Single Sampling Inspection Plans

With Specified Acceptance Probability and Minimum Average Costs.

By

A. Hald.

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INSTITUTE OF MATHEMATICAL STATISTICS
UNIVERSITY OF COPENHAGEN
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1. Introduction and summary.

The main purpose of the present paper is to give a tabulation and discussion of properties of a system of single sampling attribute plans obtained by minimizing average costs under the restriction that a point on the OC-curve has been fixed.

Inspection, acceptance, and rejection costs are assumed to be linear in $p$, the fraction defective, and lot quality is assumed to be distributed according to a double (or as a limiting case a single) binomial distribution with parameters $(p_1, p_2, w_1, w_2)$, $w_1 + w_2 = 1$ and $p_1 < p_2$.

Using average "inspection and sampling costs" as economic unit the standardized average costs become $R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 Q(p_2))$ where $(n, N)$ denote sample and lot size, respectively, and $(\gamma_1, \gamma_2)$ depend on the weights $(w_1, w_2)$ and the decision losses.

Three systems are studied corresponding to different restrictions:

(a) The LTPD system with a fixed consumer's risk, $Q(p_2) = 0.10$.

(b) The AQL system with a fixed producer's risk, $Q(p_1) = 0.05$.

(c) The IQL system with $P(p_0) = 1/2$ for $p_1 < p_0 < p_2$.

LTPD and AQL plans for a double binomial prior distribution may be found from the corresponding plans for a single binomial prior distribution by a suitable change of cost parameter.

The solution of the minimization problem and corresponding tables are given for the three systems.

Furthermore the asymptotic properties of the solution are studied.

For the LTPD and AQL systems the main properties of the sampling plans for large $N$ are the following:

1. Sample size increases linearly with the logarithm of lot size.

2. The highest allowable fraction defective in the sample converges to the fraction defective with fixed acceptance probability, the difference being of order $1/\sqrt{N}$.

3. The risk of the producer or the consumer, whichever has not been fixed, tends to zero inversely proportional to lot size.

4. The minimum costs equal sampling inspection costs plus a constant average decision loss plus a decision loss proportional to $(N - n)$ due to the restriction.

5. The sampling plans depend only on the product of one cost parameter (being a function of $\gamma_1$ and $\gamma_2$) and lot size.
For IQL plans both the consumer's and the producer's risk will tend to zero for 
$N \to \infty$, one of the risks as $O(N^{-1})$ and the other as $O(N^{-1-\delta})$, $\delta \geq 0$. For

$$p_0 = \left( \log \frac{q_1}{q_2} \right) \left( \log \frac{p_2 \alpha}{p_1 \beta} \right)$$

we have $\delta = 0$. The IQL plans are only studied in detail for this value of $p_0$.

The properties listed under (1), (2), and (5) above are also valid for these IQL plans. Furthermore we have:

(3a) The producer's and the consumer's risks are nearly equal and tend to zero inversely proportional to $N$.

(4a) The minimum costs equal sampling inspection costs plus a constant average decision loss.

The IQL plans for a double binomial prior distribution may be found with good approximation from the plans for a single binomial prior distribution.

Comparing these plans with the corresponding Bayesian plans the IQL plans have economic efficiency tending to 1 for $N \to \infty$ whereas the efficiency of the LTPD and AQL plans tends to zero.

The restrictions are mainly introduced to obtain protection against deterioration of the prior distribution and because one of the cost components may be (partly) unknown. In such cases it is recommended to use the IQL plans whereas it is not advisable to use the LTPD and AQL plans for large lots. If an upper limit has been specified for the consumer's or the producer's risk one may use the corresponding LTPD or AQL plan for small $N$ and switch over to IQL plans as soon as the condition is satisfied.

The system of sampling plans presented here contains as special cases, viz. for $w_2 = 0$ and $\gamma_1 = 1$, the Dodge-Romig system of LTPD plans, see [2], and the Weibull-Markback system of IQL plans, see [13] and [11]. It also contains for $w_2 = 0$ the asymptotic results of a previous paper [7] whereas the tables are different because hypergeometric probabilities were used for the fixed point on the OC-curve in [7] as in the Dodge-Romig tables.

2. The model.

Let $N$ and $n$ denote lot size and sample size and let $X$ and $x$ denote number of defectives in the lot and the sample, respectively. The acceptance number is denoted by $c$.

Let the costs be

$$nS_1 + xS_2 + (N - n)A_1 + (X - x)A_2$$

for $x \leq c$

and

$$nS_1 + xS_2 + (N - n)B_1 + (X - x)B_2$$

for $x > c$. 

The (prior) distribution of $X$, i.e. the distribution of lot quality, is denoted by $f_N(X)$ and it is assumed that this distribution is a mixed binomial

$$f_N(X) = \frac{1}{w} \sum_{x=0}^{N} \binom{N}{x} p^x (1-p)^{N-x} dW(p).$$

(1)

In particular $f_N(X)$ may be a double binomial, i.e. a weighted average of two binomials with parameters $p_1$ and $p_2$, $p_1 < p_2$, and weights $w_1$ and $w_2$, $w_1 + w_2 = 1$. This distribution may also be characterized by saying that $p$, the process average, has a two-point distribution.

Drawing a sample without replacement from each lot (hypergeometric sampling) and computing the average costs we find

$$K(N,n,c) = \frac{1}{w} \int K(N,n,c,p) dW(p)$$

(2)

where

$$K(N,n,c,p) = n(S_1 + S_2 p) + (N-n)((A_1 + A_2 p) F(p) + (R_1 + R_2 p) Q(p)),$$

(3)

$$P(p) = B(c,n,p) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x},$$

(4)

and $Q(p) = 1 - P(p)$.

For a detailed discussion of this model the reader is referred to Hald [8]. In the following it is assumed that the prior distribution is a double binomial distribution or as a limiting case a single binomial.

To simplify the notation we introduce the three cost functions

$$k_a(p) = A_1 + A_2 p, \quad k_r(p) = R_1 + R_2 p, \quad k_s(p) = S_1 + S_2 p,$$

(5)

and the averages

$$k = w_1 k_a(p_1) + w_2 k_s(p_2) \quad \text{and} \quad k_m = w_1 k_a(p_1) + w_2 k_r(p_2),$$

(6)

assuming that $k > k_m$. For a double binomial prior distribution $k_s$ represents the average "costs of inspection" per item and $k_m$ represents the average costs per item when all lots from process (component) No. 1 are accepted and all lots from process No. 2 are rejected. As shown in [8] $k_m$ is under certain conditions a useful reference point for average costs per item. Defining the standardized form of (2) as

$$R(N,n,c) = \frac{(K(N,n,c) - Nk_m)}{(k_s - k_m)}$$

it follows that the value of $(n,c)$ minimizing $R$ will also minimize $K$ since $k_s$ and $k_m$ are independent of $(n,c)$. The standardized average costs may be written as

$$R(N,n,c) = n + (N-n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2))$$

(7)
\[ \gamma_1 = \frac{w_1(k_r(p_1) - k_a(p_1))}{k_s - k_m}, \quad \gamma_2 = \frac{w_2(k_a(p_2) - k_r(p_2))}{k_s - k_m}. \]  

The interpretation of (7) is the following: The reduced average "costs of inspection", \( k_s - k_m \), have been used as economic unit which means that the total average costs become equal to \( n \), the average costs of inspecting the \( n \) sample items, plus the average decision loss per item times the number of items in the remainder of the lot. The term \( \gamma_1 Q(p_1) \), say, gives the probability \( (w_1) \) of a lot of quality \( p_1 \) being submitted (more precisely a lot from a process with process average equal to \( p_1 \)) times the average probability \( (Q(p_1)) \) of such a lot being rejected times the corresponding decision loss \( ((k_r(p_1) - k_a(p_1))/(k_s - k_m)) \).

The costs of accepting or rejecting all lots without inspection are \( R_a = N\gamma_2 \) and \( R_r = N\gamma_1 \) respectively.

The Bayesian solution of the inspection problem consists in choosing the procedure which leads to the lowest average costs and therefore it requires a comparison of \( R_a, R_r \), and \( \min R(N, n, c) \). This solution has been discussed and tabulated in [8]. If the Bayesian solution is sampling inspection we shall call the sampling plan minimizing \( R \) for the Bayesian (single) sampling plan.

The conditions alluded to above are that \( \gamma_1 > 0 \) and \( \gamma_2 > 0 \). For the corresponding Bayesian sampling plan we have \( n/N \to 0 \), \( Q(p_1) \to 0 \), and \( P(p_2) \to 0 \) for \( N \to \infty \) which means that \( K/N \to k_{11} \) and \( R/N \to 0 \) which is one of the reasons for standardizing the average costs in the manner above.

The conditions may also be expressed by means of the economic break-even quality \( p_r = (R_1 - \Lambda_1)/(M_2 - R_2) \), defined as the root of the equation \( k_a(p) = k_r(p) \), since \( \gamma_1 > 0 \) and \( \gamma_2 > 0 \) if and only if \( p_1 < p_r < p_2 \). If \( \gamma_1 > 0 \) and \( \gamma_2 < 0 \), say, i.e. \( p_r > p_2 > p_1 \), the Bayesian solution is acceptance without inspection.

In the present paper we shall consider sampling plans defined by minimizing the average costs under a suitably chosen restriction. The reasons for doing so and the choice of the specific restrictions will be discussed later.

Furthermore we shall also consider cases where either \( k_a(p) \) or \( k_r(p) \) is identically equal to zero so that \( \gamma_2 \) or \( \gamma_1 \) becomes negative.

One form of restriction is \( P(p_o) = 1/2 \) for \( p_1 < p_0 < p_2 \). This defines a relation between \( c \) and \( n \) with the property that \( Q(p_1) \to 0 \) and \( P(p_2) \to 0 \) for \( n \to \infty \) and consequently \( K/N \to k_m \) for \( N \to \infty \). Such sampling plans will for the right choice of \( p_o \) have similar properties as the Bayesian sampling plans for \( \gamma_1 > 0 \) and \( \gamma_2 > 0 \), see section 8.

Another form of the restriction is \( P(p_2) = 0.10 \), say. This means that \( Q(p_1) \to 0 \) for \( n \to \infty \) and \( K/N \to k_m + 0.1 \frac{w_2(k_a(p_2) - k_r(p_2))}{k_s - k_m} = k^* \), say. The restriction thus
changes the unavoidable limiting costs from \( k_m \) to \( k^*_m \) which will therefore be used in standardizing the cost function.

It should be noted, however, that \( k_m \) and \( k^*_m \) are fundamentally different because \( k_m \) depends on the prior distribution and the costs only whereas \( k^*_m \) also depends on the restriction which to some extent may be considered arbitrary.

From (7) we find

\[
R = n(1 - 0.17^2) + (N-n)\gamma_1 Q(p_1) + 0.17N
\]

leading to the (further) standardized costs

\[
\frac{R - 0.17^2 N}{1 - 0.17^2} = R_o = n + (N-n)\gamma Q(p_1)
\]

where \( \gamma = \gamma_1/(1 - 0.17^2) \). Values of \((n,c)\) minimizing \( R_o \) will be the same as those minimizing \( R \).

Similarly we shall use restrictions of the form \( Q(p_1) = 0.05 \) leading to

\[
R_o = n + (N-n)\gamma P(p_2)
\]

where \( \gamma = \gamma_2/(1 - 0.05\gamma_1) \).

Restrictions as \( P(p_2) = 0.10 \) or \( Q(p_1) = 0.05 \) are of particular interest in cases where \( k_m(p) \) or \( k_r(p) \), respectively, for some reason has been put equal to zero, i.e. \( \gamma_2 \) or \( \gamma_1 \) becomes negative.

An expression of the type (10) or (11) may, however, be obtained from \( R \) by putting \( w_2 = 0 \) or \( w_1 = 0 \). It thus follows that a restricted Bayes solution with a two-point prior distribution where the restriction fixes the acceptance probability in one of the two points may be reduced to a restricted Bayes solution with a one-point prior distribution by a suitable change of the cost parameter.

From a mathematical and numerical point of view we may therefore limit ourselves to consider the problem defined by minimizing expressions of the type given by (10) under the restriction stated.

2. Restricted Bayes solutions.

The Bayes procedure has not been widely used in practice for many reasons some of which have been listed below:

(a). It may be difficult to obtain precise information on the prior distributions and the costs.

(b). If the Bayes procedure does not lead to sampling, a running check on the assumptions regarding the prior distribution is lacking.
(c). The mathematical theory behind the Bayes solution is more difficult than for other systems of sampling plans, and adequate tables have been lacking until recently.

With respect to point (b) above it is pointed out that there are two general cases in which the Bayes principle does not lead to a sampling plan, viz. (1) if the prior distribution of p is a one-point distribution or (2) if either \( k_a(p) = 0 \) or \( k_r(p) = 0 \).

The first case is important because many investigations have been carried out on the assumption that the quality distribution under "normal conditions" is a binomial distribution. If average quality produced is better than the break-even quality then the cheapest solution will be acceptance without inspection. To obtain a sampling plan minimizing costs under this assumption it is therefore necessary to introduce some sort of restriction.

The second case is important because \( k_a(p) \) or \( k_r(p) \) are often unknown or may be considered as negligible in the short run when the costs are looked upon from the producer's or the consumer's side exclusively.

One may naturally give up the Bayes solution completely and use the minimax regret solution which depends on the cost parameters only. It seems, however, unreasonable in designing an inspection system for a series of lots from the same source not to use some plausible prior distribution based on existing inspection records and knowledge of normal market quality if only a sampling plan is used in all cases and some insurance has been built into the system against the consequences of a deterioration of the prior distribution and uncertainty in the determination of the cost parameters. This insurance may be formulated in economic terms or in technical terms only and leads to what has been called a restricted Bayes solution since the principle employed is to minimize the average costs under a suitably chosen restriction.

As indicated in section 2 we shall use restrictions which are independent of the weights in the prior distribution and the cost function. The restrictions considered consist in fixing a point on the OC-curve, i.e. specifying a quality level and a corresponding acceptance probability. Such a restriction defines a relationship between \( n \) and \( c \). Restrictions of this kind have first been used by Dodge and Romig [2] in their LTPD system of sampling plans.

The average decision loss depends on expressions of the type \( w_2(k_a(p_2) - k_r(p_2))P(p_2) \), say. If we are concerned about the stability of \( w_2 \) and the correctness of \( k_a(p_2) \) we may get some insurance against consequences of deviations from the values actually used by specifying that \( P(p_2) \) shall be small. A detailed discussion of the considerations in connection with fixing a point on the OC-curve will be
given in sections 6-8.

We shall first study LTPD and AQL sampling plans satisfying the restrictions $P(p_2) = 0.10$ and $Q(p_1) = 0.05$, respectively, and thereafter IQL sampling plans satisfying $P(p_0) = 1/2$.

4. The exact solution.

The problem consists in determining $(n,c)$ so that

$$R = n + (N-n)Q(p_1), \quad \text{(case 1)},$$

is minimized under the restriction $P(p_2) = P_2$, $P_2$ being a given number and $p_1 < p_2$, or correspondingly to minimize

$$R = n + (N-n)P(p_2), \quad \text{(case 2)},$$

under the restriction $Q(p_1) = Q_1$, $p_1 < p_2$. Since the two problems are of the same mathematical structure we shall discuss only the first one in detail.

The problem is similar to Dodge and Romig's problem for the LTPD plans and it will be solved here along similar lines as in Hald [6]. One difference should be noted however, namely that both $Q(p_1)$ and $P(p_2)$ are binomial probabilities, whereas $P(p_2)$ in Dodge and Romig's model is a hypergeometric probability.

To obtain a sampling plan as solution the costs of the plan must be smaller than the costs of complete inspection, i.e. $R < N$, which leads to the condition $Q(p_1) < 1/\gamma$ (case 1) and $P(p_2) < 1/\gamma$ (case 2). It is therefore necessary to assume that $\gamma > 0$ which is also natural from the point of view that $\gamma$ may be interpreted as the costs per item of rejection or acceptance, respectively, in the case of a one-point prior distribution.

The condition $P(p_2) = B(c,n,p_2) = P_2$ defines a relation between $n$ and $c$, $n = n_c$ say. Introducing $n = n_c$ in (17) makes $R$ a function of $c$ alone, $R(c)$ say, for any given $N$. The condition for $R(c)$ to be a local minimum is that

$$\Delta R(c-1) < 0 < \Delta R(c)$$

where $\Delta R(c) = R(c+1) - R(c)$. From (12) we have

$$R(c) = n_c + (N-n_c)\gamma (1-B(c,n_c,p_1))$$

and

$$\Delta R(c) = (1-\gamma)\Delta n_c - N\gamma B_c + \gamma \Delta (n_c B_c)$$

$$= (1-\gamma + \gamma B_c) \cdot n_c - \gamma (N-n_{c+1}) \cap B_c$$

where $B_c = B(c,n_c,p_1)$. 
Introducing the auxiliary function

\[ N_c = \frac{(i-\gamma) \Delta n + \gamma \Delta (n_{Bc})}{\gamma \Delta B_c} = n_{c+1} + \left( \frac{1}{\gamma} \cdot \frac{\Delta n}{\Delta B_c} \right). \quad (16) \]

substituting (15) into (14), and "solving" for \( N \) lead to the fundamental inequality

\[ N_{c-1} < N < N_c \quad (17) \]

together with \( \Delta B_{c-1} > 0 \) and \( \Delta B_c > 0 \) as the conditions for \((n_c, c)\) to be the optimum plan for lot size \( N \).

In case 2 the corresponding result is

\[ N_c = n_{c+1} + \left( \frac{1}{\gamma} \cdot \frac{\Delta n_c}{\Delta (1-B_c)} \right). \quad (18) \]

rogether with \( \Delta (1-B_{c-1}) > 0 \) and \( \Delta (1-B_c) > 0 \) where \( B_c = B(c, n_c, p_2) \) and \( B(c, n_c, p_2) = 1 - Q_1 \).

It has only been proved that (17) is the condition for \( R(c) \) to be a local minimum. A similar analysis may, however, be carried out by means of the difference operator \( \Delta R(c) = R(c+1) - R(c) \). The condition for \( R(c) \) to be an absolute minimum is that \( \Delta R(c) > 0 \) for \( i = 1, 2, \ldots, n_c \), and \( \Delta R(c-1) < 0 \) for \( i = 1, 2, \ldots, c \). It is easily seen that sufficient conditions for these inequalities to be fulfilled are that \( R(c) \) be a local minimum, i.e. (17) is fulfilled, and furthermore that \( N \) be a non-decreasing function of \( c \), since the inequalities \( N < N_c \leq N_{c+1} \leq \ldots \leq N_{c+i-1} \) by addition of all the numerators and denominators lead to

\[ N < \frac{(i-\gamma) \Delta n_c + \gamma \Delta (n_{Bc})}{\gamma \Delta (1-B_c)}. \quad (19) \]

i.e. \( \Delta R(c) > 0 \) for \( i > 0 \). It is conjectured that \( N_c \) is a non-decreasing function of \( c \) if \( n_c \) is considered as a continuous variable. However, in tabulating the solution only integer values of \( n_c \) has been used which means that the condition \( P(p_2) = p_2 \) (and the other similar conditions) will in most cases not be exactly fulfilled. For the three cases tabulated the cumulative binomial has been computed to six decimal places and \( n = n_c \) has been determined as

1. the smallest integer \( n \) satisfying \( B(c, n, p_2) \leq 0.10 \),
2. the integer \( n \) for which \( B(c, n, p_0) \) is nearest to 0.50,
3. the largest integer \( n \) satisfying \( B(c, n, p_1) \geq 0.95 \).

If \( N_c \) is an increasing function of \( c \) it follows from (17) that for each \((c, n_c)\) there exists an "optimum interval" \((N_{c-1}, N_c)\) so that for all \( N \) within that interval the optimum plan is \((c, n_c)\). In case \( N < N_{c-1} \) the plan \((c, n_c)\) is not optimum for any \( N \) and has to be excluded. The costs \( R(c-1) \) and \( R(c+1) \) have then to be compared.
Using $R(c+1) - R(c-1) = \Delta R(c) + \Delta R(c-1)$ it follows that $R(c-1) \leq R(c+1)$ for $N \leq N_c^*$ where
\[ N_{c-1}^* = (N_{c-1} \Delta B_{c-1} + N_c \Delta B_c)(\Delta B_{c-1} + \Delta B_c). \]

In that manner the optimum plans and the corresponding $N$-intervals may successively be determined starting from $c = 0$. The procedure is well suited for an electronic computer. The tables will be discussed in the following sections.

For large $N$ the Poisson distribution may be used as an approximation to the binomial. The original problem may also be such that the Poisson distribution is the appropriate one to use, viz. if quality is measured in number of defects per unit instead of fraction defective. For these reasons the Poisson solution has also been tabulated.

First $m = m_c$ has been determined from the relation
\[ B(c,m) = \sum_{x=0}^{m} \frac{m^x}{x!} e^{-m} = P_2, \quad m = np_2. \]

The inequality corresponding to (17) becomes
\[ M_{c-1} < M < M_c, \quad M = np_2, \]
where
\[ M_c = \frac{(1-\gamma) m_c + \gamma \Delta (m_c B_c)}{\gamma \Delta B_c} = m_{c+1} + \frac{1}{\gamma} (1-B_c) \Delta m_c \]
and
\[ B_c = \sum_{x=0}^{c} \frac{m^x}{x!} e^{-m} = \frac{P_1}{P_2} \cdot \]

Since $m = np_2$ is a function of $c$ only whereas in the binomial case $m$ is a function of both $p_2$ and $c$, it is possible to give a much more compact tabulation of the Poisson solution than of the binomial.

For small values of $N$ the solution given above need to be modified in certain cases.

For $N < n_0$ no sampling plan exists satisfying the restriction required. In such cases the solution has been given as "all" in the tables to indicate that inspection of the whole lot is necessary to obtain a protection at least as good as the one required.

In case 1 for $\gamma \geq 1$ the alternative to sampling inspection is total inspection which costs $N$. To obtain $R < N$ it is necessary that $Q(p_1) < 1/\gamma$, i.e. $B_c > 1-1/\gamma$, which may not be fulfilled for small $c$ and corresponding values of $N \in (N_{c-1}, N_c)$. In such cases the cheapest sampling plan available has nevertheless been given in the table but a "c" has been added to indicate that sampling is more costly than total inspection.
In case 1 for \( \gamma < 1 \) the alternative to sampling inspection is rejection at a cost of \( N \gamma \). To obtain \( R < N \gamma \) it is necessary that

\[
N > n_c(1 + \frac{1-\gamma}{\gamma B_c}), \quad B_c = 1 - Q(p_1). \tag{24}
\]

The corresponding result in/2 for \( \gamma \geq 1 \) is \( P(p_2) < 1/\gamma \), i.e. \( B_c < 1/\gamma \), and for \( \gamma < 1 \) with acceptance as alternative \( R < N \gamma \) which leads to

\[
N > n_c \left(1 + \frac{1-\gamma}{\gamma(1-B_c)}\right), \quad B_c = P(p_2). \tag{25}
\]

In such cases "a" has been added after the sample size to indicate that acceptance without inspection is cheaper than sampling.

It has furthermore to be taken into account that \( (c, n) \) may be used as optimum plan for \( N \) only if \( N < n_c \). If \( N < n_c \), no optimum plan exists because \( (c, n) \) is not optimum for \( N > n_c \) and \( (c+1, n+1) \) cannot be used because \( N > n_{c+1} \). It is therefore a condition for the existence of optimum plans that \( N > n_{c+1} \). From (16) and (18) follows, however, that this condition may be reduced to the one following from \( R < N \).

5. The asymptotic solution.

The procedure in arriving to an asymptotic solution giving \( c \) and \( n \) as explicit functions of \( N \) will be first to get an asymptotic expansion of \( c \) in terms of \( n \) as an expression for the condition imposed and then to eliminate \( c \) from \( R \) and solve the equation \( dR/dn = 0 \) after having replaced the binomial probability in \( R \) by an asymptotic expansion in terms of \( n \). A similar method has been used in [6].

A rather accurate solution of the equation \( B(c, n, p) = P \) may be obtained by using the expansion of Fisher and Cornish [5] which leads to

\[
c = np + u_p \sqrt{npq} + \frac{1}{6}(q-p)(u_p^2-1)-\frac{1}{2} + O(n^{-\frac{1}{2}}) \tag{26}
\]

where \( u_p \) denotes the \( P \)-fractile of the standardized normal distribution.

Writing \( h = c/n \) the condition \( P(p_2) = P_2 \) may therefore be expressed as

\[
h = p_2 + \sqrt{p_2 q_2/n} + b/n + O(n^{-\frac{1}{2}}) \tag{27}
\]

where

\[
a = u_p \quad \text{and} \quad b = \frac{1}{6} (q_2-p_2)(a^2-1)-\frac{1}{2} . \tag{28}
\]

Since \( h = p_2 + O(n^{-\frac{1}{2}}) \) we may use the following lemma which is a special case of a theorem proved by Blackwell and Hodges [1]:
For $p_1 < p_2$ and $n \to \infty$ we have

$$1 - B(c, n, p_1) = \frac{1}{\sqrt{2\pi p_2 q_2}} \frac{q_2 p_1}{p_2 - p_1} e^{-n\varphi(h, p_1)} \left( 1 + O(n^{-\frac{1}{2}}) \right)$$

(29)

where

$$\varphi(h, p) = h \ln \frac{h}{p} + (1-h) \ln \frac{1-h}{1-p}.$$  

(30)

(For $h = p_1 + O(n^{-\frac{1}{2}})$ a similar expression is valid for $B(c, n, p_2)$).

Setting

$$f(n) = \frac{n}{\gamma} e^{-n\varphi(h, p_1)}$$

and

$$\lambda = \frac{1}{\sqrt{2\pi p_2 q_2}} \frac{q_2 p_1}{p_2 - p_1}$$

we find from (12) and (29)

$$R = n + (N-n)f(n)(1 + O(n^{-\frac{1}{2}})).$$

(33)

Expanding $\varphi(h, p_1)$ in a Taylor series around $p_2$ and inserting $h-p_2$ from (27) we get

$$\varphi(h, p_1) = \varphi(p_2, p_1) + (h-p_2) \ln \frac{p_2 q_1}{p_1 q_2} + \frac{1}{2} (h-p_2)^2 + O(n^{-\frac{3}{2}})$$

$$= \varphi(p_2, p_1) + a \sqrt{\frac{p_2 q_2}{n}} \ln \frac{p_2 q_1}{p_1 q_2} + \frac{1}{n} \left( \frac{a^2}{2} + b \ln \frac{p_2 q_1}{p_1 q_2} \right) + O(n^{-\frac{3}{2}}).$$

(34)

It follows that $\tilde{m}f(n)$ tends exponentially to zero for $n \to \infty$ since $\varphi(p_2, p_1) > 0$.

From (33) we find

$$\frac{dR}{dn} = 1 + (N-n)f'(n) - f(n).$$

Solving the equation $dR/cn = 0$ for $N-n$ we get

$$N-n = -\frac{1}{f'(n)} (1 + O(n^{-\frac{1}{2}}))$$

since $f'/f \to -\varphi(p_2, p_1)$ and $f \to 0$.

Writing

$$\ln(N-n) = -\ln f(n) - \ln(\frac{f'(n)}{f(n)}) + O(n^{-\frac{1}{2}})$$

(35)

we finally have

$$\ln(N-n) = a_1 n + a_2 \sqrt{n} + \frac{1}{2} \ln n + a_3 + O(n^{-\frac{1}{2}})$$

(36)

where $a_1 = \varphi(p_2, p_1)$, $a_2 = a \sqrt{p_2 q_2} \ln \frac{p_2 q_1}{p_1 q_2}$, and

$$a_3 = \frac{n^2}{2} + b \ln \frac{p_2 q_1}{p_1 q_2} - \ln \lambda - \ln \varphi(p_2, p_1).$$
The same formula applies to case 2 if only $p_1$ and $p_2$ are interchanged, $(p_2-p_1 \text{ in (32) should be read as } |p_2-p_1|)$, and $p_2$ in (28) is replaced by $p_1 = 1-Q_1$.

This result is a generalization of the one obtained in [6] partly because it is based on the binomial instead of the Poisson distribution and partly because the model here contains a cost parameter $\gamma$ which is equal to 1 in the case previously considered.

Solving (26) with respect to $np$ gives

$$np = c + 1 - u_p \sqrt{(c+1)q} + (u_p^2 - 1)(1+p)/3 - u_p^2/2 + O(c^{-2}). \tag{37}$$

Formulas (36) and (37) give good approximations to the exact solution for $Np_2 > 15$, $p_1/p_2 \leq 0.5$, and $p_2 = 0.10$ or 0.50, and in case 2 for $Np_1 > 15$, $p_2/p_1 \geq 1.5$, and $Q_1 = 0.05$ or 0.50.

The formulas should be used as follows: For $c = 0.5, 1.5, 2.5, \ldots$ $n$ is computed from (37) and $N$ from (36) to obtain intervals for $N$ corresponding to every integer value of $c$, cf. (17). For each integer value of $c$ the appropriate sample size is determined from (37).

A formula giving the sample size directly as function of lot size may be obtained by inversion of (36) which according to the result given in [6] leads to

$$n = \beta_1 x + \beta_2 \sqrt{x} + \beta_3 \ln x + \beta_4 + \beta_5 (\ln x)/\sqrt{x} + \beta_6/\sqrt{x} \tag{38}$$

where $x = \ln N$, $\beta_1 = 1/a_1$, $\beta_2 = -a_2/a_1^{3/2}$, $\beta_3 = -1/2$, $\beta_4 = (\ln a_1 + a_2^2/a_1 - 2a_3)/2a_1$, $\beta_5 = -a_2/4$, and $\beta_6 = \beta_5 (C - 2a_1 \beta_4 + a_2^2/2a_1)$.

For a given $N$ we may compute $n$ by (38) and the corresponding $c$ by (26), round $c$ to the nearest integer and find $n$ from (37).

Numerical investigations have shown that (38) leads to rather accurate results for $p_2 = 0.10$, $p_2 \leq 0.10$, $p_1/p_2 \leq 0.5$, and $Np_2 > 15$, whereas it should not be used for $p_2 = 0.50$ or in case 2 for $Q_1 = 0.05$.

From (35) it follows that

$$\ln(N-n)f(n) = -\ln \omega(p_2,p_1) + O(n^{-2/3})$$

or

$$\frac{N-n}{\gamma Q(p_1)} = \frac{1}{\omega(p_2,p_1)} (1 + O(n^{-2/3})) \tag{39}$$

and consequently

$$\min R = n + \frac{1}{\omega(p_2,p_1)} + O(n^{-2/3}) \tag{40}$$

where $n$ is given by (38).
We have thus found the following asymptotic properties of the solution:

1. Sample size increases linearly with the logarithm of lot size, see (38).

2. The highest allowable fraction defective in the sample converges to the fraction defective with fixed acceptance probability, the difference being of order $1/\sqrt{n}$, see (26).

3. The risk of the producer or the consumer, whichever has not been fixed, tends to zero inversely proportional to lot size, see (39).

4. The minimum (standardized) costs equal sampling inspection costs plus a constant depending on $(p_1, p_2)$ only, see (40).

Analogous results have previously been given by Hald [6] for the case with $\gamma = 1$ and Poisson probabilities.

The last mentioned property means that asymptotically decision losses will be negligible as compared to sampling inspection costs.

This is true, however, only for cost functions of the form $R = n + (N-n)\gamma Q(p_2)$. If we have a cost function as (7) then $R = (1-0.1\gamma)R_0 + 0.1\gamma N$ which asymptotically equals

$$\min R = (1 - 0.1\gamma)(n + \frac{1}{\varphi(p_2, p_1)}) + 0.1\gamma N$$

where the first term is $O(\ln N)$. For large $N$ the term $0.1\gamma N$ resulting from the restriction $P(p_2) = 0.1$ becomes dominating in contrast to the result for the (unrestricted) Bayesian sampling plan where $\min R = O(\ln N)$, see [8]. For $\gamma > 0$ the economic efficiency of a restricted Bayesian sampling plan of the type above as compared to a Bayesian plan will thus tend to zero for $N \to \infty$. For $\gamma < 0$ the Bayesian solution is acceptance without inspection at a cost of $R_n = n\gamma$.

The asymptotic formulas also reveal that the cost parameter $\gamma$ influences the solution in an extremely simple manner. From (36) it will be seen that $\gamma$ only enters through $a_1$ so that $\ln(\text{cost}) = F(n)$ where $F(n)$ is independent of $\gamma$. It follows that asymptotically the sampling plan only depends on the product of lot size and cost constant so that for example the plan for lot size $N$ and cost constant $\gamma$ equals the plan for lot size $N\gamma$ and cost constant 1.

Since this property holds for large $\gamma$-intervals also for small values of $N$ it is only required to tabulate sampling plans for rather few values of $\gamma$.

Consequently it should be noted that the Dodge-Romig LTPD tables may be used to find sampling plans by entering the tables with $N/\gamma$ for $\gamma < 3$ say.
Another way of expressing the dependence of \( \gamma \) is given by

\[
n(N, \gamma) \sim n(N, 1) + \frac{\ln \gamma}{\varphi(p_2, p_1)}
\]  

(41)

which follows from (38).

Formula (39) shows another interesting result, viz. that the asymptotic value of \( Q(p_1) \) is inversely proportional to \( \gamma \), which is the reason that \( \min R \) only depends on \( \gamma \) through \( n \), see (40).

6. LTPD sampling inspection plans with minimum costs.

LTPD plans are here defined as sampling plans with a given Lot Tolerance Per Cent Defective, \( 100p_2 \), and a corresponding probability of acceptance, the consumer's risk \( P(p_2) \), which traditionally is chosen as 10 per cent.

In the discussion of sampling plans it has been found convenient for obvious terminological and pedagogical reasons to introduce a fictitious consumer and producer and concentrate attention on the corresponding two points on the OC curve, \( P(p_1) \) and \( P(p_2) \), \( p_1 < p_2 \), defining the producer's risk as \( Q(p_1) \) and the consumer's risk as \( P(p_2) \). It is useful to extend these notions also to the cost functions.

Consider a producer inspecting his own product before delivery and suppose that he has essentially two goals: (1) To make reasonably sure that lots of bid quality are not marketed. (2) To keep his inspection costs and decision losses down.

We shall in turn discuss these aspects of the problem under two different assumptions regarding the prior distribution, viz. for a one-point and a two-point distribution of \( p \).

Suppose that the producer knows his process average \( p_1 \) for "normal production" and that he occasionally produces lots of bad quality. The quality level for bad lots may be fluctuating rather much so that the producer is not willing neither to state an average quality level for these lots nor the frequency with which such lots will occur. However, the producer may be willing to select a tolerance value of the fraction defective, \( p_2 \) say, and a risk, \( P(p_2) \), of accepting lots of this quality. The choice of \( p_2 \) is difficult and rather subjective. It is based on considerations of customary market quality, the producer's own quality performance, his prestige, consequences of loss of good-will, consequences for the consumer of getting bad quality, the use of the product, etc. The consumer's risk, \( P(p_2) \), is customarily chosen as 0.10. This is perfectly arbitrary and the value of \( P(p_2) \) has therefore to be kept in mind in choosing \( p_2 \) even if ideally \( p_2 \) should be determined exclusively from technical and economical considerations.
Turning now to the costs the first question to be answered is the following: What are the producer's average costs for lots of normal quality? The answer is given by the value of the cost function $K(N,n,c,p_1)$, see (3). In many cases, however, it seems reasonable to disregard the term $(A_1 + A_2 p_1)P(p_1)$ from the producer's point of view because lots of quality $p_1$ are supposed to be satisfactory as general market quality or by mutual (tacit) agreement between the parties. Delivery of lots of quality $p_1$ will therefore not lead to (essential) complaints from the consumer, i.e. the consumer has to bear the costs due to accepted defective items. If this is so one may merely put $A_1 = A_2 = 0$ in the following formulas.

Since the producer cannot specify the quality level and the frequency of bad lots it is impossible to include the corresponding costs in the discussion. A low frequency of bad lots and the restriction $P(p_2) = 0.10$ should, however, if $p_2$ has been chosen sufficiently small, make sure that very few bad lots will be accepted so that no serious economic damage will result.

Under these circumstances it seems therefore reasonable to determine the sampling plan by minimizing the producer's costs for lots of normal quality, $K(N,n,c,p_1)$, under the restriction of a fixed consumer's risk, $P(p_2) = 0.10$. From the point of view of statistical theory this is a restricted Bayes solution with a one-point prior distribution of $p$.

Introducing the standardized costs

$$ R = (K(N,n,c,p_1) - Nk_a(p_1))/k_s(p_1) - k_a(p_1) $$

we find

$$ R = n + (N-n)\gamma_1 Q(p_1) $$

with

$$ \gamma_1 = k_s(p_1) - k_a(p_1) \quad R_1 = A_1 - (A_2 - R_2) p_1 $$

see (7) for $w_2 = 0$. This shows that the solution is the one discussed in sections 4 and 5 with cost parameter equal to $\gamma_1$ (for $w_1 = 1$).

The solution only requires knowledge of the two quality levels and the cost constants. It rests on the assumption that the quality distribution of the larger part of the lots is a binomial distribution. A weakness is the uncertainty in the determination of $p_2$ and $P(p_2)$. For practical reasons it is customary to use $P(p_2) = 0.10$ in constructing tables of the solution. The parameter left free in practice is therefore $p_2$ only ($p_1$ is assumed to be rather accurately known by the producer) and since sample size is a decreasing function of $p_2$ for given $p_1$, the producer may in case of doubt choose a small value of $p_2$ which will lead to a sharper discrimination between good and bad lots.
This system of sampling plans is a generalization of the Dodge-Romig LTPD system which may be obtained for $\gamma_1 = 1$. Dodge and Romig assume that rejection means complete inspection of the remainders of rejected lots and furthermore that the costs of complete inspection per item are the same as the costs of sampling inspection, i.e. $k_r(p) = k_s(p)$. The cost parameter $\gamma_1$ allows us to interpret "rejection" in a much wider sense than Dodge and Romig and also to take costs of acceptance into account if necessary, see the definition of $\gamma_1$ in (42). It should be noted that the consumer's risk in the tables given here has been computed as a binomial probability whereas in the previous paper [7] the hypergeometric distribution has been used as in Dodge and Romig's tables.

Suppose now that submitted lots are distributed according to a double binomial distribution with parameters $(p_1, p_2, w_2)$. If the parameters are known and the distribution is stable and if $p_1 < p_r < p_2$ the Bayes solution may be determined as in [8]. If, however, the stability of the prior distribution is doubtful and/or information on costs is incomplete the producer may prefer a restricted Bayes solution.

Firstly the producer may find it necessary to protect himself against some of the consequences of undesirable (and unknown) changes of the prior distribution and for that reason he may impose the condition $P(p_2) = 0.10$ on the plans.

Secondly the costs of acceptance may be partly unknown, e.g. because loss of goodwill is involved. In the short run the producer may regard costs of acceptance as practically negligible, i.e. $k_a(p) = 0$, if the consumer does not return an occasional bad lot but (possibly) only bad items found. If bad lots are returned by the consumer we have $k_a(p) = k_r(p)$ plus costs of delivering and returning the lots. In the long run, however, bad lots delivered will result in loss of goodwill which may be difficult to evaluate and include explicitly into the cost function.

The producer may therefore be forced to modify the model. Instead of minimizing the complete cost function without any restrictions he may choose to minimize the incomplete cost function obtained by putting $k_a(p) = 0$ under the restriction $P(p_2) = 0.10$ hoping that the resulting small frequency of bad lots accepted will reduce his (unknown) costs of acceptance sufficiently. As indicated above there may be cases where it is more reasonable to put $k_a(p_1) = 0$ and $k_a(p_2) = k_r(p_2)$.

One of the effects of fixing $P(p_2)$ may be judged by noting that on the average the ratio of number of bad lots accepted to total number of lots accepted will be $w_2 P(p_2)/(w_1 P(p_1) + w_2 P(p_2))$. For $P(p_2) = 0.10$ and $P(p_1) = 1$ this ratio will be $1/191$ for $w_2 = 0.05$ and $1/91$ for $w_2 = 0.10$.

The sampling plan is determined by minimizing the average costs, $K(N,n,c)$, under
the restriction of a fixed consumer's risk, \( P(p_2) = 0.10 \). It follows from (10) that the solution has been given in sections 4 and 5 for the following value of the cost parameter

\[
\gamma = \frac{\nu_1(k_r(p_1) - k_a(p_1))}{\nu_1(k_s(p_1) - k_a(p_1)) + \nu_2(k_s(p_2) - 0.1k_a(p_2) - 0.9k_r(p_2))} \frac{\gamma_1}{1 - 0.1\gamma_2}
\]  

(43)

where \( \gamma_1 \) and \( \gamma_2 \) are defined by (8).

For \( \nu_2 = 0 \) we have \( \gamma = \gamma_1 \) which means that from a mathematical point of view the approach leading to (42) may be regarded as a limiting case of the one above. As will be explained later in this section the same tables may therefore be used to obtain the optimum sampling plan in both cases.

It should be noted, however, that the interpretation of \( p_2 \) is different. In the first case \( p_2 \) is a tolerance fraction defective determined from technical and economical considerations whereas in the second case \( p_2 \) is a parameter in the prior distribution, viz. the average fraction defective for lots of unsatisfactory quality.

For \( \nu_2 > 0 \) the sign of \( \gamma_2 \) determines whether \( \gamma \) is smaller or greater than \( \gamma_1 \). If \( k_a(p) = 0 \) then \( \gamma_2 < 0 \) and \( \gamma < \gamma_1 \).

In case \( \nu_2 \) is known only approximately but limits for \( \nu_2 \) may be guessed at then \( \max \gamma \) can be found and used to get an upper limit for the appropriate sample size. Similarly, if \( p_2 \) is known only approximately \( \max \gamma \) can be found by choosing \( p_2 \) as small as reasonable.

It is important to notice that normally the second term of the denominator of (43) is negligible as compared to the first so that \( \gamma \) may be used as a good approximation to the cost parameter which again means that in important practical cases \( (k_a(p) = 0, k_r(p) = R_1, \text{ and } k_s(p) = S_1) \) \( \gamma \) will approximately be equal to the ratio between the costs of rejection per item and the costs of sampling inspection per item. This may be seen in the following way. For \( k_a(p) = 0 \) we find

\[
\gamma = \frac{\nu_1 k_r(p_1)}{(k_s - 0.9\nu_2 k_r(p_2))}. \quad \text{If furthermore } k_r(p) = R_1 \text{ and } k_s(p) = S_1 \text{ we have}
\]

\[
\gamma = \frac{R_1}{S_1} \left( 1 + \frac{\nu_2}{\nu_1} \left( 1 - 0.9 \frac{R_1}{S_1} \right) \right). \quad (44)
\]

For \( R_1 = S_1 \) we find \( \gamma = \nu_1/(\nu_1 + 0.1\nu_2) \). This shows that the sampling plan is rather insensitive to changes of \( \nu_2 \) unless \( R_1/S_1 \) is large.

The above discussion of the LTPD system has been carried out from a producer's point of view. For positive values of \( (\gamma_1, \gamma_2) \) similar considerations may be made by a consumer.
As mentioned in section 4 tables may be constructed with \(c\) as argument and \((n,N)\) as functions of \(c\). Such tables have, however, only been given for the solution based on Poisson probabilities because in that case it suffices to tabulate \(m = np_2\) and \(M = Np_2\) as functions of \(c\) for a given value of \(r = p_1/p_2\), see (20)-(23), which makes it possible to set up a compact and rather complete table. The two functions have been tabulated for 14 values of \(r\) \(0.05, 0.10, ..., 0.70\) and for \(c \leq 99\) with the modification that tabulation has been stopped when \(M\) exceeds 50,000. Because only an abridged version is published the last figure for \(M\) given in a column may be less than 50,000 even if \(c < 99\) which means that \(M\) exceeds 50,000 for the next entry. \(M\) has been determined to three significant figures. The optimum plan is \((c,m)\) for \(c-1 < M < M_c\). For \(\gamma > 5\) some of the smaller values of \(M\) have been underlined to indicate that total inspection for these values of \(M\) is cheaper than sampling inspection, and that the plan tabulated is the cheapest sampling plan available, i.e. \(M = m + 1\).

For practical reasons, i.e. to save space and to make the tables easier to use in practice, the tables based on binomial probabilities give \((n,c)\) as functions of \(N\). The exact solution, derived as described in section 4, has been given for \(100p_2 = 0.5, 1.2, 3.4, 5, 7, 10, 15, 20\), for five values of \(r = p_1/p_2\) chosen among the values \(r = 0.1, 0.2, ..., 0.7\), and for \(\gamma = 1\) and 5, giving a total of \(10 \times 5 \times 2 = 100\) tables. The same 20 values of \(N\) between 30 and 200,000 have been used in all the tables. Plans have been computed only for \(c < 99\). The tables also contain \(P(p_1)\) which makes it easy to compute \(R = n + (N-n)\cdot Q(p_1)\), \(R = (1-0.172)R_0 + 0.172N\), and the average costs \(K = (k_m - k_n)R + Nk_m\).

For \(\gamma > 1\) it may happen that total inspection is cheaper than sampling inspection for small lots. The cheapest sampling plan available (\(c\) as large as possible) has nevertheless been tabulated, and the letter \(t\) (for total inspection) has been added after the sample size. Such samples will be large as compared to the lot size since \(n < N < n_{c+1}\).

Since \(\log N\) is nearly a linear function of \(c\) at least for large lots, rather accurate results may be obtained by corresponding interpolation. For applications in practice it is, however, hardly worth while using logarithms, linear interpolation in \(N\) will normally suffice.

To find a sampling plan for a lot size not used as argument in the table the first step should thus be to determine \(c\) by linear interpolation with respect to \(N\) and round the result to the nearest integer. It should then be noted that \(n\) is a function of \(c\) and \(p_2\) only, i.e. \(n\) is independent of \(p_1\), so that \(n\) may in many cases be found corresponding to the given \(c\) in another column of the same LTPU table. If that is not so the nearest neighbouring values to the given \(c\) may be
found and \( n \) may be determined by linear interpolation with respect to \( c \). Another possibility is to use the formula given in section 9.

As an example consider the problem of determining the sampling plan for \( N = 1600 \), LTPD = 5%, \( p_1 = 2.5\% \), and \( \gamma = 1 \). Linear interpolation gives \( c = 12 \) and looking for \( n \) corresponding to \( c = 12 \) in another column of the same LTPD table it will be seen that \( n = 553 \). Changing \( N \) to 16,000 linear interpolation gives \( c = 27 \). The nearest values of \( c \) in the table are 25 and 28 with the corresponding \( n \)-values of 651 and 718. Linear interpolation gives \( n = 696 \). Sometimes \( n \) may be found directly in the corresponding LTPD table for \( \gamma = 5 \).

Numerical investigations have shown that the proposed method of interpolation will ordinarily give the correct value of \( c \) but may result in an error of one unit. As pointed out previously it is essential to use the right method to determine \( n \) when \( c \) has been found to secure that \( P(p_2) = 0.10 \). If the rules stated are followed the plans determined by interpolation will be optimum or very nearly so since the minimum of the cost function is rather broad.

It is customary in practice to set up rather large intervals for \( N \) and use the same sampling plan for all \( N \) within an interval. The present tabler may easily be used for constructing such intervals in two ways:

1. The tabular values of \( N \) may be considered as "midpoints" of the following intervals:

<table>
<thead>
<tr>
<th>( N )</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>85 - 150</td>
</tr>
<tr>
<td>200</td>
<td>150 - 250</td>
</tr>
<tr>
<td>300</td>
<td>250 - 400</td>
</tr>
<tr>
<td>500</td>
<td>400 - 600</td>
</tr>
<tr>
<td>700</td>
<td>600 - 850</td>
</tr>
</tbody>
</table>

2. The tabular values of \( N \) may be considered as upper endpoints of such intervals which means that too large sample sizes will be used in all cases.

Whatever procedure is applied for constructing such intervals the result will be that the sampling plan used for a certain interval will only be optimum for that part of the interval which is given by \( (N_{c-1}, N_c) \) where \( c \) is the acceptance number used. For all other parts of the interval the costs will be larger than necessary.

In a previous paper [7] similar tables have been given based on a hypergeometric consumer's risk and a binomial producer's risk as in the Dodge-Romig tables, and the relations between the solutions in the three cases (Poisson, Binomial, Hypergeometric) have been discussed. A comparison of the present tables and
the previous ones shows that in most cases a hypergeometric and a binomial consumer's risk of 10 per cent will lead to the same value of \( c \) or values of \( c \) differing only by 1. Only for \( p > 0.05 \) and \( r > 0.5 \) do the tables contain values of \( c \) differing by 2 and occasionally 3 and 4 units. One may therefore conclude that the values of \( c \) found in the present tables may also be used for the case with a hypergeometric consumer's risk. The corresponding sample size, \( n_h \), may be determined from the binomial \( n \) with good approximation from the formula

\[
\frac{n}{n-h} = n \left(1 - \frac{(np_2 - c)}{2np_2}\right).
\]  

(45)

As an example consider the problem of determining the optimum sampling plan with a 10 per cent hypergeometric consumer's risk for \( N = 200 \), LTPD = 5%, \( p_1 = 1\% \), and \( y = 1 \). The present tables show that the "binomial solution" is \( n = 77 \) and \( c = 1 \).

From (45) we find \( n_h = 77 \times 0.857 = 66 \) which actually is the correct result.

In [6] has been given a discussion of how to use the Poisson solution to obtain an approximation to both the binomial and the hypergeometric solution. One of the advantages of the present Poisson tables is that they contain the solution for 14 values of \( r \) whereas the other tables only have 5 values of \( r \). The Poisson tables are therefore useful when sampling plans are needed for values of \( p_1 \) or \( p_2 \) not contained in the other tables.

The plans have been tabulated for two values of \( \gamma \) only, \( \gamma = 1 \) and \( \gamma = 5 \). Plans for other values of \( \gamma \) may be obtained from these tables by using the result of section 5 that the optimum sampling plan asymptotically only depends on the product of lot size and \( \gamma \) parameter. This leads to the following two rules:

1. For \( \gamma \leq 3 \) and a given \( N \) compute \( N^* = Ny \) and use the plan corresponding to \( N^* \) in the table for \( \gamma = 1 \).

2. For \( 3 < \gamma < 10 \) and a given \( N \) compute \( N^* = Ny/5 \) and use the plan corresponding to \( N^* \) in the table for \( \gamma = 5 \).

Numerical investigations have shown that the two rules give remarkably good approximations to the optimum sampling plan also for small values of \( N \) which means that practically all cases for \( \gamma < 10 \) have been covered by means of the two given tables. The table for \( \gamma = 1 \) tends to give too low an acceptance number when used for \( \gamma < 1 \) and too large an acceptance number when used for \( \gamma > 1 \) and analogous results hold for \( \gamma = 5 \). In most cases, however, the correct acceptance number will be found or the error will be at most one unit. It should also be noted that the error tends to increase with \( \gamma \).

It follows that the largest deviations from the exact values of \( c \) for \( \gamma < 5 \) may be expected to occur for values of \( \gamma \) around 3. To demonstrate how the formulas work in the worst case an example has been given in Table 1 where the acceptance numbers for \( \gamma = 3 \) have been derived from both tables. It will be
seen that the values of c found deviate at most 1 from the correct values apart from one case where the deviation is 2.

Table 1.

LTPD plans with minimum costs for 100p_2 = 5 and 100p_1 = 2.

Values of c for γ = 3 computed from γ = 1 and γ = 5 compared to the exact values of c.

<table>
<thead>
<tr>
<th>N</th>
<th>Exact c</th>
<th>N = 3N</th>
<th>c</th>
<th>N = 0.6N</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>All</td>
<td>90</td>
<td>0</td>
<td>18</td>
<td>All</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>150</td>
<td>0</td>
<td>30</td>
<td>All</td>
</tr>
<tr>
<td>70</td>
<td>0</td>
<td>210</td>
<td>1</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>300</td>
<td>2</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>4</td>
<td>600</td>
<td>4</td>
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<td>2</td>
</tr>
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</tr>
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<td>-</td>
<td>-</td>
<td>60000</td>
<td>29</td>
</tr>
<tr>
<td>200000</td>
<td>32</td>
<td>-</td>
<td>-</td>
<td>120000</td>
<td>32</td>
</tr>
</tbody>
</table>

Denoting the upper limit for M = Np by M(c,γ) we have the following approximate relations for the Poisson tables: For γ ≤ 3 use M(c,γ) = M(c,1)/γ and for 3 < γ < 10 use M(c,γ) = M(c,5)5/γ, i.e. compare M = My and N = Ny/5 with the limits given in the two tables.

**Example 1.** Suppose that a producer inspects lots of 1,000 items each and that he has decided on a LTPD of 5%. His average quality under normal conditions is supposed to be 1% defectives. It is furthermore assumed that k(p) = 0 from the producer's point of view, that rejection means sorting, and that costs of sorting are the same as costs of sampling inspection per item. According to (42) these assumptions lead to γ = 1 and the corresponding optimum plan may therefore be found directly in the table as n = 132 and c = 3. If, however, sorting costs are only half of sampling inspection costs per item, i.e. γ = 0.5, then the same table should be used with N* = 0.5N = 500 which gives the optimum plan n = 105 and c = 2.
If rejection means rework of the whole lot and the costs of rework per item equals the double of sampling inspection costs, i.e. $\gamma = 2$, then the table should be entered with $N^* = 2N = 2,000$ which gives the plan $n = 158$ and $c = 4$. Had $\gamma$ been 4 instead of 2 then the table for $\gamma = 5$ should be entered with $N^* = 4N/5 = 800$ which leads to $n = 184$ and $c = 5$.

Suppose now that a prior distribution of $p$ gives probability $w_1 = 0.85$ to $p = 0.01$ and probability $w_2 = 0.15$ to $p = 0.05$, that the assumptions about the costs are as above, and that the producer wants to minimize average costs under the restriction $P(0.05) = 0.10$. From (44) we then find $\gamma = 1/1.0176 = 0.98$ as compared to $\gamma = 1$ above. Therefore we find the same sampling plan. For the other three cases we find in the same manner $\gamma = 0.46, 2.33$, and 7.39, respectively. The only important change is from 4 to 7.39 which may lead to change the sampling plan (184,5) to (209,6).

Example 2. In [8] an example with $N = 500, w_1 = 0.93, w_2 = 0.009, p_1 = 0.007, p_2 = 0.080$, $\gamma_1 = 0.567$, and $\gamma_2 = 0.166$ has been discussed in details and it has been shown that the Bayesian single sampling plan is $n = 30$ and $c = 1$. This plan, however, gives a consumer's risk of 29.6% which in certain cases may be considered unsatisfactory, and we shall therefore find the restricted Bayes solution with a consumer's risk of 10%.

As $p_1$ and $p_2$ are rather small and are not to be found in the tables with binomial probabilities we shall first derive the solution by means of the Poisson tables. Since $\gamma = 0.567/(1 - 0.0168) = 0.577$ and $M = 500 \times 0.080 = 40$ we find $M7 = 23.1$. From the table for $\gamma = 1$ and $r = 0.11$ we read $c = 1$ and $n = 3.889/0.080 = 48.6$ which gives the binomial $n_b = 47$ using the formula $n_b = n - (np_2 - c)/2$, see [6]. From a table of the binomial distribution we find $P(p_2) = 0.10104$ for $n = 47$ and $P(p_2) = 0.09455$ for $n = 48$. Using $n = 48$ and $c = 1$ (so that $P(p_2) \leq 0.10$) we find $Q(p_1) = 0.06961$ and finally $R = 48 + 25.0 = 73.0$. For the Bayes solution the corresponding results are $Q(p_1) = 0.0298$ and $P(p_2) = 0.2958$ giving $R = 30 + 31.3 = 61.3$. The price to be paid for the restriction required may thus be expressed by means of the increase in costs from 61.3 to 73.0.

7. AQL sampling inspection plans with minimum costs.

AQL plans are here defined as sampling plans with a given Acceptable Quality Level, $100p_1$, and a corresponding probability of acceptance, $P(p_1)$, which traditionally is chosen as 95 per cent.

An analysis similar to the one in the previous section may be carried out from the point of view of a consumer inspecting submitted lots. Suppose that the consumer
has the following two main objectives: (1) To make reasonably sure that lots of satisfactory quality are accepted. (2) To keep his inspection costs and decision losses down.

One may now proceed formally as in section 6, i.e. select an upper limit, $p_1$, for the acceptable process average and a corresponding risk for the producer, $Q(p_1) = 0.05$ say, and then minimize the consumer's average costs for lots of unsatisfactory quality, $K(N,n,c,p_2)$, under this restriction. This procedure is, however, not satisfactory since it corresponds to a restricted Bayes solution with a one-point distribution giving probability 1 to $p = p_2$, i.e. the consumer's costs are minimized under the assumption that all submitted lots are unsatisfactory, and this will naturally give too large samples.

We shall therefore analyse the problem under the assumption that the prior distribution of $p$ is a two-point distribution with parameters $(p_1, p_2, w_2)$.

If the parameters are known and the distribution is stable and if $p_1 < p < p_2$ the Bayes solution may be determined as described in [8]. If, however, the stability of the prior distribution is doubtful and/or information on costs is incomplete the consumer may prefer a restricted Bayes solution.

Firstly the consumer may find it necessary to protect himself against some of the consequences of a deterioration of the prior distribution and for that reason he may impose the condition $Q(p_1) = 0.05$ on the plans. This should also induce the producer to keep the main component of the prior distribution at the level $p_1$ or lower.

Secondly the costs of rejection may be (partly) unknown to the consumer because even if they may seem small in the short run rejection of good lots may in the long run involve higher prices, difficulties in getting contracts, delayed deliveries, etc.

The consumer may therefore be forced to modify the model. Instead of minimizing the complete cost function without any restrictions he may choose to minimize the incomplete cost function obtained by putting $k_r(p) = 0$ under the restriction $Q(p_1) = 0.05$ hoping that the resulting low frequency of good lots rejected will reduce his costs of rejection sufficiently.

One of the effects of fixing $Q(p_1)$ may be judged by noting that on the average the ratio of number of good lots rejected to total number of lots rejected will be $w_1Q(p_1)/(w_1Q(p_1) + w_2Q(p_2))$. For $w_2 = 0.10, Q(p_1) = 0.05$, and $Q(p_2) = 1$, say, this ratio will be about $1/3$.

The sampling plan is determined by minimizing the average costs, $K(N,n,c)$, under the restriction of a fixed producer's risk, $Q(p_1) = 0.05$. This is equivalent to
minimizing \( R_o = n + (N-n)\gamma P(p_2) \) with

\[
\gamma = \frac{w_2(k_s(p_2) - k_r(p_2))}{\frac{w_2}{2}(k_s(p_2) - k_r(p_2)) + w_1(k_s(p_1) - 0.95k_a(p_1) - 0.05k_r(p_1))} = \frac{72}{1 - 0.05\gamma} \tag{46}
\]

which problem has been solved in section 4.

Let us consider the case where \( k_r(p) = 0 \). If further \( k_s(p) = S_1 \) and \( k_a(p) = A_2 \) we may introduce the break-even quality \( p_s = S_1/A_2 \), i.e. the ratio between sampling inspection costs per item of the sample and the costs resulting from accepting a defective item. From (46) we then have

\[
\gamma = \frac{w_2p_2}{(p_2 - 0.95w_1p_1)} \tag{47}
\]

which is an increasing function of \( w_2 \) taking on the maximum value \( p_2/p_s \) for \( w_2 = 1 \).

In general, if \( w_1 \) and \( p_1 \) are known only approximately, \( w_1 \) may be chosen as small as reasonable and \( p_1 \) as large as reasonable to find an upper limit for \( \gamma \) and a correspondingly large sample size.

The tables based on Poisson probabilities give \( m = np_1 \) and \( M = Np_1 \) as functions of \( c \) so that the optimum plan is \((c, m)\) for \( N_{c-1} < M < N_c \). The two functions have been tabulated for \( r = p_2/p_1 = 1.50, 1.60, 1.80, 2.00, 2.25, 2.50, 2.75, 3.0, 3.5, 4.0, 5.0, 6.5, 10.0 \), for \( \gamma = 0.2 \) and \( 1.0 \), and for \( c \leq 99 \) with the modification that tabulation has been stopped when \( M \) exceeds 50,000. Because only an abridged version is published the last figure for \( M \) given in a column may be less than 50,000 even if \( c < 99 \) which means that \( M \) exceeds 50,000 for the next entry.

The tables based on binomial probabilities give \((n, c)\) as functions of \( N \) for 20 values of \( N \) between 30 and 200,000. Plans have been computed only for \( c \leq 99 \). As values of the parameters have been used 100\( p_1 = 0.1, 0.2, 0.5, 1, 2, 3, 4, 5, 7, 10 \), five values of \( r = p_2/p_1 \) chosen among the values 1.5, 1.7, 2.0, 2.5, 3.0, 4.0, 5.0, 6.0, 10.0 (with small modifications), and \( \gamma = 0.2 \) and 1.0, giving a total of \( 10 \times 5 \times 2 = 100 \) tables. The tables also contain \( P(p_2) \) which makes it easy to compute \( R_o, R = (1 - 0.05\gamma)R_o + 0.05N\gamma \), and the average costs \( K = (k_s - k_m)R + Nk_m \).

For \( \gamma < 1 \) it may happen that acceptance without inspection is cheaper than sampling inspection for small lots. In such cases the cheapest sampling plan available (\( c \) as small as possible) has nevertheless been tabulated and the letter \( a \) (for acceptance) has been added after the sample size.

The same methods of interpolation as described for the LTPD plans should be used here.
For $\gamma \leq 0.6$ plans may be found from $N^* = N_{y/0.2}$ and the tables for $\gamma = 0.2$, whereas for $0.6 < \gamma < 2.0$ the tables for $\gamma = 1.0$ should be used with $N^* = N_{y}$.

"Interval-tables" for $N$ may be constructed as indicated for the LTPD tables.

Applications of the AQL system of sampling plans with fully specified prior distribution and cost parameters do not cause any difficulties, see Examples 3 and 4.

A comparison of the present system with other AQL systems is rather difficult since the other systems only partly are based on explicitly formulated mathematical assumptions. The systems chosen for comparison are the SRG system [12], the SMS (Swedish Military Standard) system [10], and the system recently proposed by Dodge [4]. The Military Standard 105 has not been included because the acceptance probability at the AQL value is not constant but an increasing function of lot size.

To carry out such a comparison it is obviously necessary to simplify the present system in particular with respect to the cost parameters because the other systems do not have explicitly formulated assumptions regarding costs. Using the assumptions leading to (47), and assuming furthermore (arbitrarily) for the break-even quality that $p_s = \sqrt{p_1p_2}$ we find for small values of $w_2$

$$\gamma = \frac{w_2p_2}{(p_s - p_1)} = w_2\frac{r}{(\sqrt{r}-1)}, \quad r = p_2/p_1.$$ 

The function $r/(\sqrt{r}-1)$ attains its minimum which is equal to 4 for $r = 4$ and does not exceed 5 for $2 \leq r \leq 13$ which is the domain of interest in practice. Under the assumptions stated we may therefore use $4w_2$ as a rough approximation to $\gamma$.

Table 2 contains comparisons of acceptance numbers for 5 AQL values and 7 lot sizes (sample size is the same function of acceptance number for the four systems). For the three other systems the recommended normal inspection level has been used. The present system has been denoted RB (Restricted Bayes) and the parameters have been chosen as $r = 4$ and $w_2 = 0.1$ giving $\gamma = 0.4$. The acceptance numbers have been found by entering the Poisson table for $\gamma = 0.2$ with $N = 2N_1$.

**Table 2.**

Comparison of acceptance numbers.

<table>
<thead>
<tr>
<th>AQL</th>
<th>0.1%</th>
<th>0.4%</th>
<th>1.0%</th>
<th>4.0%</th>
<th>10.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SRG</td>
<td>D</td>
<td>SMS</td>
<td>RB</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>A:1</td>
<td>All</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0</td>
<td>A:1</td>
<td>All</td>
<td>0</td>
<td></td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
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<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
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<td>3</td>
<td>18</td>
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<td>1</td>
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<td>1</td>
<td>4</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>SMS</td>
<td>RB</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
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<td>7</td>
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</tr>
<tr>
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<td>6</td>
<td>22</td>
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<tr>
<td>100000</td>
<td>9</td>
<td>10</td>
<td>8</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>
It will be seen that the SMS and the present system give nearly the same results whereas the SRG and Dodge's systems have smaller acceptance numbers for small AQL's and larger for large AQL's. (The same is true for the SMS but to a much smaller degree). One way of obtaining similar results as the SRG and Dodge within the present framework is to make \( r \) a function of \( p_1 \). From a practical point of view it seems a reasonable explanation that the SRG and Dodge have implied that the ratio between the typical bad and good quality level decreases with increasing values of the quality level itself. Looking for the values of \( r \) which will give the acceptance numbers in Table 2 we find approximately the following results:

<table>
<thead>
<tr>
<th>( 100p_1 )</th>
<th>SRG and D</th>
<th>SMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>( 100p_2 )</td>
<td>( r )</td>
</tr>
<tr>
<td>0.1</td>
<td>10.0</td>
<td>6.5</td>
</tr>
<tr>
<td>0.4</td>
<td>6.5</td>
<td>2.6</td>
</tr>
<tr>
<td>1.0</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>4.0</td>
<td>2.5</td>
<td>10.0</td>
</tr>
<tr>
<td>10.0</td>
<td>2.0</td>
<td>20.0</td>
</tr>
</tbody>
</table>

The same idea has actually been built into the tables based on binomial probabilities since the solution has been tabulated for values of \( r \) between 2 and 10 for \( 100p_1 = 0.1 \) decreasing to values of \( r \) between 1.5 and 3 for \( 100p_1 = 10.0 \).

It thus seems that the other systems have a simple interpretation within the present model. The arbitrary relationship between lot size and sample size in these systems may be converted to an (arbitrary) relationship between \( p_1 \) and \( p_2 \) which, however, is easier to interpret and understand. It will normally be much easier to reach a motivated decision with respect to the choice of \( p_2 \) than with respect to "inspection level".

Another way of influencing the amount of inspection is by varying \( \gamma \), i.e. \( w_2 \), which has the simple effect of changing the "effective lot size". If \( w_2 \) is changed from 0.10 to 0.05 the same table should be entered with a lot size half the original one.

The other systems may possibly be obtained from the present one in various other ways, but the above model seems to give one of the simplest and most useful interpretations containing only two \( (p_2 \) and \( w_2 \)) adjustable parameters. (The other systems possess a number of simple properties valuable from an administrative point of view which, however, have not been included in the discussion above).

Example 3. Suppose that a consumer inspects lots of 3,000 items each coming from a process with probability \( w_1 = 0.05 \) for \( p = 0.01 \) and probability \( w_2 = 0.15 \) for \( p = 0.03 \). It is furthermore assumed that \( k_r(p) = 0 \) from the consumer point of view, that sampling inspection costs are 0.15 units per item, and that the costs of accepting a defective are 10.0 units, which gives a break-even quality of \( p = 0.015 \). According to (47) these assumptions lead to \( \gamma = 0.650 \). The sampling plan may therefore be found
in the table for γ = 1 with N* = 3000 x 0.650 = 1950 which gives n = 399 and c = 7. The same result may be obtained from the table for γ = 0.2 with N* = 3000 x 0.650/0.2 = 9750.

**Example 4.** Using the data from Example 2, but changing the condition from 
P(p_2) = 0.10 to Q(p_1) = 0.05, we find γ = 0.168/(1-0.05 x 0.567) = 0.173. From 
M = 500 x 0.009 = 4.5 we find M* = 4.5 x 0.173/0.2 = 3.89 which should be compared to M_\text{c} in the Poisson table for γ = 0.2 and r = 0.080/0.009 = 8.9. The result is 
c = 1 and n = 0.3555/0.009 = 39.5. A table of the binomial distribution shows that 
Q(p_1) = 0.04818 for n = 39 and Q(p_1) = 0.05042 for n = 40 so that n = 39 must be preferred if the condition Q(p_1) ≤ 0.05 has to be respected. As P(p_2) = 0.16995 we find 
R = 39 + 25.8 = 64.8 which exceeds the Bayesian costs by 3.5.

8. IQL sampling inspection plans with minimum costs.

IQL plans are here defined as sampling plans with a given Indifference Quality Level, 100\%_0, and a corresponding probability of acceptance P(p_0) = 1/2.

The IQL plans are particularly well suited for use in cases where the producer and the consumer are parts of the same firm. The reasons for using the restriction 
P(p_0) = 1/2 are of a similar nature as those discussed in the two previous sections.

It is clear that all the results regarding LTPD plans with a one-point distribution of p may be used analogously for IQL plans based on minimization of \( R = n + (N-n)Q(p_1) \). Such plans are generalizations of the plans discussed by Weibull [13] and tabulated by Markblick [11] in the same sense as the LTPD plans are generalizations of the Dodge-Romig plans.

For a two-point prior distribution, however, new problems arise since it is not possible to reduce \( R = n + (N-n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2)) \) as for the LTPD and AQL plans because the restriction P(p_0) = 1/2, p_i < p_0 < p_2, cannot be used to "eliminate" Q(p_1) or P(p_2) from \( R \). The restriction leads to the desirable result that both the producer's and the consumer's risks tend to zero with increasing sample size.

The restriction

\[ P(p_0) = B(c, n, p_0) = 1/2 \]  

defines a relation \( n = n_\text{c} \) between \( n \) and \( c \). Proceeding as in section 4 we find that the plan \((c, n_\text{c})\) is optimum for \( N_{c-1} < N < N_\text{c} \) where

\[ N_\text{c} = n_{c+1} - (1-\gamma_1 Q(p_1) - \gamma_2 P(p_2))\Delta n_\text{c}/(\gamma_1 Q(p_1) + \gamma_2 P(p_2)), \]

\[ Q(p_1) = 1 - B(c, n_\text{c}, p_1), \text{ and } P(p_2) = B(c, n_\text{c}, p_2). \] Optimum plans may therefore be tabulated by a similar procedure as described in section 4 the only essential difference being
that the plans here depend on five parameters \((p_0, p_1, p_2, \gamma_1, \gamma_2)\) instead of three \((p_1, p_2, \gamma)\).

The asymptotic properties of the system may be found as in section 5. The condition (48) corresponds to

\[
c = np_0 - \frac{1}{3}(2 - p_0) + O(\frac{1}{n}),
\]

see (26), or

\[
\frac{c}{n} = h = p_0 + \frac{b}{n} + O\left(\frac{1}{n^2}\right),
\]

where \(b = -(2 - p_0)/3\).

Setting

\[
f(n, p) = \frac{\lambda}{\sqrt{n}} e^{-n\varphi(h, p_1)}
\]

and

\[
\lambda = \frac{q_0}{\sqrt{2\pi p_0 q_0}} \frac{\gamma_1 p_1}{|p_0 - p_1|}
\]

we find by means of formulas analogous to (29) and (34)

\[
f(n, p) = \varphi(p_0, p_1) + \frac{b}{n} \ln \frac{p_0 q_1}{q_0 p_1} + O\left(\frac{1}{n^2}\right)
\]

and

\[
\varphi(h, p_1) = \varphi(p_0, p_1) + \frac{b}{n} \ln \frac{p_0 q_1}{q_0 p_1} + O\left(\frac{1}{n^2}\right).
\]

For \(n \to \infty\) one of the exponential terms will be infinitely small as compared to the other depending on which of the two coefficients \(\varphi(p_0, p_1)\) and \(\varphi(p_0, p_2)\) is the larger. The solution will therefore have the same asymptotic properties as those previously studied with the exception that instead of having one risk fixed and the other inversely proportional to \(N\), both risks will here tend to zero, one inversely proportional to \(N\) and the other inversely proportional to \(N\), say, if \(\varphi(p_0, p_2) > \varphi(p_0, p_1)\).

This lack of symmetry will normally be considered unreasonable, and unless there exist strong reasons to the contrary \(p_0\) should be chosen so that \(\varphi(p_0, p_1) = \varphi(p_0, p_2)\) which gives

\[
p_0 = \left(\log \frac{q_1}{q_2}\right) \left(\log \frac{q_1 p_2}{q_2 p_1}\right).
\]

For this value of \(p_0\) we find

\[
R = n + (N-n) \frac{\lambda}{2n} e^{-n\varphi_0} (1 + O\left(\frac{1}{n^2}\right))
\]

where \(\varphi_0 = \varphi(p_0, p_1) = \varphi(p_0, p_2)\) and

\[
\lambda = \lambda_1 e^{-b\varphi_1} + \lambda_2 e^{-b\varphi_2}, \quad \varphi_1 = \ln\left(p_0 q_1/q_0 p_1\right).
\]
Minimization with respect to \( n \) gives

\[
\ln(N-n) = \varphi_o n + \frac{1}{2} \ln n - \ln(\lambda_o \varphi_o) + o(1)
\]  
(57)

and

\[
\min R = n + 1/\varphi_o + o(1).
\]  
(58)

For the risks we have

\[
Q(p_1) = \frac{1}{\lambda_o \varphi_o} \frac{1}{N-n} + o\left(\frac{1}{N}\right),
\]  
(59)

and a similar expression for \( P(p_2) \) which means that

\[
P(p_2)/Q(p_1) \rightarrow \frac{p_2(p_2 - p_1)}{p_1(p_2 - \rho)} e^{b(\tilde{\phi}_1 - \tilde{\phi}_2)} = \rho,
\]  
(60)

say. Since \( b = -2/3 \) we find

\[
\rho = \frac{p_0 - p_1}{p_2 - \rho} \left( \frac{p_2}{p_1} \right) ^{1/3} \left( \frac{q_2}{q_1} \right) ^{2/3}.
\]  
(61)

To find an approximation to \( \rho \) for small values of \( (p_1, p_2) \) we let \( p_1 \rightarrow 0 \) for fixed \( r = p_2/p_1 \) which leads to

\[
\rho \rightarrow \frac{r - 1 - \ln r}{r \ln r - r + 1} r^{1/3}.
\]  
(62)

The limiting value of \( \rho \) increases slowly from 1.00 to 1.06 for \( r \) increasing from 1 to 20. For most purposes it will be sufficiently accurate to use \( \rho = 1 \).

By means of (49) and (57) we may find good approximations to the IQL plans.

There exists, however, another possibility of approximating these plans by making use of the property (60). Writing

\[
R = n + (N-n)\frac{Q(p_1)(\gamma_1 + \gamma_2P(p_2)/Q(p_1))}{Q(p_1)}
\]

and noting that (60) does not depend on the minimization but only on the restriction \( P(p_0) = 1/2 \) we have for large \( N \) that

\[
R \sim n + (N-n)(\gamma_1 + \rho \gamma_2)Q(p_1).
\]  
(63)

It seems therefore reasonable to use the IQL plan defined by the parameters \( (p_0, p_1, \gamma) \), \( \gamma = \gamma_1 + \rho \gamma_2 \), i.e. a plan based on a one-point distribution of \( p \), as approximation to the IQL plan defined by \( (p_1, p_2, \gamma_1, \gamma_2) \).

The above result may also be derived formally from the asymptotic expressions for \( R_c \).

Consider the problem of minimizing \( R_c = n + (N-n)\lambda_0(\tilde{\phi}_1) \) for an arbitrary \( \gamma \) under the
restriction \( P(p_0) = 1/2 \). Proceeding as in section 5 we find 

\[
R_o = n + (N-n) \frac{1}{\gamma_1} e^{-b \gamma_1} \frac{1}{\sqrt{n}} e^{-n \theta} (1 + O(n^{-1/2})),
\]

which is analogous to (i3). Comparing (64) and (55) it will be seen that the two expressions are identical for

\[
\frac{\lambda_1 \gamma_1 e^{-b \gamma_1}}{\gamma} = \theta
\]

and solving for \( \gamma \) we find \( \gamma = \gamma_1 + \alpha \gamma_2 \).

It should furthermore be noticed that the IQL plans and the Bayesian plans defined by \( (p_1, p_2, \gamma_1, \gamma_2) \) have the same asymptotic properties, see [8]. For the Bayesian plans we have that \( c = np_0 + o(1) \) which means that asymptotically \( P(p_0) = 1/2 \). The asymptotic form of \( R \) which has to be minimized with respect to \( n \) to find the Bayesian plan is identical to (55) with \( \theta_0 \) substituted for \( b \). The essential difference between the Bayesian sampling plan and the corresponding IQL plan lies therefore in the different constant terms of the linear relations between \( c \) and \( n \). A good approximation to the Bayesian plan may therefore be found by looking up the value of \( c \) in the table of the corresponding IQL plan and computing \( n \) from \( c \) by means of the correct (Bayesian) relation, see the examples later in this section.

It follows that the IQL plans have economic efficiency 1 for \( N \to \infty \) as compared to the Bayesian plans.

The IQL plans have thus very desirable properties:

1. The restriction \( P(p_0) = 1/2 \) corresponds practically to a linear relation between \( n \) and \( c \).
2. The relation between \( n \) and \( N \) is approximately equal to 
   \[ \ln(N-n) = \varphi_0 n + \frac{1}{2} \ln n - \ln(\lambda_0 \omega_0). \]
3. The producer's and consumer's risks are nearly equal and tend to zero inversely proportional to \( N \).
4. Asymptotically the minimum costs are \( n + 1/\omega_0 \), i.e. decision losses tend to zero as compared to sampling inspection costs.
5. The plans for a double binomial prior distribution may be found approximately from the plans for a single binomial distribution which reduces the necessary tables greatly.
6. The IQL plans have asymptotic efficiency equal to 1 as compared to the Bayesian plans.
7. The IQL plans may be used to find good approximations to the
The tables of IQL plans are based on a one-point prior distribution and correspond to the previously discussed tables of LTPD plans. The rules given in section 6 for interpolation, construction of "interval-tables", and change of constituent parameter may therefore be applied.

The tables based on Poisson probabilities give \( M(c, \gamma) \) for \( \gamma = 1 \) and \( c \leq 99 \) for \( r = p_1/p_o = 0.10, 0.15, \ldots, 0.80 \).

The tables based on binomial probabilities show the optimum plans for \( 100p_o = 0.5, 1, 2, 3, 4, 5, 7, 10, 15 \); for five values of \( r \) chosen among the values \( 0.2, 0.3, \ldots, 0.8 \), and for \( \gamma = 1 \), giving a total of 45 tables.

Tables are given for \( \gamma = 1 \) only since these may be used to find plans for all \( \gamma < 10 \) by intererating the tables with \( N^* = N\gamma \).

On the basis of the asymptotic theory above it has been postulated that the tabulated IQL plans which are based on a one-point prior distribution may be used to find the IQL plans for a two-point prior distribution with good approximation also for small values of \( N \). This has been confirmed by numerical investigations, and a few typical examples based on Poisson probabilities are shown below for \( K_r(p) = K_s(p) \), \( p_r = 0.01 \), and \( w_2 = 0.05 \).

**Table 3.**
Comparisons of acceptance numbers for equivalent IQL plans based on one- and two-point prior distributions.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( p_1 = 0.004 )</th>
<th>( p_o = 0.02 )</th>
<th>( p_1 = 0.0050 )</th>
<th>( p_o = 0.010 )</th>
<th>( p_1 = 0.006 )</th>
<th>( p_o = 0.010 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>700</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1000</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2000</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3000</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5000</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7000</td>
<td>3</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>10000</td>
<td>4</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Under the assumptions stated \( \gamma_1 = 1 \) and \( p_o, p, \) and \( \gamma \) have been computed from (54), (61), and \( \gamma = \gamma_1 + p\gamma_2 \). It will be seen that the approximation is excellent also for small values of \( N \) even in cases where \( p_2/p_1 \) is quite small.

It has also been postulated that the acceptance number for the Bayesian sampling plan based on a two-point distribution is approximately equal to the acceptance number for the "equivalent" IQL plan for a one-point
distribution. Numerical investigations have confirmed that the approximation is good for \( p_2/p_1 > 5 \), whereas deviations of 1 or 2 may occur for \( 3 < p_2/p_1 < 5 \). For \( p_2/p_1 < 3 \) the approximation is usually poor. The following typical examples are based on the same assumptions as in Table 3.

### Table 5

Comparisons of acceptance numbers for Bayesian plans and equivalent IQL plans.

<table>
<thead>
<tr>
<th>N</th>
<th>( p_1 = 0.004 )</th>
<th>( p_1 = 0.004 )</th>
<th>( p_1 = 0.0050 )</th>
<th>( p_1 = 0.006 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p_0 = 0.02 )</td>
<td>( p_0 = 0.010 )</td>
<td>( p_0 = 0.010 )</td>
<td>( p_0 = 0.010 )</td>
</tr>
<tr>
<td></td>
<td>( p_2 = 0.06 )</td>
<td>( p_2 = 0.020 )</td>
<td>( p_2 = 0.0175 )</td>
<td>( p_2 = 0.015 )</td>
</tr>
<tr>
<td></td>
<td>( p = 0.96 )</td>
<td>( p = 0.915 )</td>
<td>( p = 0.904 )</td>
<td>( p = 0.9079 )</td>
</tr>
<tr>
<td></td>
<td>( \gamma_2 = 0.539 )</td>
<td>( \gamma_2 = 0.858 )</td>
<td>( \gamma_2 = 0.0790 )</td>
<td>( \gamma_2 = 0.66 )</td>
</tr>
<tr>
<td></td>
<td>( \gamma_1 = 1.416 )</td>
<td>( \gamma_1 = 1.089 )</td>
<td>( \gamma_1 = 1.079 )</td>
<td>( \gamma_1 = 1.071 )</td>
</tr>
<tr>
<td>300</td>
<td>1</td>
<td>0</td>
<td>Accept</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>1</td>
<td>0</td>
<td>&quot;</td>
<td>0</td>
</tr>
<tr>
<td>700</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1000</td>
<td>2</td>
<td>1</td>
<td>&quot;</td>
<td>2</td>
</tr>
<tr>
<td>2000</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3000</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>5000</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>1000</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3000</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>10000</td>
<td>6</td>
<td>6</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

**Example 5.** The data are as in Example 1 with the modification that the producer has decided on an IQL of 3% instead of an LTPD of 5%. For \( p_1 = 0.01 \), \( p_0 = 0.03 \), \( \gamma = 1 \), and \( N = 1000 \) we find the IQL plan in the table as \( n = 89 \) and \( c = 2 \). If \( \gamma \) had been equal to 1/2 instead of 1 the table should have been entered with \( N^* = 500 \) which gives the plan \( n = 56 \) and \( c = 1 \).

Suppose now that there exists a prior distribution of \( p \) with probability \( w_1 = 0.85 \) for \( p = 0.01 \) and probability \( w_2 = 0.15 \) for \( p = 0.05 \) and that the producer wants to minimize average costs under the restriction \( P(p_1) = 1 \) where \( p_0 \) is determined by (54), i.e. \( p_0 = 0.0250 \). For \( k_r(p) = k_s(p) \) and \( k_a(p) = 0 \) we find \( \gamma_1 = 1 \) and \( \gamma_2 = -0.176 \). From (61) it follows that \( \rho = 0.998 \) so that \( \gamma = \gamma_1 + \rho \gamma_2 = 0.824 \). For \( N^* = 824 \) the tables for IQL = 2% and 3% give \( c = 1 \) and 2 respectively. Consulting the Poisson table for \( r = p_1/p_0 = 0.40 \) and \( M^* = 25 \times 0.824 = 20.6 \) it will be seen that \( c = 2 \) is to be preferred. From (49) we then find \( n = 106 \).

**Example 6.** To find the IQL plan for the data in Example 2 we first compute \( p_0 = 0.033 \) by means of (54), \( p = 1.001 \) from (61), and \( \gamma = \gamma_1 + \rho \gamma_2 = 0.735 \). Entering the IQL Poisson table with \( M^* = 500 \times 0.033 \times 0.735 = 12.1 \) and \( r = 0.009/0.033 = 0.27 \) we find \( c = 1 \) and \( n = 1.678/0.033 = 51 \). The binomial probability \( P(p_0) = 0.4996 \). From \( Q(p_1) = 0.07732 \) and \( P(p_2) = 0.07733 \) we find \( N = 51 + 25.5 = 76.5 \).

Using \( c = 1 \) to find the Bayesian \( n_c \) we compute \( \alpha = (\log \frac{\gamma_2}{\gamma_1})/(\log \frac{q_1}{q_2}) = -16.4 \),
\[ \beta = 1/p_0 = 30.3, \text{ and } n_c = -16.4 + 30.3 \times 1.5 = 29 \text{ as compared to the exact solution } n = 30. \]

The results found in Examples 2, 4, and 6 have been summarized in the following table.

<table>
<thead>
<tr>
<th>Plan</th>
<th>c</th>
<th>n</th>
<th>R</th>
<th>L00Q(p₁)</th>
<th>L00P(p₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTPD</td>
<td>1</td>
<td>48</td>
<td>73.0</td>
<td>7.0</td>
<td>9.5</td>
</tr>
<tr>
<td>AQL</td>
<td>1</td>
<td>39</td>
<td>64.8</td>
<td>4.8</td>
<td>17.0</td>
</tr>
<tr>
<td>IQL</td>
<td>1</td>
<td>51</td>
<td>76.5</td>
<td>7.7</td>
<td>7.7</td>
</tr>
<tr>
<td>Bayes</td>
<td>1</td>
<td>30</td>
<td>61.3</td>
<td>3.0</td>
<td>29.6</td>
</tr>
</tbody>
</table>

9. The OC-curve.

Let the solution of the equation \( L00P(p) = \alpha, 0 < \alpha < 100, \) be denoted by \( p_\alpha \). From (49) we have with good approximation

\[ p_50 = \frac{(c + \frac{2}{3})}{(n + \frac{1}{3})}, \]

so that \( p_50 \) may be easily found for any given sampling plan.

In [6] it has been shown that an approximate solution to the equation \( L00B(c,n,p) = \alpha \) may be found by first solving the corresponding Poisson equation \( L00B(c,m) = \alpha \) with respect to \( m \) and then computing

\[ p = m/(n + \frac{m-c}{2}). \]

The accuracy of this approximation has been checked numerically for \( p_{10} \) and \( p_{95} \). The relative error is normally a decreasing function of \( c \) and an increasing function of \( p \).

For \( p_{10} \) the relative error is less than 0.5% for all \( c \) and for all \( p < 0.20 \) which means that the formula gives \( p_{10} \) to three significant figures in practically all cases.

For \( p_{95} \) the relative error is less than 0.5% for all \( c \) and all \( p < 0.05 \), less than 1% for \( p = 0.10 \), and less than 2% for \( p = 0.20 \). (For \( p = 0.20 \) the statement does not hold for \( c = 1 \) where the relative error is 4%).

Values of \( m \) as function of \( c \) may be found in the LTPD and AQL Poisson tables in the Appendix, or in a table of \( X^2 \)-fractiles.

By using this simple formula and the information given in the sampling tables it follows that at least four points on the OC-curves/known or easily found.

Consider for example the LTPD plan for \( p_2 = 0.10, p_1 = 0.04, \gamma = 1, \) and \( N = 300 \), which give \( (n,c) = (78,4) \). The table gives \( P(p_1) = 0.798 \) and \( P(p_2) = 0.10 \). The formulas above give \( p_{50} = 0.060 \) and \( p_{95} = 1.970/(78-1.02) = 0.0256 \).

It should also be noted that the same plan may occur in other columns of the same
LTPD table or in the corresponding table for \( \gamma = 5 \) in which case further values of \( 100P(p) \) may be read from the table. In the example above we find in the table for \( \gamma = 5 \) and \( p_1 = 0.03 \) the result \( P(p_1) = 0.915 \).

Consider the plan for \( N = 500 \) instead of \( N = 300 \). The plan is \((n,c) = (91,5)\) and it occurs in all columns of the table either for \( \gamma = 1 \) or \( \gamma = 5 \). Therefore 6 points on the OC-curve are given directly by the table.

As a further example consider the IQL plan for \( p_o = 0.05, p_1 = 0.03, \gamma = 1 \) and \( N = 1000 \), which give \((n,c) = (113,5)\). The table directly gives \( P(0.015) = 0.993 \), \( P(0.02) = 0.974 \), \( P(0.03) = 0.875 \), and \( P(0.05) = 0.50 \). The formula gives \( P_{0.05} = 2.613/111.8 = 0.0234 \) and \( P_{10} = 9.275/115.1 = 0.0806 \). Furthermore the relation \( P(p_1) + P(p_2) = 1 \) may be used to obtain the three approximate values of \( P(p_2) \) corresponding to the given values of \( P(p_1) \) since \( p_2 \) has been tabulated as function of \( p_o \) and \( p_1 \) on p. 31 in the Appendix.

It should finally be noted that in case \( c \) and \( p \) are known, the formula may be used to find \( n \) as

\[
n = \frac{m}{p} - \frac{m - c}{2}.
\]

10. A generalization of the AOQL system of sampling inspection plans.

Suppose that the quality distribution of the main part of lots submitted for inspection is a binomial distribution with parameter \( p_1 \) and that the prior distribution otherwise is unknown. Suppose further that the cost functions are \( k_s(p) = S_1 \), \( k_x(p) = R_1 \), and \( k_a(p) = A_2 p \). The average costs for lots of quality \( p \) due to accepted defective items then become \((1-n/N)A_2 p P(p)\) per item of the lot. In an attempt to control the damage resulting from accepted defective items one might specify an upper limit, \( k_L \) say, for these costs instead of choosing \( p_2 \) and \( P(p_2) \) as in the first part of section 6.

As a reasonable principle for determining a sampling plan one may then choose to minimize the average costs for lots of normal quality, i.e. \( R = n + (N-n)\gamma Q(p_1) \) where

\[
\gamma = \frac{(R_1 - A_2 p_1)}{(S_1 - A_2 p_1)},
\]

under the restriction that \( \max \{(1 - n/N)A_2 p P(p)\} = k_L \).

One of the advantages of this system as compared to the LTPD system is that the (arbitrary) choice of two parameters, viz. \( p_2 \) and \( P(p_2) \), is replaced by the choice of one parameter, \( k_L \), which in most cases also will be more meaningful. Furthermore the system has the property that both the producer's and the consumer's risks tend to zero for \( n \to \infty \).

It will be seen that \( k_L/A_2 = p_L \) is identical to the AOQL in Dodge and Romig's terminology [3]. For \( \gamma = 1 \) we obtain the Dodge-Romig AOQL system.
The asymptotic properties of the present system are therefore identical to the properties of the AOQL system, see Hald and Kousgaard [9], with one addition which takes into account that $\gamma$ may be different from 1. Comparing the proof in [9] with the corresponding one for the LTPD system given in section 5 it follows immediately that the result regarding $\gamma$ is valid in both cases, i.e. the sampling plan for lot size $N$ and cost parameter $\gamma$ is for large $N$ equal to the plan for lot size $N' = N\gamma$ and cost parameter 1. It is conjectured that this property holds also for small $N$ with good approximation if only $\gamma < 3$. The Dodge-Romig AOQL tables may therefore be used in such cases.

If rejected lots are rectified $p_L$ has the usual AOQL interpretation. In cases with unknown $A_2$ and rectification one may therefore specify $p_L$ and minimize $R$ with $\gamma = R_1/S_1$ since $A_2p_1$ normally is small.


We shall here compare the three systems of sampling plans and the Bayesian solution under the assumption that $p_1 < p_r < p_2$. Furthermore some comments on the three systems are given for the case where $p_r$ is unknown because one of the components, $k_3(p)$ or $k_1(p)$, of the cost function is unknown. We shall, however, always assume that $p_1$ represents a satisfactory and $p_2$ an unsatisfactory quality level so that lots of these qualities ideally should be accepted and rejected, respectively.

For a given prior distribution and given costs the optimum solution is the Bayesian one which for small $N$ often will be acceptance (or rejection) without inspection. However, if the assumption of a stable prior distribution fails and there is no inspection heavy losses may be incurred before the change will be detected. Therefore a need exists for supplementing the Bayesian solution for small $N$ with a sampling plan or for replacing the Bayesian solution in general by a system with similar properties as the Bayesian for large $N$ and leading to a reasonable sampling plan also for small $N$. It follows from the discussion in section 8 that the IQL system has the desired properties, i.e. the IQL plan with parameters $(p_0, p_1, \gamma_1, \gamma_2)$ is recommended as a substitute for the Bayesian solution with parameters $(p_1, p_2, \gamma_1, \gamma_2)$ if the possibility of a deterioration of the prior distribution has to be taken into account.

The LTPD and the AQL system may also be used for small lots but not for large lots since the economic efficiency of these plans as compared to the IQL and Bayesian plans tends to zero for $N \to \infty$. This is due to the fact that the fixed risk introduces a term proportional to $N$, $0.172N$ and $0.05\gamma_1N$ respectively, into the average costs and this term will for large $N$ dominate over the sampling inspection costs $N = O(ln N)$ and the remaining decision losses which tend to a constant, see section 5. From an economic point of view it is therefore not advisable to use the
LTPD and the AQL system for large lots. These systems will tend to give too small sample sizes and too large costs because of the fixed risk.

In view of the conclusion above it seems reasonable to try to reformulate the ideas behind the LTPD and the AQL system so that the fixed risk required only becomes of importance for small lots. This might be done by minimizing the average costs under the restriction \( P(p_2) = \beta \) for \( N \leq N_0 \) and \( P(p_2) = \beta N_0 / N \) for \( N > N_0 \) (or similarly \( Q(p_1) = \alpha \) for \( N \leq N_0 \) and \( Q(p_1) = \alpha N_0 / N \) for \( N > N_0 \)). Theory and tables for such plans may easily be developed along similar lines as in the present paper, but they have the drawback of depending on two arbitrary parameters, \((\beta, N_0)\) or \((\alpha, N_0)\).

It is possibly not worth while pursuing this idea further because a similar effect may be obtained by switching over from the LTPD (or AQL) system to the IQL system for a certain value of \( N, N_0 \) say. If for some specific reason an upper limit of 10% has been fixed for the consumer's risk one may use the LTPD system for \( N \leq N_0 \) and the IQL system for \( N > N_0 \) where \( N_0 \) is determined so that IQL plans for \( N > N_0 \) all have \( P(p_2) < 0.10 \).

As discussed previously another reason for introducing restrictions on the Bayes solution may be lack of detailed knowledge of one of the cost components, \( k_3(p) \) or \( k_r(p) \). This does not, however, change the results of the above discussion if only it is clear that \( p_1 \) represents a satisfactory and \( p_2 \) an unsatisfactory quality level. Since the economic consequences of wrong decisions are more serious for large than for small lots it is necessary that the risk of wrong decisions decreases with increasing lot size. This condition is satisfied by the IQL system but not by the LTPD and AQL systems.

Acknowledgements.

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References.


Appendix of Tables
---------------------

LTPD tables with consumer's risk of 10%

<table>
<thead>
<tr>
<th>Table Type</th>
<th>Pages</th>
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</thead>
<tbody>
<tr>
<td>Description of tables</td>
<td>2 - 3</td>
</tr>
<tr>
<td>Tables with binomial risks. γ = 1.</td>
<td>4 - 8</td>
</tr>
<tr>
<td>Tables with binomial risks. γ = 5.</td>
<td>9 - 13</td>
</tr>
<tr>
<td>Tables with Poisson risks. γ = 1.</td>
<td>14</td>
</tr>
<tr>
<td>Tables with Poisson risks. γ = 5.</td>
<td>15 - 16</td>
</tr>
</tbody>
</table>

AQL tables with producer's risk of 5%

<table>
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<td>Tables with binomial risks. γ = 0.2.</td>
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</tr>
<tr>
<td>Tables with binomial risks. γ = 1.</td>
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</tr>
<tr>
<td>Tables with Poisson risks. γ = 0.2.</td>
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<td>Tables with Poisson risks. γ = 1.</td>
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IQOL tables with risk of 50%

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<tr>
<td>Tables with Poisson risks. γ = 1.</td>
<td>38 - 39</td>
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<tr>
<td>Tables of p₀.</td>
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</table>
LTPD single sampling tables with consumer's risk of 10% and minimum average costs.

The tables on pp. 4 - 13 are based on a binomial consumer's risk of 10%, \( P(p_2) = 0.10 \), and a binomial producer's risk, \( Q(p_1) = 1 - P(p_1) \). The sampling plans given minimize the average costs \( R_0 = n + (N - n) \gamma Q(p_1) \).

The same plans minimize the average costs \( R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2)) \) for \( P(p_2) = 0.10 \) since \( R = (1 - 0.1 \gamma_2) R_0 + 0.1 \gamma_2 N \) with \( \gamma = \gamma_1/(1 - 0.1 \gamma_2) \).

The condition \( P(p_2) = 0.10 \) has been fulfilled as nearly as possible in the way that \( n \) has been determined as the smallest integer satisfying \( B(c, n, p_2) \leq 0.10 \).

The tables give \( n, c \) and 100 \( P(p_1) \) as functions of \( N \) for \( \gamma = 1 \) and 5, and for the following 50 combinations of 100 \( p_2 \) and 100 \( p_1 \):

<table>
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<tr>
<th>( 100p_2 )</th>
<th>( 100p_1 )</th>
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<td>0.05</td>
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<tr>
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<tr>
<td>3</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
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<tr>
<td>7</td>
<td>2.1</td>
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<tr>
<td>10</td>
<td>3.0</td>
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<td>15</td>
<td>4.5</td>
</tr>
<tr>
<td>20</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Methods of interpolation have been discussed in section 6.
The tables may be used for $\gamma \ge 1$ and $\gamma \ge 5$ in the following way: For $\gamma \le 3$ compute $N^* = N\gamma$ and use the plan for $N^*$ and $\gamma = 1$. For $3 < \gamma < 10$ compute $N^* = N\gamma/5$ and use the plan for $N^*$ and $\gamma = 5$.

For $\gamma > 1$ total inspection is cheaper than sampling inspection if $Q(p_1) > 1/\gamma$. In such cases the letter $t$ has been added after the sample size.

If the consumer's risk is defined as a hypergeometric instead of a binomial probability a good approximation to the solution may be obtained by using the binomial $c$ and correcting the binomial $n$ to $n_h = n\left[1 - (np_2\cdot c)/(2Np_2)\right]$.

The tables on pp. 14 - 16 are based on the same assumptions with the only modification that the consumer's and the producer's risks have been computed from Poisson probabilities. The functions $m = np_2$ and $M = Np_2$ have been tabulated for $M < 50,000$ with $c$ and $r = p_1/p_2$ as arguments for $c \le 99$ and $r = 0.05, 0.10, \ldots, 0.70$, and for $\gamma = 1$ and 5. The optimum plan is $(c,m)$ for $M(c) < H < M(r)$.

For $\gamma \le 3$ use $M^* = M\gamma$ and the table for $\gamma = 1$. For $3 < \gamma < 10$ use $M^* = M\gamma/5$ and the table for $\gamma = 5$.

Underlining of $M$ in the table for $\gamma = 5$ means that total inspection is cheaper than sampling inspection.

An approximation to the "binomial solution" may be obtained by using $c$ from the Poisson table and correcting the corresponding $n$ to $n_h = n - (np_2\cdot c)/2$. 
Single Sampling Tables for $LTPD = 0.5$ per cent and $\gamma = 1$

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Single Sampling Tables for $LTPD = 1.0$ per cent and $\gamma = 1$

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### Single Sampling Tables for LTPD = 2.0 per cent and γ = 1

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<td>265 2 64.4</td>
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### Single Sampling Tables for LTPD = 3.0 per cent and γ = 1

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<td>76 0 40.0</td>
<td>76 0 31.7</td>
</tr>
<tr>
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<td>125 1 31.4</td>
</tr>
<tr>
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<td>176 2 64.6</td>
<td>176 2 50.5</td>
<td>176 2 38.6</td>
</tr>
<tr>
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<td>125 1 94.2</td>
<td>176 2 78.8</td>
<td>176 2 64.6</td>
<td>176 2 50.5</td>
<td>176 2 38.6</td>
</tr>
<tr>
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<td>176 2 64.6</td>
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</table>

*Note: The table continues with similar entries for different sample sizes and LTPD values.*
Single Sampling Tables for LTPD = 4.0 per cent and $\gamma = 1$

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<tr>
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<td>198 4 78,7</td>
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<tr>
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<tr>
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Single Sampling Tables for LTPD = 5.0 per cent and $\gamma = 1$

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### Single Sampling Tables for LTPD = 10.0 per cent and γ = 1

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All values are percentage of defective units allowed in a sample for conformance with the LTPD specification.
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### Single Sampling Tables for LTPD = 20.0 per cent and \( \gamma = 1 \)

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### Single Sampling Tables for LTPD = 0.5 per cent and \( \gamma = 5 \)

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### Single Sampling Tables for LTPD = 1.0 per cent and \( \gamma = 5 \)

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N 为样本大小，n 为样本中不合格产品的数量，c 为允许不合格产品的数量。
### Single Sampling Tables for LTPD = 2.0 per cent and \( \gamma = 5 \)

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### Single Sampling Tables for LTPD = 3.0 per cent and \( \gamma = 5 \)

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### Additional Data

- For LTPD = 2.0 per cent and \( \gamma = 5 \)
- For LTPD = 3.0 per cent and \( \gamma = 5 \)
- Tables include values for various sample sizes and LTPD percentages.
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Single Sampling Tables with Consumer’s Risk of 10% 

\( B(c,m) = 0.10, r = p_1/p_2, m = n p_2, M = N p_2, \gamma = 1. \)
Single Sampling Tables with Consumers Risk of 10%

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AQL single sampling tables with producer's risk of 5 %
and minimum average costs.

The tables on pp. 19 - 28 are based on a binomial producer's risk of 5 %, \( Q(p_1) = 0.05 \), and a binomial consumer's risk, \( P(r_2) \). The sampling plans given minimize the average costs \( R = n + (N - n)\gamma P(p_2) \).

The same plans minimize the average costs \( R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2)) \) for \( Q(p_1) = 0.05 \) since \( R = (1 - 0.05\gamma_1)R_o + 0.05\gamma_1N \) with \( \gamma = \gamma_2/(1 - 0.05\gamma_1) \).

The condition \( Q(p_1) = 0.05 \) has been fulfilled as nearly as possible in the way that \( n \) has been determined as the largest integer satisfying \( B(c,n,p_1) \geq 0.95 \).

The tables give \( n, c \) and \( 100P(p_2) \) as functions of \( N \) for \( \gamma = 0.2 \) and \( 1.0 \), and for the following 50 combinations of \( 100p_1 \) and \( 100p_2 \):

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<th>( 100p_1 )</th>
<th>( 100p_2 )</th>
</tr>
</thead>
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</tr>
<tr>
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<td>0.4</td>
</tr>
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<tr>
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</tr>
<tr>
<td>7.0</td>
<td>10.5</td>
</tr>
<tr>
<td>10.0</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Methods of interpolation have been discussed in section 6.

The tables may be used for \( \gamma \leq 0.2 \) and \( \gamma \geq 1.0 \) in the following way: For \( \gamma \leq 0.6 \) compute \( N^* = Ny/0.2 \) and use the plan for \( N^* \) and \( \gamma = 0.2 \). For \( 0.6 < \gamma < 2 \) compute \( N^* = Ny \) and use the plan for \( N^* \) and \( \gamma = 1 \).
For $\gamma < 1$ it may happen that acceptance without inspection is cheaper than sampling inspection for small lots. In such cases the letter a has been added after the sample size.

The tables on pp. 29 - 30 are based on the same assumptions with the only modification that the consumer's and the producer's risks have been computed from Poisson probabilities. The functions $m = np_1$ and $M = Np_1$ have been tabulated for $M < 50,000$ with $c$ and $r = p_2/p_1$ as arguments for $c \leq 99$ and $r = 1.50, 1.60, 1.80, 2.00, 2.25, 2.50, 2.75, 3.0, 3.5, 4.0, 5.0, 6.5, 10.0,$ and for $\gamma = 0.2$ and 1.0. The optimum plan is $(c, m)$ for $M(c-1) < M < M(c)$.

For $\gamma \leq 0.6$ use $M^* = My/0.2$ and the table for $\gamma = 0.2$. For $0.6 < \gamma < 2$ use $M^* = My$ and the table for $\gamma = 1$.

Underlining of $M$ in the table for $\gamma = 0.2$ means that acceptance without inspection is cheaper than sampling inspection.

An approximation to the "binomial solution" may be obtained by using $c$ from the Poisson table and correcting the corresponding $n$ to $n_b = n + (c - np_1)/2$. 
### Single Sampling Tables for AQL = 0.1 per cent and γ = .2

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<td>n c 100P</td>
<td>n c 100P</td>
<td>n c 100P</td>
<td>n c 100P</td>
</tr>
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<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
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<td>51a 0</td>
<td>59.9</td>
<td>51a 0</td>
<td>81.5</td>
<td>51a 0</td>
</tr>
<tr>
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<td>51a 0</td>
<td>59.9</td>
<td>51a 0</td>
<td>81.5</td>
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<td>51a 0</td>
<td>81.5</td>
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</tr>
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<td>51a 0</td>
<td>81.5</td>
<td>51a 0</td>
</tr>
<tr>
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<td>51a 0</td>
<td>59.9</td>
<td>51a 0</td>
<td>81.5</td>
<td>51a 0</td>
</tr>
<tr>
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<td>59.9</td>
<td>51a 0</td>
<td>81.5</td>
<td>51a 0</td>
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<tr>
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<td>51a 0</td>
<td>59.9</td>
<td>51a 0</td>
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<td>51a 0</td>
</tr>
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<td>59.9</td>
<td>51a 0</td>
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</tr>
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<td>51a 0</td>
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</tr>
<tr>
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<tr>
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<td>51a 0</td>
<td>59.9</td>
<td>51a 0</td>
<td>81.5</td>
<td>51a 0</td>
</tr>
<tr>
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<td>51a 0</td>
<td>81.5</td>
<td>51a 0</td>
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### Single Sampling Tables for AQL = 0.2 per cent and γ = .2

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<td>n c 100P</td>
<td>n c 100P</td>
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<td>25a 0</td>
<td>81.8</td>
<td>25a 0</td>
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<td>25a 0</td>
<td>81.8</td>
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## Single Sampling Tables for AQL = 0.5 per cent and \( y = 0.2 \)

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## Single Sampling Tables for AQL = 1.0 per cent and \( y = 0.2 \)

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## Single Sampling Tables for AQL = 2.0 per cent and $\gamma = .2$

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### Additional Data for Various AQLs

- **Single Sampling Tables for AQL = 4.0 per cent and \( y = 0.2 \)**
- **Single Sampling Tables for AQL = 5.0 per cent and \( y = 0.2 \)**
### Single Sampling Tables for AQL = 7.0 per cent and \( \gamma = .2 \)

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| 3000       | 56 9 | 1.3 | 71 11 | 3.8 | 119 17 | 7.0 | 177 24 | 13.0 | 211 28 | 27.7 |
| 4000       | 56 9 | 1.3 | 79 12 | 2.5 | 152 21 | 3.1 | 211 28 | 8.5 | 289 37 | 16.8 |
| 7000       | 63 10 | 0.8 | 94 14 | 1.3 | 152 21 | 3.1 | 254 33 | 4.9 | 368 46 | 10.0 |
| 10000      | 63 10 | 0.8 | 102 15 | 1.3 | 177 24 | 1.7 | 280 25 | 3.5 | 421 52 | 7.0 |

<p>| 20000      | 71 11 | 0.4 | 119 17 | 0.3 | 194 26 | 5.1 | 522 41 | 1.9 | 511 62 | 5.7 |
| 30000      | 71 11 | 0.4 | 119 17 | 0.3 | 211 28 | 0.7 | 559 45 | 1.2 | 574 69 | 2.4 |
| 40000      | 86 13 | 0.1 | 127 18 | 0.2 | 223 30 | 0.5 | 405 50 | 0.7 | 638 76 | 1.5 |
| 70000      | 86 13 | 0.1 | 135 19 | 0.1 | 254 33 | 0.2 | 421 52 | 0.5 | 702 63 | 0.7 |
| 100000     | 94 14 | 0.1 | 143 20 | 0.1 | 283 35 | 0.2 | 443 55 | 0.4 | 757 68 | 0.6 |
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Single Sampling Tables with Producer's Risk of 5%.

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### Single Sampling Tables with Producer's Risk of 5%.

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<td>( M )</td>
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<td>7</td>
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</tbody>
</table>
IQL single sampling tables with minimum average costs.

The tables on pp. 33 - 37 are based on a binomial risk of 50% for lots of quality \( p_0 \), i.e. \( r(p_0) = 0.50 \), and a binomial producer's risk, \( Q(p_1) = 1 - P(p_1) \).

The sampling plans given minimize the average costs \( R_0 = n + (N - n)Q(p_1) \).

The same plans will with good approximation minimize the average costs
\[
R = n + (N - n)(\gamma Q(p_1) + \beta P(p_2)) \quad \text{for} \quad P(p_1) = 0.50 \quad \text{where} \quad \gamma = \gamma_1 + \gamma_2 \\
p_0 = (\log \frac{q_1}{q_2}) / (\log \frac{p_1}{p_2 q_1}).
\]

The condition \( P(p_0) = 0.50 \) has been fulfilled as nearly as possible in the way that \( n \) has been determined as the integer for which \( I(c, n, p_0) \) is nearest to 0.50.

The tables give \( n, c, \) and 100 \( P(p_1) \) as functions of \( N \) for \( \gamma = 1 \) and for the following 45 combinations of 100 \( p_0 \) and 100 \( p_1 \) (100 \( p_2 \) has been added in parenthesis after 100 \( p_1 \)):

<table>
<thead>
<tr>
<th>100p_0</th>
<th>100p_1 (100p_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.1 (1.42) 0.15 (1.18) 0.2 (1.01) 0.25 (0.877) 0.3 (0.773)</td>
</tr>
<tr>
<td>1</td>
<td>0.2 (2.84) 0.3 (2.35) 0.4 (2.01) 0.5 (1.75) 0.6 (1.55)</td>
</tr>
<tr>
<td>2</td>
<td>0.4 (5.63) 0.6 (4.68) 0.8 (4.01) 1.0 (3.50) 1.2 (3.09)</td>
</tr>
<tr>
<td>3</td>
<td>0.6 (8.39) 0.9 (6.99) 1.2 (6.00) 1.5 (5.24) 1.8 (4.62)</td>
</tr>
<tr>
<td>4</td>
<td>1.2 (9.27) 1.6 (7.97) 2.0 (6.96) 2.4 (6.16) 2.8 (5.49)</td>
</tr>
<tr>
<td>5</td>
<td>1.5 (11.5) 2.0 (9.92) 2.5 (8.69) 3.0 (7.69) 3.5 (6.85)</td>
</tr>
<tr>
<td>7</td>
<td>2.8 (13.8) 3.5 (12.1) 4.2 (10.7) 4.9 (9.58) 5.6 (8.60)</td>
</tr>
<tr>
<td>10</td>
<td>4.0 (19.5) 5.0 (17.2) 6.0 (15.3) 7.0 (13.7) 8.0 (12.3)</td>
</tr>
<tr>
<td>15</td>
<td>6.0 (28.7) 7.5 (25.4) 9.0 (22.7) 10.5 (20.4) 12.0 (18.4)</td>
</tr>
</tbody>
</table>

Methods of interpolation have been discussed in section 6.

The tables may be used for \( \gamma < 10 \) by computing \( N^* = N\gamma \) and finding the plan for \( N^* \) and \( \gamma = 1 \).
The tables on pp. 38 - 39 are based on the same assumptions with the only modification that the risks have been computed from Poisson probabilities. The functions \( m = np_0 \) and \( M = Np_0 \) have been tabulated for \( M < 50,000 \) with \( c \) and \( r = p_1/p_0 \) as arguments for \( c \leq 99 \) and \( r = 0.10, 0.15, \ldots, 0.80 \), and for \( \gamma = 1 \). The optimum plan is \( (c,m) \) for \( M(c-1) < M < M(c) \).

For \( \gamma < 10 \) use \( M^* = M \gamma \) and the table for \( \gamma = 1 \).

An approximation to the "binomial solution" may be obtained by using \( c \) from the Poisson table and correcting the corresponding \( n \) to \( n_b = n - 1/3 \) or by computing \( n_b \) directly as \( n_b = (c + (2 - p_0)/3)/p_0 \).

An auxiliary table of \( p_0 \) as function of \( p_1 \) and \( r = p_2/p_1 \) has been given on pp. 40 - 41.
### Single Sampling Tables for IQL = 0.5 per cent and $\gamma = 1$

<table>
<thead>
<tr>
<th>$100p_1$</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
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<td>c 100P</td>
<td>c 100P</td>
<td>c 100P</td>
<td>c 100P</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>100</td>
<td>138</td>
<td>87.1</td>
<td>81.9</td>
<td>79.7</td>
<td>75.9</td>
</tr>
<tr>
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<td>81.9</td>
<td>79.7</td>
<td>75.9</td>
</tr>
<tr>
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<td>87.1</td>
<td>81.9</td>
<td>79.7</td>
<td>75.9</td>
</tr>
<tr>
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<td>87.1</td>
<td>81.9</td>
<td>79.7</td>
<td>75.9</td>
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<tr>
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<td>81.9</td>
<td>79.7</td>
<td>75.9</td>
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<tr>
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<td>138</td>
<td>87.1</td>
<td>81.9</td>
<td>79.7</td>
<td>75.9</td>
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### Single Sampling Tables for IQL = 1.0 per cent and $\gamma = 1$

<table>
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<th>0.40</th>
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<th>0.60</th>
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</tr>
<tr>
<td>All</td>
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<td>-</td>
<td>-</td>
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<td>69</td>
<td>87.1</td>
<td>81.9</td>
<td>79.7</td>
<td>75.9</td>
</tr>
<tr>
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<td>79.7</td>
<td>75.9</td>
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### Single Sampling Tables for IQL = 2.0 per cent and \( \gamma \geq 1 \)

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<td>87.3</td>
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### Single Sampling Tables for IQL = 3.0 per cent and \( \gamma \geq 1 \)

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- All \( \gamma \) values and \( \gamma \geq 1 \) for 2.0 and 3.0 per cent IQL are provided in the tables.
### Single Sampling Tables for IQL = 4.0 per cent and \( \gamma = 1 \)

<table>
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<td>( c )</td>
<td>( 100p )</td>
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<td>( c )</td>
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<td>0</td>
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<tr>
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### Single Sampling Tables for IQL = 5.0 per cent and \( \gamma = 1 \)

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Single Sampling Tables with Risk of 50% for Lots of Indifference Quality

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