STUDY OF MICROWAVE GENERATION
UTILIZING DOUBLE-STREAM INSTABILITIES IN ANISTROPIC MEDIA

Report No. 1
Contract No. DA 36-039 AMC-03424 (E)
DA Project No. 1G6-22001-A-055-04

First Quarterly Progress Report
4 November 1963 to 31 January 1964

U.S. Army Electronics Laboratories
Fort Monmouth, New Jersey

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Microwave Electronics Corporation
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Microwave Electronics Corporation
3165 Porter Drive
Palo Alto, California
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Object

To determine the feasibility of utilizing the plasma oscillations in anistroic materials such as pyrolytic graphite for microwave power generation and amplification

Prepared by
F.A. Olson
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PURPOSE

The purpose of this program is to theoretically and experimentally study the possible means of utilizing double-stream instabilities for microwave power generation and amplification. The processes to be investigated are those double-stream instabilities created when the negative and positive charges in a solid drift at different rates. The investigation is to determine the feasibility of utilizing the plasma oscillations in anisotropic materials, such as pyrolytic graphite, for microwave generation and amplification. The work is to be done at any convenient microwave frequency, but it is intended that the results be applicable to the generation and amplification at millimeter and submillimeter wavelengths.
ABSTRACT

A physical basis for the amplification of an rf signal by the interaction between counterstreaming beams of negative and positive charges is presented in Section I of this report. The description and analysis for this discrete-velocity double-stream amplification process provides a simplified analogy to the operation of the double-stream amplifier, utilizing pyrolytic graphite, currently under investigation. Using the analogy, and taking care to avoid having the mathematics of statistical distributions obscure the physical phenomena, the double-stream interaction is considered both qualitatively and quantitatively. Conditions for the existence of instabilities become apparent in terms of the physical parameters of the material. The results of these analyses are in agreement with the results of formal statistical calculations.

The characteristics of pyrolytic graphite that are important to the existence of double-stream instabilities are discussed in Section II. The material offers a two-component plasma with electron and hole densities of $6 \times 10^{18} \text{ cm}^{-3}$ and mobilities on the order of $10^5 \text{ cm}^2 \cdot \text{volt}^{-1} \cdot \text{sec}^{-1}$ at room temperature. The effective masses of the holes and electrons are extremely small so that the long relaxation times required for the two-stream instability process are possible. Calculations reveal the relaxation times of the carriers in pyrolytic graphite to be approximately $10^{-12} \text{ sec}$, which is of the same magnitude as found in indium antimonide. It is shown that carrier mobilities and densities can be determined from three separate measurements. Measurements of conductivity in the initial samples of pyrolytic graphite have been correlated with others. Arrangements have been made for obtaining additional samples which have been annealed at a range of temperatures; thus, a range of material parameters will be available for the double-stream instability experiments.
PUBLICATION, LECTURES, REPORTS AND CONFERENCES

During the first quarter there were no publications, lectures or reports resulting from research carried on under this contract. Two conferences were held.

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2. January 17, 1964 at Microwave Electronics Corporation, Palo Alto, California.
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FACTUAL DATA

STUDY OF MICROWAVE GENERATION
UTILIZING DOUBLE-STREAM INSTABILITIES IN ANISOTROPIC MEDIA

INTRODUCTION

The possibility of generating millimeter waves by means of double-stream instabilities in anisotropic media is being investigated. The millimeter-wave generation technique being investigated is the coherent excitation of plasma oscillations, utilizing the two-component plasma available in pyrolytic graphite. The oscillations result from a two-stream instability created when the electrons drift with respect to the holes, as an electric field is applied. Transfer of energy is from the carriers to the oscillations and, hence, the source of energy is the dc electric field that produces the drift of the carriers. The process involved is essentially the same as that used to create double-stream instabilities in electron-beam studies, but much higher densities are now available from the drifting streams of holes and electrons in solid-state materials such as pyrolytic graphite.

The medium is termed a two-component plasma because both the electrons and holes have high mobility. This is in contrast to a gaseous plasma where only the electrons are really mobile, since the slow-moving ions act only as a uniform background of positive charge.

A detailed general theoretical analysis of the behavior of solid-state plasmas has been carried out by Pines and Schrieffer, showing that, under certain conditions, high frequency oscillations can be excited.

Pyrolytic graphite has been chosen as the primary material for investigation due to the high carrier concentrations and the mobilities available in this material. These high concentrations and mobilities are necessary to meet the conditions required for the excitation of high frequency oscillations.
PART I. DOUBLE-STREAM AMPLIFIER PROCESSES

A. Introduction

This section presents the physical basis for amplification of an rf signal by the interaction between a beam of negative charges and a beam of positive charges having equal and opposite discrete velocities. The physical description and the analysis of this discrete-velocity double-stream amplifier is useful in that it provides a simplified analogy to the operation of the solid-state plasma amplifier. In the solid-state amplifier, the interaction takes place between drifting streams of holes and electrons which have a displaced Maxwellian distribution of velocities so that the mathematics obscure the physical phenomena.

B. Qualitative Description of Space Charge Waves

The phenomena of space-charge waves in electron beams is well known. A brief description which applies to either positive or negative charges is given in order to establish a background of physical concepts and definitions of terms. The basic phenomena will be described in terms of a stationary cloud of electrons. The same description applies to a moving cloud of electrons provided that a coordinate system is used which moves along with the average or dc velocity of the electrons. The electron cloud is initially assumed to be in a steady state condition with uniform charge density so that the Coulomb forces acting on the electrons are equal to zero. If the uniform charge density is perturbed by a displacement of some of the electrons in the cloud, an electric field is produced which tends to restore the electrons to their original position. However, as the electrons return, they overshoot the original position and assume a displacement in the opposite direction. Since the restoring force is proportional to the displacement from the equilibrium condition, a simple harmonic motion or oscillation results, as in a pendulum. The oscillation is referred to as a plasma oscillation and the corresponding frequency of oscillation is the plasma frequency ($f_p$).
A plasma wavelength can be defined by \( \lambda_p = \frac{f_p}{v_o} \) in the case of an electron beam which is moving at a constant velocity \( (v_o) \). It should be noted that the plasma oscillation is a local phenomenon and wavelike motion occurs only for a moving stream.

By applying Gauss's law, \( \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \), and the force equation, \( \mathbf{E} = ma \), it can be shown that the plasma frequency is given by

\[
\omega_p^2 = \frac{\rho e}{\varepsilon_0 m}
\]

where \( \rho \) is the charge density, \( e \) is the electron charge, \( \varepsilon_0 \) is the dielectric constant and \( m \) is the electron mass.

Consider now, that a sinusoidal disturbance in velocity at the radian frequency \( \omega \) is applied to an initially uniform stream of electrons moving at constant velocity by means of a modulating gap as in a klystron. Bunching of the beam will then take place due to the fast electrons overtaking the slow electrons. The distance between the bunches will be \( \lambda_s \) where \( \lambda_s = \frac{2\pi}{\omega v_o} \) as shown in Fig. 1. Figure 1 also shows pictorially the bunching for both positive and negative charges as related to the electric field in the modulating gap.

The arrows at the top of the figure show the direction of the force due to the applied field when the charges a through f are considered as positive charges. The arrows at the bottom are the forces when charges a through f are negative charges. The effect of the forces is to form an excess negative charge at point b and g for both the positive and negative-charge beam case, as shown by the solid circles. Similarly, an excess positive charge forms at point e for both types of beam, as shown by the empty circles. That is, a counter-streaming beam of positive and negative particles bunches the same as a single stream in which the charge density is the sum of the magnitudes of the charge density of the individual streams.
Fig. 1. The bunching of beams of positive and negative charges by a modulating electric field.
The net result, then of the application of a sinusoidal perturbation or signal of radian frequency, \( \omega \), is a sinusoidal variation of the charge density with distance along the direction of the beam motion.

C. Qualitative Description of the Double-Stream Interaction

The behavior of individual streams, only, has been considered up to this point. The interaction of two closely adjacent counterstreaming beams of positive and negative charge will now be considered.

This phenomena is, of course, closely related to the electronic double-stream amplifier in which two electron streams travel in the same direction with slightly differing velocities. In these tubes the space charge waves grow in amplitude as a function of distance, thus providing a useful gain mechanism.

Consider the countermoving streams of positive and negative charges bunched at a given instant of time, \( t_0 \), as shown in Fig. 2a. The solid circles represent bunches of excess negative charge (or deficit positive charge) and the open circles represent bunches of excess positive charge (or deficit negative charge). The arrows show the direction of the coulomb forces on the particular positive charge labeled A in beam 2 and on the negative charge labeled B in beam 1 when both A and B are near a region of excess negative charge. The solid arrows show the force on a charge produced by bunches in its own stream; the dashed line arrows show the force on a given charge due to the influence of the other stream. Note that both the dashed line and solid line forces act in the same direction on a particular charge in Fig. 2a.

One half of a plasma frequency period later, when \( t = t_0 + \frac{T_p}{2} \), charge A and charge B are in regions of excess positive charge as shown in Fig. 2b. For this case, it is assumed that the beams are stationary with respect to each other. Note that the dashed-line and solid-line forces on a given charge still act together, but are of opposite sense as compared with Fig. 2a.
Fig. 2c. $t = t_0 + \frac{\tau_d}{2}$ and the beams have moved a distance $\lambda_s/2$ with respect to each other. That is, the positive charge beam has moved $\lambda_s/4$ to the right and the negative charge beam has moved $\lambda_s/4$ to the left.

Fig. 2. Diagram of the forces on particular charges in counter-moving streams of positive and negative charges.
That is, the total forces on a charge fluctuate in direction so that the charges oscillate back and forth about their equilibrium position as previously described.

Consider now the case where the streams are in relative motion. Assume that the streams have slipped the distance, $\lambda_s/2$, apart during the time interval between $t = t_o$ and $t = t_o + T_p/2$. This situation is depicted in Fig. 2c where the positive charge beam has moved $\lambda_s/4$ to the left as compared to its position in Fig. 2b, and the negative charge beam has moved $\lambda_s/4$ to the right of its position in Fig. 2b. Note that the dashed-line and solid-line forces on a particular charge are now in opposite directions. The important point is that the dashed line forces on charge A at $t = t_o$ (Fig. 2a) are in the same direction as at $t = t_o + T_p/2$ (Fig. 2c). That is, the force on a charge in beam 1 due to beam 2 remains in the same direction as a function of time. Similarly, the force on a charge in beam 1 due to beam 2 is in the same direction. Note that when $t = t_o + T_p/4$, the bunches have disappeared and the forces on the charges are zero. The net result of this action of the coupled counterstreaming beams is to continually increase the magnitude of the bunches. That is, the bunches grow as a function of time and thus provide a gain mechanism.

According to this qualitative description it has been established that a growing space-charge wave is possible when $\lambda_s = \Delta v_o T_p$ or when

$$\lambda_s = \Delta v_o \frac{2\pi}{\omega_p}, \tag{2}$$

where $\lambda_s$ is the distance between bunches, $\Delta v_o$ is the relative velocity one stream is moving with respect to the other, and $T_p$ is the period of a plasma frequency. Since the streams are assumed to have equal and opposite velocities, let the magnitude of the velocity of beam 1 be equal to $+v_o$ and that of beam 2 to be equal to $-v_o$. The relative velocity difference is then $\Delta v_o = 2v_o$. 

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Assume that both streams have the same charge density so that, from Eq. (1), the plasma frequencies are the same for both beams. Substituting $\lambda_s = 2\pi v_0/\omega$ and $\Delta v_0 = 2v_0$ into Eq. (2), the condition established qualitatively for growing space-charge waves is

$$\frac{\varepsilon}{\rho} = \frac{1}{2}$$ (3)

This result will be derived quantitatively in the next section.

D. Quantitative Analysis of the Double-Stream Interaction

Consider two modulated beams flowing in the same region in the z-direction. Assume that the cross section of each beam is infinite so that the rf magnetic field is zero and a potential, $V$, can be introduced which is related to the coulomb electric field, $E$, by

$$E = -\nabla V.$$ (4)

The following notation will be used:

- $\rho_t$ = charge density,
- $v_t$ = particle velocities, and
- $i_t$ = convection current density.

The quantities of interest in beams 1 and 2 can be expressed as:

$$\rho_{t1} = \rho_{o1} + \rho_1$$ (5)
$$\rho_{t2} = \rho_{o2} + \rho_2$$ (6)
$$v_{t1} = v_{o1} + v_1$$ (7)
$$v_{t2} = v_{o2} + v_2$$ (8)

$$i_{t1} = i_{o1} + i_1 = i_{o1} + \rho_{o1}v_1 + \rho_1v_{o1} + \rho_1v_1$$ (9)

$$i_{t2} = i_{o2} + i_2 = i_{o2} + \rho_{o2}v_2 + \rho_2v_{o2} + \rho_2v_2$$ (10)
where the subscript, \( t \), refers to total quantities; the subscript, \( o \), refers to zero order or dc quantities; \( \rho, v \) and \( i \) are first order or rf quantities; and the subscripts, 1 and 2, refer to beam 1 and beam 2, respectively.

The one dimensional equation relating the charge densities, velocities, currents, and potentials in the beam is as follows:

\[
- \nabla \cdot E = \nabla^2 V = - \frac{\rho_1 + \rho_2}{\varepsilon_0} = \frac{\partial^2 V}{\partial z^2}
\]  

which is Poisson's equation relating the potential and the rf charge densities.

The equation of continuity is

\[
\frac{\partial i_1}{\partial z} + \frac{\partial \rho_1}{2 \tau} = 0.\tag{12}
\]

The equation of motion is

\[
\frac{d v_1}{d t} = \frac{\partial v_1}{\partial t} + (v_{o1} + v_1) \frac{\partial v_1}{\partial z} = - \frac{e_1}{m} \frac{\partial V}{\partial z}.\tag{13}
\]

Similar equations apply for beam 2.

Note that the potential, \( V \), in Eq. (13) is given by Eq. (11) which contains the rf charge for both beams. Poisson's equation therefore introduces the interaction between the two beams. The equations can be most easily solved by assuming that rf quantities vary as \( e^{j \omega t + \Gamma z} \) where \( \Gamma \) is the propagation constant. Equation (11) then becomes

\[
l = - \frac{\rho_1 + \rho_2}{\varepsilon_0 \Gamma^2 V}.
\]  

Using (9) and (10), Eqs. (12) and (13) can be algebraically manipulated to yield
Equations (14) and (15) provide two values for the ratio \(-\left(\rho_1 + \rho_2\right) / \varepsilon_o \mu^2 v\) so that the dispersion equation for the assumed propagation constant, \(\mu\), becomes the quartic equation

\[ 1 = \frac{\beta_{p1}^2}{(j \mu - \beta_{e1})^2} + \frac{\beta_{p2}^2}{(j \mu - \beta_{e2})^2} \]  

Since Eq. (16) is quartic there will be four roots or four values for \(\mu\). These values of \(\mu\) can be obtained graphically by plotting Eqs. (14) and (15) versus \(j \mu\) as shown in Fig. 3. The allowed values of \(\mu\) are obtained by the intersection of the two curves.

Two cases are shown by the dashed and solid line curves in Fig. 3. The solid line curve shows four intersections or four real values for \(j \mu\); thus \(\mu\) is imaginary. From the dependence \(e^{j \omega t + \mu \gamma z}\), it is apparent that no growing waves are obtained. The dashed curve shows a case where two intersections are obtained so that only two real roots are obtained for \(j \mu\). The two roots which do not appear are complex, so that \(\mu\) has two real values corresponding to growing and decaying waves.

Equation (16) can also be solved directly for the case of interest where it is assumed that one beam is formed of positive charges and the other is formed of negative charges. Assume equal and opposite drift velocities; then, if \(v_{o1} = v_o\) and \(v_{o2} = -v_o\):

\[ \beta_{e1} = -\beta_{e2} \]

where \(\beta_e = \omega / v_o\).
Fig. 3. Plot of $-\frac{\rho_1 + \rho_2}{\epsilon_0 \Gamma^2 \sqrt{V}}$ as a function of $\beta_p$. Eq. (14) and (15). The dashed curve is for larger values of $\beta_p$.
Assume further that $\omega_{p1} = \omega_{p2}$ so that $\beta^2_{p1} = \beta^2_{p2}$ and $\beta_{p1} = \beta_{p2}$.

$\beta_{p2} = -\beta_{p1}$.

Equation (16) then becomes

$$1 = \frac{\beta^2_p}{(j \beta_{p2})^2} + \frac{\beta^2_p}{(j \beta_{p1} + \beta_{p2})^2}. \quad (17)$$

The solution for $\Gamma$ then is

$$j \Gamma = \pm \beta_p \left[ \left( \frac{\beta_e}{\beta_p} \right)^2 + 1 \pm \left( 4 \left( \frac{\beta_e}{\beta_p} \right)^2 + 1 \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \quad (18)$$

Since rf quantities vary as $e^{j\omega t} + \Gamma z$, $\Gamma$ must be real to obtain a growing wave so the right hand side of Eq. (18) must be imaginary.

Since $(\beta_e/\beta_p)^2$ is positive, the right hand side will have two real roots when the (+) sign is chosen inside the large brackets. For this choice of signs $\Gamma$ is purely imaginary and the waves are constant in amplitude. For the choice of the negative sign inside the brackets, the right hand side becomes imaginary when

$$\left[ 4 \left( \frac{\beta_e}{\beta_p} \right)^2 + 1 \right]^{\frac{1}{2}} > \left( \frac{\beta_e}{\beta_p} \right)^2 + 1$$

or if

$$0 < \left( \frac{\beta_e}{\beta_p} \right)^2 < 2, \quad (19)$$

This relationship is thus the criterion for obtaining gain.
For the negative charge beam \((\beta_e / \beta_p)^2 = (\beta_{e1} / \beta_{p1})^2 = (\omega_p / \omega)^2\) and for the positive charge beam \((\beta_e / \beta_p)^2 = (-\beta_{e2} / -\beta_{p2})^2 = (\omega_p / \omega)^2\). The required relationship thus becomes

\[
0 < \left(\frac{\omega_p}{\omega}\right) < \sqrt{2}.
\]  

This analytical result is in agreement with the previous criterion given by Eq. (3), which was derived qualitatively to be \(\omega_p / \omega = 2\), and with the results of the formal calculations of Pines and Schrieffer.

In order to get high frequency operation large values of \(\omega_p\) are required which in turn requires large values of charge density and low values of particle mass, since \(\omega_p^2 = \rho e / \epsilon_0 m\). The suitability of the respective quantities in pyrolytic graphite for high frequency operation is discussed in the following sections.
PART II. SIGNIFICANT PROPERTIES OF PYROLYTIC GRAPHITE

A. Introduction

Pyrolytic graphite is an intrinsic semiconductor or semimetal that is prepared by cracking methane, then vapor-depositing the carbon on a flat surface. Graphitization or crystallization is accomplished by a heat treatment at temperatures ranging from 2000 to 3500 °K.

Pyrolytic graphite is a material that is characterized by a highly oriented microscopic crystallite structure that is formed in closely spaced planes called the a-planes or basal planes. The material is anisotropic both thermally and electrically. The properties of primary interest in the application of the material to solid state plasmas is the high carrier density and the high mobility in the basal plane.

Pyrolytic graphite offers a two-component plasma with electron and hole densities of $6 \times 10^{18} \text{cm}^{-3}$ and mobilities on the order of $10^4 \text{cm}^2 \cdot \text{volt}^{-1} \cdot \text{sec}^{-1}$ at room temperature. Thus, the effective masses of the holes and electrons are extremely small so that the long relaxation times required for the two-stream instability process are possible.

B. Quantities Important to the Instability Process

A detailed theoretical analysis of the solid-state plasma instability has been carried out by Pines and Schrieffer, showing that high frequency oscillations can be excited under certain conditions. The important material quantities which must be considered are the mobilities, $\mu_-$ and $\mu_+$; the densities, $n_-$ and $n_+$; the temperatures $T_-$ and $T_+$; the effective masses, $m_+$ and $m_-$; the thermal velocities, $v_-$ and $v_+$; the drift velocities, $v_d-$ and $v_d+$; and the relaxation times, $\tau_-$ and $\tau_+$ where the subscripts (-) and (+) refer to electrons and holes, respectively. Conditions favorable to oscillations include (1) large mobilities for both electrons and holes, (2) large densities for both electrons and holes, (3) a high ratio of electron to hole
temperatures, (4) small effective masses for both electron and holes, (5) a large ratio of drift velocities to thermal velocities for both electrons and holes, and (6) long relaxation times for both holes and electrons.

When considering the properties of pyrolytic graphite it is important to note that the values depend upon the temperature at which the material is annealed, as well as the ambient temperature. To illustrate this dependence, the average carrier mobility, defined by \( \mu_{\text{ave}} \), is shown in Fig. 4 as a function of absolute temperature for pyrolytic graphite heat treated at the three different temperatures\(^2\). Note that the mobility increases with the heat treatment temperature, especially at low ambient temperatures.

The total carrier density, \( n_+ + n_- \), is shown in Fig. 5 as a function of temperature for pyrolytic graphite heat treated at temperatures in the range from 2500°C to 3000°C.

C. Calculation of Pyrolytic Graphite Plasma Properties

The range of values for the plasma properties of pyrolytic graphite heat-treated at 3000°C are shown in Table I. The mobilities and the densities can be calculated from measurements of the resistivity, the Hall coefficient, and the magnetoresistance as discussed in the next section.

The effective masses are determined from quantum mechanical considerations to be \( m_- = 0.03 \, m \) and \( m_+ = 0.06 \, m \) where \( m \) is the usual free electron mass.

If it is assumed that the inter-electron and inter-hole collisions dominate the scattering mechanisms, then the conservation of momentum holds for holes and electrons so that \( m_- v_- \) equals \( m_+ v_+ \) and \( v_- / v_+ \) equals 2.
Fig. 4. Average carrier mobility, defined as $(\mu_-\mu_+)^{\frac{1}{2}}$ versus temperature for pyrolytic graphite heat treated at three different temperatures. (From Klein, Ref. 2)
Fig. 5. Total carrier concentration, $n_− + n_+$, as a function of temperature for pyrolytic graphite heat treated at temperatures in the range from 2500°C to 3000°C. (From Klein, Ref. 2).
TABLE I

Plasma Properties of Pyrolytic Graphite Heat Treated at 3000°C

Mobility

\[ \frac{\mu_-}{\mu_+} = 1.25 \text{ where } \mu_- \sim 1.125 \times 10^5 \text{ cm}^2/\text{volt-sec} \]

Density

\[ \frac{n_-}{n_+} = 1 \text{ where } n_- = 6 \times 10^{18} \text{ cm}^{-3} \]

Effective Masses

\[ \frac{m_+}{m_-} = 2 \text{ where } m_- = 0.03 \text{ m} \]

Thermal Velocities

\[ \frac{v_-}{v_+} = 2 \]

Drift Velocities

\[ \frac{v_{d-}}{v_{d+}} \sim 2 \]

Temperatures

\[ \frac{T_-}{T_+} \sim 2 \]

Relaxation Times

\[ \frac{\tau_-}{\tau_+} = 0.625 \text{ where } \tau_- \sim 2 \times 10^{-12} \text{ sec} \]
The electron and hole drift velocities and the electron and hole temperatures depend upon the applied electric field, $E_o$, which produces the drift. For large values of $E_o$, the electron temperature becomes larger than the lattice temperature. The values of the temperatures and drift velocities as a function of $E_o$ have not been determined as yet for pyrolytic graphite. However, for small $E_o$, the thermal values can be taken as a lower limit for the temperature ratio, $T_-/T_+$. 

From $m \nu_\pm^2 = KT_\pm$, then $T_-/T_+$ is approximately equal to $\pi\nu$ where $K$ is Boltzmann's constant.

The relaxation times can be determined from the definition of mobility

$$\nu_\pm = e\tau_\pm/m_\pm$$

so that

$$\frac{\tau_-}{\tau_+} = \frac{m_-\nu_-}{m_+\nu_+}$$

Assuming $m_+/m_-$ equals two and $\nu_-/\nu_+$ equals 1.25, then $\tau_-/\tau_+$ equals 0.625. Values of $\tau$ can, of course, be calculated from the mobility equation above.

For $\nu = 10^5 \text{ cm}^2 \text{ - volt}^{-1} \text{ sec}^{-1}$ and $m_-/m = 0.03$, the value $\tau \approx 2 \times 10^{-12} \text{ sec}$ is obtained. This value for the relaxation time is suitable for the oscillations process and is comparable to the relaxation time of a "highly regarded" material such as indium antimonide. The long relaxation time of pyrolytic graphite is a particularly important result since it is necessary that $\omega\tau_\pm \geq 1$ in order to obtain the desired instability at the radian frequency, $\omega$. 

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D. Measurement of Properties

The carrier mobilities and densities of pyrolytic graphite can be determined by measurement of the resistivity, $\rho_0$; the Hall coefficient, $R_0$; and the magnetoresistance coefficient, $A_0$. The equations relating these parameters to mobility, $\mu$, and charge concentration, $n$ are:

\[
\frac{1}{\rho_0} = \sigma = en_+ \mu_+(a + b), \tag{23}
\]

\[
R_0 = \frac{1}{en_+} \frac{a - b^2}{(a + b)^2}, \tag{24}
\]

and

\[
A_0 = \frac{\Delta \rho}{\rho_0 H^2} = \frac{\mu^2}{10^{16}} \frac{a b (1 + b)^2}{(a + b)^2}, \tag{25}
\]

where $\sigma$ is the conductivity, $n_-$ and $n_+$ are the electron and hole concentrations, $\mu_-$ and $\mu_+$ are the electron and hole mobilities, $e$ is the electron charge, $a = n_+/n_-$, $b = \mu_-/\mu_+$, and $H$ is the magnetic intensity in gauss.

The above three equations involve the four unknowns $\mu_+$, $\mu_-$, $n_+$ and $n_-$ so that they cannot be determined explicitly. However, assuming a pure pyrolytic graphite material, the electron to hole concentration ratio "a" is approximately unity. For $a = 1$ Eqs. (21) through (23) become

\[
\frac{1}{\rho_0} = \sigma = en_+ \mu_+(1 + b) \tag{26}
\]

\[
R_0 = \frac{1}{en_+} \frac{1 - b}{1 + b} \tag{27}
\]

\[
A_0 = \frac{b}{10^{16}} \frac{\mu^2}{\mu_+ \mu_-} = \frac{\mu_+ \mu_-}{10^{-16}} \tag{28}
\]

These three equations can be solved for $\mu_+$, $\mu_-$, and $n_-$. 

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The results of the solutions are listed below where $\sigma = \frac{1}{\rho_o}$ is used.

\begin{equation}
2\mu_+ = R_0 \sigma + \left[ (R\sigma)^2 + 4(10^{16})A_o \right]^{1/2}, \tag{29}
\end{equation}

\begin{equation}
2b = 2 \frac{\mu_+}{\mu_-} = C + (C^2 - 4)^{1/2} \tag{30}
\end{equation}

where

\begin{equation}
C = \left[ \frac{(R\sigma)^2}{10^{16} A_o} \right] + 2.
\end{equation}

After solving for $\mu_+$ and $b$ from Eqs. (27) and (28), the electron concentration is given from (24) as

\begin{equation}
n_- = n_+ = \frac{\sigma}{e \mu_+(1+b)} \tag{31}
\end{equation}

The units for mobility are cm$^2$/volt -sec; the units for $n$ are cm$^{-3}$; $\rho_o$ is in ohm-cm, $R_o$ is in cm$^3$/coulomb, and $A_o$ is in gauss$^{-2}$.

Preliminary measurements of resistivity have been made by use of the four-probe technique$^{13}$ on a sample of pyrolytic graphite obtained from High Temperature Materials, Inc., Boston, Massachusetts. This graphite sample was deposited at about 2100°C and heat treated at 2500°C. A value of $280 \times 10^{-6}$ ohm-cm was obtained for the basal plane resistivity at room temperature. This compares to $240 \times 10^{-6}$ ohm-cm obtained by Klein$^2$.

Further samples will be obtained from High Temperature Materials and heat treated at temperatures up to 3000°C at Raytheon, Advanced Materials Department, Waltham, Massachusetts.
CONCLUSIONS

A special case of the double-stream instability has been considered both quantitatively and qualitively in order to establish physical concepts. The case considered was that of two coupled counterstreaming beams having equal and opposite discrete velocities and equal plasma frequencies. In the actual solid-state plasma the velocities for the hole and the electrons both have a displaced Maxwellian distribution and the plasma frequencies are not necessarily the same. The result of the discrete velocity analysis was that the oscillation frequency is approximately equal to the plasma frequency of either beam. This is in good agreement with the more detailed analysis of Pines and Schrieffer.

Methods of measuring the plasma properties of pyrolytic graphite have also been considered. The basic properties of hole and electron concentrations and mobilities can be determined by measurement of the conductivity, the Hall coefficient, and the magnetoresistance coefficient. Calculations of the plasma properties of pyrolytic graphite, based on the measured value of Klein indicate that this material is highly suitable for the study of solid-state plasma instabilities.
PROGRAM FOR THE NEXT QUARTER

Analysis of the solid-state plasma instability will be continued for the more general case of displaced Maxwellian velocity distribution for the holes and electrons. The hole and electron temperatures and drift velocities will be calculated as a function of the applied electric field.

Pyrolytic graphite samples annealed at various temperatures will be procured and dc measurements of the plasma properties will continue. Experimental microwave circuits will also be designed to measure the pyrolytic graphite properties at microwave frequencies.

The measured properties will be used to calculate the oscillation frequencies and the growth rates for the best samples.

The optimum method for observing the instabilities experimentally will be determined.
REFERENCES

IDENTIFICATION OF KEY TECHNICAL PERSONNEL

Key technical personnel and the respective man-hours devoted to this contract are listed below. Biographies of these men appear on the following pages.

F.A. Olson, Senior Research Engineer
Solid-State Research and Development Group
123 hours

L.D. Buchmiller, Senior Research Engineer
Solid-State Research and Development Group
162 hours
Lyle D. Buchmiller, Senior Engineer, Research and Advanced Development

Mr. Buchmiller holds a B. A. in Physics from Union College, Lincoln, Nebraska, and an M.S.E.E. degree (Electronic Science) from Stanford University.

During the period from 1950 to 1962, Mr. Buchmiller was a graduate student, research assistant, and research associate at Stanford Electronics Laboratories, Stanford, California. His work on research and development projects in the electron devices area has included cyclotron resonance tubes, general traveling-wave tube amplifiers, low-noise traveling-wave amplifiers, multiple-input traveling-wave tube phase-shifters, and a cathode-ray frequency meter and signal analyzer device.

Mr. Buchmiller joined MEC in 1962 as a Senior Engineer and is currently doing development work in the field of masers, solid-state devices, and microwave tubes. He has been involved in the design, fabrication, and testing of the traveling-wave maser, the RF components, and the ultra-stable temperature-compensated magnet for the NASA (JPL) S-band field-operational maser system.

Mr. Buchmiller is the author of nine technical papers. His memberships in technical organizations include Sigma Xi and the Institute of Radio Engineers.


Frank A. Olson, Senior Engineer, Research and Advanced Development

Dr. Olson holds a B.S. degree in E.E. from Oregon State University and an M.S. degree and a Ph.D. in E.E. from Stanford University.

After receiving his B.S. in 1955, Dr. Olson spent four years with the Microwave Physics Laboratory of Sylvania Electric Products, Inc., doing research on plasma oscillations, control of microwave energy via plasmas, crossed field electron devices, and parametric amplifiers. During this period, he completed the requirements for the M.S. degree at Stanford University under the Sylvania Honors Cooperative Program.

In 1958, he was appointed Research Assistant at the Stanford Electronics Research Laboratory, where he did research on parametric phenomena associated with semiconductor diodes and was active in the development of a Sylvania TW parametric amplifier. Parametric circuits and their use as microwave limiters was described in his thesis: "The Large-Signal Properties of Microwave Cavity-Type Parametric Circuits."

In 1960, Dr. Olson was assigned as a USAF First Lieutenant to the Cambridge Research Laboratories. As a Research Engineer and Group Leader in the Microwave Physics Section of the Electromagnetic Radiation Laboratory, he did research on interaction of microwave radiation with solid-state materials, principally in ferromagnetic, ferroelectric, and phonon processes and their application to the control and generation of microwave energy. He published papers on ferromagnetic resonance and studies of magnetoelastic interactions of microwave phonons and spin-waves in ferrimagnetic single crystals. Dr. Olson also was engaged in teaching advanced mathematics at Northeastern University and solid-state physics and microwave electronics at Lowell Technological Institute.

Since joining Microwave Electronics Corporation in late 1962, Dr. Olson has initiated a research and development program in single-crystal solid-state devices with emphasis on magneto-acoustic and electro-acoustic devices for microwave delay lines and solid-state microwave amplifiers.

Dr. Olson is the author of eleven technical papers, is a member of the Institute of Radio Engineers, Sigma Xi, Phi Kappa Phi, Tau Beta Pi, and Eta Kappa Nu and has received the Westinghouse Achievement Award and the Air Force Commendation Medal for research contributions.
Frank A. Olson, Bibliography


"Microwave Magnetoelastic Measurements by Parallel Pumping," IRE Transactions on Ultrasonics Engineering (to be published).
Microwave Electronics Corp.
Palo Alto, Calif.

STUDY OF MICROWAVE GENERATION UTILIZING DOUBLE-STREAM INSTABILITIES IN ANISOTROPIC MEDIA, First Quarterly rept. for 4 Nov 63-31 Jan 64, by F.A. Olson and L.D. Buchmiller. 29 p.
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