Analytical Approximations

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Analytical Approximation

Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_0^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .00037 over \((0, \infty)\),

\[ q(0.5, 0.5 + y) \approx 1 - \frac{0.1045}{[1 + 0.129y + 0.079y^2 + 0.056y^3]^4} \]

The parametric form used is convenient for approximating fixed-\( R \) semi-cross-sections of the \( q(R, R+y) \) surface for any \( R > 0 \) and for \( y \) ranging over \((0, \infty)\).

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Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{R}^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0007 over \((0, \infty)\),

\[ q(1, 1+y) \approx 1 - \frac{.267}{\left[ 1 + .203y + .079y^2 + .062y^3 \right]^4} \]

The parametric form used is convenient for approximating fixed-\( R \) semi-cross-sections of the \( q(R, R+y) \) surface for any \( R \geq 0 \) and for \( y \) ranging over \((0, \infty)\).

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Offset Circle Probability Function: We consider the function

\[ q(R, x) = \int_{R}^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0011 over \((0, \infty)\),

\[ q(4, 4+y) \approx 1 - \frac{.45}{[1 + .227y + .064y^2 + .065y^3]^4} \]

The parametric form used is convenient for approximating fixed-\( R \) semi-cross-sections of the \( q(R, R+y) \) surface for any \( R > 0 \) and for \( y \) ranging over \((0, \infty)\).

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Offset Circle Probability Function: We consider the function

\[ q(R,x) = \int_{R}^{\infty} e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho \, d\rho \]

in which \( I_0(z) \) is the usual Bessel function.

To better than .0013 over \((0,\infty)\),

\[
\lim_{R \to \infty} q(R,R+y) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt
\]

\[ \approx 1 - \frac{5}{\left[1 + .209y + .061y^2 + .062y^3\right]^4} \]

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the \( q(R,R+y) \) surface for any \( R > 0 \) and for \( y \) ranging over \((0,\infty)\).

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Offset Circle Probability Function: We consider the function

\[
q(R, x) = \int_{x}^{\infty} e^{-\frac{1}{2} \left( \rho^2 + x^2 \right)} I_0 \left( \rho x \right) \rho \, d\rho
\]

in which \( I_0 (z) \) is the usual Bessel function.

To better than .006 over \((0, \infty)\),

\[
\lim_{R \to 0} \frac{1 - q(R, R+y)}{1 - q(R, R)} = e^{-\frac{1}{2} y^2}
\]

\[
= \frac{1}{\left[ 1 + .015y + .076y^2 + .040y^3 \right]^2}
\]

The above gives information concerning a degenerate limiting case in the approximating of fixed-\(R\) semi-cross-sections of the \(q(R, R+y)\) surface for \(y\) ranging over \((0, \infty)\).

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