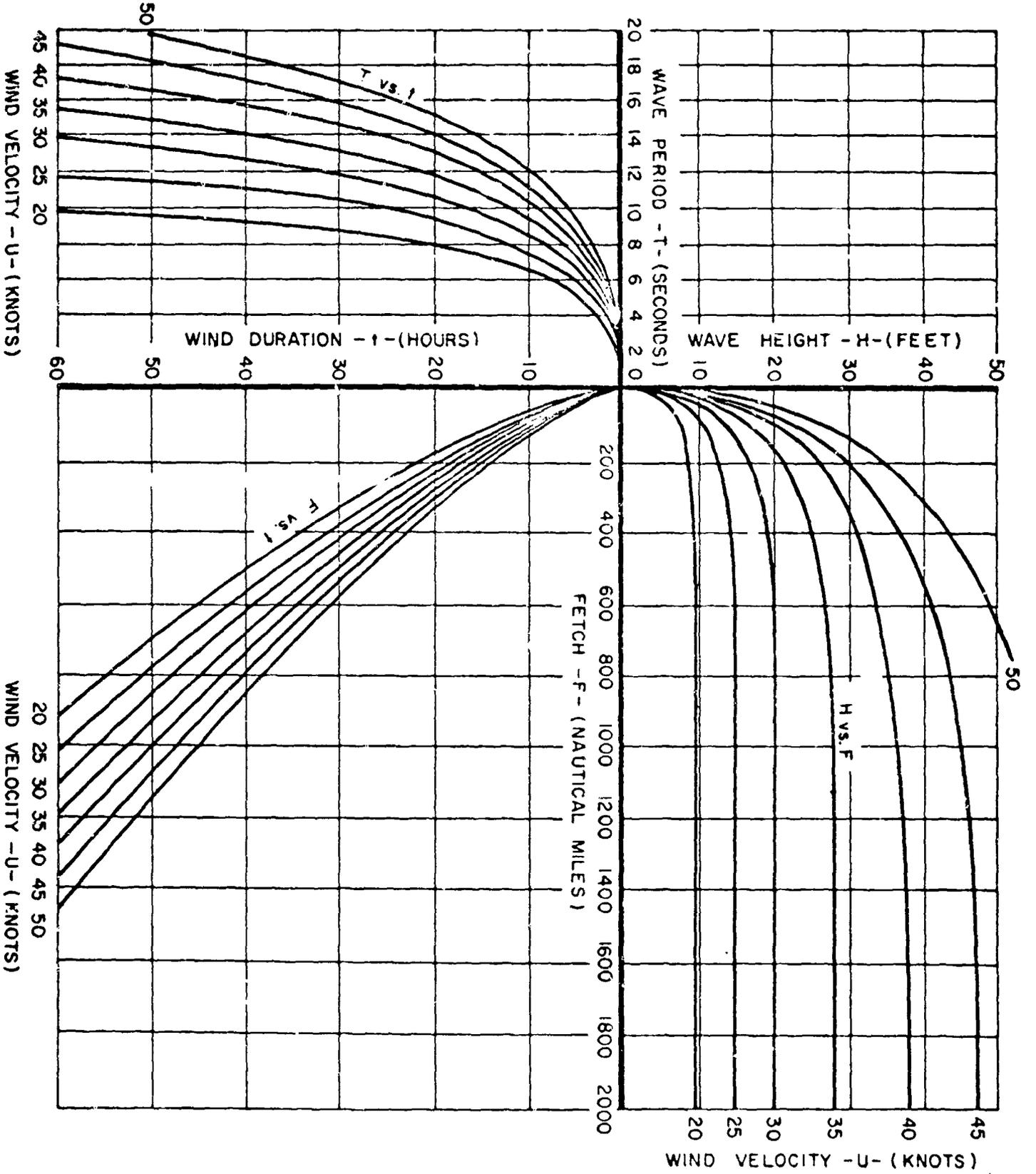
$$p(H) dH = \frac{\pi}{2} \frac{H}{(\bar{H})^2} e^{-\frac{\pi}{4} \left(\frac{H}{\bar{H}}\right)^2} dH \quad (21)$$

and the cumulative distribution is

$$P(H) = 1 - e^{-\frac{\pi}{4} \left(\frac{H}{\bar{H}}\right)^2} \quad (22)$$

where H is the individual wave height and  $\bar{H}$  is the average wave height.

The average wave height  $\bar{H}$ , the significant wave height  $H_{33}$ , and the average of the highest ten percent of the waves  $H_{10}$ , are related as follows:



**FIGURE 7 H.T.F.T - DIAGRAM FOR FORECASTING WIND-GENERATED WAVES AFTER WILSON (1955)**

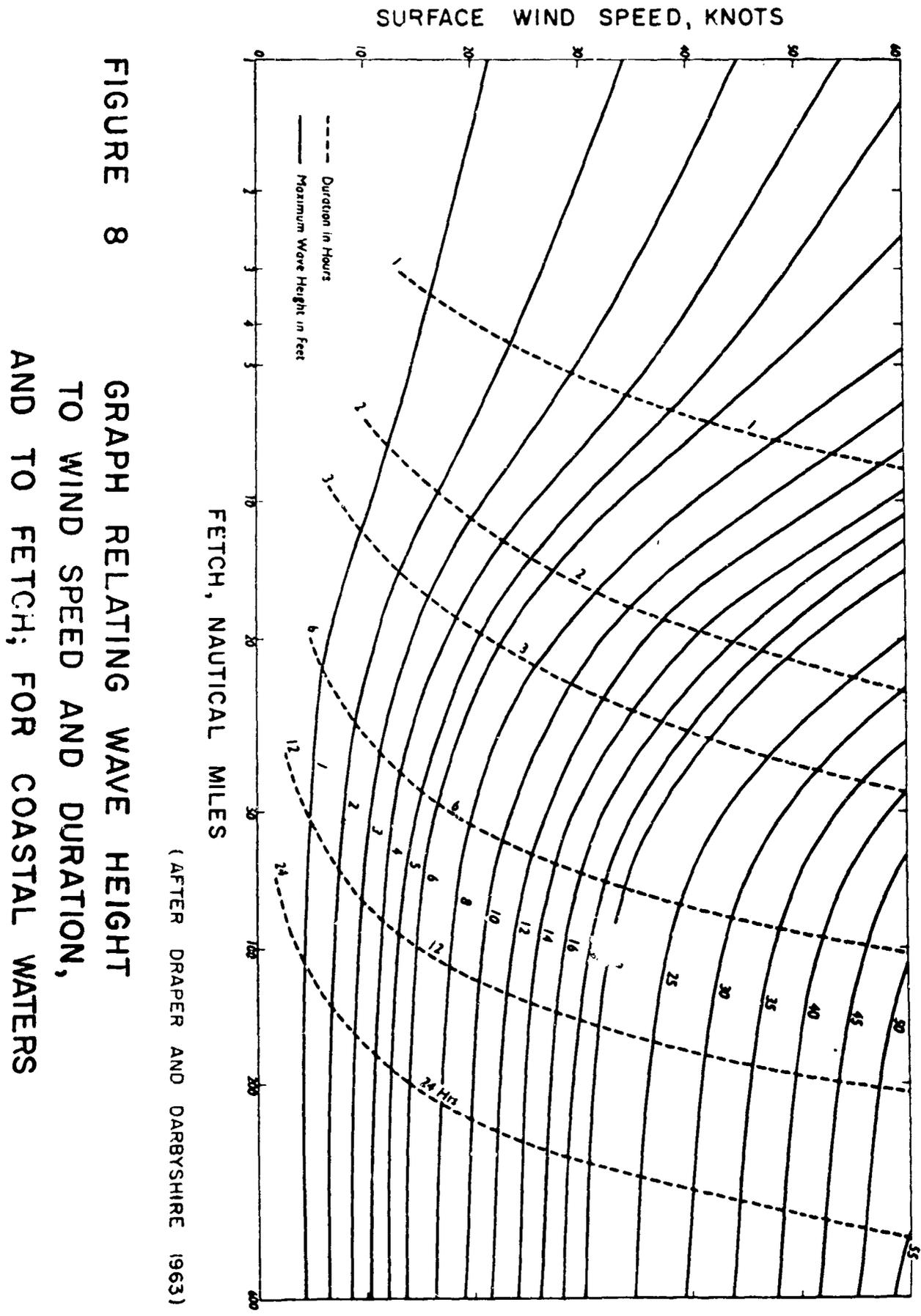
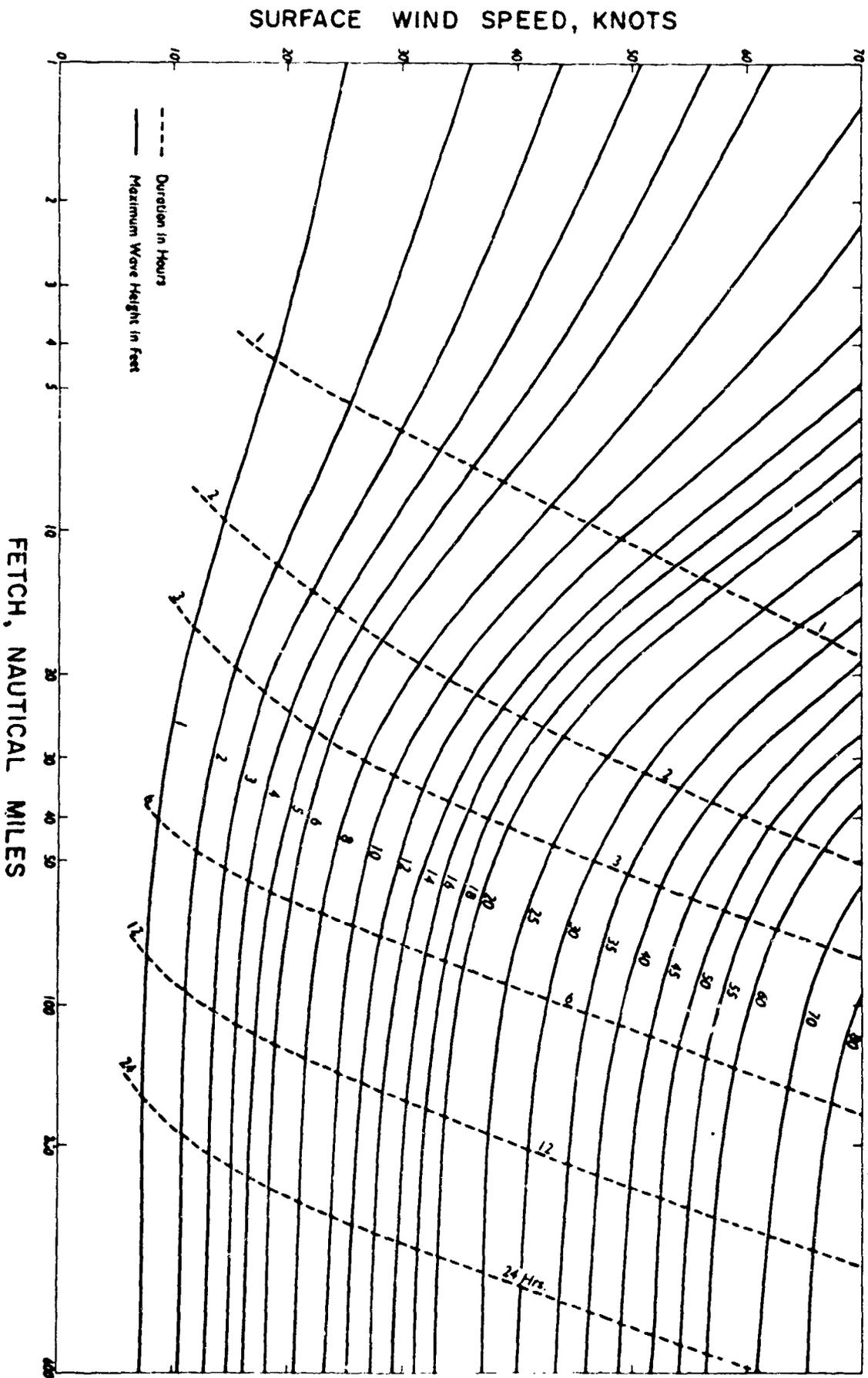


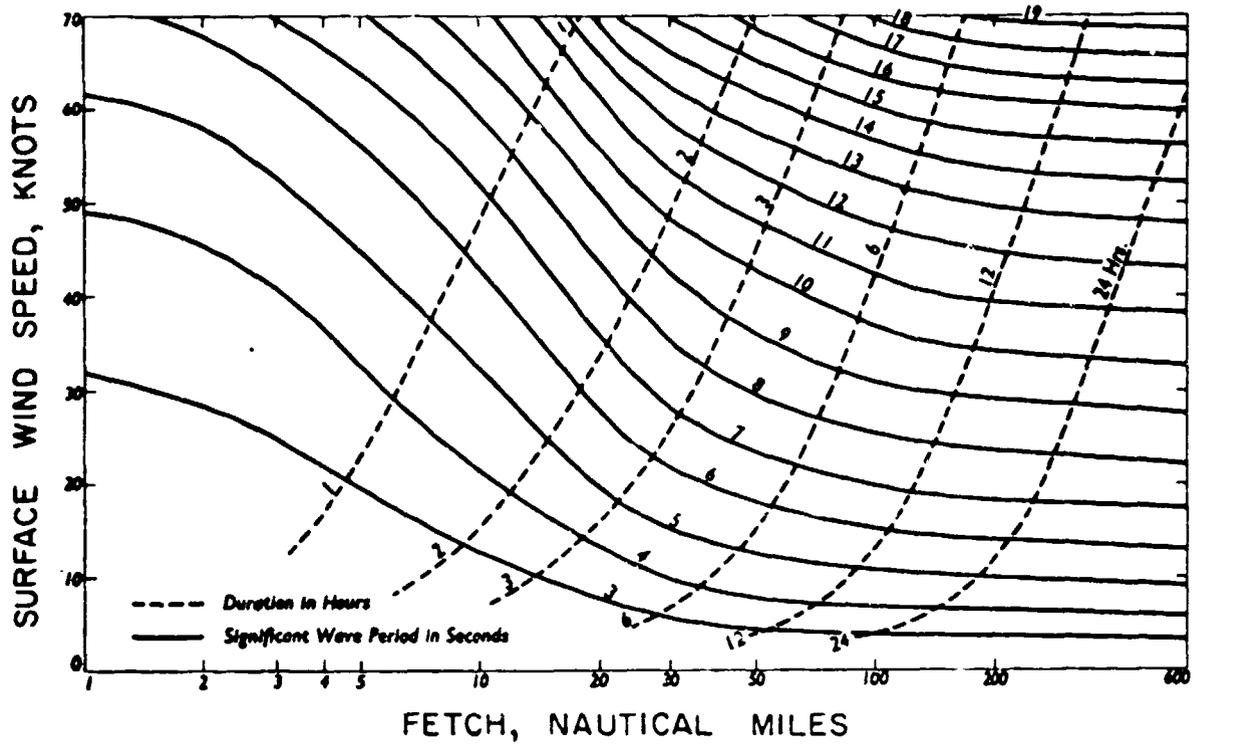
FIGURE 8 GRAPH RELATING WAVE HEIGHT TO WIND SPEED AND DURATION, AND TO FETCH; FOR COASTAL WATERS





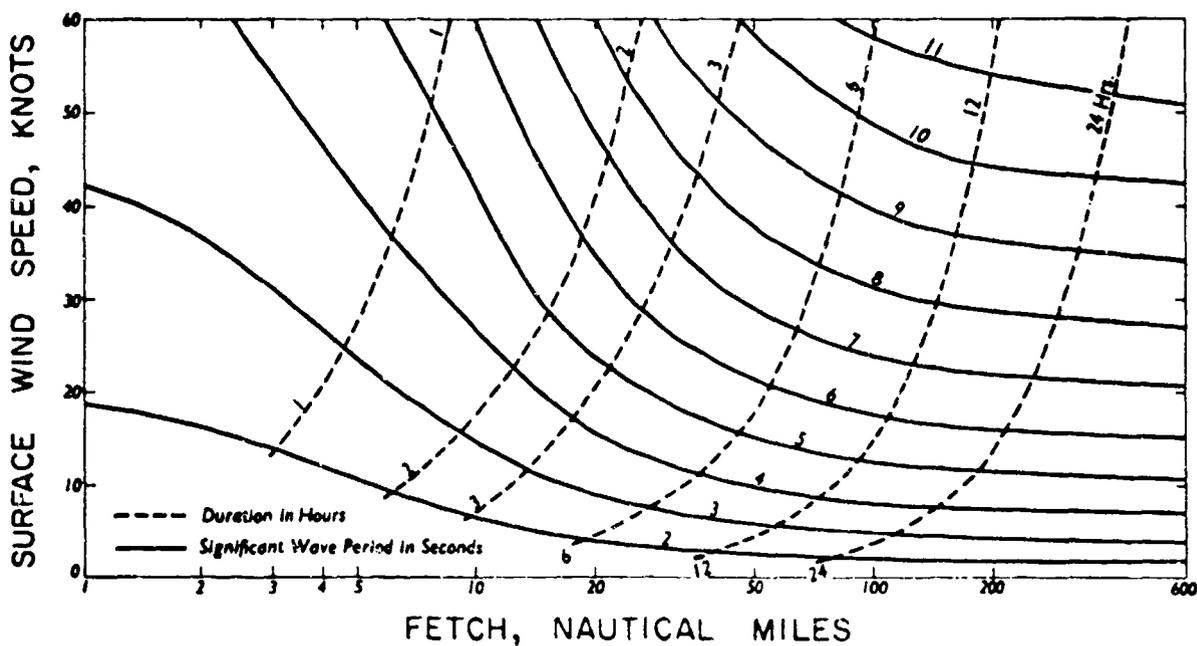
(AFTER DRAPER AND DARBYSHIRE 1963)

FIGURE 9 GRAPH RELATING WAVE HEIGHT TO WIND SPEED AND DURATION, AND TO FETCH; FOR OCEANIC WATERS



(AFTER DRAPER AND DARBYSHIRE 1963)

FIGURE 10 GRAPH RELATING  
WAVE PERIOD TO WIND SPEED  
AND DURATION, AND TO  
FETCH; FOR OCEANIC WATERS



(AFTER DRAPER AND DARBYSHIRE 1963)

FIGURE 11 GRAPH RELATING  
WAVE PERIOD TO WIND SPEED  
AND DURATION, AND TO  
FETCH; FOR COASTAL WATERS

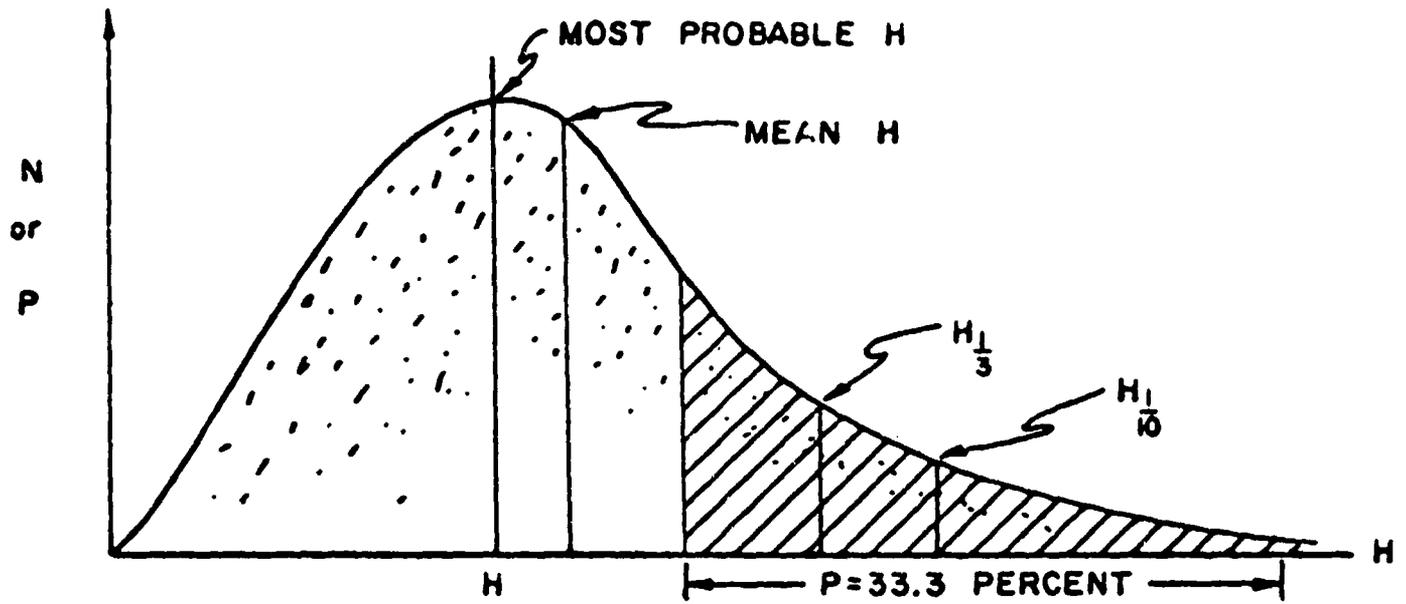


FIGURE 12 STATISTICAL DISTRIBUTION OF HEIGHTS

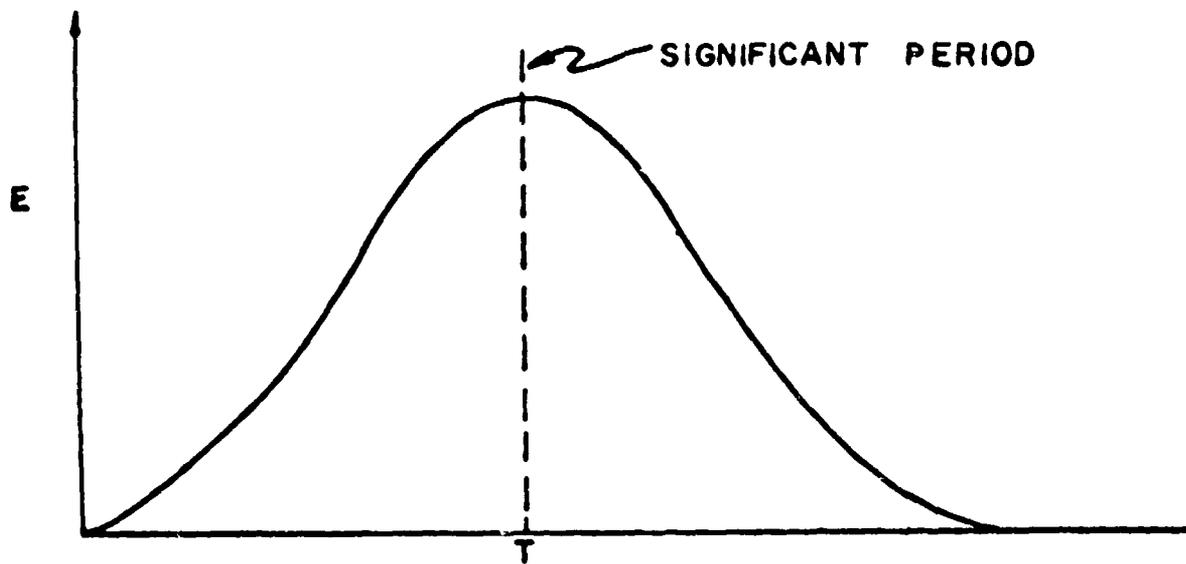


FIGURE 13 PERIOD SPECTRUM

$$\bar{H} = .025 H_{33} \quad (23)$$

$$H_{10} = 2.3 \bar{H} = 1.27 H_{33}$$

The cumulative distribution  $P(H)$  is given in Figure 14.

c. Distribution of Wave Periods. The significant wave period is not a statistical parameter of the wave period distribution function. The significant wave period has statistical significance only when both wave heights and wave periods are considered. Putz (1952) obtains a Gamma-type distribution function for wave period variability. Krylov (1956 and 1958) postulates the Rayleigh distribution also for the wave lengths, and Bretschneider (1959) shows that the distribution of the square of the wave periods (proportional to the deep water wave lengths) can be represented approximately by the Rayleigh distribution, a transformation of which leads to the following distribution function for wave periods:

$$p(T) dT = 2.7 \frac{T^3}{(\bar{T})^4} e^{-0.675 \left(\frac{T}{\bar{T}}\right)^4} dT \quad (24)$$

and the cumulative distribution is

$$P(T) = 1 - e^{-0.675 \left(\frac{T}{\bar{T}}\right)^4} \quad (25)$$

The cumulative distribution  $P(T)$  is also given in Figure 14. Korneva (1961) gives results which verify the probability distribution functions of wave height and period.

d. The Weibull Distribution Function. In many cases it may be desirable to represent empirical data by means of a simple distribution function. Weibull (1951) proposes a simple analytical distribution function for use in certain civil engineering problems as follows:

$$P = 1 - e^{-BX^m} \quad (26)$$

where  $P$  is the cumulative distribution

$B$  and  $m$  are constants

$X \geq 0$  is the variate, i. e.  $H$ ,  $L$ , or  $T$  as the case may be.

The probability density is given by

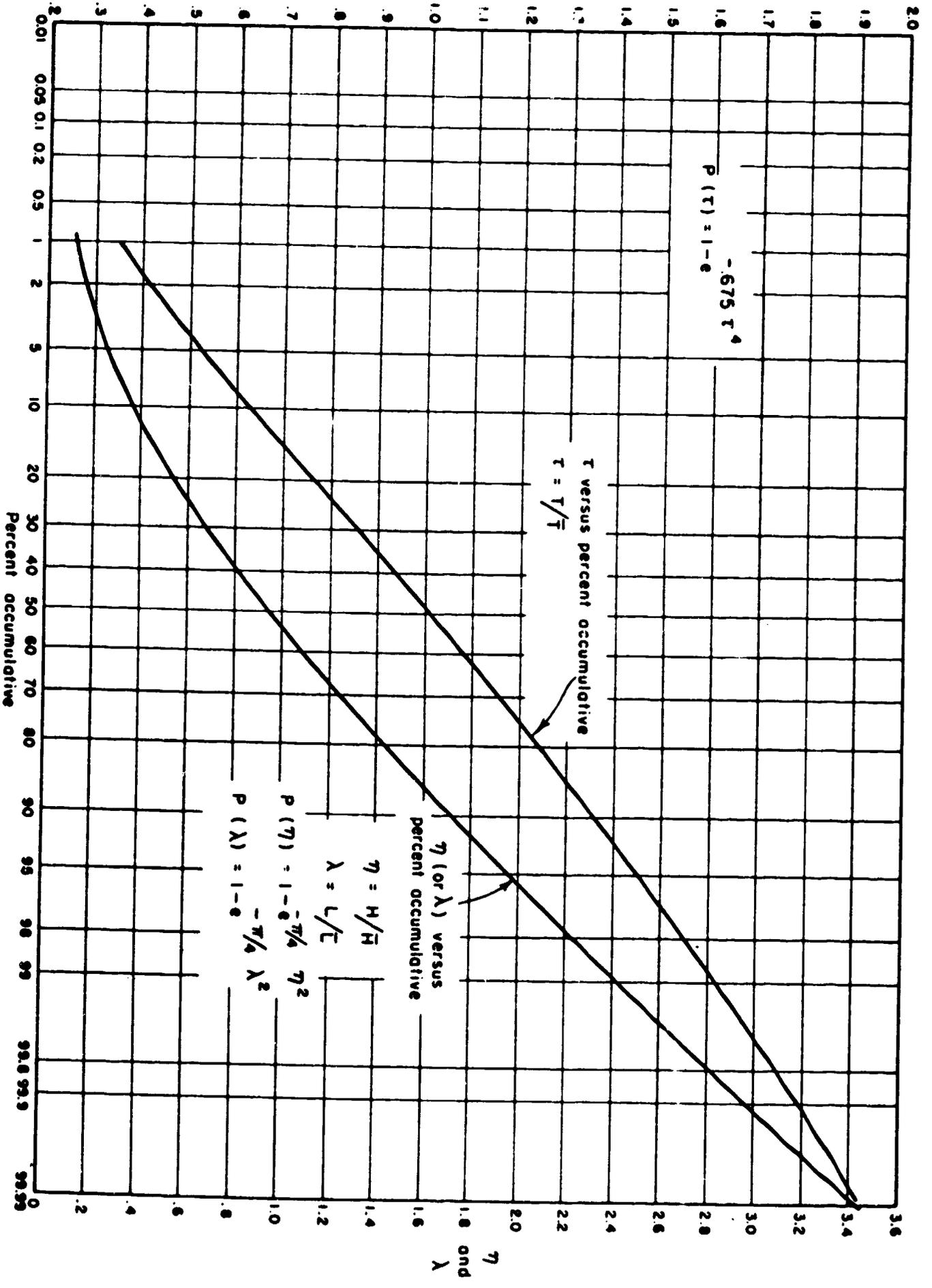


FIGURE 14 DISTRIBUTION FUNCTIONS FOR PERIOD VARIABILITY AND HEIGHT VARIABILITY

$$p(X) = \frac{d(P)}{dX} = mB X^{m-1} e^{-BX^m} \quad (27)$$

It can be seen that when  $m = 2$ , the Weibull distribution function is the same as the Rayleigh distribution (Eq. 21 or 22) and when  $m = 4$  the Weibull distribution function is the same as Eq. 24 or 25. Thus the Weibull distribution function might be suitable for investigating wave variability when the data deviate somewhat from the Rayleigh distribution.

The moments can be generated from

$$m_n = \int_0^{\infty} X^N p(X) dX = \left(\frac{1}{B}\right)^{\frac{N}{m}} \Gamma\left(1 + \frac{N}{m}\right) \quad (28)$$

where  $\Gamma$  represents the Gamma function.

If the terms of Eq. (26) are rearranged and the logarithm is taken twice, one obtains

$$\ln \ln \left( \frac{1}{1-P} \right) = \ln B + m \ln X \quad (29)$$

Eq. (29) is that of a straight line. When  $\ln \ln \frac{1}{1-P}$  is plotted against  $\ln X$ , the intercept becomes  $\ln B$  and the slope of the line is given by  $m$ .  $P$  is the cumulative probability distribution, calculated from the data and  $B$  is related to the mean wave height or mean wave period as the case may be.

From Eq. (28),  $m_0 = 0$ ,  $m_1 = \bar{X}$ ,  $m_2 = \bar{X}^2$ , etc. It then follows that

$$\bar{X} \equiv \bar{H} \equiv \bar{T} = \left(\frac{1}{B}\right)^{\frac{1}{m}} \Gamma\left(1 + \frac{1}{m}\right) \quad (30)$$

or

$$B = \left[ \frac{\Gamma\left(1 + \frac{1}{m}\right)}{\bar{X}} \right]^m$$

Thus Eqs. (26) and (27) become

$$P = 1 - e^{-\left[ \frac{X}{\bar{X}} \Gamma\left(1 + \frac{1}{m}\right) \right]^m} \quad (31)$$

$$P(X) = m \left[ \frac{X}{\bar{X}} \Gamma \left( 1 + \frac{1}{m} \right) \right]^{m-1} \frac{1}{X} e^{- \left[ \frac{X}{\bar{X}} \Gamma \left( 1 + \frac{1}{m} \right) \right]^m} \quad (32)$$

By use of Eq. (31), Eq. (29) becomes

$$\ln \ln \frac{1}{1-P} = m \ln \left[ \Gamma \left( 1 + \frac{1}{m} \right) \right] + m \ln \left( \frac{X}{\bar{X}} \right) \quad (33)$$

It must be emphasized that the Weibull distribution should be used only where the trend of the data shows a nearly linear relationship according to Eq. (29) or Eq. (33). If such a linear relationship becomes apparent from the data, then a graphical solution is possible, and a more accurate solution can be obtained by the statistical method of least squares. Although the Weibull distribution has no theoretical basis, the function does have a wide range for practical applications. For example, Figures 15 and 16 represent an analysis of wave data according to the Weibull distribution. These data are based on long wave records obtained at Lake Texoma by U. S. Army engineers (1953) and are summarized in a report by Bretschneider (1959). The straight lines given in Figures 15 and 16 were fitted visually, although a least squares fit might have been made to obtain better accuracy.

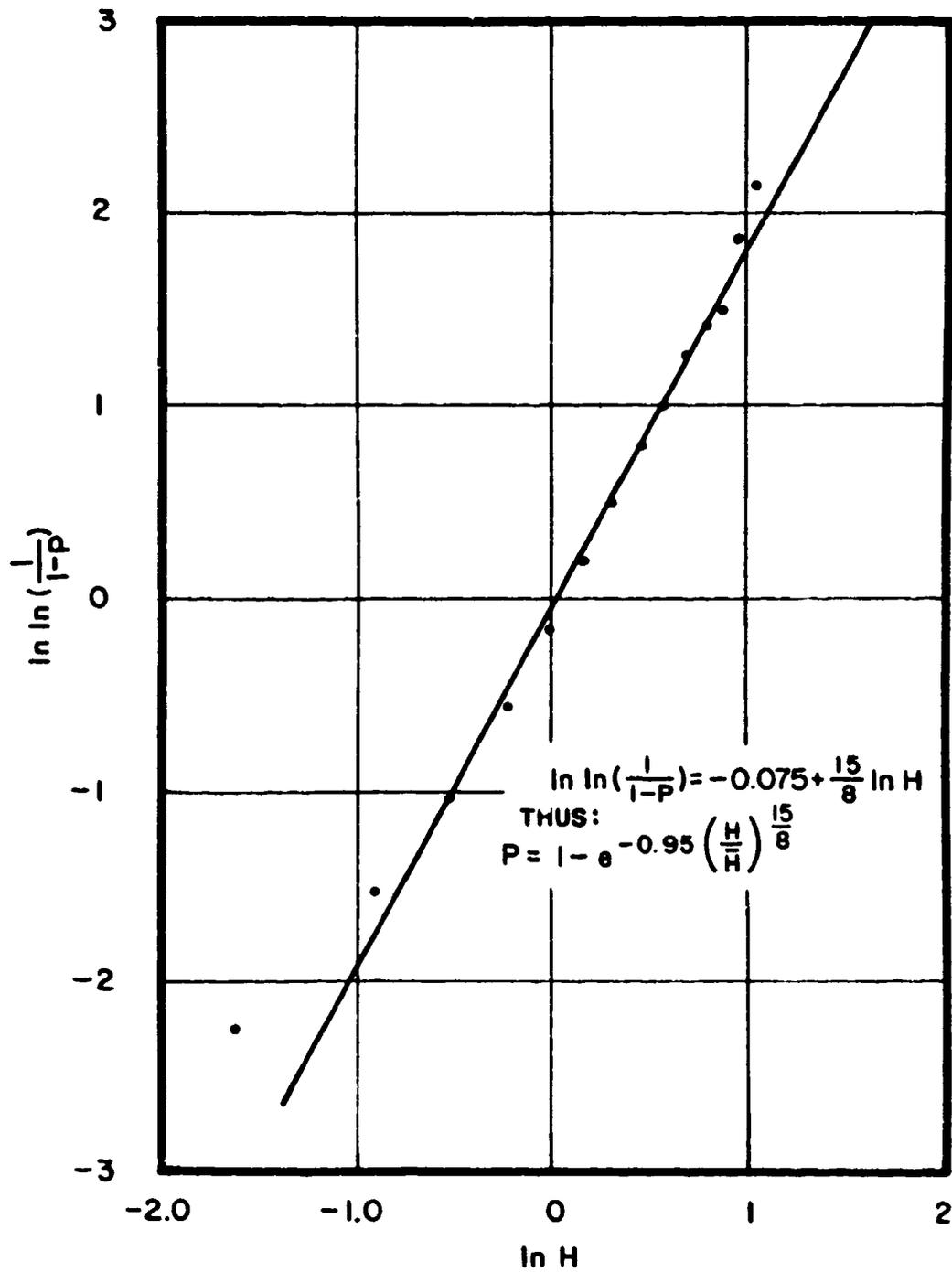
e. Joint Distribution of Wave Heights and Periods. The joint distribution of wave heights and periods is quite complex, except for the special case of zero correlation between wave height and wave period. Figure 17 is a scatter diagram of  $\eta = H/\bar{H}$  and  $\lambda = T^2/(\bar{T})^2$  for a case very nearly zero correlation. If the number of wave heights is summed independently of the wave periods, one obtains the marginal distribution of wave heights, and if the number of wave periods squared is summed independently of the wave heights, one obtains the marginal distribution of wave periods squared. Both of the above distributions are represented approximately by the Rayleigh distribution. For the case of zero correlation between wave height and wave period, the joint distribution is given directly by

$$p(H, T) = p(H) \cdot p(T) \quad (34)$$

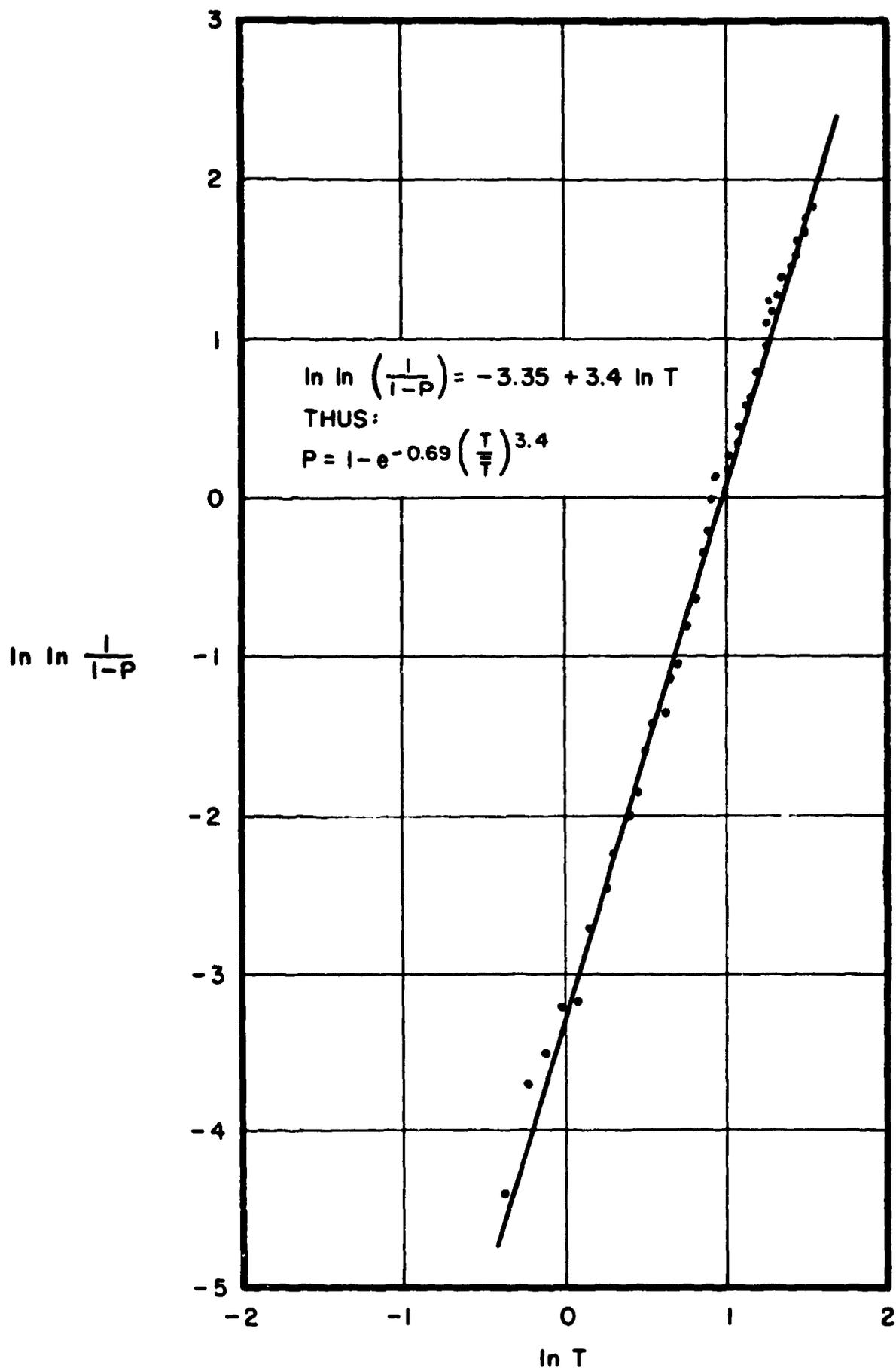
Based on Eqs. (21), (24), and (34), the joint probability distributions have been calculated for various ranges of  $H/\bar{H}$  and  $T/\bar{T}$  and are summarized in Table I. This table assumes zero correlation between  $H$  and  $T^2$ . Gumbel (1960) discusses the bivariate exponential distributions, taking into account correlation coefficients different from zero.

Eq. (34) can be considered approximately correct for practical engineering uses, except that the limit of breaking waves should always be checked. The breaking wave limit is given theoretically by Michard (1954) as follows:

$$\frac{H}{L} = \frac{1}{7} \tanh \frac{2\pi d}{L} \quad (35)$$



**FIGURE 15 SAMPLE WEIBULL DISTRIBUTION DETERMINATION FOR WAVE HEIGHT**  
 (DATA FROM U.S. ARMY ENGINEERS, 1953)



**FIGURE 16 SAMPLE WEIBULL DISTRIBUTION DETERMINATION FOR WAVE PERIOD**  
 (DATA FROM U.S. ARMY ENGINEERS, 1953)

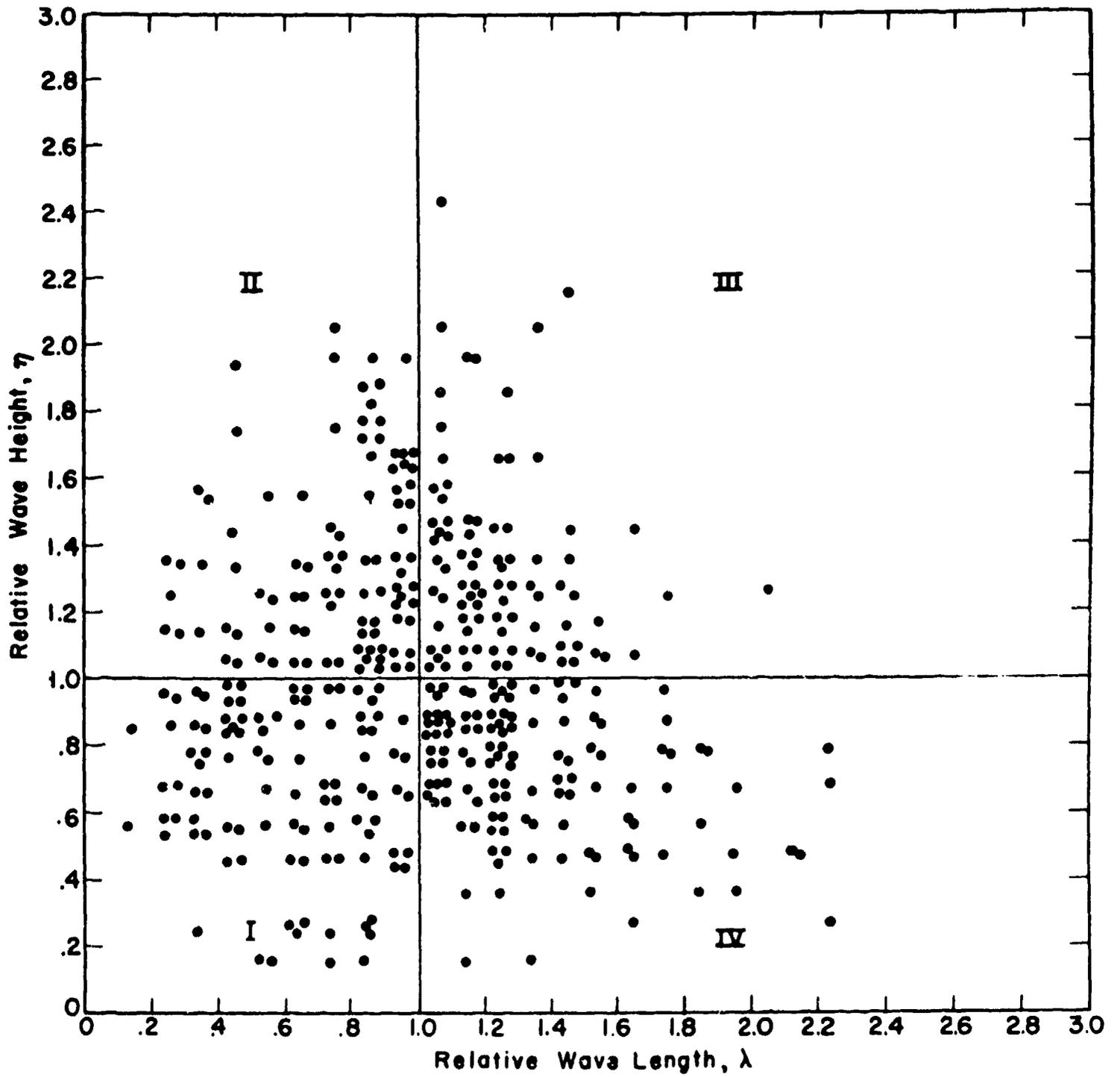


FIGURE 17 SCATTER DIAGRAM OF  $\eta$   
AND  $\lambda$  FOR 400 CONSECUTIVE  
WAVES FROM THE GULF OF MEXICO.

TABLE 1

JOINT DISTRIBUTION OF H AND T FOR ZERO CORRELATION

Number of Waves Per 1,000 Consecutive Waves for Various Ranges in Height and Period

Range in Relative Height H/H̄	RANGE IN RELATIVE PERIOD																Cumulative
	0-	0.2-	0.4-	0.6-	0.8-	1.0-	1.2-	1.4-	1.6-	1.8-	2.0-	0-	2.0	0-	2.0	tive	
0-0.2	0.03	0.50	2.05	4.86	7.68	8.09	5.31	1.92	0.34	0.03	30.81	30.81					
0.2-0.4	0.10	1.41	5.81	13.78	21.76	23.92	15.05	5.44	0.98	0.07	88.32	119.13					
0.4-0.6	0.14	2.06	8.54	20.23	31.95	33.65	22.10	7.99	1.44	0.11	128.21	247.34					
0.6-0.8	0.16	2.40	9.91	23.48	37.08	39.06	25.65	9.27	1.67	0.12	148.80	396.14					
0.8-1.0	0.16	2.40	9.92	23.51	37.13	39.11	25.69	9.28	1.67	0.12	148.99	545.13					
1.0-1.2	0.15	2.14	8.87	21.02	33.19	34.97	22.96	8.30	1.49	0.11	133.20	678.33					
1.2-1.4	0.12	1.74	7.21	17.07	26.96	28.40	18.65	6.74	1.21	0.09	108.19	786.52					
1.4-1.6	0.09	1.30	5.37	12.72	20.09	21.16	13.90	5.02	0.90	0.07	80.62	867.14					
1.6-1.8	0.06	0.90	3.72	8.82	13.93	14.67	9.64	3.48	0.63	0.05	55.90	923.04					
1.8-2.0	0.03	0.48	1.99	4.72	7.45	7.85	5.15	1.86	0.33	0.03	29.39	952.93					
2.0-2.2	0.03	0.42	1.72	4.09	6.45	6.80	4.47	1.61	0.29	0.02	25.90	978.83					
2.2-2.4	0.01	0.18	0.76	0.80	2.84	2.99	1.97	0.71	0.13	0.01	11.40	990.23					
2.4-2.6	0.01	0.09	0.39	0.93	1.47	1.55	1.02	0.37	0.07		5.90	996.13					
2.6-2.8		0.04	0.18	0.43	0.67	0.71	0.47	0.17	0.03		2.70	998.83					
0-3.0	1.09	16.06	66.44	157.16	248.65	262.93	172.03	62.16	11.18	0.83							
Cumulative	1.09	17.15	83.59	241.05	489.70	752.63	924.66	986.82	998.00	998.83							

where the limit in shallow water is given by the solitary wave theory

$$\frac{H}{L} = 0.78 \frac{d}{L} \quad (36)$$

Based in part on theory and in part on empirical data, Bretschneider (1960) presents a relationship of the breaking wave limit which is as follows:

$$\frac{H}{L} = 0.124 \tanh \frac{2\pi d}{L} \left[ 1 + 0.152 \tanh \frac{2\pi d}{L} \right] \quad (37)$$

Values of H and T obtained by use of Table I should always be checked by use of the above equations.

### C. WAVE SPECTRUM CONCEPTS

The original wave spectrum concept for forecasting waves is that due to Neumann (1952). The wave spectrum method resulting from this concept is that due to Pierson, Neumann and James (1955). Much work has also been done with regard to a wave spectrum method by Darbyshire (1952 and 1955). Bretschneider (1959) has added a wave spectrum approach to the significant wave method. A more recent spectrum concept has been proposed by Pierson (1964), the form of which is in agreement with that derived by Bretschneider (1959). Whereas the significant wave method is commonly called the S-M-B method, the Pierson, Neumann and James method is commonly called the P-N-J method.

Just as the S-M-B method is based on empirical wave data, so it is with the P-N-J method. As mentioned before, any suitable wave forecasting method must be calibrated by use of wave data. The P-N-J method can be used to predict the spectrum of waves from which one may obtain the significant wave height as well as the statistical distribution of the waves. The S-M-B method is used to predict the significant wave height, from which it is possible to obtain the wave spectrum and the statistical distribution of the waves. Both methods utilize the distribution function derived theoretically by Longuet-Higgins (1952). This distribution function is in very close agreement with the empirical relationships given by Putz (1952) based on the analysis of 25 ocean wave records. Consequently, when both S-M-B and P-N-J methods predict exactly the same significant wave height, then the two methods result in exactly the same distribution of waves. It must be remembered that both methods are based on actual wave data. However, the wave data are not the same utilized in the two methods, and the method of analysis of the data is different. The analysis of the data for the S-M-B method consists of determining the significant wave height and period, which in turn are related to wind speed, fetch length, and wind duration.

Prior to the development of the P-N-J method, Neumann (1952) proposed a theoretical wave spectrum based on a great abundance of

individual wave observations. This spectrum should be considered as a semi-theoretical or semi-empirical spectrum in the strict sense of the definition. From his wind and wave observations Neumann (1952) computed the parameters  $H/\tilde{T}^2$  and  $\tilde{T}/V$ , and arrived at the following relation:

$$\frac{H}{g\tilde{T}^2} = \text{const } e^{-\frac{g\tilde{T}}{2\pi V}} \quad (38)$$

(See figure 18.) The parameter  $H/\tilde{T}^2$  is directly related to the wave steepness and  $\tilde{T}/V$  the wave age.  $H$  is the individual height.  $\tilde{T}$  is the individual wave period, which he called the apparent wave period, defined as the time between two successive up-crossings of the still water surface by the wave form.

From this empirical relationship (Eq. 38) and other considerations, Neumann (1952) derived his theoretical wave spectrum of energy. From these results he concluded for unlimited fetch length and duration (fully developed sea) that the wave height was equal to some constant times the wind velocity to the 5/2 power, and the energy at high frequency was  $f^{-6}$ . Because of the method of solution, a dimensional constant was introduced, but the equation has been evaluated by considering that the total energy under the curve of the spectrum was proportional to the energy of the equivalent root mean square wave height. The dimensional constant has created objections by various individuals in the field of wave forecasting, particularly those who adhere to dimensional homogeneity. However, if one accepts the relations as empirical rather than theoretical, then these relations ought to be suitable for wave forecasting, at least over the range of parameters from which the data were analyzed. The important thing in his development of the spectrum is the fact that total energy of the spectrum is correct, assuming that the work of Longuet-Higgins is correct, and this appears to be the case. The actual distribution of the energy with respect to wave period or frequency might not be exactly correct, but perhaps is in close agreement with the actual data from which it was derived. Not until much additional data become available will it be possible to reconcile any differences between Neumann's wave spectrum and a more correct spectrum.

Roll and Fischer (1956) suggested a different derivation of the Neumann spectrum. In this derivation there resulted no dimensional constant, the wave height for fully developed sea became proportional to  $U^2$ , and the energy at high frequency was  $f^{-5}$ .

The concept of the wave spectrum is certainly a great advance in trying to understand the nature of wave generation. It is shown, for example, in spite of the objections to the Neumann spectrum, that the waves generate from the high frequency end of the spectrum, and with this concept Neumann (1952) derived from his spectrum relationships

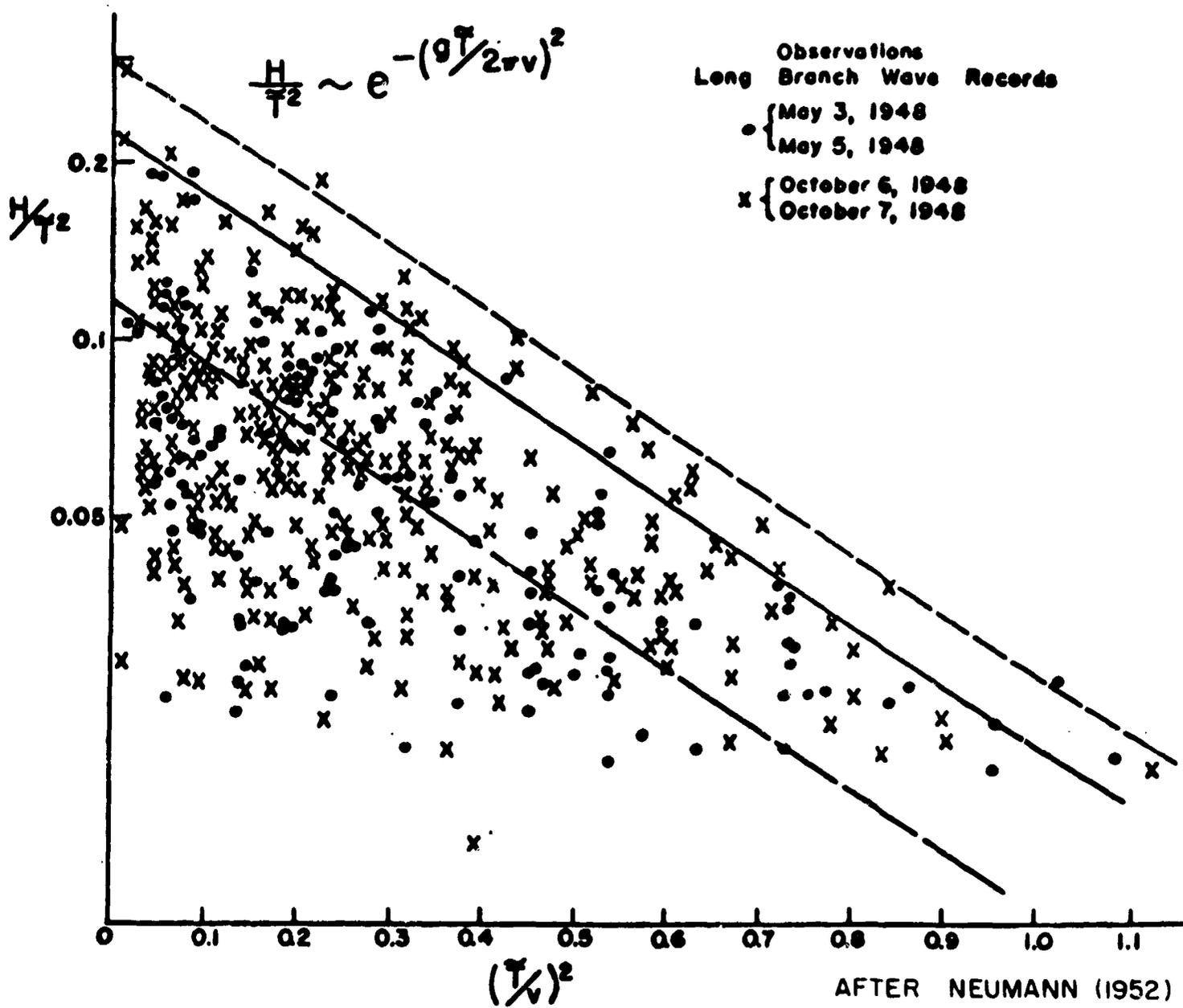


FIGURE 18 RATIO OF WAVE HEIGHTS TO THE SQUARE OF APPARENT WAVE PERIODS,  $H/\tau^2$

for the so-called young or transient state of sea. That is, relations called co-cumulative power spectra are developed from which it is possible to predict E-values, where E is related to the generated wave energy.

The theoretical wave distribution derived by Longuet-Higgins (1952) is directly related to the E-values, as this condition was utilized in the development of the wave spectrum. The significant wave height, mean wave height, etc. are related to E as follows:

$$\begin{aligned} H_{ave} &= 1.772 \sqrt{E} \\ H_{1/3} &= 2.832 \sqrt{E} \\ H_{1/10} &= 3.600 \sqrt{E} \end{aligned} \quad (39)$$

Typical examples of wave forecasting relationships based upon the above concept are given in Figures 19 and 20.

The spectrum proposed by Bretschneider (1959) was obtained by use of a theoretical function for the joint probability distribution of wave heights and periods. The period spectrum was obtained as follows:

$$S_E(T) = \int_0^{\infty} H^2 p(H, T) dH \quad (40)$$

where the integration is over all wave heights as a function of wave period. Assuming no correlation between H and T :

$$p(H, T) = p(H) \cdot p(T), \text{ whence} \quad (41)$$

$$S_E(T) = p(T) \int_0^{\infty} H^2 p(H) dH = H^2 p(T) = \frac{4}{\pi} (\bar{H})^2 p(T) \quad (42)$$

By use of Eq. (24) one obtains the period spectrum:

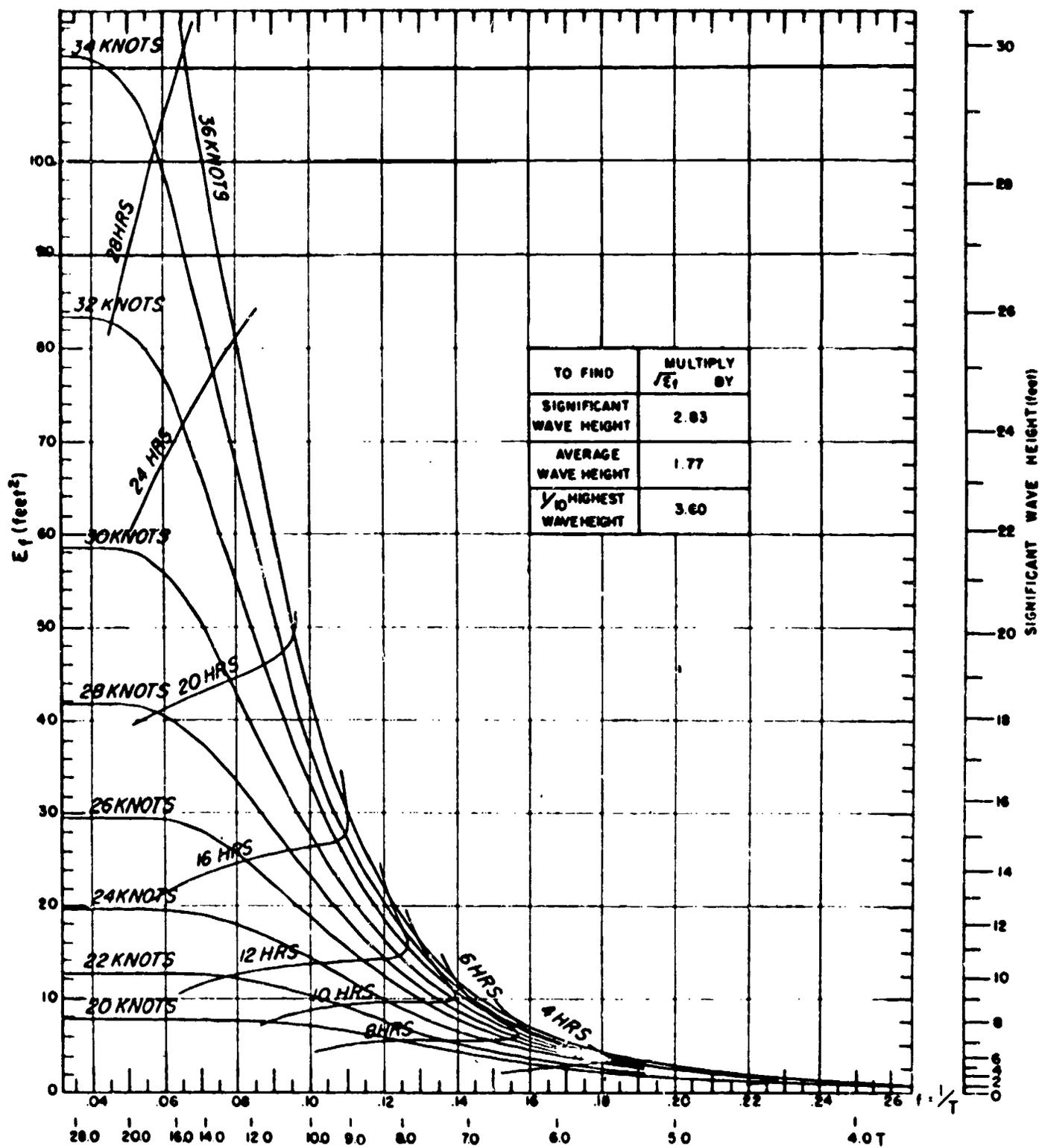
$$S_E(T) = 3.43 (\bar{H})^2 \frac{T^3}{(T)^4} e^{-.675 \left(\frac{T}{\bar{T}}\right)^4} \quad (43)$$

For the frequency spectrum:

$$T = \frac{1}{f} \text{ and } dT = -\frac{1}{f^2} df$$

from which the frequency spectrum becomes:

$$S_E(f) = 3.43 \frac{(\bar{H})^2}{(\bar{T})^4} f^{-5} e^{-.675 (f\bar{T})^{-4}} \quad (44)$$



( H.O. PUB 603, 1957 )

FIGURE 19 DURATION GRAPH  
CO-CUMULATIVE SPECTRA FOR  
WIND SPEEDS FROM 20 TO 36 KNOTS  
AS A FUNCTION OF DURATION

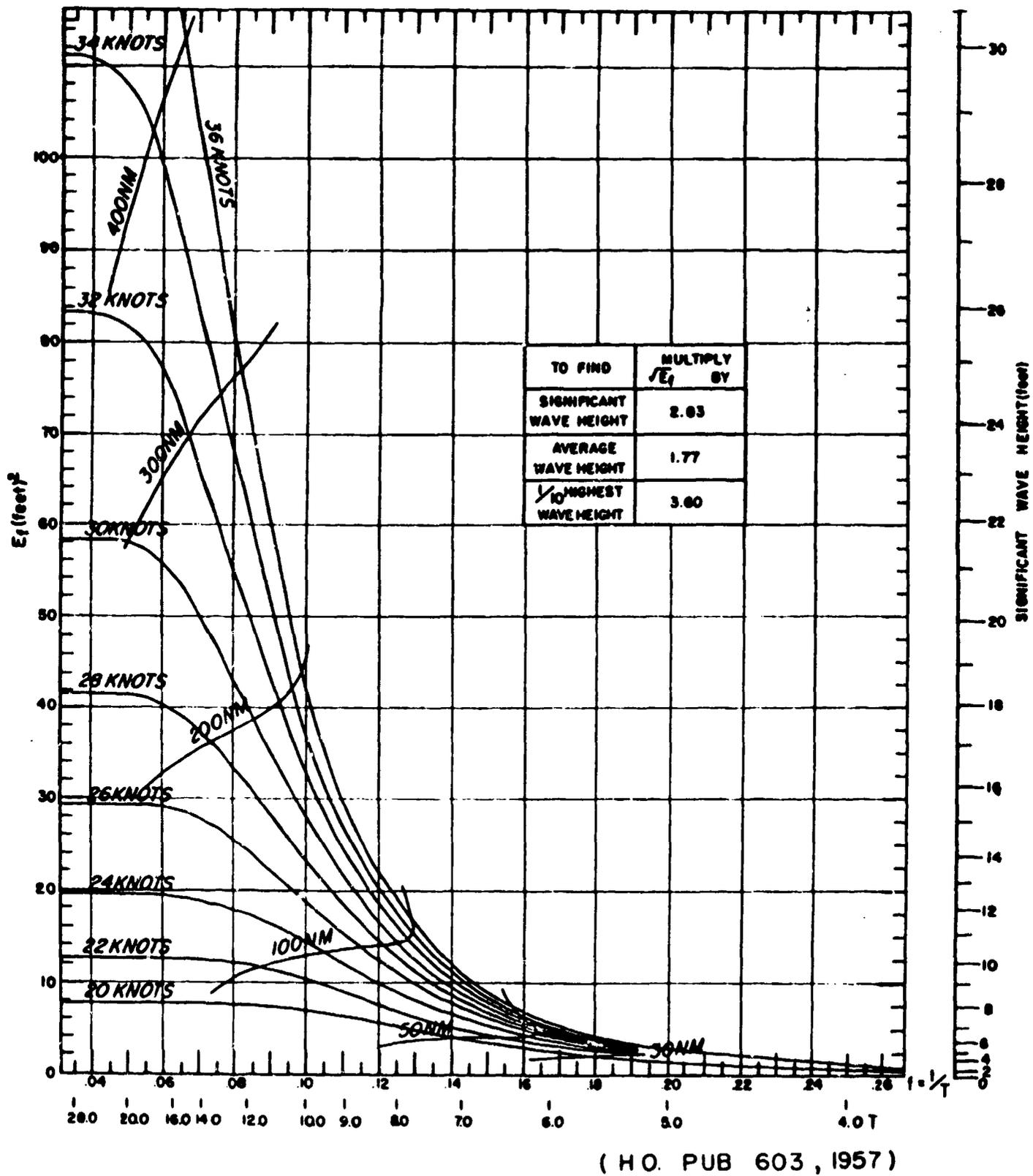


FIGURE 20 FETCH GRAPH  
 CO-CUMULATIVE SPECTRA FOR  
 WIND SPEEDS FROM 20 TO 36 KNOTS  
 AS A FUNCTION OF FETCH

If one lets

$$F_1 = \frac{g\bar{H}}{U^2} = \frac{.625 gH_{33}}{U^2} \quad (45)$$

and

$$F_2 = \frac{g\bar{H}}{2\pi U} = \frac{gT_{1/3}}{2\pi U} \quad (46)$$

it then follows that

$$S_E(f) = 3.43 \left[ \frac{g F_1}{(2\pi F_2)^2} \right]^2 f^{-5} e^{-\frac{.675}{F_2^4} \left( \frac{2\pi Uf}{g} \right)^{-4}} \quad (47)$$

For the high frequency end of the spectrum  $S_E(f)$  is proportional to  $f^{-5}$ . This is in agreement with the results of Roll and Fischer (1956) and Phillips (1957) as well as the newly proposed spectrum of Pierson and Moskowitz (1963).

Other forms of the wave spectra have been suggested by Bretschneider in the Proceedings of the Conference on Ocean Wave Spectra (1963). The corresponding period and frequency spectra are as follows:

$$S(T) = a T^m e^{-bT^n} \quad (48)$$

or

$$S(f) = a f^{-m-2} e^{-bf^{-n}} \quad (49)$$

For the Weibull distribution function ( $m = n - 1$ ) one can propose the following form of wave spectra:

$$S(f/f_0) = (f/f_0)^{-m-1} e^{-\frac{m+1}{m} \left[ (f/f_0)^{-m} - 1 \right]} \quad (50)$$

where the peak of the spectrum occurs at  $f = f_0$ . For the cumulative form the above equation becomes

$$S(f) = -E \left[ 1 - e^{-\frac{m+1}{m} (f/f_0)^{-m}} \right] \quad (51)$$

where

$$E = \int_{\infty}^0 S(f) df \quad (52)$$

Pierson and Moskowitz (1963), based on the similarity theory of Kitaigorodski (1961) propose the following form of wave spectra for a fully developed sea:

$$S(\omega) d\omega = \frac{\alpha g^2}{\omega^5} e^{-\beta(\omega_0/\omega)^4} d\omega \quad (53)$$

where

$$\begin{aligned} \omega &= 2\pi f \\ \omega_0 &= g/U_{19.5} \\ \alpha &= 8.1 \times 10^{-3} \\ \beta &= .74 \end{aligned}$$

$U_{19.5}$  was the measured wind speed at 19.5 meters above mean sea level, and was used directly instead of making an adjustment to the standard anemometer level of 10 meters. This was to avoid selecting someone else's drag coefficient. According to the Weather Bureau criteria, the wind speed over water at elevation 19.5 meters is about 1.1 times that at elevation 10 meters.

If Eq. (53) is integrated to obtain the total energy and related to the significant wave height, one obtains

$$gH/U_{19.5}^2 = .21$$

or

$$gH/U_{10}^2 = 0.254$$

which is very nearly equal to  $gH/U^2 = .26$  used by Sverdrup and Munk (1947).

Eqs. (47) and (53) are directly related, any differences resulting only from the empirical relations for  $\omega$ ,  $\omega_0$ ,  $F_1$  and  $F_2$  as functions of wind speed, and the corresponding interpretations of the wind speed.

#### D. FROUDE SCALING OF THE WAVE SPECTRUM

Assuming that the Froude law applies for wave spectra, it is possible to estimate design wave spectrum by the concept of Froude scaling. The design wave parameter, for example, the significant wave height, can be obtained from the analysis of the compiled wave statistics (wave statistics can be compiled by long term measurements or by use of wave hindcasts). Measured wave spectra are available for certain storm conditions, for example, Bretschneider, et. al. (1962). For the design storm, once in 100 years, for example, the significant wave height can be considerably larger than that corresponding to a measured wave spectrum.

In a report by Bretschneider and Collins (1964), the most severe hurricane which might occur in the Atlantic Ocean can generate a 62-foot significant wave height. The measured wave spectrum of Bretschneider, et. al. (1962), was used to estimate the corresponding wave spectrum for the 62-foot significant wave height. The following table gives significant wave heights based on five measured wave spectra. For conversion to the 62-foot significant wave height the Froude length scale is obtained from  $\lambda = 62 \sqrt{H \text{ measured}}$  and the corresponding time scale is

$$\tau = \sqrt{\lambda}.$$

<u>Date</u>	<u>Time</u>	<u>Table II</u>		<u><math>\tau = \sqrt{\lambda}</math></u>
		<u><math>H_S</math></u>	<u><math>\lambda = 62 \div H_S</math></u>	
17 Dec 1959	0600	35.2	1.76	1.33
"	0900	34.4	1.80	1.34
"	1500	33.9	1.83	1.35
"	1800	39.7	1.56	1.25
18 Dec 1959	0000	35.3	1.76	1.33

To convert the measured frequency spectrum to design spectrum the ordinate  $S(f)$ , having dimension  $\text{ft}^2 \text{sec}$ , must be multiplied by  $\lambda^{5/2}$  and the abscissa  $f$ , having dimension  $\text{sec}^{-1}$ , must be multiplied by  $\lambda^{-1/2}$ .

To convert the corresponding period spectrum to design spectrum the ordinate  $S(T)$ , having dimension  $\text{ft}^2 \text{sec}^{-1}$ , must be multiplied by  $\lambda^{3/2}$  and the abscissa  $T$ , having dimension  $\text{sec}$ , must be multiplied by  $\lambda^{1/2}$ .

If the design significant wave were different from  $H = 62$  feet, then  $\lambda$  and  $\tau$  would change accordingly.

(Data tabulated by frequency analysis is often given for intervals of  $\Delta f$ . In order to convert such data to a design spectrum, one must make an additional conversion by multiplying  $\Delta f$  by  $\lambda^{-1/2}$ .)

For a number of reasons the period spectrum has a more practical application than the frequency spectrum. A simple operation converts the frequency spectrum to the period spectrum as follows:

$$S(T) dT = - S(f) df \quad (54)$$

where  $f = 1/T$   
 $df = -1/T^2 dT$

Thus

$$S(T) = f^2 S(f) \quad (55)$$

Figure 21 shows the Froude scaling of the period spectrum for  $H_S = 62$  feet as given in Table II. The Froude scaled spectra in Figure 21 are shown superimposed on the theoretical spectrum given by Eq. (43). The significant variability at some frequencies is apparent. The theoretical spectrum by Eq. (43) can be considered as yielding the mean spectral energy density in any frequency band, but within such a band an individual ordinate will vary according to some skew-type distribution. The areas under all spectra given in Figure 21 are identical and are related to the same value of the significant wave height, but only the individual ordinates vary. For example, if a given ordinate varies according to the Rayleigh distribution, then the upper and lower 95% confidence levels would be respectively almost twice and half of the mean values.

In Figure 21 the large instabilities in the scaled observed spectra at low periods (high frequencies) are not quite so apparent in the frequency spectrum. These instabilities are probably caused by the introduction of noise in the original record, either by sample rate or instrument insensitivity, or, most likely a combination of both. Since there is a smoothing process (related to  $\Delta f$ ) for the frequency spectrum, it becomes apparent that a similar smoothing process (related to  $\Delta T$ ) should also be applied to the period spectrum. This is because  $\Delta T$  is not linearly related by a constant to  $1/\Delta f$ . Additional studies are required to investigate the disparity observed between spectra in terms of  $\Delta f$  and those in terms of  $\Delta T$ .

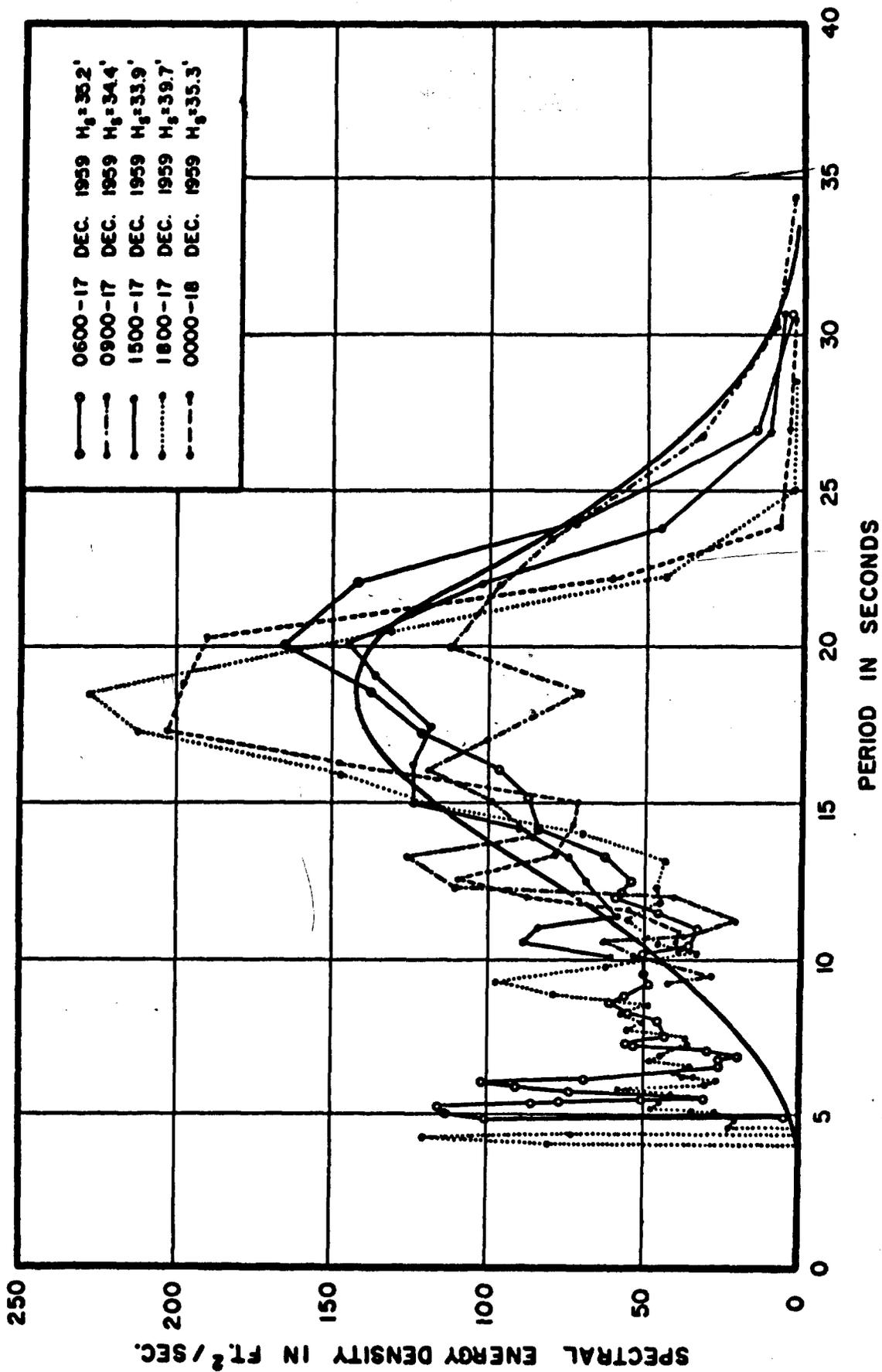


FIGURE 21 COMPARISON OF PERIOD SPECTRUM  
 FOR 62-FOOT SIGNIFICANT  
 WAVE WITH STATION "J" DATA

### III. PROPAGATION OF WAVES AND SWELLS INTO SHALLOW WATER

When waves or swells propagate into shallow water, a number of modifications take place: refraction, shoaling and energy losses. These factors will be discussed subsequently, considering only a simple sinusoidal wave. The fact that waves travel in groups of variable amplitude and frequencies -- the wave spectrum -- entails additional considerations discussed later.

If the waves are long-crested and are moving obliquely towards a straight shoreline whose depth contours are also straight and parallel to the shore, those portions of the wave front which effectively feel bottom first are retarded first; therefore, the wave becomes subject to a progressive curving or refraction which, in its overall effect, tends to align the wave front to the depth contours. This is in analogy to simple light rays of a particular wave length when traveling from one media to a media of different density. The light is bent or refracted. When all the wave lengths for the spectrum of light travel from one media to a media of greater density, each wave length is refracted to a different degree, which results in sorting of the colors. Similarly, when the spectrum of ocean waves enters a refracting area, the different frequencies or wave lengths are sorted, and the resulting spectrum is changed accordingly. Figure 22 illustrates the effect of refraction for a simple wave.

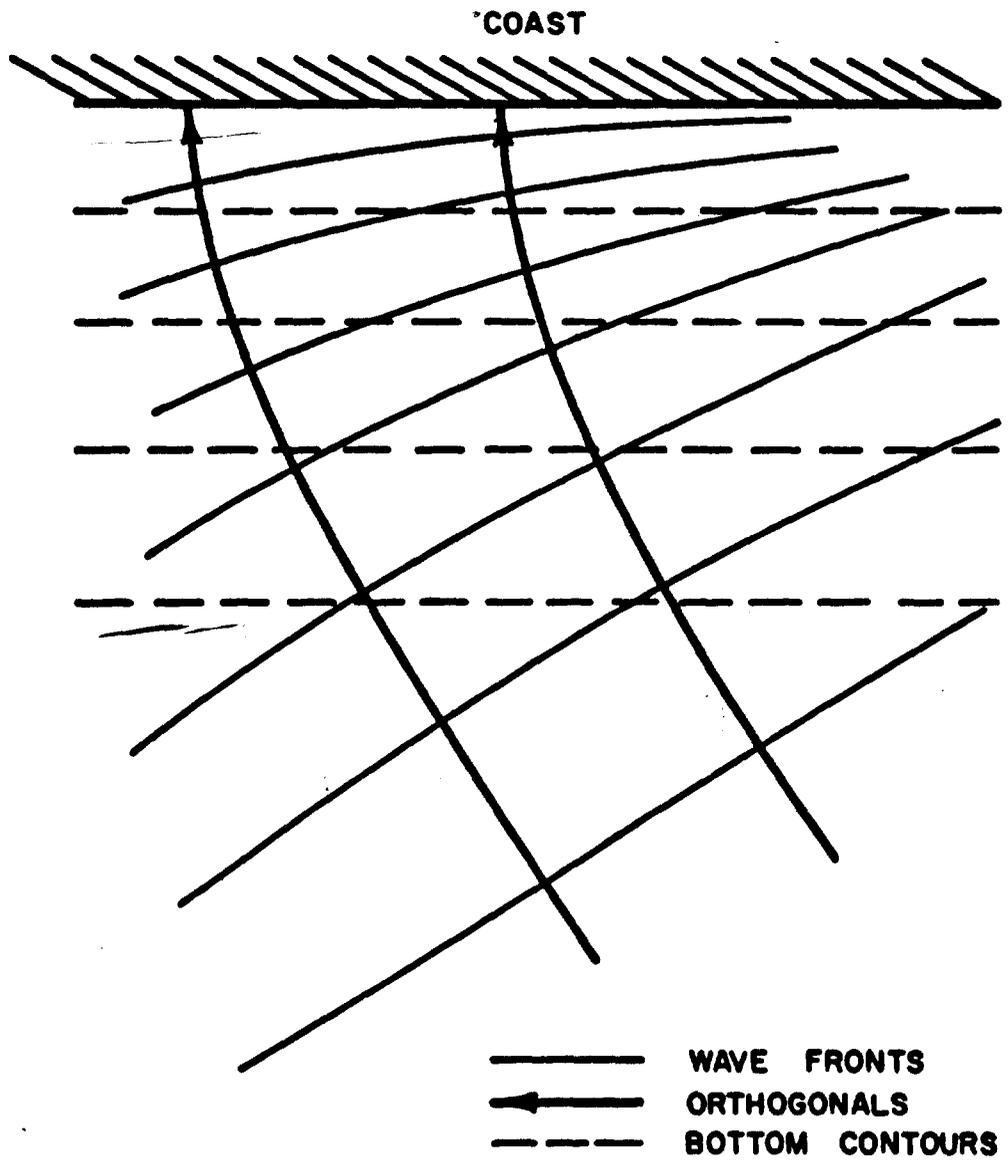
The orthogonals (wave rays) represent the direction that the wave fronts are taking. They become curved in the process of refraction and in general may tend to diverge or converge. Over a submarine canyon (Figure 23) the waves will always tend to diverge, and conversely, over a submarine ridge (Figure 24), the waves will always tend to converge.

It is generally assumed that the wave energy contained between orthogonals remains constant as the wave front progresses: this supposes that there is no dispersion of energy laterally along the front, no reflection of energy from the rising bottom, and none lost by other processes. If  $b_0$  represents the distance between orthogonals in deep water, and  $b_x$  the distance between orthogonals somewhere in shallow water (where the corresponding wave heights are  $H_0$  and  $H_x$ ), since the energy of the wave is proportional to the square of the wave height, it follows that

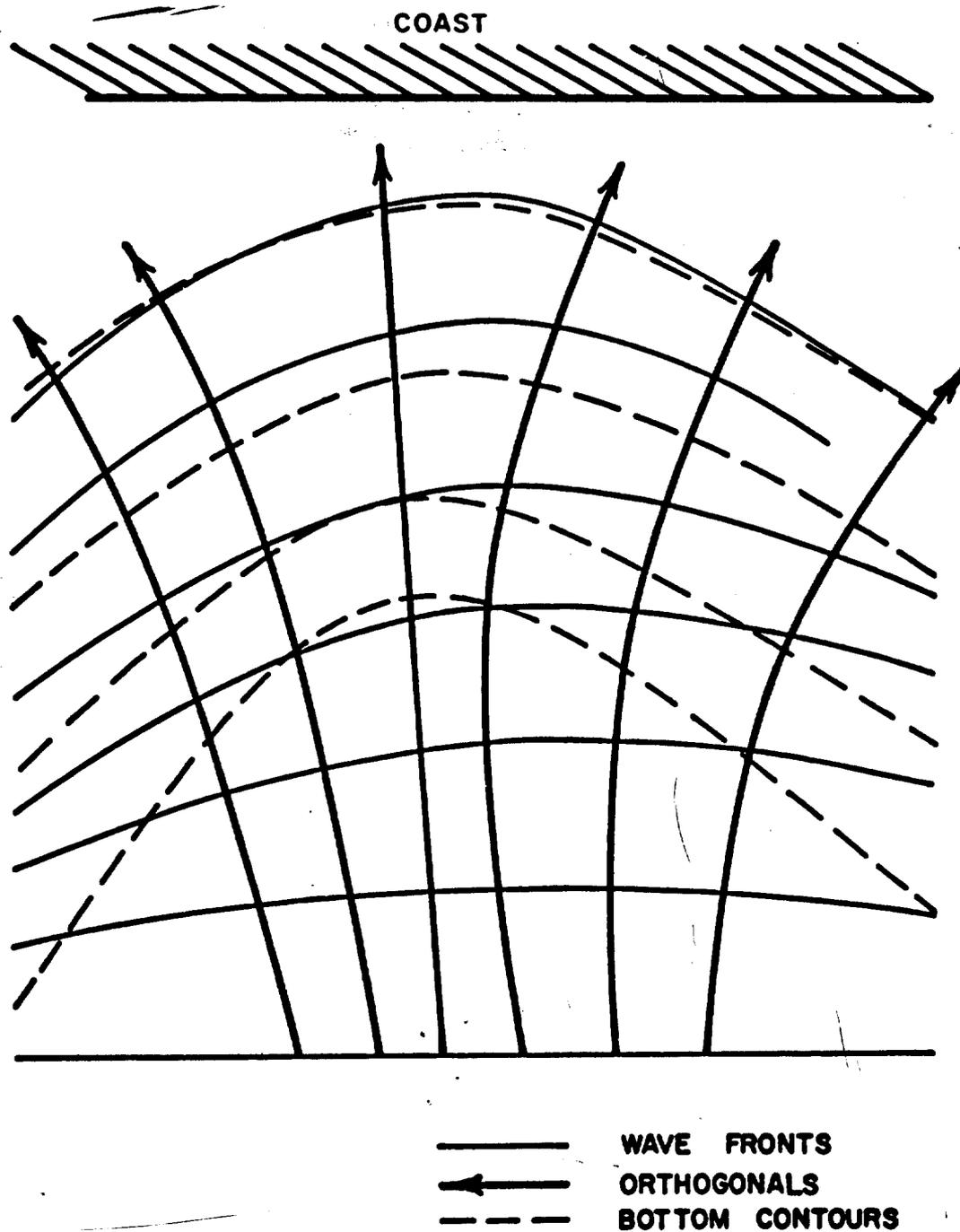
$$b_0 H_0^2 C_{g_0} = b_x H_x^2 C_{g_x} \quad (56)$$

or

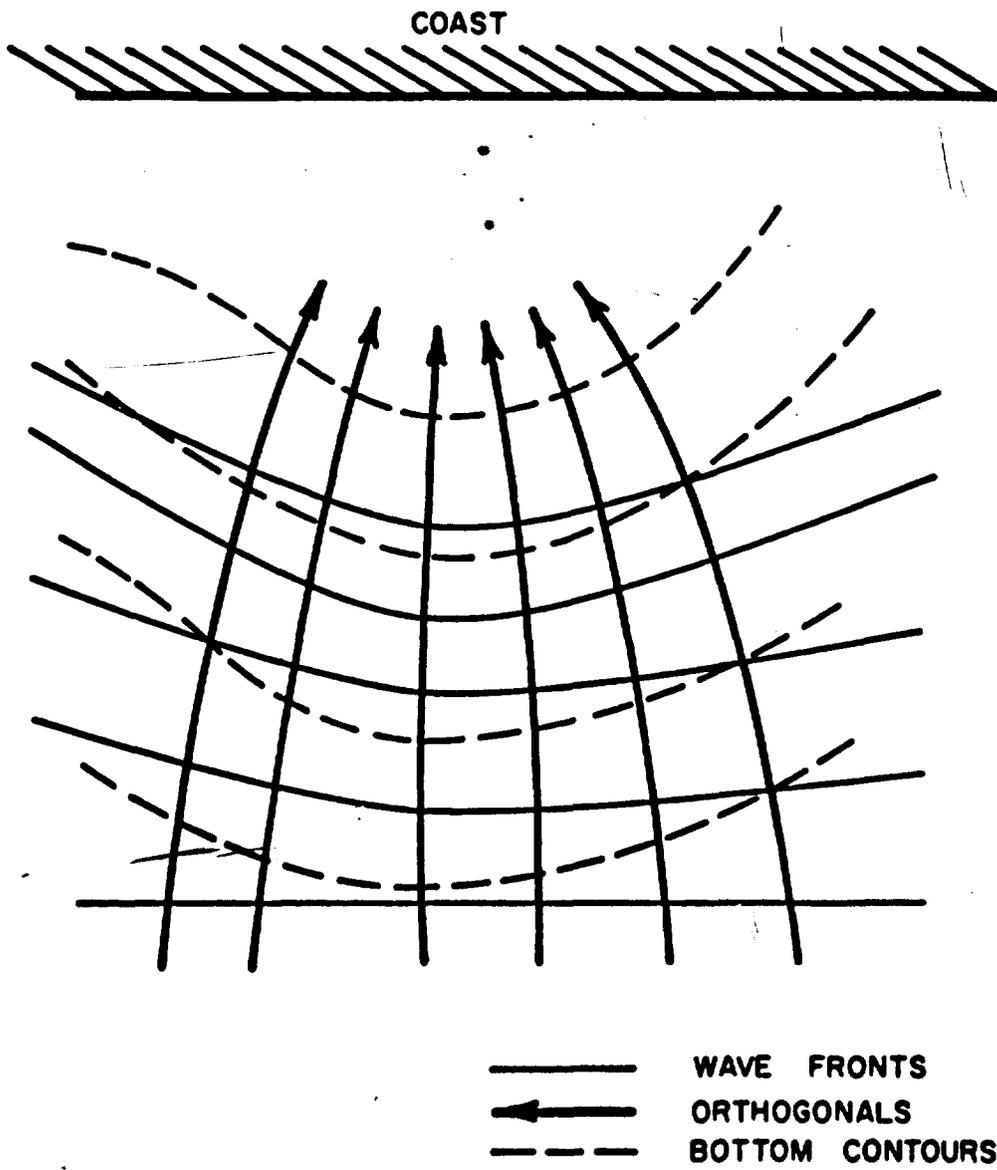
$$H_x = H_0 \sqrt{b_0/b_x} \sqrt{C_{g_0}/C_{g_x}} \quad (57)$$



**FIGURE 22 REFRACTION EFFECT**



**FIGURE 23      DIVERGING      ORTHOGONALS**



**FIGURE 24 CONVERGING ORTHOGONALS**

where  $\sqrt{b_o/b_x} = K_r$ , the refraction coefficient. The above is a simplified example and the general mechanics of transformation for a simple wave follow.

Assuming that the gradient of the sea bed is not so steep that wave energy is reflected, and that there is no gain or loss of energy due to lateral diffraction or dispersion, then the wave energy transmitted between orthogonals remains the same, except for loss due to bottom friction and percolation in the permeable bed. Frictional losses represent the energy expended in overcoming resistance, shear stresses or drag forces on the sea bottom. Percolation losses represent energy lost through actual infiltration of the wave motion into the semi-fluid mass of certain finely suspended sediments of the permeable sea bottom.

If  $P_f$  and  $P_p$  represent the frictional and percolation energy losses per unit area of bottom per unit time, respectively, then from Figure 25, which shows the elemental length  $\delta x$  of rising sea bottom in the direction of motion, the energy loss (time rate), in terms of power ( $P$ ) transmitted across the vertical section  $bh$  between the orthogonals will be:

$$(P_f + P_p) b \delta x = [Pb - \delta(Pb)] - Pb \quad (58)$$

whence

$$\frac{d(Pb)}{dx} = -(P_f + P_p) b \quad (59)$$

The wave energy  $E$  per unit surface area is given by

$$E = \frac{1}{8} \rho g H^2 \quad (60)$$

which is transmitted across the unit area with the group velocity  $C_g$ ; i. e.  $P = E C_g$ . The group velocity changes from deep water into shallow water according to

$$C_g = nC \quad (61)$$

where  $C$  is the wave celerity and  $n$  is the transmission coefficient and, according to linear wave theory, is given by

$$n = \frac{1}{2} \left[ 1 + \frac{2kh}{\sinh 2kh} \right] \quad (62)$$

where  $k = 2\pi/L$ . It then follows that the power of transmission is

$$Pb = Eb C_g = \frac{1}{8} \rho g H^2 b (nC) \quad (63)$$

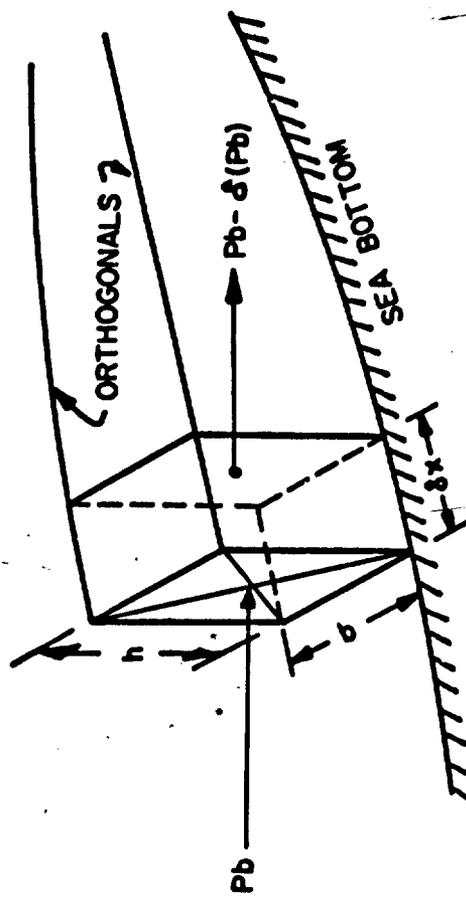


FIGURE 25 EXPERIMENTAL LENGTH  $\delta x$  OF  
 RISING SEA BOTTOM IN DIRECTION OF MOTION

In deep water, just before the waves are due to feel the effects of depth, Eq. (63) becomes

$$P_o b_o = \frac{1}{8} \rho g H_o^2 b_c n_o C_o \quad (64)$$

The ratio of Eqs. (63) and (64) results in

$$\frac{H}{H_o} = \left( \frac{b_o}{b} \right)^{1/2} \left( \frac{n_o C_o}{nC} \right)^{1/2} \left( \frac{Pb}{P_o b_o} \right)^{1/2} \quad (65)$$

which may be expressed as

$$\frac{H}{H_o} = K_r K_s K_{fp} \quad (66)$$

where  $K_r$ ,  $K_s$  and  $K_{fp}$  are known as the refraction coefficient, the shoaling coefficient, and the friction-percolation coefficient, respectively.

The refraction coefficient is separately defined as

$$K_r = \sqrt{\frac{b_o}{b}} \quad (67)$$

the square root of the ratio of the widths between orthogonals in deep and shallow water. According to linear theory,  $K_r$  depends only on wave period, water depth contours and initial deep water wave direction. In practical coastal engineering problems, refraction coefficients can be determined by one of two methods: wave front and direct ray or orthogonal methods, both of which are discussed in B. E. B. T. R. 4 (1961).

The shoaling coefficient is given completely, according to linear wave theory, by:

$$K_s = \sqrt{\frac{n_o C_o}{nC}} \quad (68)$$

The wave celerity  $C$  in shallow water is related to  $C_o$  in deep water by

$$C = C_o \tanh kh \quad (69)$$

and in deep water  $n_o \rightarrow 1/2$ , whence from Eqs. (63), (68), and (69)

$$K_s = \left[ \left( 1 + \frac{2 kh}{\sinh 2 kh} \right) \tanh kh \right]^{-1/2} \quad (70)$$

$K_s$  is then given explicitly as a function of  $h/L$  or  $h/L_o$  and may be obtained from tables by Wiegel (1954) where  $K_s = H/H_o$ .

The friction-percolation coefficient  $K_{fp}$  is defined as

$$K_{fp} = \left( \frac{P_b}{P_o b_o} \right)^{1/2} \quad (71)$$

Returning to Eq. (59) and substituting  $P_b$  from Eq. (71), one obtains

$$P_o b_o \frac{d(K_{fp}^2)}{dx} = -(D_f + D_p) b \quad (72)$$

or

$$\frac{d(K_{fp})}{dx} = - \frac{1}{P_o b_o} \frac{1}{2 K_{fp}} (D_f + D_p) b \quad (73)$$

According to Putnam and Johnson (1949)

$$D_f = \frac{4}{3} \pi^2 \frac{\rho_f H^3}{T^3 (\sinh kh)^3} \quad (74)$$

and according to Putnam (1949)

$$D_p = \frac{4 \pi^2}{\nu T^2} \frac{g p H^2}{\sinh 2 kh} \quad (75)$$

In Eqs. (74) and (75)

- $\rho$  = mass density of water
- $f$  = dimensionless friction factor
- $\nu$  = kinematic viscosity of sea water
- $p$  = permeability coefficient of Darcy's Law having the dimensions of (length)<sup>2</sup>

Eq. (75) is applicable for  $h/L > 0.3$  but becomes more complicated for  $h/L < 0.3$ .

Returning to the differential Eq. (73) and in view of Eqs. (74) and (75), one recognizes, after careful examination, that this is of the same general form as Bernoulli's nonlinear differential equation of the first order; i. e.

$$\frac{dy}{dx} + M_y = N y^n \quad (76)$$

Without going into the algebra, one may write Eq. (73) as follows:

$$\frac{d}{dx} (K_{fp}^{-1}) - \frac{F_p}{K_{fp}} = F_f \quad (77)$$

and the solution is:

$$K_{fp}^{-1} = e^{-\int F_p dx} \left[ \int e^{-\int F_p dx} F_f dx + \text{Const} \right] \quad (78)$$

In general, numerical integration of Eq. (78) is required, but a number of special cases have been investigated by Bretschneider and Reid (1954). Of most interest at present are two simple cases: (1) no bottom friction, and (2) no percolation.

The first case for  $F_f = 0$  results in  $K_{fp} = K_p$  where

$$K_p = e^{-\int_{x_0}^x F_p dx} \quad (79)$$

The second case for  $F_p = 0$  results in  $K_{fp} = K_f$  where

$$K_f = \left[ 1 + \int_0^x F_f dx \right]^{-1} \quad (80)$$

For a bottom of constant depth Eqs. (79) and (80) become respectively

$$K_p = e^{-F_p x} \quad (81)$$

and

$$K_f = \left[ 1 + F_f x \right]^{-1} \quad (82)$$

Eqs. (81) and (82) can be used for practical engineering solutions for a bottom of variable depth where small increments of  $\Delta x$  are selected, each increment having a bottom of constant depth equal to the mean depth over the increment; i. e.

$$K_{fp} = \sum \Delta K_f \cdot \Delta K_p \quad (83)$$

where

$$\Delta K_p = e^{-F_p \Delta x} \quad (84)$$

and

$$\Delta K_f = \left[ 1 + F_f \Delta x \right]^{-1} \quad (85)$$

Graphs presented by Bretschneider and Reid (1954) can be used to facilitate computations by the numerical method.

Bottom friction and shoaling modifications can be obtained by use of Figures 26, and 27. Figure 26 is based on a bottom of constant depth, but can be used for a variable bottom by representing the bottom by a series of sections, each having an average bottom depth.

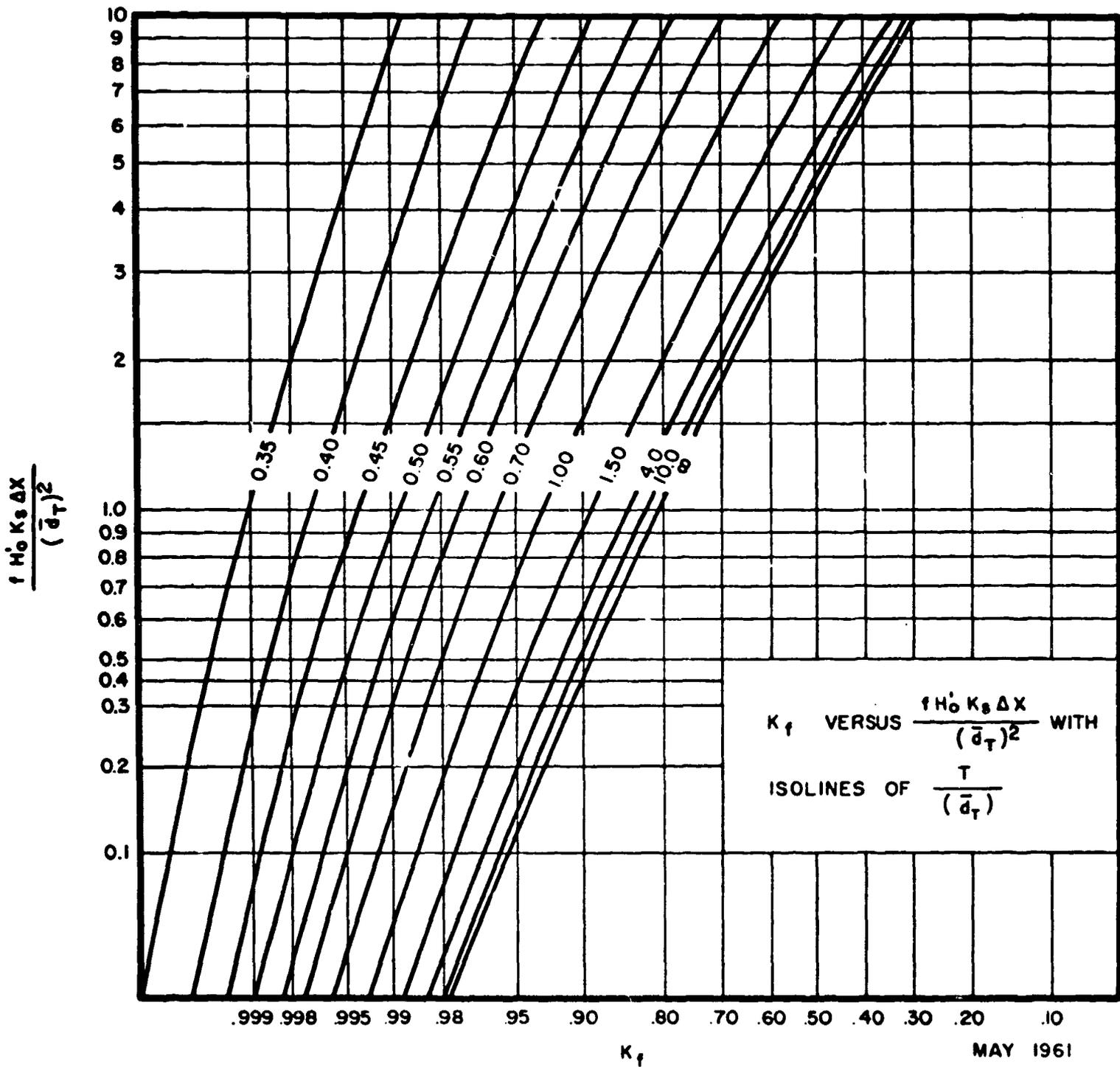


FIGURE 26 RELATIONSHIP FOR FRICTION LOSS OVER A BOTTOM OF CONSTANT DEPTH

$K_s$

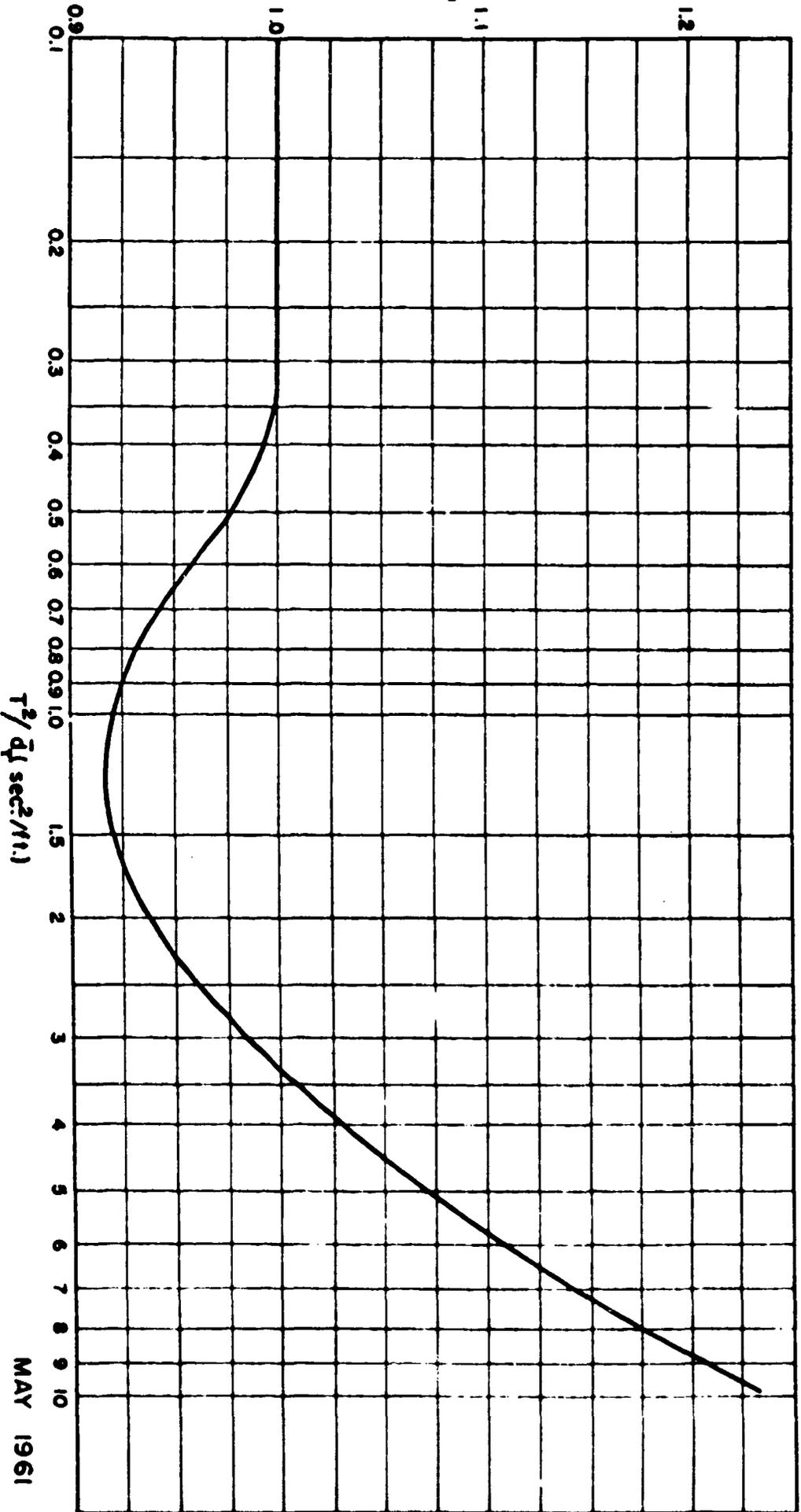


FIGURE 27

$K_s$  VERSUS  $T^2/d_t$

MAY 1961

#### IV. GENERATION OF WIND WAVES IN SHALLOW WATER

Less information is available on wind waves in shallow water than in deep water. This is true in regard to both theory and available data. The first information on this subject was given by Thijssse (1949), based on limited data. Additional data and relationships were brought forth by Dr. Garbis Keulegan at the National Bureau of Standards, although never published to the knowledge of the author. The U. S. Army Corps of Engineers, Jacksonville District (1955), performed an extensive field investigation on wind, waves and tides in Lake Okeechobee, Florida. Based on the hurricane wind and wave data from Lake Okeechobee, and some ordinary wind wave data from the shallow regions of the Gulf of Mexico, Bretschneider (1954) was able to establish a numerical procedure for computing wind waves in shallow water taking bottom friction into account. A friction factor of  $f = .01$  appears satisfactory. Presently these techniques are used for the continental shelf, but may require further calibration when more wind and wave data are available.

##### A. GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT DEPTH

If  $d/T^2 < 2.5 \text{ ft/sec}^2$ , then the waves effectively "feel bottom" and the depth and bottom conditions enter as additional factors with respect to the heights and periods of waves which can be generated. The effect of frictional dissipation of energy at the bottom for such waves limits the rate of wave generation and also places an upper limit on the wave heights which can be generated by a given wind speed and fetch length.

Figure 4, the dimensionless deep water wave forecasting relationships, in effect represents the generation of wave energy in deep water as a function of  $F$ ,  $U$  and  $t$ , since the energy is proportional to  $H^2$ , whereas Figure 26 represents the dissipation of wave energy due to bottom friction. Figures 4 and 26 were combined by a numerical method of successive approximation to obtain relationships for the generation of waves over an impermeable bottom of constant depth. Best agreement between wave data and the numerical method was obtained when a bottom friction factor of  $f = .01$  was selected. Perhaps a "calibration friction factor" is a more appropriate term since it would take into account other influential factors not normally included in the friction factor term such as energy loss by "whitecaps." The curves of figure 28 are the results of these computations. The curve of  $gT/U$  versus  $gd/U^2$  is based on the wave data, whereas the curves of  $gH/U^2$  versus  $gd/U^2$  and  $gF/U^2$  are based on the numerical computations. The curves of these figures are not too much different from those presented by Thijssse and Schijf (1949), reproduced in Figure 30. Figure 29, based on Figure 28, gives wave forecasting curves for shallow water of constant depth and unlimited wind duration and fetch length. Figure 28 may be used when both the fetch length as well as the depth are restricted.

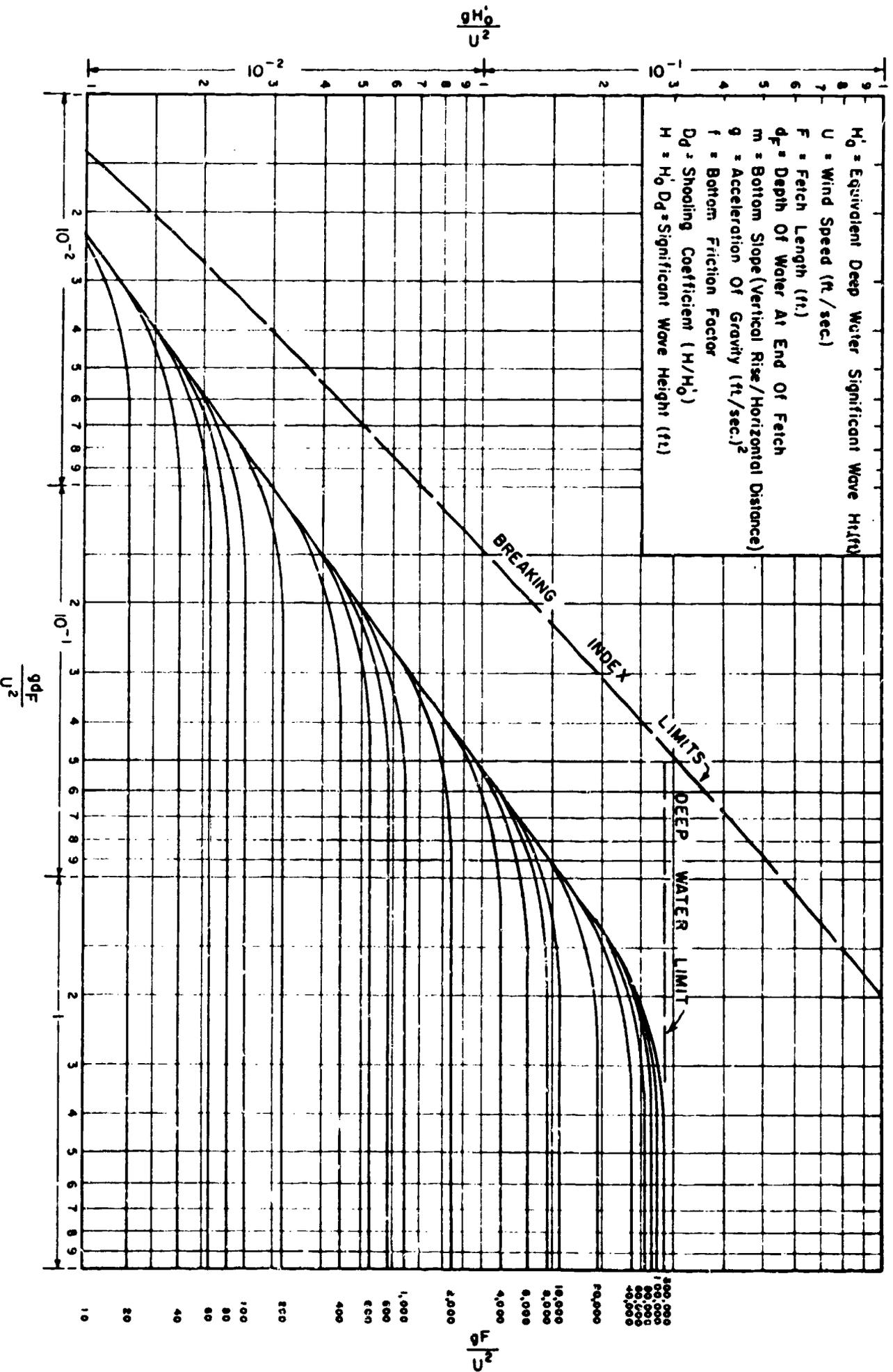
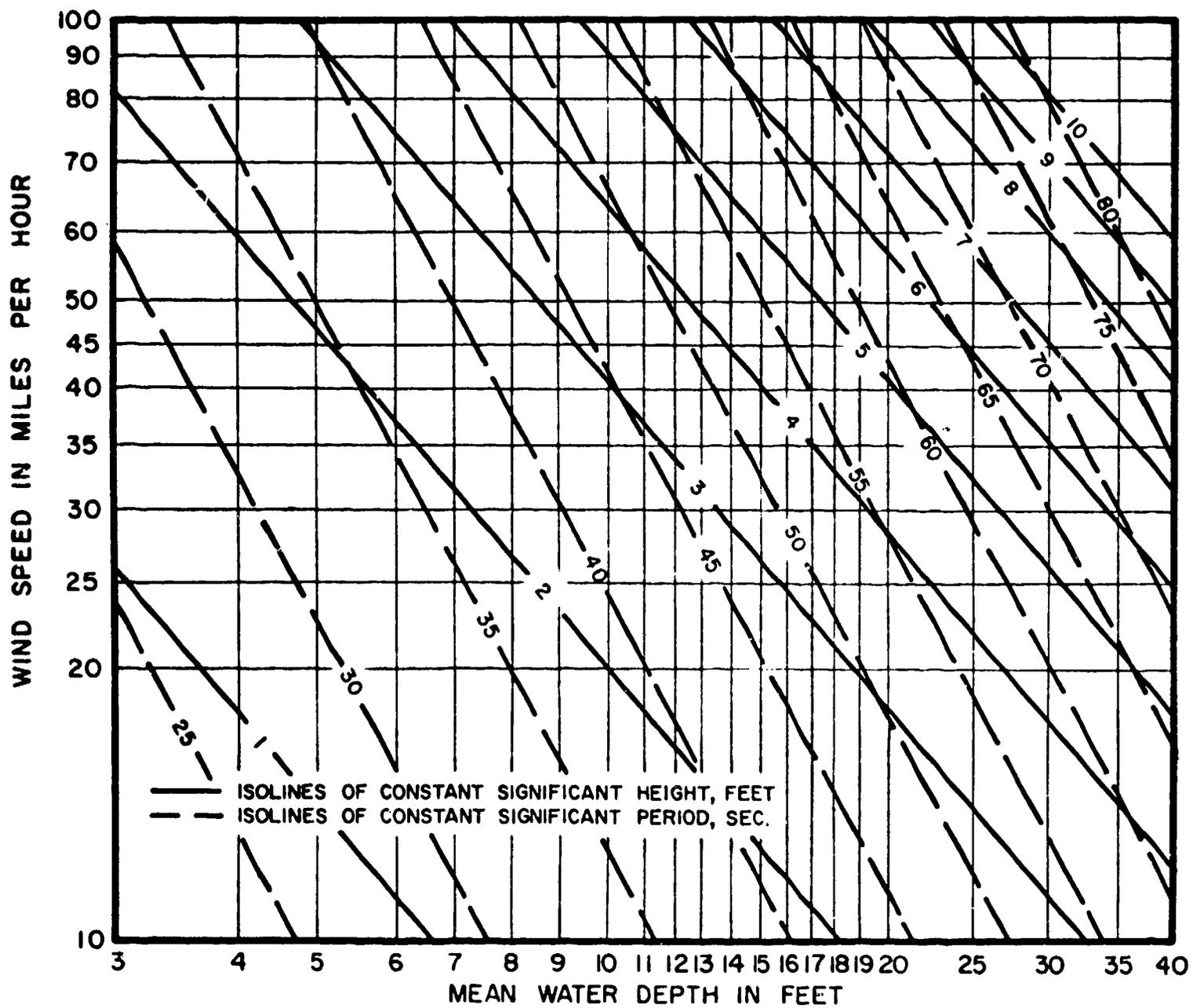


FIGURE 28 GENERATION OF WIND OVER A BOTTOM OF CONSTANT DEPTH FOR UNLIMITED WIND DURATION REPRESENTED AS DIMENSIONLESS PARAMETERS



**FIGURE 29 WAVE FORECASTING RELATIONSHIPS FOR SHALLOW WATER OF CONSTANT DEPTH**

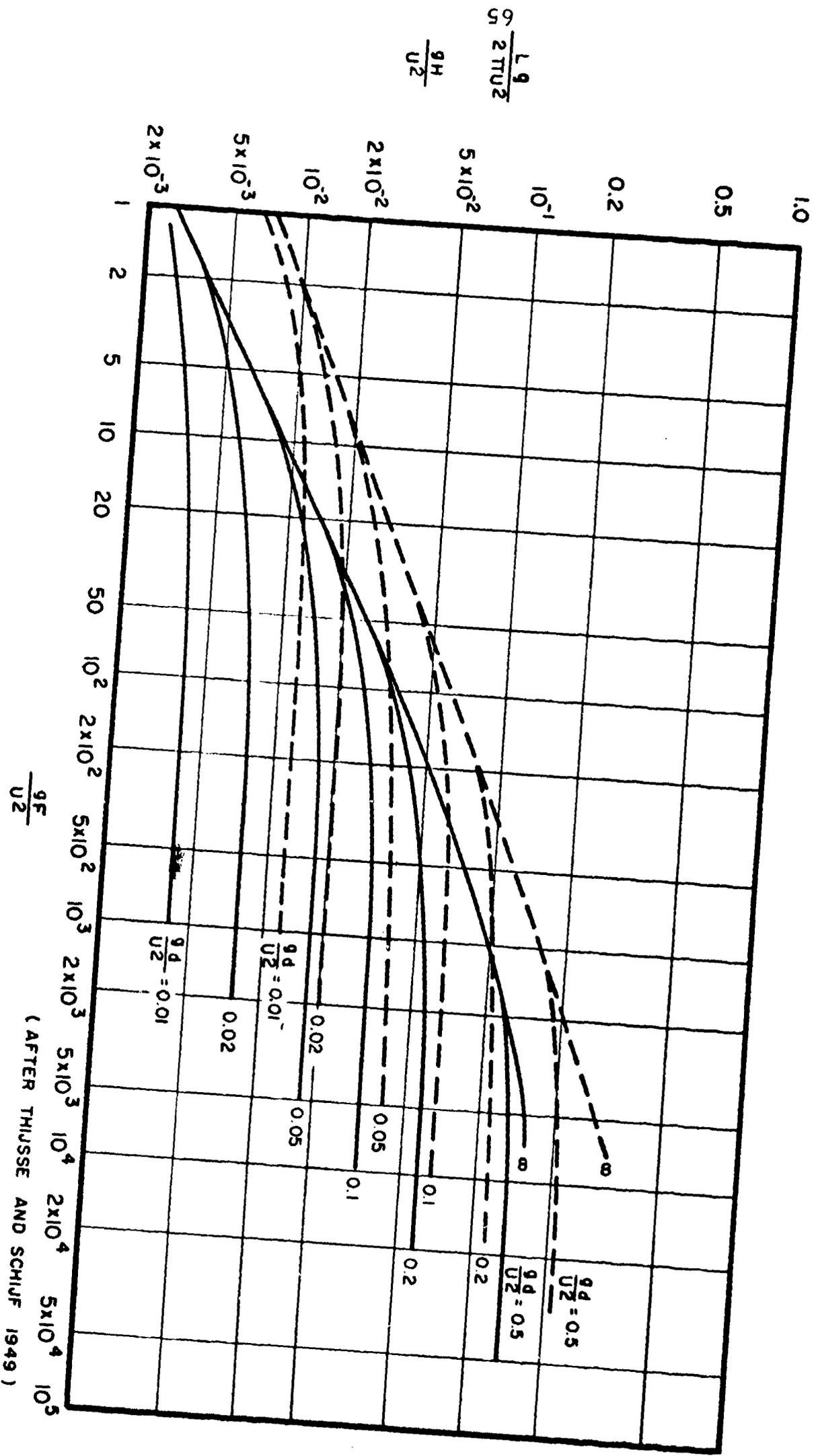


FIGURE 30 GROWTH OF WAVES IN A LIMITED DEPTH

(AFTER THUISSE AND SCHUIF 1949)

The important fact from the above material, however, is the establishment of a numerical procedure for computing wind waves in shallow water of constant depth which can be verified by use of wave data. This procedure can be extended to a bottom of constant slope wherein the bottom is segmented into elements, each element having a mean depth assumed to be constant. Sample computations for a typical continental shelf are given by Bretschneider (1957).

Presently, the system of calculations is being practiced on generation of wind wave spectra over the continental shelf. It will be some time in the future before the system is perfected. However, Figure 31, reproduced from Bretschneider (1964), is a typical example resulting from calculation of wave spectra in shallow water, taking into account the combined effect of bottom friction and wind action. The hindcast deep water wave spectra is based on the method of Bretschneider (1957) and bottom friction is taken into account according to the method of Bretschneider (1963) wherein the wind effect is added.

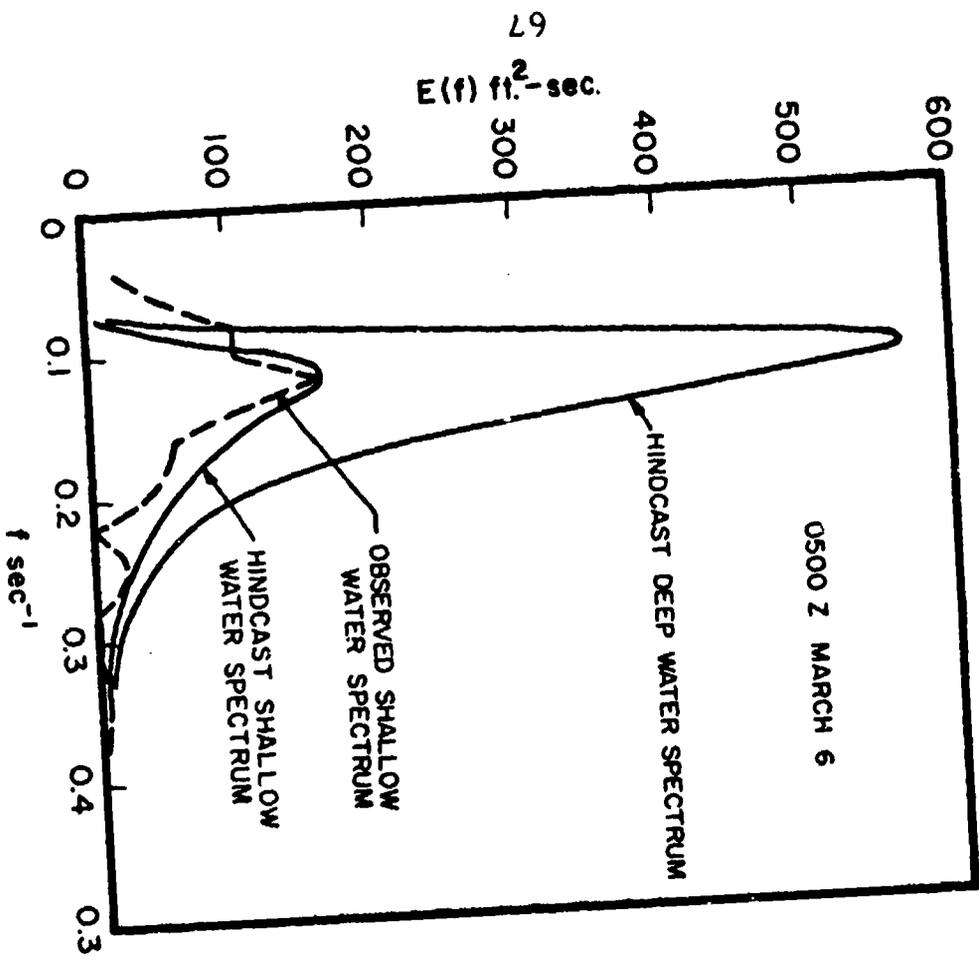
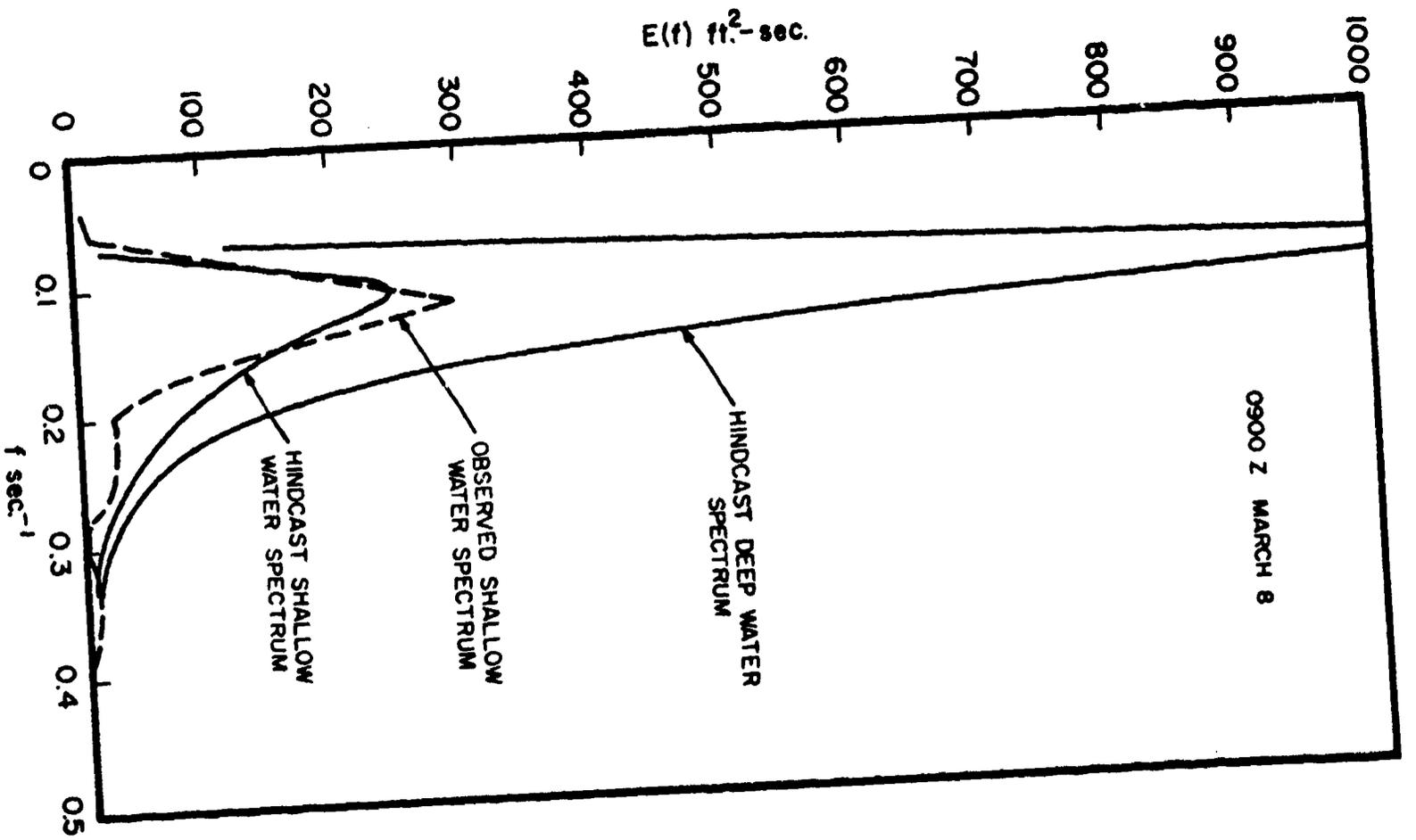


FIGURE 31 WAVE SPECTRA  
FOR ATLANTIC CITY, N.J.



## V. DECAY OF WAVES IN DEEP WATER

When waves leave a generating area and travel through a calm or an area of light winds, a transformation takes place. At any particular time and with respect to decay distance, the significant wave height decreases and the modal period shifts to the longer period waves, resulting in an apparent increase in significant wave period. However, at a particular decay distance the modal period decreases. To understand the significance of a period shift, one must refer to the spectrum of waves and the original work of Barber and Ursell (1948). Recent work of Munk (1964) shows that most of this transformation takes place in the first 1000 miles of decay. Based on field tests of waves arriving from the southern hemisphere and traversing a great circle to Alaska, Munk (1964) shows that there is a very rapid loss or transformation of energy during the first 1000 miles and a shift of peak frequency to about  $0.6$  to  $0.7 \text{ sec}^{-1}$ , after which little change occurs.

The work of Wiegel and Kimberly (1950) and Bretschneider (1950) was included in the revisions in forecasting decay of swell. Empirical relationships were developed, based on the correlation of wave data obtained off Southern California and wind and fetch characteristics obtained from weather charts of the Southern Hemisphere, from where the waves had originated. The rather meager data were recorded in shallow water and had to be transformed for deep water according to the shoaling coefficients given by Figure 27. In spite of the above difficulties, empirical forecasting curves for wave decay were determined by Bretschneider (1951) and are presented in Figure 32. The scatter of data on which these curves are based indicates that the above relations can deviate by as much as  $\pm 50$  percent. Although these curves are not completely satisfactory, they are the best available to date. Further refinements are certainly necessary.

Assuming average conditions for a severe storm, a wave spectra hindcast was made for the end of the fetch, and assuming that Figure 32 applied for  $H_D/H_F$  as a function of  $T$  for decay of the wave spectra, computations were made for various elements of the period spectrum. The results of these calculations are shown in Figure 33. The curve for  $D = 0$  is zero decay distance or the end of the fetch. Other values of the decay distance  $D$  are given in nautical miles. The short period waves disappear rapidly, leaving only the longer period waves to decay more slowly.

The areas under each spectrum are related to the square of the corresponding significant wave height, and the modal period is close to the significant wave period. Figure 34 shows the change in significant wave height and period as a function of decay distance. From this figure it is seen that most of the decay takes place during the first 1000 miles

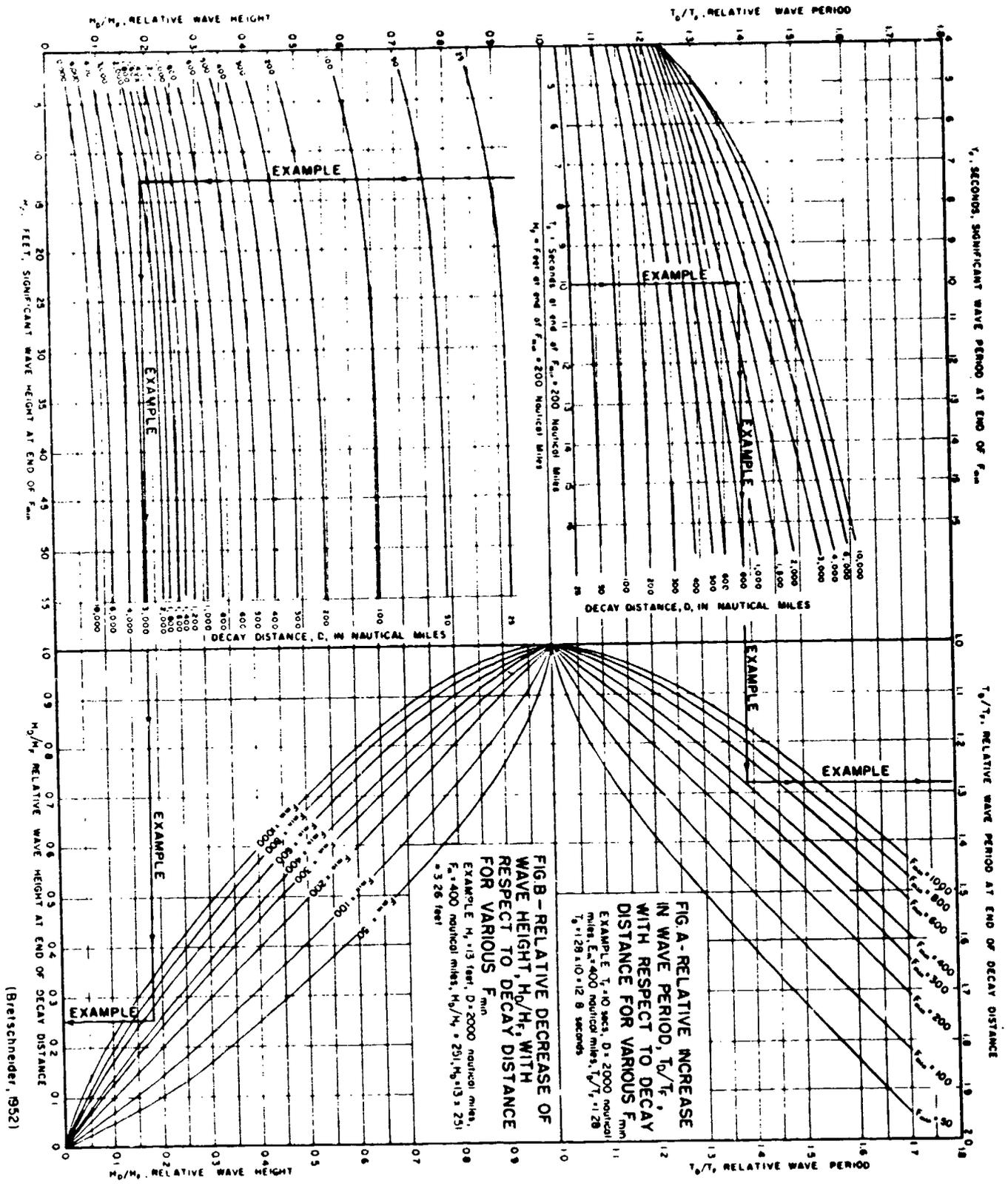


FIGURE 32 FORECASTING CURVES FOR WAVE DECAY

(Breitschneider, 1952)

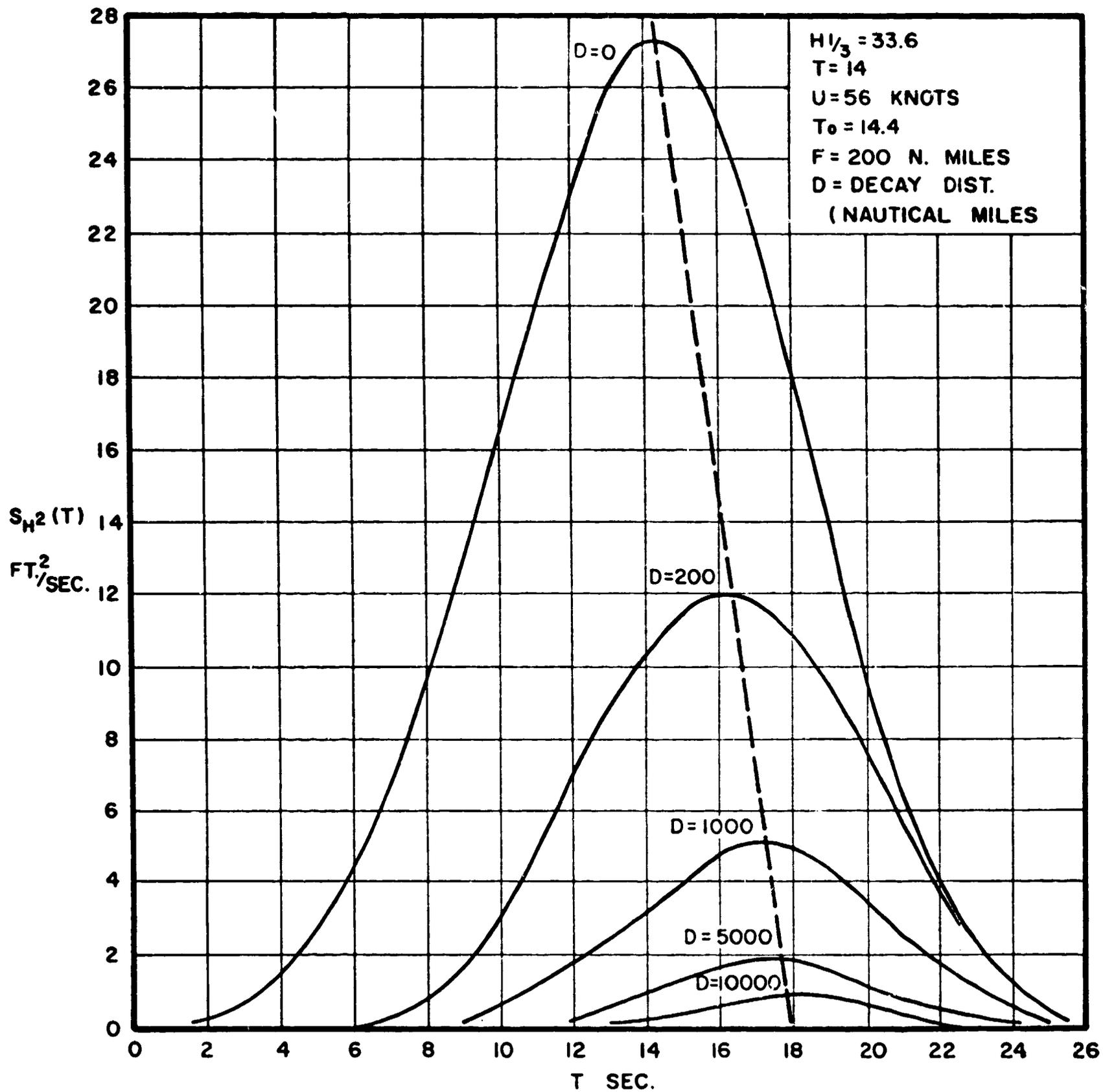


FIGURE 33

DECAY OF WAVE SPECTRA WITH DISTANCE

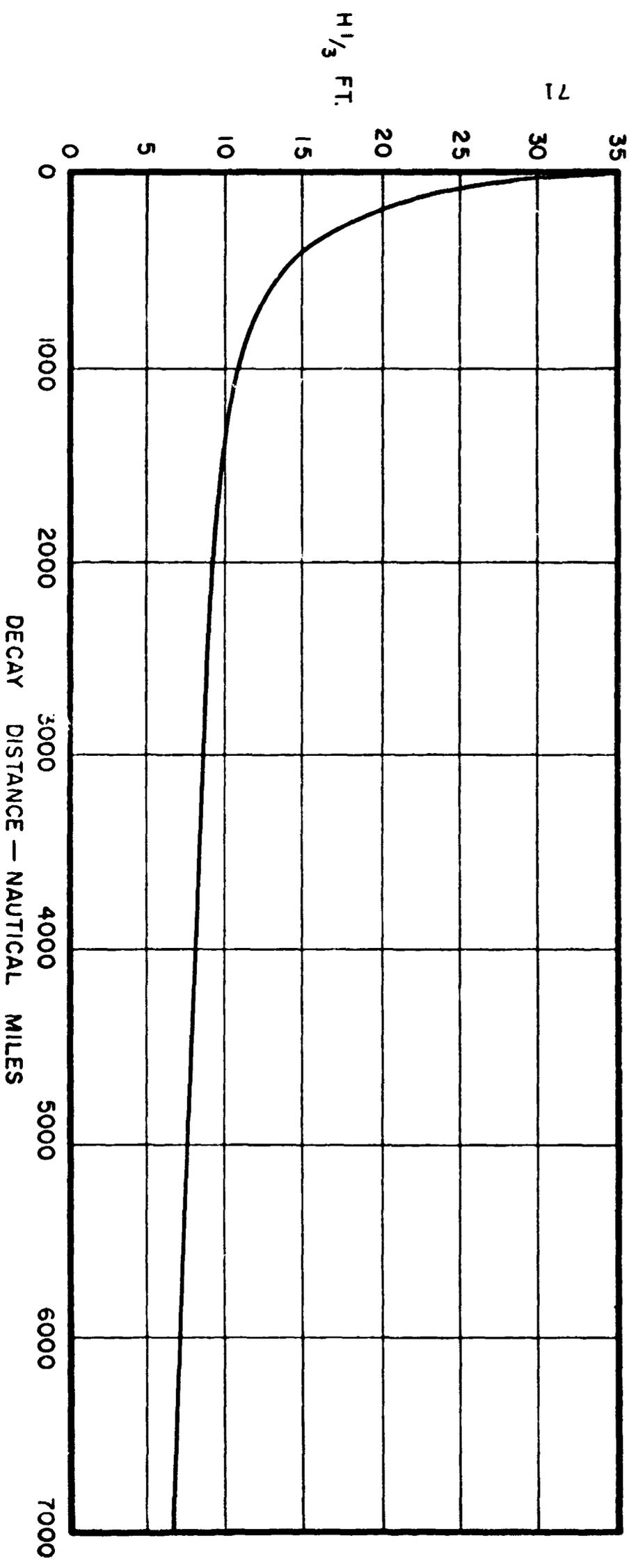
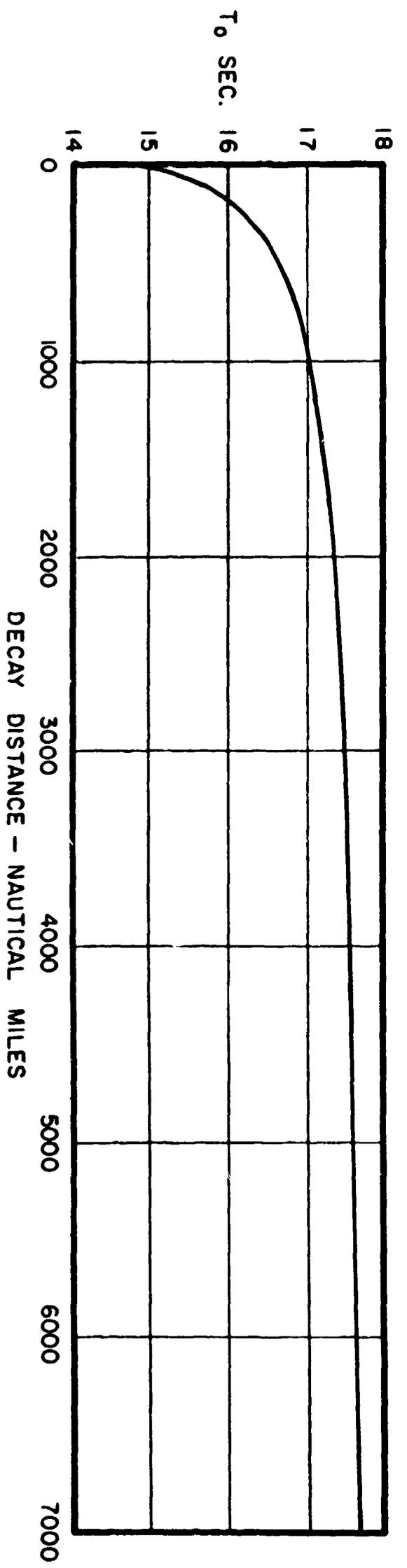


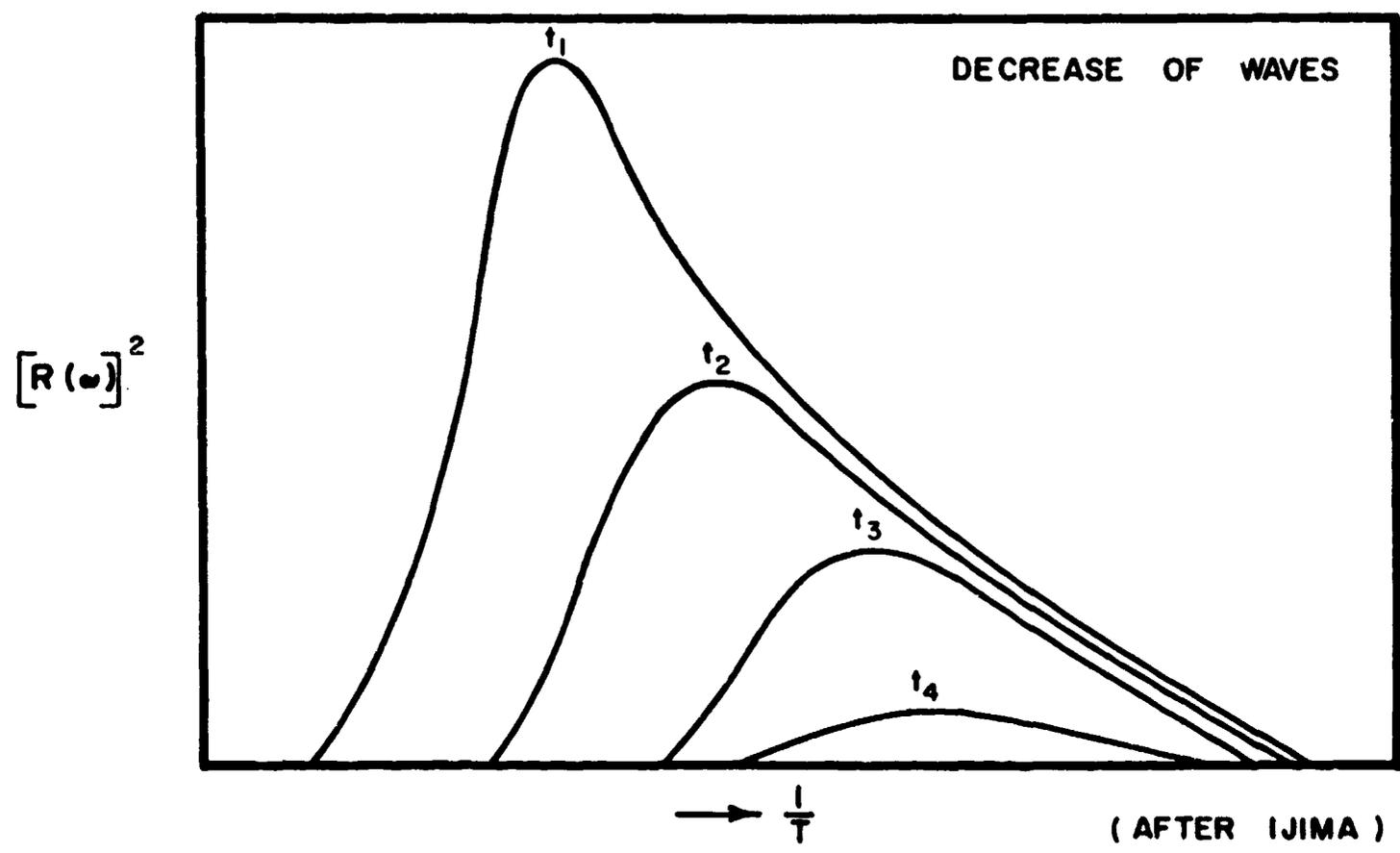
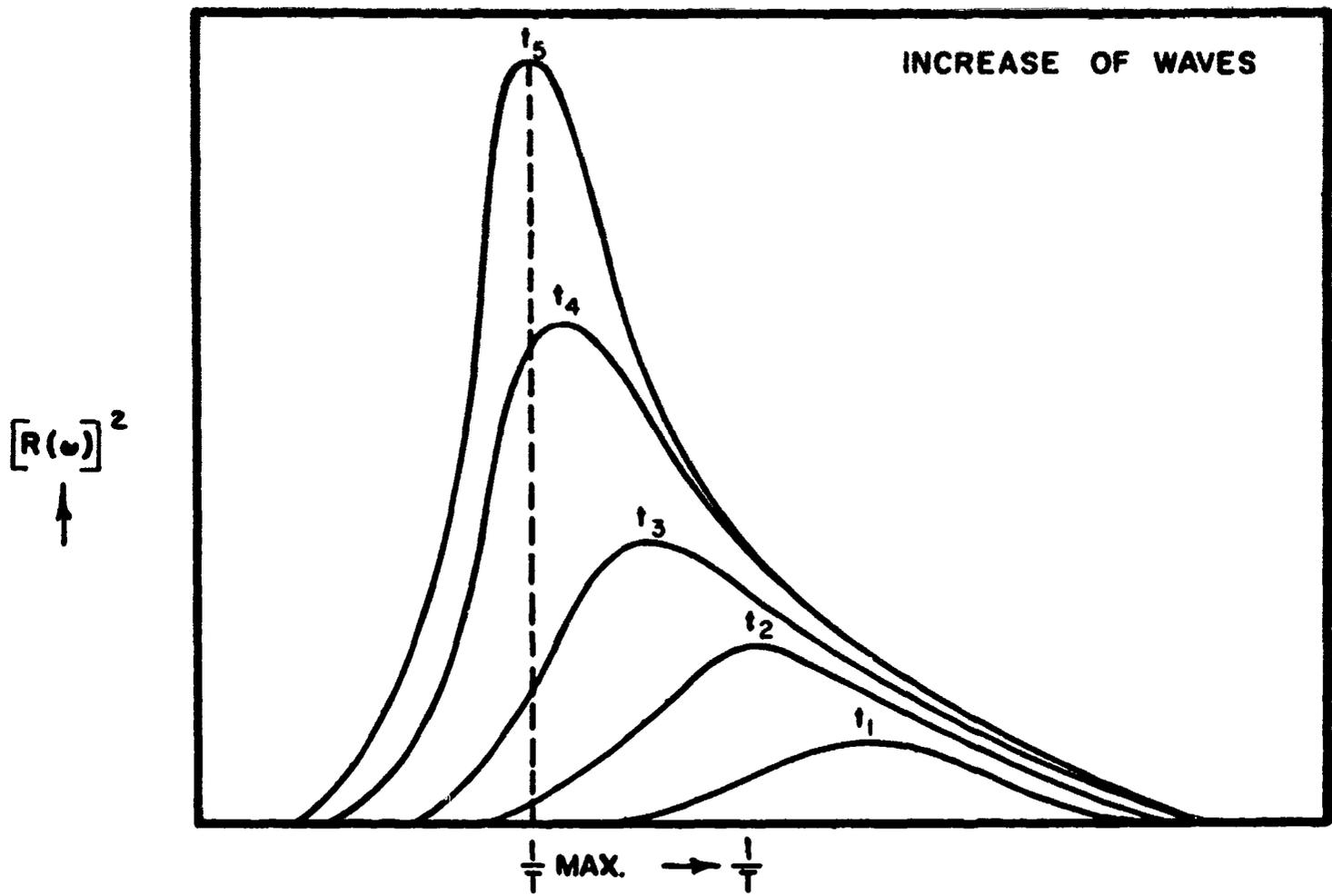
FIGURE 34 DECAY OF SIGNIFICANT WAVES WITH DISTANCE

after which little change occurs, which is in agreement with the idea proposed by Munk (1964). The relationships of Figure 34, having been based on empirical data, do not indicate how the energy is dispersed, dissipated or transformed from one frequency to another.

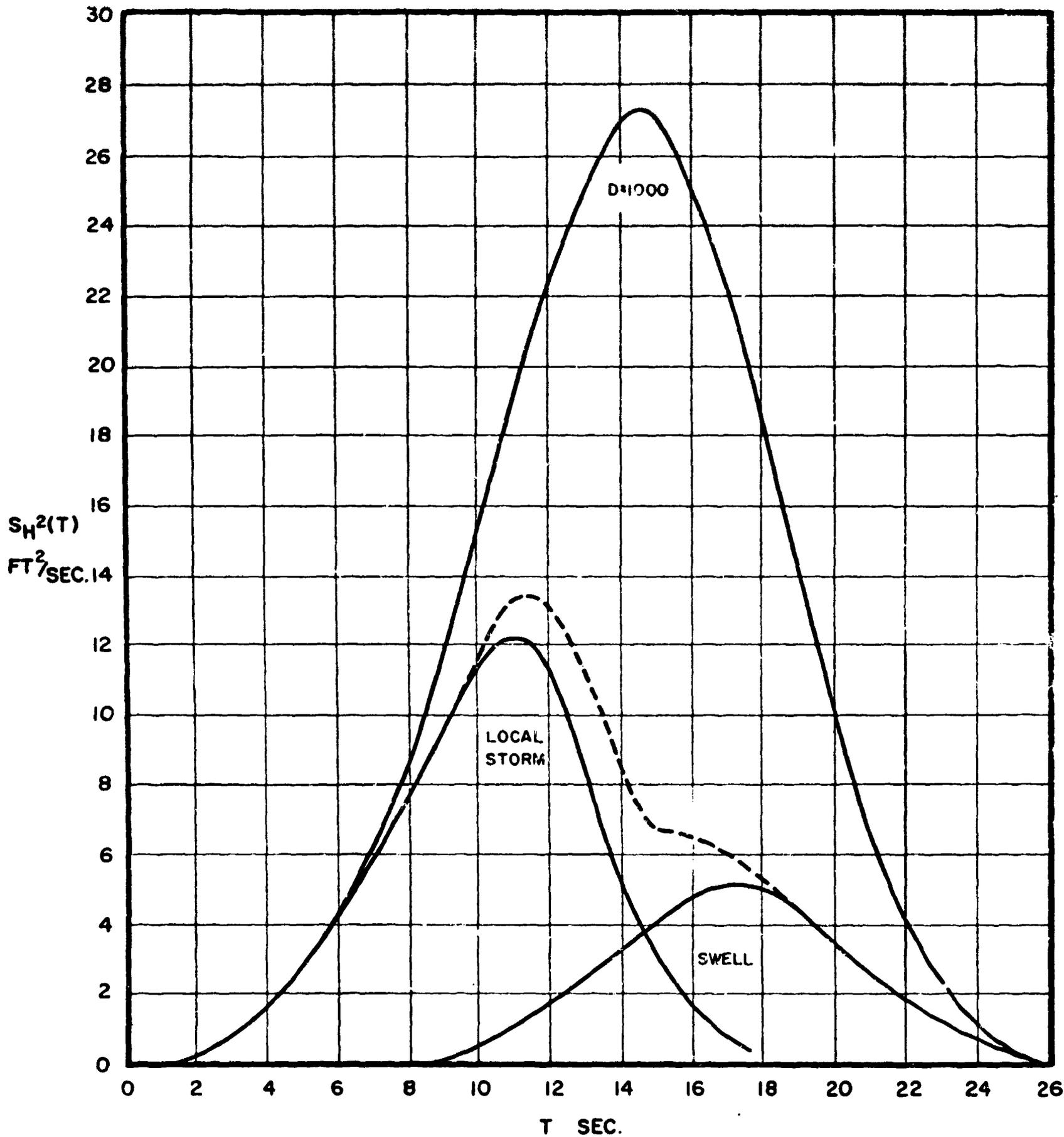
No relationships have been developed to show how the wave spectrum decays with respect to time at a particular location. The wave spectra data of Ijima (1957) show that the shift in modal period is opposite to that given in Figure 33, as should be expected. Figure 35 represents frequency spectrum for increase of waves and for decrease of waves, reproduced from Ijima (1957). The growth of wave energy is from high frequency (low period) and the decay of wave energy is from low frequency (long period).

The combination of wind waves and swell can lead to a multi-peaked spectrum. This is particularly true for hurricane conditions where the hurricane winds act at nearly a right angle to the direction of swell propagated ahead of the storm as the storm moves over the swell. A typical illustration of these conditions is shown for Hurricane Donna by Bretschneider (1961) in an appendix to the paper by Caldwell and Williams (1961) as published in Ocean Wave Spectra (1963). A demonstration of this phenomenon is shown illustratively in Figure 36 for the period spectrum and which, when transformed to the frequency spectrum, is shown in Figure 37, remembering that  $S(f) = T^2 S(T)$ . It might be noticed that a careful inspection of Figures 36 and 37 reveals the fact that for certain applications the period spectrum has certain advantages over the frequency spectrum. It appears that if the frequency spectrum were plotted on a log scale, there would be a clarification of the apparent difficulties.

Figure 32 is based entirely on meager empirical data. These relationships take into account the generating fetch length, and as a result give a family of relationships for wave decay, whereas the original work of Sverdrup and Munk (1947) gives a single relationship for wave decay. It should be mentioned that the work of Pierson, Neumann and James (1955), as outlined in Hydrographic Office Publication 603, offers the best scientific means for describing the decay of waves. Further research and collection of field data is necessary before the filter techniques proposed in H. O. publication 603 can be implemented for practical applications.



**FIGURE 35 TYPICAL CHANGE OF WAVE ENERGY SPECTRUM IN THE BUILD-UP AND DECAY OF WAVES**



**FIGURE 36**                      **EXAMPLE OF PERIOD**  
**SPECTRA OF COMBINED**  
**LOCAL STORM AND SWELL**

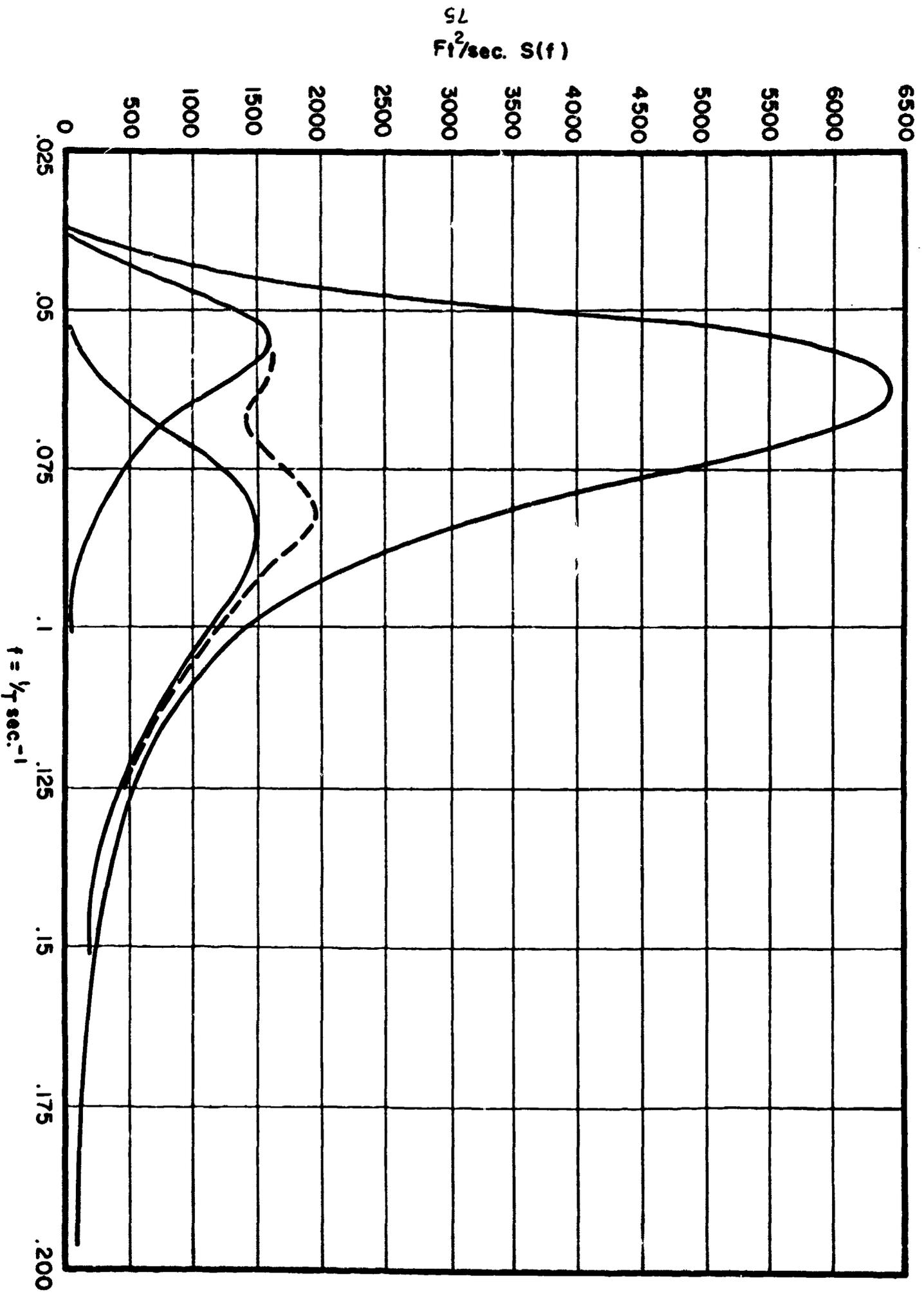


FIGURE 37 EXAMPLE OF FREQUENCY SPECTRUM OF COMBINED LOCAL STORM AND SWELL

## VI. WAVE STATISTICS

Wave statistics are defined in terms of probability of occurrence or recurrence intervals, for example the average number of years required for a particular value of the significant wave height to be equalled or exceeded. This definition does not necessarily have the same statistical meaning as wave variability. Wave variability is reserved for wave height or period distribution for a particular continuous wave record.

In this section probability is defined as the percent of time a particular event is expected to occur. The cumulative probability is the percent of time a particular event is equalled or exceeded. The recurrence interval is the time required for a particular event to recur. In order to determine cumulative probability and recurrence intervals of wave heights from severe storms which might be prognosticated from climatological events, several methods of approach might be utilized. If data were measured over a sufficient length of time, this would be useful. Otherwise, wave hindcasts from past meteorological weather maps can be made for presentation of the data. The method of Beard (1952) has been used successfully for small samples -- 20 to 40 years of records -- by hydrologists and hydraulic engineers to predict recurrence intervals for peak floods in engineering studies of watersheds, reservoir capacity and dam construction.

The equations from Beard (1952) for determining probabilities and recurrence intervals are as follows:

$$P = 100 \frac{s - 1/2}{S} \quad (86)$$

and

$$I = \frac{100Y}{SP} \quad (87)$$

where  $S$  = total number of occurrences on record.

$s$  = the summation of occurrences, beginning with the lowest value to any successive higher value until  $s = S$ .

$P$  = the cumulative probability that an event is equal to or less than a particular value.

$Y$  = the number of years of record.

$I$  = the recurrence interval in years corresponding to the probability  $P$  and the number of years of record  $Y$ .

Generally, the probability  $P$  versus the magnitude of the event is plotted on probability paper and a smooth curve is constructed. From the smooth curve values of the event and the corresponding probability are determined, and the recurrence interval  $I$  is then determined from Eq. (87).

Instead of using the graphical plot on probability paper, one could use to a better degree of accuracy the Weibull distribution function, Eq. (33):

$$\left\{ \begin{array}{l} \ln \ln \left( \frac{1}{1-P} \right) \\ Y \end{array} = \begin{array}{l} m \ln \left[ \Gamma \left( 1 + \frac{1}{m} \right) \right] \\ A \end{array} + \begin{array}{l} m \ln \left( \frac{X}{\bar{X}} \right) \\ MX \end{array} \right\} \quad (88)$$

If the data follow a nearly linear relationship according to the above equations, then a graphical determination can be made. If required, a statistical least squares technique can be used to obtain the best fit parameters  $A$  and  $M$ . Once the proper analytical equation has been determined for the probability  $P$ , then the recurrence interval  $I$  can be determined as before.

Jasper (1956) presented statistical distributions of significant wave heights based on about 5-1/2 or 6 years of Weather Bureau data for certain stations of the North Atlantic. These data resulted in a linear relationship on log normal probability paper. Figure 38 is reproduced from Jasper (1956).

More recent data on wave statistics have been presented in tabular form by M. Darbyshire (1963). These data are in terms of the maximum wave measured and the corresponding wave period.

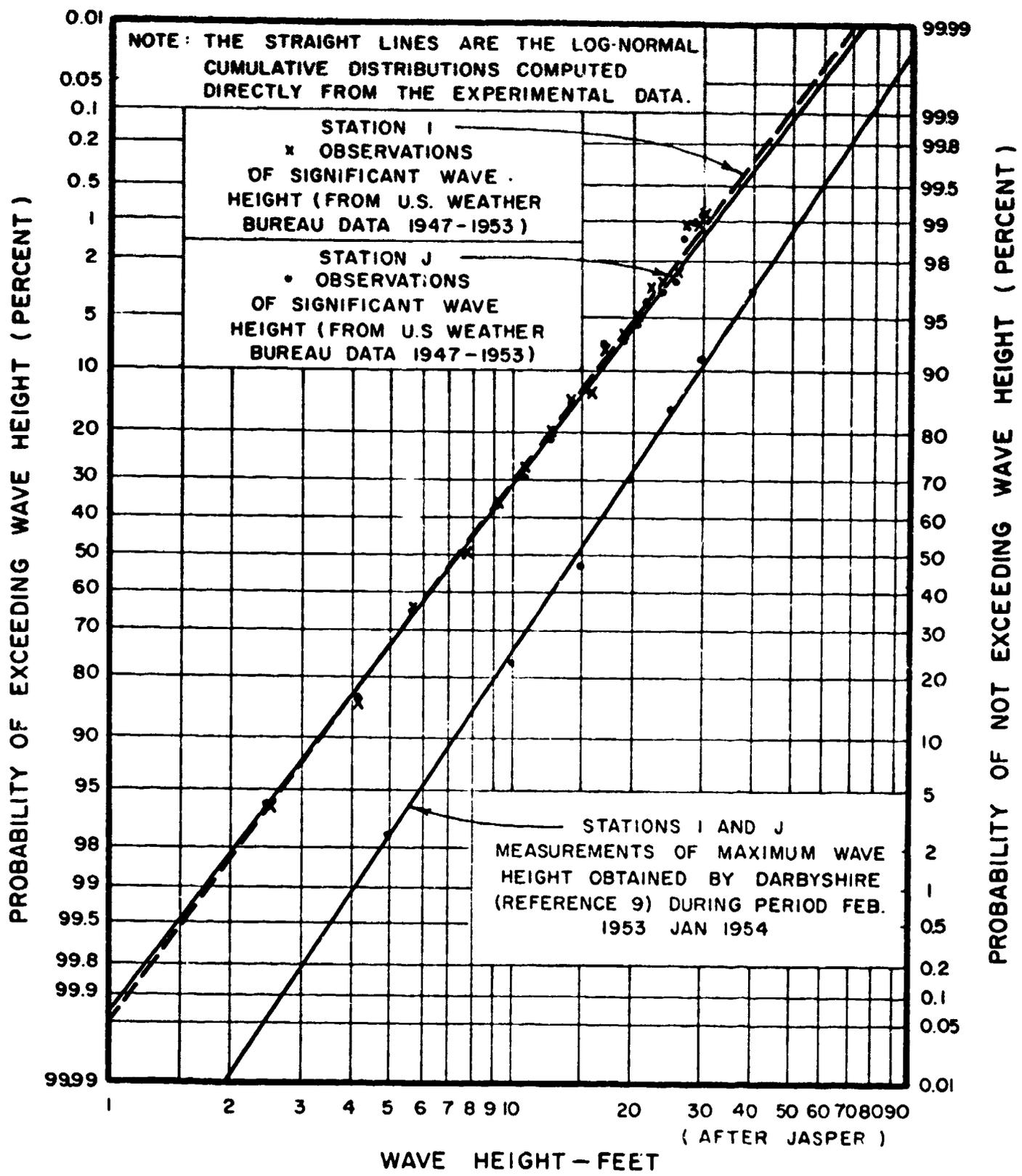


FIGURE 38 COMPARISON OF WAVE HEIGHT DISTRIBUTIONS DERIVED FROM VISUAL OBSERVATIONS AND FROM MEASUREMENTS OF WAVE HEIGHTS AT ATLANTIC OCEAN STATIONS I AND J

## VII. WIND SPEED VERSUS WIND SPEED

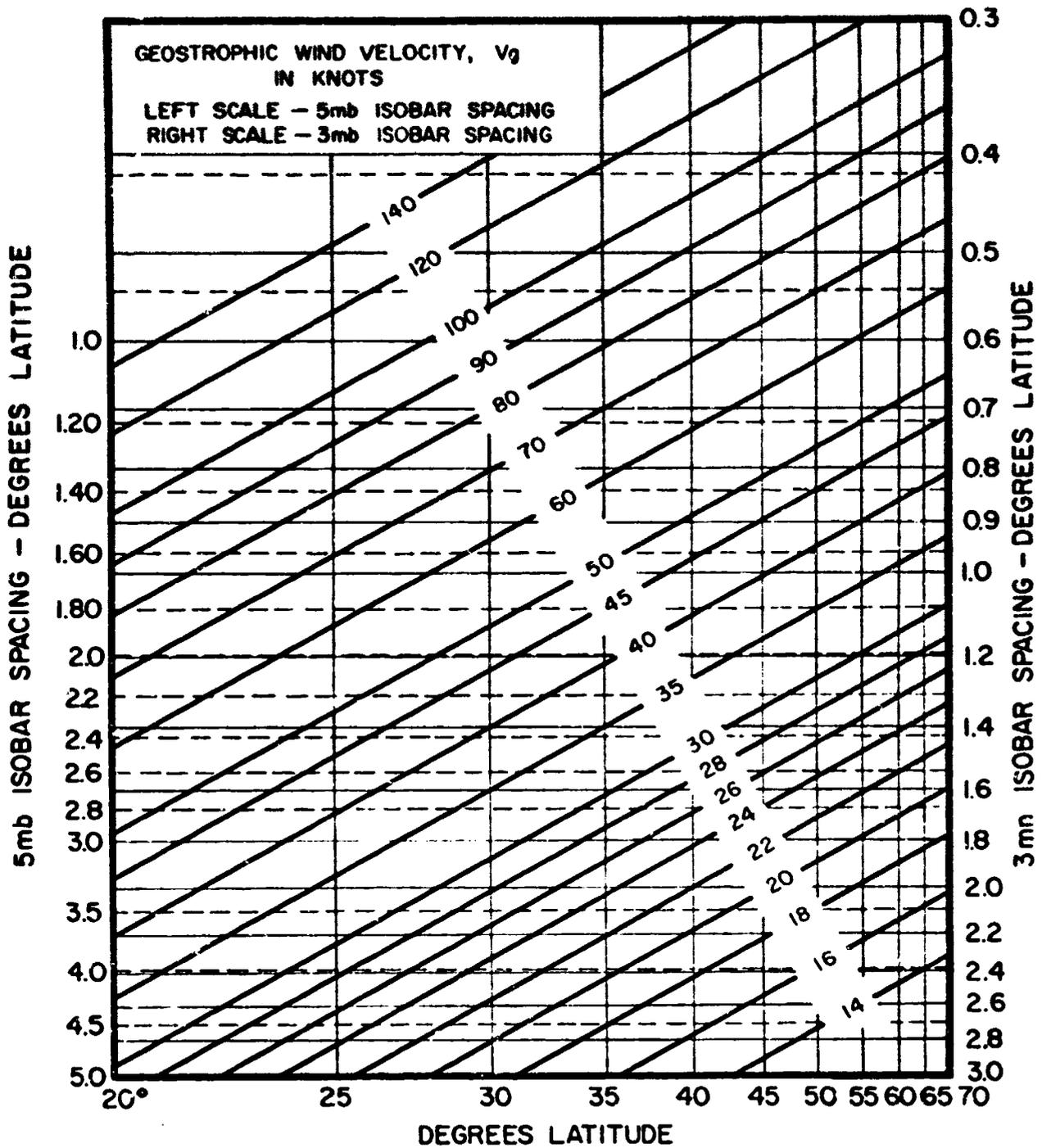
This section on wind speed versus wind speed is indeed a subject about which much discussion can be promoted. Although there are controversies about use of the proper wind speed in wave forecasting, there can be no disagreement that much research is required to standardize wind speed as related to wave generation. It appears that each author of a particular wave forecasting technique has his own definition of wind speed, whereas the user of the wave forecasting techniques sometimes uses a mixed definition. It is no wonder that the various methods have been criticized by the users.

Pierson (1963) for the first time gave a reminder that the wind data obtained by Neumann (1948 and 1952) was actually measured at 7.5 meters above the sea, aboard a ship, which itself was bobbing in the sea. A hand anemometer extended on a pole away from the ship was used to record the wind speed, and, at the same time, the waves were observed. The type of average wind speed determined is difficult to know. Perhaps some of the scatter of the data given in Figure 18, reproduced from Neumann (1952), is due to conditions under which the measurements were made. However, the averages can still be of importance.

An attempt by Pierson (1963) was made to interpret wave spectra in terms of the wind profile instead of wind measured at a constant elevation, and Pierson and Moskowitz (1963) proposed a wave spectra based on wind data measured at a constant height of 19.5 meters above mean sea level. This was done to avoid using drag coefficients by other authors for reducing the wind speed to some other standard elevation. Pierson (1963) also attempted to reconcile the differences between the various spectra on the basis of the elevation at which winds were measured, and he advocated longer averages for wind speed in order to obtain better correlation with the averaged wave data.

Darbyshire (1952, 1955) related wave data with the geostrophic wind speed calculated from the surface pressure gradient. Since the isobars were smoothed through the recorded pressure data, the geostrophic wind represented some type of a longer time average.

Bretschneider (1951, 1959) calculated a surface wind speed from the geostrophic wind speed, taking into account curvature and air-sea temperature differences. The corrections due to curvature and air-sea temperature differences were based on the original empirical Scripps data given by Arthur (1947). These data were correlated with the observed or recorded one-minute average wind speeds. Figure 39, based on the geostrophic wind duration, and figure 40, based on the Scripps data of Arthur (1947), can be used to calculate a mean surface wind speed, presumably for an elevation of 10 meters above the mean

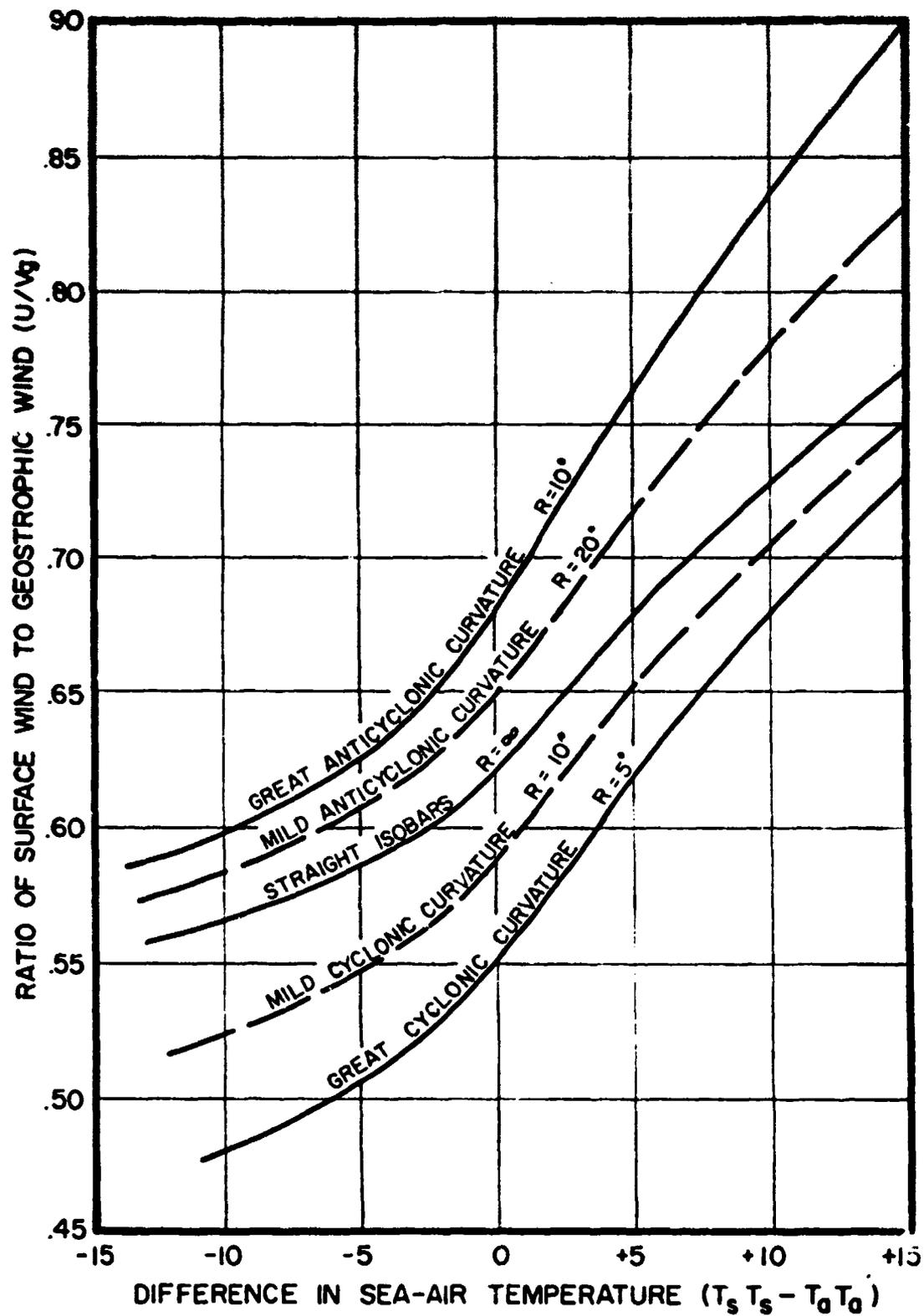


(BRETSCHNEIDER, 1952)

**FIGURE 39 GEOSTROPHIC WIND SCALE**

$$V_g = \frac{1}{2\Omega\rho\sin\phi} \quad \text{FOR } \Delta p = 5\text{mb \& } 3\text{mb}$$

$\Delta n = \text{DEGREES LATITUDE}$   
 $p = 1013.3\text{mb}$   
 $T = 10^\circ\text{C}$   
 $\rho = 1.2 \times 10^{-3}\text{gm/cm}^3$



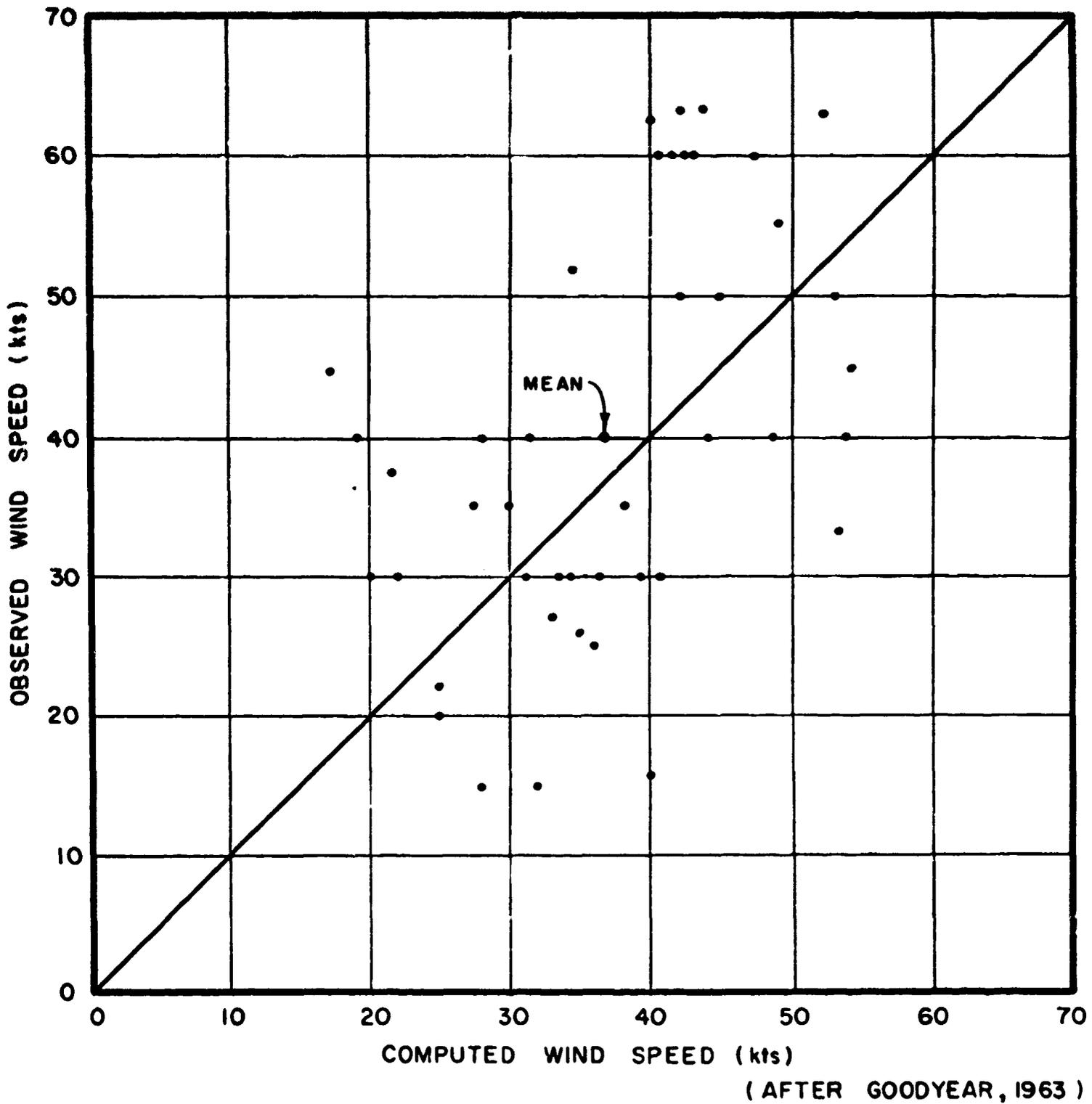
**FIGURE 40 SURFACE WIND SCALE**  
(BRETSCHNEIDER 1952)

sea surface. Sometimes the pressure gradients are not too well defined, particularly for lighter winds, but in any case the calculated wind speeds should agree with the measured wind speeds to within  $\pm 15\%$ .

The data of Goodyear (1963) show a considerable amount of scatter between the calculated wind speed and the observed wind speed, and states that the "calculated wind speeds represent a 10 to 15 minute average" whereas the measured wind speeds are at the most several minutes duration. Figure 41, reproduced from Goodyear (1963), shows the deviations between observed one- or two-minute average wind speed and the calculated 10 to 15 minute average wind speed from pressure gradients. It is no wonder that Wilson (1963) could not calculate wind speeds from the pressure gradients and obtain a correlation with the reported wind speeds. Instead Wilson (1963) uses the reported wind speeds to calculate isolines of constant wind speed, but this must also entail an averaging process, an average with respect to distance. Further, when using the space-time wind field concept there is still another average: a time average from one weather map to the next. Some of the ship reports of winds are winds measured at elevations different from 19.5 meters above the sea. One can hardly help wonder what wind speed is being talked about.

Certainly there are directional spectra of winds (which vary with elevation), just as there are directional spectra of waves. Horizontal variation in wind speed and direction must be considered of great importance in wave generation, particularly when one discusses effective fetches and durations as well as fully-developed seas. Until more care is taken in reporting winds and defining wind variability, one cannot expect any novel advances in wave forecasting. Even then it will be required to correlate new wave spectra data with new wind speed determinations.

Presently the forecaster must use the existing tools for calculating waves. He must use the type of wind analysis for making the forecast which is compatible to the methods used to develop the forecasting relationships and procedures. A change in the procedures on which the tools are originally based should also require a change in the tools, or else the accuracy of the forecasts will become worse instead of better.



**FIGURE 41    COMPUTED vs. OBSERVED  
 SURFACE WIND SPEED FOR 43  
 RANDOMLY SELECTED POINTS**

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