ESD-TR-64-677

AD610121

CORTEX: A COMPUTER-BASED SYSTEM FOR AIDING DECISION MAKING

Emir H. Shuford

DECEMBER 1964

DECISION SCIENCES LABORATORY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 4690, Task 469003

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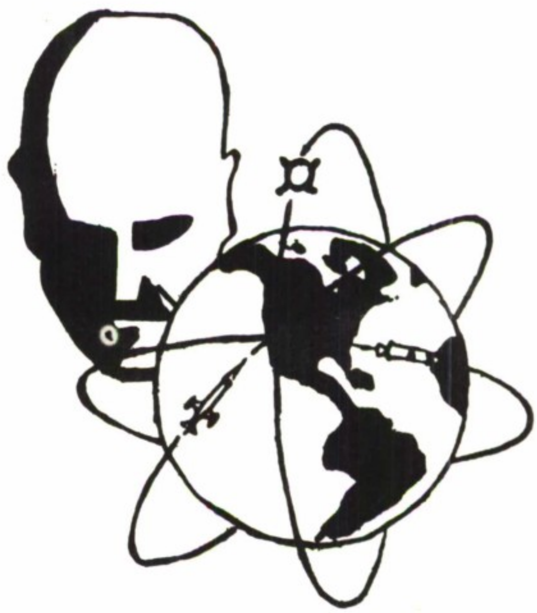

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FOREWORD

This paper was presented at the Second Congress on the Information System Sciences; it appears in INFORMATION SYSTEM SCIENCES: PROCEEDINGS OF THE SECOND CONGRESS, published by Spartan Books, Incorporated (Washington, D.C.), 1965.

ABSTRACT

Decision theory is the contemporary manifestation of the mathematics of the decision process and thus can be viewed as a primary aid to the human decision process. The costs and gains of applying the concepts and algorithms of decision theory are considered in some detail. A man/computer system is described which is designed to make the concepts and algorithms of decision theory available to a decision maker at a greatly reduced personal cost. This is achieved, in large part, by significantly reducing the special knowledge required of the decision maker. Thus, the decision maker needs no knowledge of computer programming and a minimal knowledge of decision theory and mathematics in order to begin using the system in his day-to-day decision making activities.

PUBLICATION REVIEW AND APPROVAL

This Technical Documentary Report has been reviewed and is approved.

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Acting Chief, Decision Techniques Division
Decision Sciences Laboratory

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CORTEX: A Computer-Based System for Aiding Decision Making

Emir H. Shuford, Jr.

In a broadly conceived, and somewhat trivial, sense most applications of computers can be interpreted as attempts to improve the quality of decisions. Information systems for the military as well as information retrieval systems for the scientist and engineer can be viewed as attempts to improve the quality of decisions by providing an increased amount of timely information to the decision maker. Computer-controlled production processes and accounting procedures can be seen as attempts to improve the effectiveness of decisions by providing for the reliable and timely realization of plans previously selected by a decision maker. Thus, computers are used as aids both in the information gathering stage and in the operational stage of decision making. In addition, computers are used as aids to the decision process itself and it is in this sense that I want to discuss aiding decision making.

An individual, e.g., a military commander or staff officer, a business manager, a design engineer, or a scientist, has a problem to solve or a plan to prepare. How can he be helped in the actual performance of the decision process itself? Are there any mathematics available that might be used to give logical consistency to his decisions and to aid his understanding of the nature of the decision problem? What role might a computer be expected to perform in this process? These are the questions that I would like to consider in some detail.

The Mathematics of the Decision Process.

Again in a broadly conceived, but not trivial, sense the use of applied mathematics can be viewed as an aid to decision making. By using mathematics the decision maker can obtain a very special type of information which comes from applying a logically consistent interpretation to some part of his problem. Though this logical consistency is considered desirable because it seems to lead to a certain generality and workability of the solutions, this is no place to digress into a discussion of the possibly subtle reasons why this is so.

It will be useful, however, to consider in some detail the historical development of one area of applied mathematics, statistics. The earliest stage of development of statistics was concerned primarily with use of numerical indices to summarize data. Decisions were then made on the basis of these indices. Statisticians soon began using mathematics in the form of probability theory to provide theoretical models for the sampling and data-generating processes frequenty encountered in practice. These models proved to be particularly useful as guides in the design of experiments and in the selection of indices. Further, knowledge of the underlying theoretical model resulted in the index being more informative to the decision maker. This development of theoretical models for data-generating processes continues to this day and, of course, is a very vital and useful part of statistics.

Another, quite important, line of development in statistics, that of relating the statistical procedure more and more closely to the decision problem of the user of statistics, received great impetus from the work of Neyman and Pearson and somewhat later from the work of Wald. The trend continued until we now have contemporary decision theory[1] as the most complete and unified mathematical treatment of the decision process yet achieved. It is a significant extension of the earlier statistical theories in that by explicitly incorporating the decision maker's judgment and experience (in the form of probabilities and utilities) into the formulation of the decision problem it is able to deal not only with decisions made on the basis of sampling data but also with decisions which must be made in the absence of data obtained from sampling and, of course, with intermediate decisions as to whether or not to obtain additional information before making a terminal decision. In this sense, therefore, decision theory can be seen to provide the applied mathematics of the decision process.*

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* Some traditionally trained statisticians object to this decision-theoretic extension of classical statistical theory because, in their view, it makes their methods of analysis seem less objective. Thus, there is controversy among statisticians as to whether to use a decision-theoretic or a "classical" procedure in a particular application. Since we are considering the use of mathematics to aid the human decision process, it is important to evaluate the implications of this controversy about statistical practice.

First, it should be appreciated that in those cases in which a decision must be made without the assistance of data obtained by sampling there is no conflict. Classical statistical theory does not apply. Second, in those cases in which sampling data is available there is <u>no logical inconsistency</u> between procedures recommended by decision theory and by classical statistical theory. The procedures can be viewed as differing only with respect to the type of intuitive judgment required of the decision maker, e.g., the specification of probabilities and utilities for the decision-theoretic analysis versus the specification of error rates or confidence coefficients for the Neyman-Pearson analysis.[2] Thus, if we agree that a decision maker should be allowed to express those intuitive judgments that he finds most natural and to deal with his problem in those terms, there remains no basic conflict between the statistical theories.

Discrepancy between Potential and Actual Use of Decision Theory

Thus, decision theory is a natural development of the growth of statistical theory but more importantly it represents nothing more, nor less, than the extension of logic and mathematics into the domain of choice and decision. It deals with the choice of the proper course of action to achieve a certain end result. The scope of application of decision theory is constantly expanding through the definition and solution of new classes of decision problems. The problems that we routinely approach today were considered immensely complex just a few years ago. New formulations and accretions to decision theory will undoubtedly aid in the continued expansion of the domain of application of this area of applied mathematics.

In summary, the conceptual tools of decision theory are undergoing continual improvement and it is difficult to perceive any insurmountable obstacle to the continuing increase in the scope of potential application of the mathematics. Notice that I used "potential" to qualify application. Here I refer to the obvious, but sometimes forgotten, fact that the mere existence of a certain tool does not guarantee its use in practice.

The Annals of Mathematical Statistics represents a vast pool of refined techniques for the statistical analysis of data. To one with any appreciation of the body of potential techniques represented by the Annals, an examination of the statistical techniques actually used in practice by engineers, scientists, and even statisticians must be a rather discouraging experience. Each of us probably has his own favorite estimate of the lag existing between the development of a new statistical technique and its general utilization by practitioners. My favorite is three decades, but I won't quibble. One decade or ten decades, the lag seems excessive.

So, in contrast to a rather encouraging view of the future development of the mathematical theory of optimal decision making, we have a very discouraging picture of the extent of future applications of this theory. If conditions remain as in the past, decision theory will not be widely applied in this generation or by this generation. As a new generation of managers, engineers, and scientists is trained in the universities to use this new theory we will see more application, but even granted this training, the use of decision theory will be restricted to the more complicated, the more important problems. And this is as it should be, unless--.

The Cost and Gains of Applying Decision Theory

The concepts and methods of decision theory can, of course, be applied to the decision of whether or not to use the mathematics of decision theory as an aid in making a particular decision. Or, possibly, the choice is between a simpler or a more complete formulation and analysis of the particular decision problem. In either case, one can formulate the choice in terms of two marginal costs, the marginal cost of foregoing the decision-theoretic analysis and the marginal cost involved in carrying out the decision-theoretic analysis. In both cases, especially the first, there is always a loss involved in foregoing a decision-theoretic analysis. Given the formulation of the particular decision problem, the decision-theoretic analysis must recommend the course of action yielding the largest possible expected payoff whereas an intuitive or a less complete analysis will result either in the choice of this optimal course of action or in one which yields a smaller expected payoff. In this latter case, a non-zero cost is incurred in foregoing the decision-theoretic analysis. In other words, one cannot do better, by definition, than to choose the course of action recommended by the decision-theoretic analysis.*

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* Of course this is conditional upon one accepting the particular formulation and values of the decision problem under analysis.

Though it is known that typically there will be a loss incurred by foregoing a decision-theoretic analysis, the actual magnitude of this loss is unknown unless the decision-theoretic analysis is performed and the results compared with that yielded by an intuitively chosen course of action. Thus, a reasonable strategy to adopt in this kind of situation is to try to minimize the other cost, the marginal cost involved in carrying out the decision-theoretic analysis. Ideally, this cost should be reduced to as near zero as possible. Then the best choice would be to conduct the decision-theoretic analysis in all cases in which the cost of application were zero or insignificant.

In considering the reduction of this marginal cost of application it is useful to analyze the components that make up the total cost of carrying out a full-fledged decision-theoretic analysis of a decision problem. It will be convenient to divide this cost into two components; personal cost and non-personal cost, where personal cost represents the marginal cost to the decision maker while non-personal cost represents the marginal cost to the organization, institution, or society.

As to the personal marginal cost to the decision maker, consider what he must be able to do in order to carry out a decision-theoretic analysis. He must learn the concepts and techniques of decision theory and related mathematics. He must learn how to utilize the devices and techniques he has available for carrying out the decision-theoretic analysis. He must spend time and effort formulating the particular decision problem, assessing the probabilities and utilities, and applying the various computations necessitated by the theory. Obviously, the application of decision theory to a specific decision probelm is typically a very costly process for the decision maker.

The non-personal marginal cost to the organization, institution, or society is in many respects related to the personal costs described above. For example, the several years of higher education that may be required for mastery of decision theory may be considered a cost to society. The time required for a manager, say, to apply decision theory to one of his management problems may be seen as a cost to his organization.

Some non-personal costs have the characteristic that they can be used to reduce other costs of application, both personal and non-personal. For example, the institution may incur the cost of employing a consulting statistician in order to reduce the training requirements on other employees. The installation of a computer facility along with a programming staff serves as another example.

The use of a computer can result in fantastic reduction in the cost of computation though the costs of programming have tended to restrict its use to very extensive and complex computations in scientific applications or to highly repetitive routine operations in business applications. In the case of very extensive computations, the advantage of using a computer appears not in a reduction in the cost of an on-going operation but in allowing for more complex and thorough analyses which would never have been performed (both for economic and other reasons) without the aid of the computer. In the case of highly repetitive, routine operations, the advantage of using a computer lies in its low cost of computation plus the fact that the intial programming investment can be amortized over many repeated applications.

Reducing Costs of Application Through the Use of Computers

Now computers have certainly been used to perform decision-theoretic computations. However, due to the great cost of programming, these applications have been restricted to the analysis of rather complex and important decision problems. Furthermore, because of the apparent rarity of occurrence of these complex decision problems there seems to be no possibility of amortizing the cost of programming over many different applications. Thus, the cost of a decision-theoretic analysis remains quite high.

Suppose that we consider increasing the investment in programming in order to increase the scope of application of the programmed techniques. It may be possible to so increase the number of situations in which the techniques can be used that the programming and hardware costs per application become quite negligible.

Two ingredients seem to be necessary to the success of such an attempt. First, the programmed techniques need to be of such generality that they may be sensibly applied in a wide variety of situations. Second, the programmed techniques must be such that they result in a significant reduction in the personal cost of application.

Some of the potentialities inherent in such an approach are illustrated by STATPAC, a scope directed on-line system for data analysis, designed by John B. Goodenough of the Decision Sciences Laboratory.[3] The original data from, say, a psychological experiment is entered into the computer by means of punched paper tape. The psychologist using scope and light pen can then perform a wide variety of mathematical operations on his data. Results of interest are either typed or punched out at the option of the psychologist.

One significant achievement of this system is that the only special knowledge required of the user is that he know how to get his data punched on paper tape in the proper format, how to start the computer and read in paper tape, and how to use the light pen. In particular, the user does not need to know anything at all about computer programming nor does he need to learn the vocabulary and grammar of a "problem-oriented" language. The user does need to know something about elementary statistical analysis of data though even this requirement seems to be minimal since a user, given that he knows enough to suspect that using STATPAC may help him, may "learn by doing" in using the system.* However, this type of learning is clearly limited to the more elementary operations. The operations involved and the interpretation and meaning of a derived inferential value like Student's <u>t</u> cannot be learned in this manner. In summary, STATPAC makes the power of the computer available for data analysis under the control of a non-programming scientist.

* STATPAC has complete program protection from all user errors along with diagnostics for many illegal operations. Thus, a user may attempt to perform any operation in any order and, if successfully completed, examine the result on the scope display.

It is important to realize that STATPAC is not suited to the complex and extensive data analyses of the type usually programmed for computer solution.* Its domain of application lies among those computations that would commonly be performed by hand or with the aid of a desk calculator. The use of a computer has generally not been competitive with manual computation unless the problem has required hundreds or thousands of hours of human computations. Thus, another significant achievement of STATPAC is that, by eliminating marginal costs of programming and by reducing to a minimum the requirement for special knowledge on the part of the potential user, the user of the computer offers advantages even in the case of data analysis problems requiring only a few hours of hand computation.

* It appears that even some of the more extensive data analysis techniques might benefit from programming to reduce the cost of application. For example, scope directed programs for analysis of variance and for information-theoretic analysis of data are being developed by Charles R. Brown at the Decision Sciences Laboratory.

The net effect of STATPAC has been to bring in a new group of users of our computer, i.e., those staff members who have not mastered computer programming and operation. Furthermore, many experiments are more thoroughly analyzed and much more quickly analyzed than ever before.

A decision-theoretic analysis of a problem has much in common with the statistical analysis of data. They both employ mathematical procedures of varying degrees of sophistication. Both are potentially applicable to a wide variety of situations. Thus, considering the success of STATPAC it appears reasonable to apply similar principles to the development of a system for the on-line analysis of decision problems.

CORTEX, Computer-based Optimization Routines and Techniques for Effective X (= command, management, engineering design, research,...), is just such a system under development and study at the Decision Sciences Laboratory.

CORTEX in Operation

What follows is a brief description of how CORTEX as presently conceived operates. The description is brief both because it is best to avoid getting lost in the details and because many details are subject to change as we attempt to make CORTEX easier to use. However, some of the operations, particularly the assessment of probabilities, are discussed in some detail in later sections.

A potential user, hereafter called the decision maker, sits down before a remote computer-driven scope and light pen station. All communication between the computer and the decision maker occurs through the light pen and scope except for certain final results which are needed in hard-copy form. The decision maker requests CORTEX by throwing a switch on the console. If the computer is not busy, CORTEX is now available and the scope and light pen are active.

I. Formulation of the basic decision problem.

A. CORTEX asks the decision maker to specify the courses of action he wishes to consider. If there is a small number of courses of action, the decision maker may identify them by "typing" them on the face of the scope with the light pen. If there is a large number of potential courses of action, e.g., when the problem is to choose a certain amount to purchase, stock, employ, or deploy, the decision maker may identify them by specifying the range and step size. It should be noted that the selection of these courses of action is tentative since the decision maker at a later stage may delete some and/or add others and reformulate the decision problem where necessary.

B. CORTEX asks the decision maker to identify those states of the world which determine the desirability or undersirability of the courses of action. CORTEX queries the decision maker to insure that the states are logically mutually exclusive and exhaustive. The identification of the states of the world is again tentative and may be changed by the decision maker at a later stage.

C. CORTEX provides two classes of methods to aid the decision maker in assessing the probabilities of the states of the world. If the decision maker chooses to use both methods the results are checked for consistency and if the decision maker is concerned at the lack of agreement between the two sets of probabilities he may choose either to resolve the inconsistency in some manner, to retain the two sets for a sensitivity analysis at a later stage, or to repartition the states of the world so that he may more easily assess the probabilities.

D. The consequences of each possible outcome, i.e., the joint occurrence of a course of action and a state of the world, are assessed numerically by the decision maker. CORTEX provides tests to determine whether monetary costs and gains are adequate or whether more general utilities are needed.* CORTEX tests automatically for dominated courses of action, i.e., a course of action which yields no more desirable consequences than another course of action for each of the possible states and yields a less desirable consequence for at least one state, and calls these to the attention of the decision maker for possible elimination from consideration or for re-evaluation. The decision maker may find that he has partitioned the states in a manner that makes it difficult for him to specify the consequences. In this case he may choose to collapse some of the states, in which case the new probability is automatically computed by CORTEX, or to partition further one or more states, in which case the additional probabilities are requested by CORTEX.

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* In those applications where the consequences can be expressed in monetary terms, simple reference contracts are used to determine whether or not the decision maker's utilities are linear with

respect to monetary gains and losses over the range necessitated by the particular decision problem.[4] If approximate linearity is found then the monetary values can serve as the numerical values of the consequences. If, however, significant departures from linearity are indicated then an approximate utility function must be determined for the decision maker or the decision problem must be reformulated since such deviations may be due to the partial formulation of interdependent decision problems[4] or to the effect of induced utilities[5].

E. Once the decision maker is satisfied with the problem formulation developed above he can request CORTEX to compute and to list (or, in some problems, to graph) the expected utilities for each course of action. The optimal course of action yielding the largest expected utility is emphasized by additional intensification on the scope. In addition, the expected loss or cost of uncertainty for the optimal course of action is displayed. At the option of the decision maker, these results may be obtained in a hard-copy form for further reference.

F. If the decision maker feels any uncertainty about the decision problem or if he would like to understand better the nature of the decision problem, he may perform a sensitivity analysis by going back to earlier stages of CORTEX, changing the courses of action, the states of the world, the numerical values of the consequences, or the probabilities of the states, and then observing the resulting changes in the expected utilities and the optimal course of action. If the decision maker judges that the effect of these changes is slight, he need feel no concern about his uncertainty. If, on the other hand, the decision maker finds that the changes produce major effects in the solution of his decision problem, he might consider ways of obtaining additional information in order to reduce his uncertainty.

II. Evaluation of Potential Information about the States.

A. Uncertainty about the states of nature can, of course, be reduced by gathering additional information about the states, possibly but not necessarily through sampling. Therefore, at this stage CORTEX displays again the cost of uncertainty, reminds the decision maker that this value represents the largest possible gain that could result from obtaining additional information about the states, and then queries the decision maker about his desire to consider the value of additional information. If the cost of uncertainty appears large to the decision maker relative to what he thinks that additional information might cost, the decision maker may choose to consider the possibility of obtaining additional information on the states of the world.

B. CORTEX then queries the decision maker about the nature of the additional information that may be obtained. A major distinction is made between sampling experiments involving repeated, independent observations and other non-sampling types of information. In the case of sampling, the decision maker may assess the conditional probabilities of the data given each state of the world by using aids similar to those described in the section on the assessment of probabilities or he may choose to specify the type of data-generating process and the parameters of the process.* In the case of non-sampling information, there are no data-generating processes available so the decision maker must assess at least some of the conditional probabilities.

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* CORTEX contains the standard data-generating processes such as the binomial, multinomial, Pascal, Poisson, normal, and multivariate normal.

Once the conditional probabilities have been specified, the decision maker may request a consistency check based on hypothetical decisions in a simplified experiment as described in the section on the assessment of probabilities.

C. CORTEX interrogates the decision maker as to the estimated cost of the potential information. In the case of sampling, the decision maker specifies the marginal, or if he prefers the cumulative, cost of sampling and the maximum feasible sample size. The cost of sampling may be a fixed amount per observation, a fixed amount per group of observations, or in general any function of the cumulative number of observations. CORTEX provides a graphical response mode for the more complex functions so that the decision maker can draw the function on the face of the computer-driven scope.

D. Now that all of the necessary information is available, CORTEX computes and displays all of the relevant expected utilities. In the case of sampling, for example, CORTEX displays a graph showing the expected utility of the optimal strategy for each fixed sample size from one up to and including the largest feasible sample size. The largest expected utility, corresponding to the optimal sample size for fixed sampling, is emphasized by additional intensification and the location of a pointer. The expected utility associated with optimal sequential sampling is displayed as a horizontal dashed line which may or may not intersect the expected utility function for fixed sampling. If it does then fixed sampling is to be preferred; otherwise sequential sampling is better.* In addition to these expected utilities for sampling experiments, CORTEX displays along the vertical axis the expected utilities for the best and worst courses of action under the condition of no sampling information. By this means the decision maker is provided with information sufficient to determine whether or not it is worthwhile to obtain additional information about the states of the world through use of a sampling experiment. Additionally, the decision maker is provided with information about the expected performance of fixed and sequential sampling procedures as a basis for choosing the best experiment.

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* Sequential sampling is no worse and frequently better than fixed sampling when cost of sampling is proportional to sample size. However, in the case of more complicated cost structures, fixed sampling may be superior to sequential sampling.

E. If the decision maker feels any uncertainty about his specification of the conditional probabilities of the data given the states of the world or of the marginal costs of sampling and the maximum sample size, he may as before request a hard-copy summary of the results of the current analysis and then return to an earlier stage of CORTEX to perform a sensitivity analysis. In an analogous manner, the decision maker may return to an earlier stage and insert new probabilities and costs of sampling in order to evaluate an alternative experimental design in the expectation of finding a procedure yielding better performance.

F. Once the decision maker is satisfied with the analysis of the decision problem he may request a hard-copy record of the results of the analysis including a complete statement of the optimal decision rules and expected utilities conditional upon the observed data. If sequential sampling is indicated, the decision maker may request CORTEX to compute, display, and record the probability distribution of optimal sample sizes and the expected optimal sample size. This information can be used to guide the conduct of the sampling experiment that is under consideration.

The Assessment of Probabilities

The assessment of probabilities is certainly one of the more critical and less developed aspects of decision making. It is through probabilities that much of the decision maker's judgement and experience is given numerical expression and brought to bear upon the mathematical solution of his decision problem. Thus, it may prove worthwhile to consider in some detail a number of the methods that are available for assisting the decision maker in expressing his probabilities.

First, take the case of a discrete probability distribution. Let there be a set of mutually exclusive and collectively exhaustive events each of which corresponds to a different state of the world, say, or to one of the possible outcomes of a sampling study. At the given moment of assessment, the decision maker may not know the actual state of the world but, since the set of events is mutually exclusive and collectively exhaustive, one and only one state of the world must exist. In order to utilize CORTEX, or in fact any method based on the theory of probability, the decision maker must assign a probability value to each event.

One method used in CORTEX is based on having the decision maker express his preference between two simple hypothetical experiments, one having outcomes dependent upon the *original* set of events, the other having outcomes dependent upon a *reference* set of events. Rather than have the decision maker deal simultaneously with all of the events in the original set we have the decision maker deal with the events two at a time by contrasting the occurrence of a particular event with the non-occurrence of the particular event. Thus, CORTEX begins by describing two hypothetical experiments for the decision maker's consideration. The first experiment yields $100 if a specified <u>original</u> event occurs and $0 if it does not occur. The second experiment yields $100 if a specified <u>reference</u> event occurs and $0 if it does not occur.

The reference event used by CORTEX is based upon a well-understood data-generating process and is explained to the decision maker. A line is displayed on the scope. The line is divided into left-hand and right-hand segments by a movable index. One of the points on the line will be chosen at random, i.e., each point is as likely to be selected as any other point. The decision maker may observe this process operating at the rate of several selections per second for any length of time he desires. The reference event occurs when the random point falls to the left of the movable index. Thus, the second experiment yields $100 if the random point is located on the line segment to the left of the index and $0 if located on the line segment to the right of the index. It should be noted that the probability of the reference event is proportional to the length of the line segment extending from the left end of the line over to the index.

Now, it should be clear that the expected value to the decision maker of the first experiment is proportional to the probability of the specified original event while the expected value of the second experiment is proportional to the probability of the specified reference event. The decision maker prefers, of course, the experiment possessing the largest expected value. Since the expected value of the second experiment depends upon the location of the index, there will exist at least one setting of the index at which the decision maker will not prefer the first experiment and there will exist at least one setting of the index at which he will not prefer the second experiment.* More importantly, there will exist a setting of the index or a range of settings at which the decision maker will be indifferent between the two experiments. Once this index setting representing practical indifference has been determined the ratio of the length of the line segment to the left of the index setting to the length of the total line is taken to be the probability of the original event.

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* This sentence is worded in this somewhat awkward manner in order to include the possibility that the probability of the original event is zero or one.

The decision maker can use one or both of two modes of operation of CORTEX to arrive at this index setting. In the first mode of operation the probability of the origninal event is determined by a method of successive approximation. CORTEX sets the index at the middle of the line corresponding to a probability of the reference event equal to one-half and queries the decision maker as to his preference between the two experiments. If the decision maker prefers the first experiment (indicating that the probability of the original event is greater than one-half), CORTEX sets the index at the three-fourths position. If, on the contrary, the decision maker prefers the second experiment (indicating that the probability of the original event is less than one-half), CORTEX sets the index at the one-fourth position. Now, the decision maker indicates his preference between the first experiment and the new second experiment based on the adjusted index setting. CORTEX continues this search procedure until the decision maker judges that his preferences are becoming sufficiently difficult to express and/or that the probability of the original event has been determined with sufficient accuracy and indicates his wish to discontinue the assessment.

In the second mode of operation the decision maker uses his light pen to move the index to any desired position along the line. The decision maker can utilize this mode of operation at any time, either before or after starting in the successive approximation mode. Once the decision maker feels that he has achieved a satisfactory setting of the index, he indicates his wish to discontinue the assessment. Whereas the first mode of operation is particularly useful for giving a decision maker unfamiliar with the assessment of probabilities a clear understanding of the nature of the process, the second mode of operation offers certain advantages in terms of speed and ease of operation to a decision maker experienced in the operation of this portion of CORTEX.

The decision maker and CORTEX then continue on in this manner to obtain a probability for each one of the original events. Immediately upon the completion of the last assessment, CORTEX checks to determine whether or not the probabilities sum to one, informs the decision maker of this condition, and if required offers a number of ways for the decision maker to adjust the probabilities so that they do in fact sum to one.

This first method of assessing a discrete probability distribution has many advantages in its favor, e.g., the choices required of the decision maker are of a particularly simple nature and are independent of the utility values that the decision maker may attach to the amounts of money involved.[6] However, the introduction of a reference set of events, external to the decision problem at hand, may sometimes lead to inconsistent and biased measurements. An extensive discussion of this problem may be found in the articles[7,8] dealing with what I have called[5] the Chipman-Ellsberg-Fellner paradox. In essence, the two sets of experiments may under certain conditions prove to be noncomparable in such a way that the decision maker will encounter great difficulty in getting the probabilities to sum to one. In this case, the decision maker might choose either to reformulate his decision problem or else to utilize the assessment procedure described below.

CORTEX, at present, makes available one other method for the assessment of discrete probability distributions. The list of events originally generated by the decision maker is displayed on the scope. Adjacent to the label of each event CORTEX displays a line segment, each initially set to be of the same length. The decision maker uses the light pen to adjust the lengths of these lines but CORTEX constrains the total length of the lines so that it always equals a fixed value. Adjacent to the label of each event CORTEX also displays a hypothetical payoff value which changes value automatically as the corresponding line length is changed by the decision maker. Each payoff value represents the hypothetical amount of money that the decision maker would earn if the corresponding event did in fact occur. Thus, the decision maker adjusts the lengths of the lines until he is satisfied that the corresponding set of payoff values is to his best advantage.

Now the critical thing to realize is that the payoff values have a very special relation to the lengths of the lines such that the decision maker obtains his maximum expected payoff if and only if the length of the line corresponding to each and every event is proportional to the probability of that event.* In other words, by accurately adjusting the lines so as to honestly reflect his probabilities, the decision maker maximizes the expected payoff from the hypothetical experiment. It should be mentioned that the payoffs do not have to remain hypothetical. They could, for example, represent an actual bonus or fee to be paid to the decision maker by his employer.**

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* This characteristic of the payoff function has been termed the matching property ty Toda.[9] Only three different payoff functions have been found which have this property. They are the spherical gain[9,10], the logarithmic loss[9,11], and the quadratic loss[9,12].

** Grayson in his study of the application of decision theory to drilling decisions by oil and gas operators[13] suggests the potential usefulness of similar bonus schemes to obtain unbiased probability estimates from consulting geologists. He finally concludes however that "...the only way a geologist (or any decision maker) will ever give estimates of probabilities that approximates his true beliefs will be through understanding and acceptance of the basic idea." I tend to agree.

It should be apparent that this second method for the assessment of discrete probability distributions differs from the first method in a number of important respects. A possibly major difference is that the second method is self-contained in the sense that it does not require the introduction of an external set of reference events as does the first method. The accuracy of the probabilities yielded by this second method does depend upon the values or utilities that the decision maker places on the amounts of money involved in the hypothetical payoffs whereas in the first method it does not. Another operational difference is that in the second method the decision maker deals concurrently with all of the probabilities under the automatic constraint that they sum to one.

I am not pointing out these differences to bemoan the lack of a perfect method. On the contrary, I expect that some decision makers will find one method more appropriate and natural in some situations while other decision makers will prefer the other method. Additionally, a decision maker might choose to use both of the methods to assess the probabilities. If there is considerable disagreement between the two sets, the decision maker may have cause for concern and may consider reformulating his decision problem or he may choose to use both sets of probabilities in a sensitivity analysis. Finally, experience with both methods of assessment should contribute to the decision maker's understanding of the nature of probability and uncertainty.

Consider now the assessment of a continuous probability distribution. This case arises when the events under consideration are quite numerous, these events represent different amounts or magnitudes, and it is mathematically convenient to deal with continuous variables. As before, CORTEX offers the decision maker two methods of assessment.

One of these methods is quite similar in spirit to the first method described for the assessment of discrete probability distributions. In fact, it employs the same reference experiment which is represented as before by a line divided into two segments by an index. If a point to the left of the index is selected the decision maker earns $100 while if a point to the right of the index is selected he earns $0. CORTEX initially sets the index at the position corresponding to a probability of one-fourth. However, in this method of assessment the decision maker has no control over the position of this index.

CORTEX displays a second line above the line representing the reference experiment. This new line represents the range of values that the continuous events may assume and is so labeled. There is also an index on this line which thus represents a hypothetical experiment based upon the probability distribution under assessment. If one of the events located to the left of this index actually occurs the decision maker earns $100 while if one of the events located to the right of this index occurs he earns $0. Therefore, the expected value of this experiment to the decision maker depends upon the probability of obtaining, in the actual decision problem, an amount or magnitude less than that specified by the index setting.

This probability is assessed, as before, by determining the index setting at which the decision maker is practically indifferent between the two hypothetical experiments. In this case, however, the index is varied not in the reference experiment but in the experiment based on the events in the decision problem under consideration. Again, CORTEX has two modes of operation available for this determination, a successive approximation mode and a continuous adjustment mode. Either may be used at will by the decision maker.

Once the decision maker has achieved a satisfactory setting of the index, he expresses his wish to discontinue the procedure and the value of the final setting is interpreted by CORTEX as the first quartile of the continuous probability distribution. CORTEX then places the index for the reference experiment at the position corresponding to a probability of three-fourths and the procedure is repeated once more to determine the third quartile of the continuous probability distribution.

The decision maker may use CORTEX to determine the second quartile or median and, in fact, any fractile of the distribution. However, if the decision problem is such that the decision maker judges that the continuous distribution may be approximated by one of a family of distributions such as the beta, gamma, and normal then the two quartiles are sufficient information for CORTEX to determine the complete distribution which is then displayed along with the corresponding values of the parameters commonly used to describe the distribution.* If the decision maker is dissatisfied with the appearance or with some characteristic of this distribution, he may go back and read just one or both of the quartiles and observe the resulting changes in the shape of the distribution until he is satisfied.

* If, for example, a beta distribution were being employed, CORTEX would display the two basic parameters p and q, an alternative parameterization in terms of r and n^1, and, in addition, the mean, median, mode, and standard deviation of the beta distribution implied by the two quartiles provided by the decision maker.

It should be apparent that this method has many of the advantages and disadvantages that characterize the corresponding method for assessing discrete probability distributions described above. For example, while the choices required of the decision maker are rather simple, the introduction of an external reference experiment may produce complications. Thus, CORTEX provides a second method for assessing continuous probability distributions which, interestingly, has many of the properties of the alternative method provided in the case of discrete probabilities.

CORTEX displays a line representing the range of values for the continuous events in the decision problem under consideration. The line is segmented by two indices that may be adjusted by the decision maker. The hypothetical experiment is such that the decision maker earns a certain constant amount of money if the actual event assumes a value lying between the two indices and he earns nothing if the actual event falls outside of this interval. In any case, however, the decision maker must pay an amount of money which is proportional to the length of this interval bounded by the two indices. The decision maker may adjust the locations of the indices in order to maximize the expected value of the hypothetical experiment. Though CORTEX explains and demonstrates the optimal strategy to the decision maker, I will not attempt to describe it here.* The essential thing is that the decision maker's settings can be used to solve for the complete distribution which, as before, is displayed along with a set of relevant parameters. Again, the decision maker may readjúst until he is satisfied with the continuous probability distribution.

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* This method has been derived and described in some detail by Toda[9].

The Future Development and Evaluation of CORTEX

As implied above, the total value of the application of a system such as CORTEX to the operations of an organization can be thought of in terms of the total marginal gains minus the total marginal costs of utilizing the system. Thus the decision whether or not to have such a system depends upon whether this difference is positive or negative. Now, these marginal gains of application are seldom, if ever, measured in an unequivocal manner and, in addition, are quite expensive to measure. Since these marginal gains are very difficult to measure in application to particular decision problems, the total marginal gain resulting from the application to many individual decision problems is practicably unmeasurable and must remain uncertain. Thus, the total effectiveness of CORTEX is as uncertain as is the total effectiveness of, say, linear programming, matrix algebra, or statistics.

Of course, most of us feel that mathematics can be a useful tool if intelligently used. This judgement gives us a basis for assuming that some gains are non-zero under certain conditions. Therefore, we might attempt to increase the total effectiveness of mathematics both by increasing the number of intelligent applications of the mathematics and by reducing the cost of application of the mathematics. CORTEX represents an attempt to do just this for the mathematics of decision making.

By any criterion, CORTEX cannot be successful if it is not used to solve decision problems. It must be called upon quite frequently to solve a large number of decision problems. Whether or not it will be used to solve a particular problem will be decided, in effect, by the answers to each of two questions. Are the techniques provided by CORTEX relevent to my decision problem? Is CORTEX easy enough to use in working with this decision problem? The frequency of positive replies to the first question depends in large part upon the generality and variety of the algorithms incorporated into CORTEX* while the frequency of positive replies to the second question depends in large part upon the effectiveness of CORTEX in reducing the personal costs of application referred to previously. Thus the development of CORTEX should evolve in the direction of including more and more algorithms and in the direction of requiring less and less special knowledge on the part of the decision maker.

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* It also depends to an unknown but possibly great extent upon whether or not the decision maker has learned to view the world through the eyes of decision theory.[14] Lest an opponent of decision theory rejoice too quickly, I hasten to add that though mathematics may be afflicted with tunnel vision it yields images with a very sharp focus.

The attempt to reduce the amount of special knowledge required of the decision maker seems certain to lead sooner or later to a system that has so many explanations, queries, prompts, etc. that a decision maker experienced in the use of the system will find it quite annoying to use. At this point, preferably earlier, CORTEX should be modified so that it has, say, three levels of operation: Level I for a decision maker relatively unfamiliar with decision theory and with the operation of CORTEX, Level II for a decision maker moderately experienced in the operation of CORTEX, and Level III for a decision maker highly experienced in both decision theory and the use of CORTEX. The decision maker could then specify the desired level immediately after calling CORTEX.

While CORTEX must be used in order to be considered a success, it is interesting to consider whether or not frequency of usage is an unambiguous measure of effectiveness. In a very special sense it is not. Suppose that we observe a decision maker who utilizes CORTEX quite extensively for the first year or so and then uses it less and less. He may be becoming less satisfied with the system or he may be becoming more and more convinced that it is not really helping him in his job. Suppose that we take a closer look and find that this decision maker is confining his use of CORTEX to the analysis of those decision problems which require only the most complex and least intuitively comprehensible algorithms. One possible cause of this behavior on the part of the decision maker just might be that through using CORTEX he has learned the concepts of decision theory and the operation of the algorithms so well that he know longer needs to use the system to arrive at a satisfactory understanding of and solution to many of his decision problems. Thus, the decision maker has internalized the operation of CORTEX and can now make many decisions, certainly at less cost and more rapidly, than the computer-based system which is now partially outmoded and no longer needed.

This line of reasoning suggests that an effective system for aiding decision making may prove to be an effective system for teaching intuitive decision making. Though the mechanism by which individuals through extensive experience with a mathematical system gain a profound appreciation of the behavior of the system is not well-understood, it certainly exists. Further, it seems reasonable to expect that this internalization of mathematics is more likely to occur for some processes than for other processes, probably for the "simpler" processes. Since CORTEX is not conceived as a static system but rather as one which evolves to incorporate the future advances in decision theory and thus will include more and more "complicated" processes, it is unlikely that a decision maker through internalization will ever achieve complete independence from CORTEX. However, the internalization by some decision makers of some of the processes provided by CORTEX should not be surprising and should be considered a thoroughly desirable consequence of the use of CORTEX.

REFERENCES

[1]Raiffa, H. and Schlaifer, R., Applied Statistical Decision Theory (Harvard Business School, Division of Research, Boston, 1961)

[2]Schlaifer, R., Introduction to Statistics for Business Decisions (McGraw-Hill Book Company, Inc., New York, 1961)

[3]Goodenough, J. B., "A program for lightpen-controlled on-line data analysis," Electronic Systems Division Technical Documentary Report Number ESD-TDR-64-515, (August 1964).

[4]Schlaifer, R., Probability and Statistics for Business Decisions (McGraw-Hill Book Company, Inc., New York, 1959).

[5]Toda, M. and Shuford, E. H., Jr., "Utility, induced utility, and small worlds," Electronic Systems Division Technical Documentary Report Number ESD-TDR-63-622, (October 1963).

[6]Savage, L. J., The Foundations of Statistics (John Wiley & Sons, Inc., New York, 1954).

[7]Chipman, J. S., "Stochastic Choice and Subjective Probability," in Wilner, D. (Ed.), Decisions, Values and Groups (Pergamon Press, New York, 1960).

[8]Symposium published in the Quarterly Journal of Economics, 75 (1961), pp. 643-694, containing:

Ellsberg, D., "Risk, ambiguity, and the Savage axioms,"

Fellner, W., "Distortion of subjective probabilities as a reaction to uncertainty,"

Raiffa, H., "Risk, ambiguity, and the Savage axioms: comment."

[9]Toda, M., "Measurement of subjective probability distributions," Electronic Systems Division Technical Documentary Report Number ESD-TDR-63-407, (July 1963).

[10]Roby, T. B., "Belief states: a preliminary empirical study," Electronic Systems Division Technical Documentary Report Number ESD-TDR-64-238, (March 1964).

[11]Van Naerssen, R. F., "A scale for the measurement of subjective probability," Acta Psychologia, (1961), pp. 159-166.

[12]De Finetti, B., "Does it make sense to speak of 'good probability appraisers'?," in Good, I. J. (Ed.), The Scientist Speculates (Basic Books, Inc., New York, 1962).

[13]Grayson, C. J., Jr., Decisions under Uncertainty (Harvard Business School, Division of Research, Boston, 1960) Part 2, Chap. 9, p. 263.

[14]Toda, M. and Shuford, E. H., Jr., "Logic of systems: introduction to the formal theory of structure," Electronic Systems Division Technical Documentary Report Number ESD-TDR-64-193, (January 1964).

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Decision Sciences Laboratory, Electronic Systems Division, L. G. Hanscom Field, Bedford, Massachusetts, 01731	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP n/a

3. REPORT TITLE

CORTEX: A Computer-Based System for Aiding Decision Making

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

n/a

5. AUTHOR(S) *(Last name, first name, initial)*

Shuford, Emir H.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1964	38	14

8a. CONTRACT OR GRANT NO. In-House	9a. ORIGINATOR'S REPORT NUMBER(S) ESD-TR-64-677
b. PROJECT NO. 4690	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* None
d.	

10. AVAILABILITY/LIMITATION NOTICES

Qualified requesters may obtain from DDC.
Copies available from OTS

11. SUPPLEMENTARY NOTES None	12. SPONSORING MILITARY ACTIVITY Electronic Systems Division, Air Force Systems Command, United States Air Force, L. G. Hanscom Field, Bedford, Massachusetts, 01731

13. ABSTRACT

Decision theory is the contemporary manifestation of the mathematics of the decision process and thus can be viewed as a primary aid to the human decision process. The costs and gains of applying the concepts and algorithms of decision theory are considered in some detail. A man/computer system is described which is designed to make the concepts and algorithms of decision theory available to a decision maker at a greatly reduced personal cost. This is achieved, in large part, by significantly reducing the special knowledge required of the decision maker. Thus, the decision maker needs no knowledge of computer programming and a minimal knowledge of decision theory and mathematics in order to begin using the system in his day-to-day decision making activities.

DD FORM 1 JAN 64 1473

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14. KEY WORDS	LINK A		LINK B		LINK C	
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