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THE ELECTRICAL CONDUCTIVITY OF AIR UP TO 24,000°K

By

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I. INTRODUCTION

The electrical conductivity of air in thermodynamic equilibrium at temperatures up to about 7,000°K is known with considerable reliability from shock tube experiments reported by Lamb and Lin*). These data have been used to compute the conductivity directly as a function of temperature and density**). In this case, the contribution of the positive ions to the total electron collision probability is still relatively small, but becomes increasingly important at higher temperatures.

It appears that no detailed experimental data are available at present above 7,000°K. On the other hand, many of the contemplated applications of magnetoaerodynamics involve considerably higher temperatures. In this paper estimates of the conductivity up to 24,000°K are computed. This upper limit was decided upon because it coincides with the limit of Gilmore's computations of the equilibrium composition of air***).

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*) Lamb, L. and Lin, S. : Electrical Conductivity of Thermally Ionized Air Produced in a Shock Tube. Jr. of Appl. Phys., Vol. 28, No. 7, 1957.

**) Bush, W B. · Calculation of Electrical Conductivity of Air. The Ramo-Wooldridge Corporation, Physical Research Laboratory Report ARL-7-61, 1957.

***) Gilmore, F. R. :Equilibrium Composition and Thermodynamic Properties of Air to 24,000°K. Rand Corp. Research Memo, RM-1543, 1955.

II. LIST OF SYMBOLS

B	magnetic induction
C	rms value of velocity
$\bar{C}$	average velocity
e	electronic charge
e^-	number of free electrons per "air atom"
H	height above sea level
k	Boltzmann's constant
ℓ	characteristic length
m	particle mass
n	number density
n_A	number density of "air atoms"
N, N_2, 0	number of (dissociated) N-atoms, N-molecules, 0-atoms, per "air atom"
Q	parameter
$\bar{Q}$	Maxwell-averaged total electron collision cross section.
T	temperature
u	velocity behind normal shock
V	flight velocity
ν	collision frequency
ρ	density
σ	electric conductivity
ω	cyclotron frequency

Subscripts

e	electrons
i	ions
n	neutrals
o	standard temperature and pressure
∞	free-stream condition

III. THE ELECTRICAL CONDUCTIVITY OF AIR

The usual practice in computing the conductivity of partially ionized gases is to add the Maxwell-averaged total electron collision cross sections (weighted with the number density) of the neutral molecules to a corresponding equivalent cross section of the ions, the latter being computed on the basis of a theory which applies to fully ionized gases. The degree of approximation involved in this assumption is not known. In the case of air, a further element of uncertainty is introduced by the fact that the cross sections of some of the neutral species, notably atomic nitrogen, are not known with certainty.

The conductivity σ is expressed by

$$\sigma = \frac{n_e e^2}{m_e \bar{C}_e \sum_j n_j \bar{Q}_j} \qquad (1)$$

where n_e is the number density of free electrons, e the electronic charge, m_e the electronic mass, n_j the number density of the species j, $\bar{Q}_j$ its Maxwell-averaged total electron collision cross section, and where

$$\bar{C}_e = \left(\frac{8kT}{\pi m_e} \right)^{1/2}$$

(k = Boltzmann's constant, T = temperature) is the mean speed of the electrons. Thermodynamic equilibrium with an electron temperature equal to the gas temperature is assumed.

The summation in Eq. (1) is extended over those species, neutrals as well as ions, which contribute appreciably to the total cross section. An inspection of the equilibrium composition of air in the range from 6 000°K to 24,000°K and a density ratio ρ/ρ_0 (ρ_0 = standard density) varying from 10^{-3} to 10 (Gilmore, 1955) indicates that the neutrals of major importance are N_2, N and 0. In view of the approximate nature of the computation, the inclusion of 0_2, N0 and other, even less frequent species, would not be warranted. The cross sections for N_2, N and 0 above an electron temperature of $6,000^{\circ}K$ are believed to be quite similar, and nearly independent of the energy in the range considered (cf. Lamb and Lin, 1957). At much higher temperatures, the effect of the neutrals becomes negligible. The computations can be based therefore on a common cross section of the neutrals, $\overline{Q}_n$, which was taken as 0.80×10^{-15} cm^2. In view of the uncertainty in the cross sectional data of N and 0, a more detailed computation would not be justified.

The sum over all cross sections in Eq. (1), can therefore be expressed as

$$\sum_j n_j \overline{Q}_j = n_n \overline{Q}_n + n_i \overline{Q}_i$$

$$= n_{A,o} \frac{\rho}{\rho_o} \left[(N_2 + N + 0) \overline{Q}_n + e^- \overline{Q}_i \right] \qquad (2)$$

where n_n and n_i are the number densities of neutrals and ions, respectively. $n_{A,o}$ is the number density of "air atoms" at standard condition; i.e. $n_{A,o} = 5.38 \times 10^{19}$ cm^{-3}. N_2, N, 0 and e^- are the number of N-molecules, N-atoms, 0-atoms, and free electrons, respectively per "air atom". The values computed by Gilmore have been used for these fractions.

Within the limits of temperature and density considered here, the effect of doubly ionized ions on the conductivity can be neglected.

Similarly, the effect of 0^- is negligible. Consequently, for a neutral plasma,

$$n_i = n_e = n_{A,o} \frac{\rho}{\rho_o} e^- \tag{3}$$

a result, which was already utilized in the derivation of Eq. (2).

Spitzer and Härm's*) results for the conductivity of a fully ionized gas, if written in terms of an equivalent cross section $\overline{Q}_i$ for the positive ions, give

$$\overline{Q}_i = \left(\frac{\pi e^2}{4kT}\right)^2 \frac{\ln(qC_e^2)}{\gamma_E} \tag{4}$$

for singly ionized ions, where

$$q = \frac{m_e}{2e^3}\left[\frac{kT}{2\pi n_e}\right]^{1/2}$$

and $\gamma_E = 0.582$. C_e is the rms electron velocity,

$$C_e = \left(\frac{3kT}{m_e}\right)^{1/2}$$

The results of a computation of σ, based on Eqs. (1) to (4)

*) Spitzer, L. and Härm, R. : Transport Phenomena in a Completely Ionized Gas. Phys. Rev., Vol. 89, No. 5, 1953.

Cf. also Cohen, R. S., Spitzer, L. and Routly, P. : The Electrical Conductivity of an Ionized Gas. Phys. Rev., Vol. 80, No. 2, 1950.

are given in Fig. 1. It is noted that the dependence upon density is relatively weak, and that its gradient reverses within the considered interval of temperature. At the lower temperatures, the largest contribution to the total cross section comes from the neutrals. At the higher temperatures considered, the situation is reversed; i.e. the effective cross section of the positive ions give the principal contribution. In the latter case, the dependence of σ upon the density is due to the logarithmic term in Eq. (4). Above $24,000^{\circ}K$ the effect of the second ionization becomes increasingly important.

IV. COMPARISON WITH EXPERIMENTAL VALUES

At the lower end of the temperature interval, the computed conductivity agrees with the experimental data of Lamb and Lin. The computed values fall inside the experimental scatter. However, this agreement is to be expected, since cross sectional values were used, which are derived from these data.

At indirect check of the conductivity at a higher temperature is obtained from the experimental data reported by Ziemer and Bush*), which would indicate an approximate agreement between the computed and experimentally determined conductivity for the conditions of these tests ($T = 17{,}000^\circ K$ to $18{,}000^\circ K$, $\rho/\rho_0 = 2 \times 10^{-2}$).

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*) Ziemer, R. W. and Bush, W. B.: Magnetic Field Effects on Bow Shock Stand-Off Distance. The Ramo-Wooldridge Corp., Phys. Res. Lab. Report GM-TR-0127-00396, 1958.

V. CONDUCTIVITY OF AIR BEHIND A NORMAL SHOCK

Gilmore's results for the thermodynamic variables of state of the air behind a normal shock resulting from hypersonic velocities, have been used to calculate the conductivity, for different flight velocities V and altitudes H above sea level (Fig. 2). For comparison, the velocity of a satellite in a circular orbit of 7×10^6 m radius, and the escape velocity, have also been indicated.

In many applications of magnetoaerodynamics, the parameter

$$Q = \sigma B^2 \ell / \rho u \tag{5}$$

plays an important role, since it is a measure for the ratio of the magnetic forces to the inertial forces. σ, ρ and u are defined as the conductivity, density and velocity behind a normal shock. B is the magnetic induction, and ℓ a characteristic dimension associated with the body producing the shock wave. In general, Q must be at least of order one, if the magnetic field is to produce an appreciable effect.

In order to permit a quick estimate of the typical field strength required in magnetoaerodynamics (assuming thermodynamic equilibrium), Fig. 3 has been drawn. In this figure, the required field strength is given for an assumed $Q = 1$ and $\ell = 1$m. Since from conservation of mass

$$\rho u = \rho_\infty V$$

where $\rho_\infty = \rho_\infty(H)$ is the free-stream density, the computation of B as a function of H and V is straight-forward.

Eq. (1) gives σ correctly only in the case of weak fields. Otherwise, the conductivity is no longer described by a scalar quantity. This occurs if the cyclotron frequency ω_e of the electrons is comparable or larger than their collision frequency ν_e.

From

$$\omega_e = \frac{Be}{m_e}$$

and

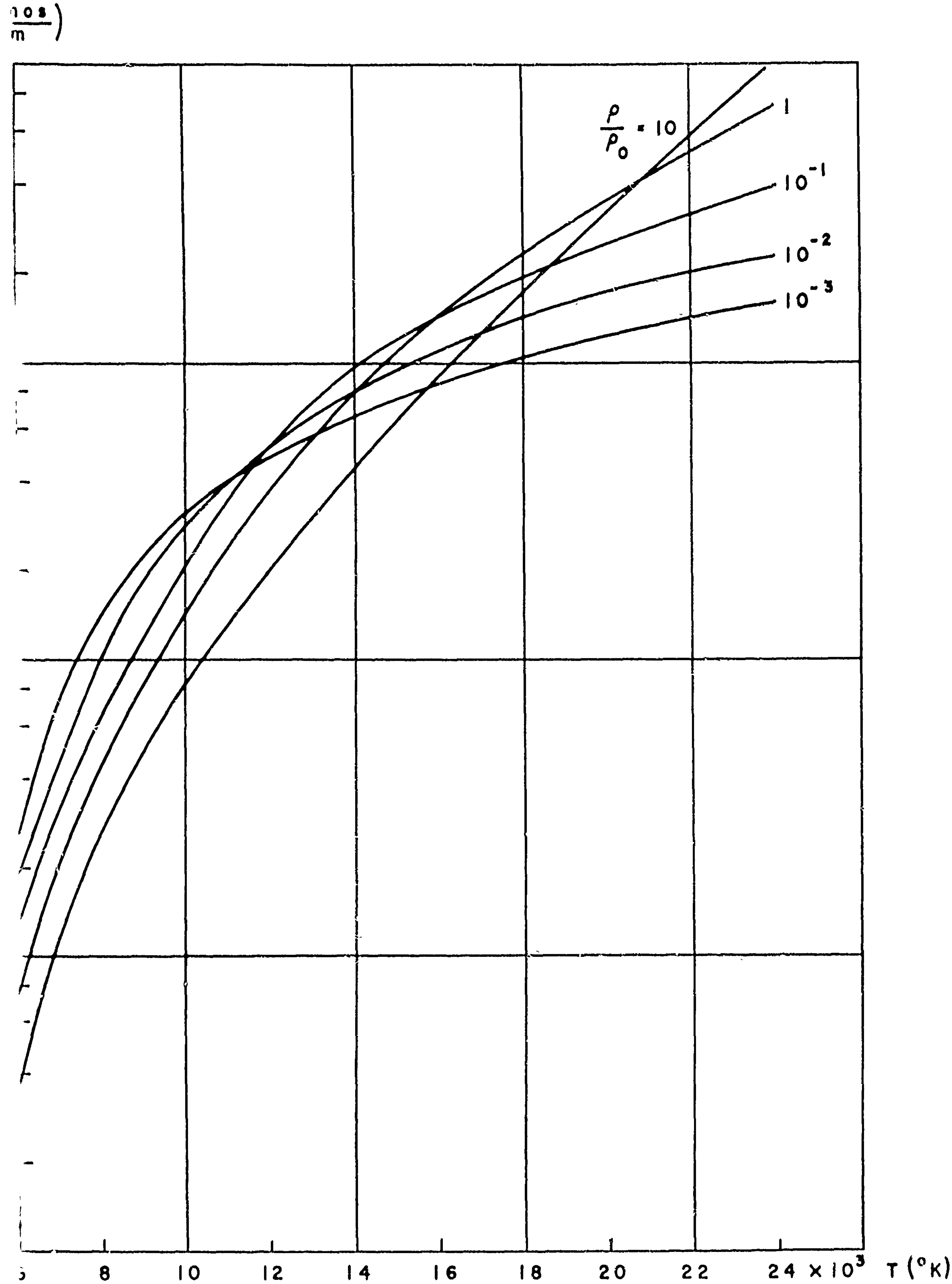
$$\sigma = \frac{n_e e^2}{m_e \nu_e}$$

it follows from equating ω_e and ν_e, that

$$B' = \frac{n_e e}{\sigma} \tag{6}$$

where B' can be considered as a rough estimate of the maximum field strength B for which Eq. (1) is still valid. The locus of points in the H-V diagram for which this limiting condition exists, is indicated in the figure. To the left of this curve, $\omega_e > \nu_e$.

The writer is indebted to Dr. Burton D. Fried for a number of helpful discussions.



nos
m
$\frac{\rho}{\rho_0}$ = 10
1
10^{-1}
10^{-2}
10^{-3}
6
8
10
12
14
16
18
20
22
24×10^3
T (°K)

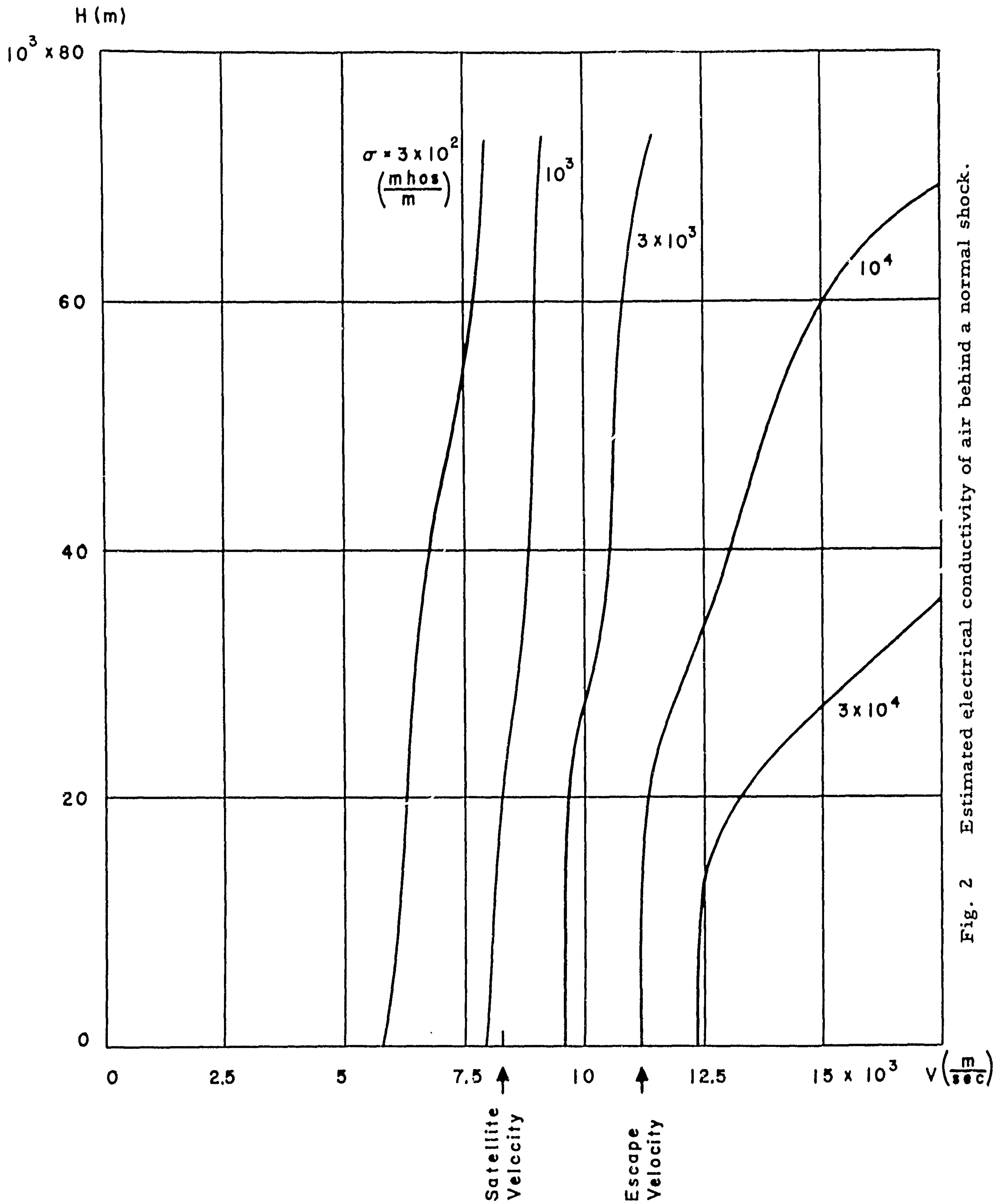


Fig. 2 Estimated electrical conductivity of air behind a normal shock.

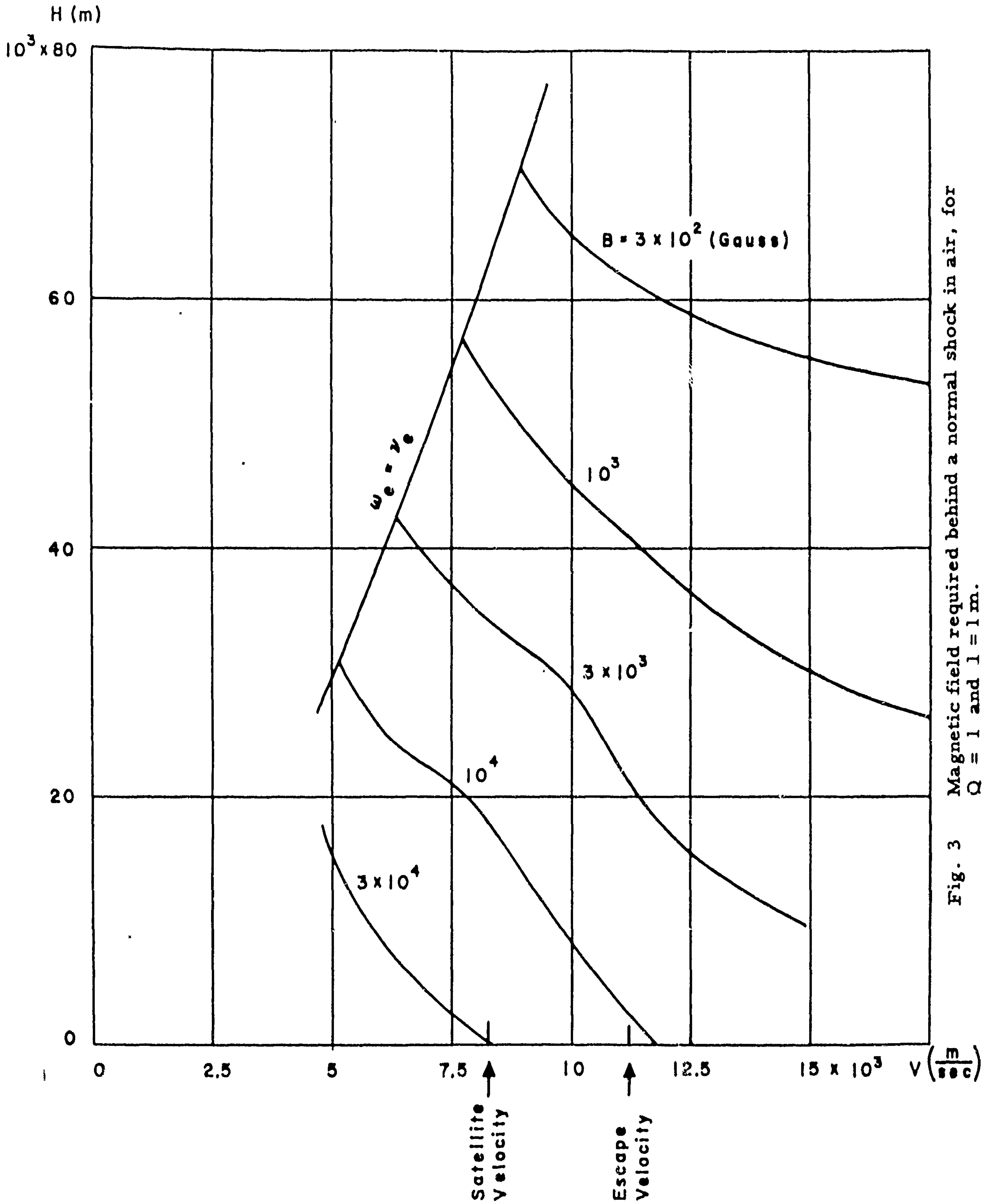


Fig. 3 Magnetic field required behind a normal shock in air, for $Q = 1$ and $l = 1$ m.