LIMITATIONS IN REPRODUCTION QUALITY

1. WE REGRET THAT LEGIBILITY OF THIS DOCUMENT IS IN PART UNSATISFACTORY. REPRODUCTION HAS BEEN MADE FROM BEST AVAILABLE COPY.

2. A PORTION OF THE ORIGINAL DOCUMENT CONTAINS FINE DETAIL WHICH MAY MAKE READING OF PHOTOCOPY DIFFICULT.

3. THE ORIGINAL DOCUMENT CONTAINS COLOR, BUT DISTRIBUTION COPIES ARE AVAILABLE IN BLACK-AND-WHITE REPRODUCTION ONLY.

4. THE INITIAL DISTRIBUTION COPIES CONTAIN COLOR WHICH WILL BE SHOWN IN BLACK-AND-WHITE WHEN IT IS NECESSARY TO REPRINT.

5. LIMITED SUPPLY ON HAND: WHEN EXHAUSTED, DOCUMENT WILL BE AVAILABLE IN MICROFICHE ONLY.

6. LIMITED SUPPLY ON HAND: WHEN EXHAUSTED DOCUMENT WILL NOT BE AVAILABLE.

7. DOCUMENT IS AVAILABLE IN MICROFICHE ONLY.

8. DOCUMENT AVAILABLE ON LOAN FROM CFSTI (TF DOCUMENTS ONLY).

9. DOCUMENT IS AVAILABLE IN MICROFICHE ONLY.
ANALYTICAL APPROXIMATIONS
VOLUME 13

Cecil Hastings, Jr.
James P. Weng, Jr.

P-441

9 October 1953

Approved for OTS release.

COPY OF
HARD COPY $1.00
MICROFICHE $0.50

The RAND Corporation

Copyright 1953
The RAND Corporation
Analytical Approximation

Unnamed Definite Integral: To better than .00055 over \((0, \infty)\),

\[
N(x) = \frac{30}{\pi^4} \int_0^\infty \frac{e^{-x} t^7}{1 + 3.0302x^6 - 0.5565x^7 + 0.3479x^8 - 0.10369x^9 + 0.01245x^{10}} dt
\]

Cecil Hastings, Jr.
James P. Wong, Jr.
RAND Corporation
Copyright 1953
Analytical Approximation

Pearson Cosine Transformation: To better than .00017 over (0, 1),

\[ r(x) = \cos \left( \frac{\pi}{1+\sqrt{x}} \right) \]

\[ = \frac{-1-4.828 \eta + 7.866 \eta^2 - 2.038 \eta^3}{1+5.560 \eta - 4.985 \eta^2 + .385 \eta^3}, \eta = \frac{x}{.16+.04x} \]

\[ r(x^{-1}) = -r(x) \] can be used to obtain function values over (1, \( \infty \)).

Cecil Hastings, Jr.
James P. Wony, Jr.
RAND Corporation
Copyright 1953
Analytical Approximation

Bessel Function: To better than .00008 over \((0, \infty)\),

\[
e^{-xI_1(x)} = \frac{x}{\sqrt{15.4 + 74.8x + 67.2x^2 + 235.8x^3 + 43.5x^4 + 59.4x^5 + 39.6x^6}}
\]

Cecil Hastings, Jr.
James P. Wong, Jr.
RAND Corporation
Copyright 1953
Analytical Approximation

Mach Number in Terms of Pressure Ratio: To better than \(0.0021\) over \(0.3 \leq M \leq 3\), the inverse of the function defined by

\[
x = \frac{P_s}{P_R} = \left[ 1 + \left(\frac{\gamma-1}{2}\right) M^2 \right]^{-\frac{\gamma}{\gamma-1}}
\]

over \(0.3 \leq M \leq 1\) and

\[
x = \frac{P_s}{P_R} = \frac{\left(\frac{2\gamma}{\gamma+1}\right) M^2 - \left(\frac{\gamma-1}{\gamma+1}\right) \frac{1}{\gamma-1}}{\left(\frac{\gamma+1}{2}\right) M^2 \frac{1}{\gamma-1}}
\]

over \(1 \leq M \leq 3\) where \(\gamma = 1.4\), is given by

\[
M = \frac{8.11 + 23.60x - 39.66x^2 + 8.98x^3}{1 + 28.70x - 15.99x^2 - 5.74x^3}
\]

Cecil Hastings, Jr.
James P. Wong, Jr.
RAND Corporation
Copyright 1953
Analytical Approximation

Natural Addition Logarithm: To better than .00026
over $0 \leq x \leq \infty$,

$$\ln(1 + e^{-x}) = \frac{\ln 2}{\left(1 + .3581x + .1151x^2 + .0094x^3 + .0052x^4\right)^2}.$$
Analytical Approximation

Natural Addition Logarithm: To better than .000,045 over $0 \leq x \leq \infty$.

$$\ln(1+e^{-x}) \approx \frac{\ln 2}{(1+0.36123x+1.0204x^2+0.02411x^3-0.00055x^4+0.00069x^5)^2}$$

Cecil Hastings, Jr.
James P. Wong, Jr.
RAND Corporation
Copyright 1953
Analytical Approximation

Natural Addition Logarithm: To better than .000,008 over $0 \leq x \leq \infty$,

\[
\ln(1+e^{-x}) = \frac{\ln 2}{(1+.360571x+.105546x^2+.018760x^3+.002545x^4-.00100x^5+.000066x^6)^2}
\]

Cecil Hastings, Jr.
James P. Wong, Jr.
RAND Corporation
Copyright 1953