FINAL REPORT

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OPERATIONS RESEARCH STUDY OF COMMAND AND CONTROL TRADE-OFFS IN AN AMPHIBIOUS ENVIRONMENT

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CONTENTS

ILLUSTRATIONS

FIGURE

ILLUSTRATIONS (Continued)

INTRODUCTION

This study was performed in response to an ONR Contract No. Nonr-3983(00) awarded to Space Technology Laboratories, Inc. in November 1962. The proposal for this study was motivated by a paper titled "A Model for the Survival of Tactical Units on the Battlefield" that was presented at the Military Operations Research Symposia (MORS) at Ft. Monroe, Virginia, in April 1962, by Paul D. Chaiken[3]. This paper demonstrated the use of analytical techniques to quantitatively describe the interrelationships between mobility, dispersion, surveillance, and fire power as it affects the survival of tactical units on the battlefield. The Office of Naval Research indicated interest in the methods utilized in the paper and requested a repetition of the MORS presentation in Washington. It was after this briefing that the proposal was generated. The resulting study attempts to analyze significant problems from both Navy/Marine Corps amphibious operations and Marine Corps tactical operations. In order to help STL formulate such problems, Mr. Irving Dow of the Naval Warfare Research Center of Stanford Research Institute (Pasadena) and the staff members of the Office of Naval Research (Washington) were consulted.

The problems were formulated in game-theoretic terms in an attempt to obtain basic mathematical structure rather than specific numerical analyses. By mathematical structure, it is meant the inherent strategies, mixed or pure (in a game-theoretic sense), which appear in the course of the analysis that allude to the decision making events associated with the military scenario from which the problem was defined. Hopefully, these strategies can then be functionally related to real world inputs in order to obtain better insight into the nature of the specific requirements to be made of the limited warfare system used as a basis for the model. The definitizing of tactical system requirements affecting surveillance, communications, command and control, logistics, fire power, force size, dispersion, defense, and ancillary support equipment, utilizing mathematical models, was the major objective of the approach described above.

The approach taken by this study will be to mathematically abstract specific problems (see Figure 1.) from the area of Navy/Marine Corps amphibious opera tions and Marine Corps tactical operations. This approach presents the analyst with dilemma. If the problem abstracted reflects too many real world variables, a succes ful solution to the problem can become impossible. On the other hand, if the proble is too much of a simplification of the real world situation, the solution will yield results that are trivial. Significant analyses, i.e., non-trival solvable problems, can only be accomplished by a full understanding of the problem area. Thi understanding allows the analyst to determine the important parameters affecting the structure of the solution and even more important allows him to pick measures of effectiveness or criteria such that these parameters can be related.

There are many parameters or factors which have a great deal of importance in the real world situation. However, the true test of their importance when abstracting this real world situation into a mathematical model depends upon how these parameters, factors or variables affect the structure of the solution. An example of this was illustrated in the MORS[3] paper where the parameters of mobility, dispersion and surveillance completely defined the structure of the mathematics develop whereas fire power, a very important real world parameter, could not be introduced except as a scaler quantity completely independent of the mathematical structure On the other hand, the same military situation utilizing the Lanchester Model completely inverts the above parameter relationships to mathematical structure and yields an entirely different approach to the same problem. Which model formulation should be used can only be answered by asking what type of parameter trade-offs (outputs) is the analyst interested in. The formulation that involves parameters that define mathematical structure usually (but not always) yields more hidden or less obvious results concerning these parameters than those formulations where the same parameters are independent of the mathematical structure. Which model formulation is the best is simply answered by stating that the model that works the best is the best! By 'best' it is meant the formulation which yields outputs or answers that more closely correspond to the real world outputs in a similar real world situation.

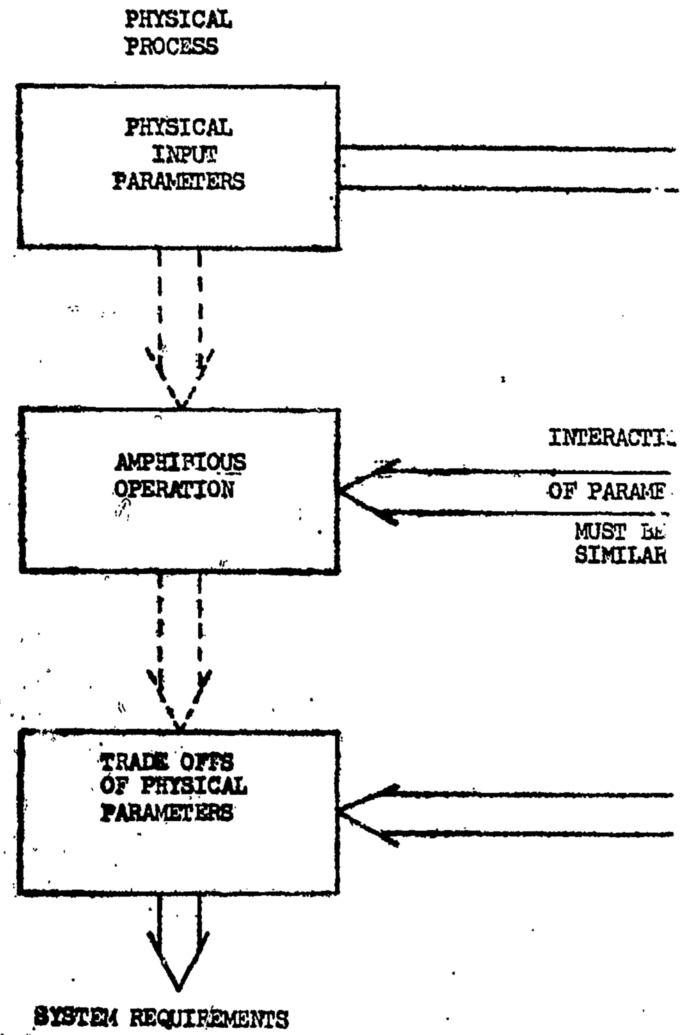
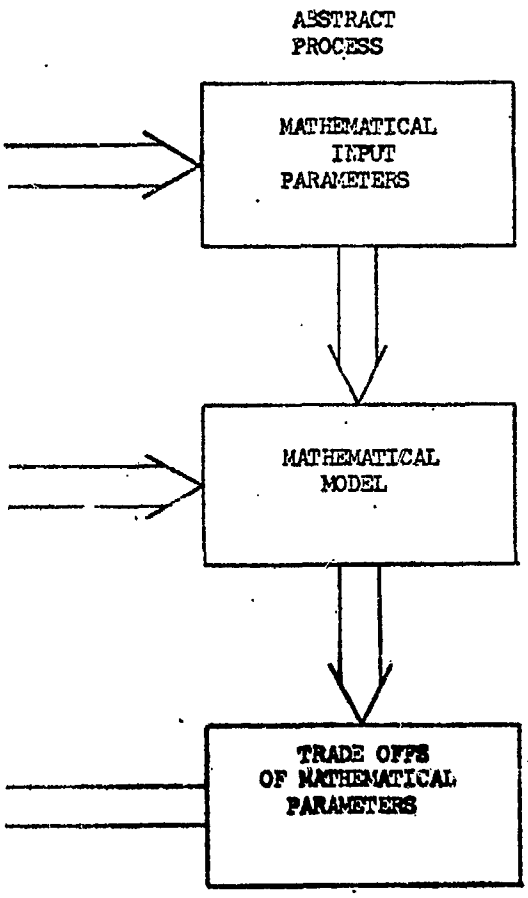


Figure 1. The Mathematical Abstraction c



Amphibious Operation

I. SUMMARY AND CONCLUSIONS

This study demonstrates the use of analytical techniques to quantitatively describe the interrelationships between mobility, dispersion, surveillance, and fire power as they affect the survival of tactical units on the battlefield. The purpose of such a study was to emphasize the possible use of analytical models to explore areas of Marine Corps/Navy advanced warfare military systems and operations in which outputs obtained from such analyses could lead, by implication, to recommendations for requirements for surveillance, fire power, force size, logistics, and command and control subsystems.

The basic problem analyzed in this study can be summarized as follows:

> An amphibious landing force x, at sea, is to assault a limited area defended by a force y. The landing force x is to be split up into an air mobile force x_a and a surface mobile force x_s. The defending force y, in turn, commits its forces by allocating part of y, β_1 y, in defense of the beach and the remaining force, β_2 y, to defend against the air mobile attack.

Some pertinent questions based upon this amphibious operations scenario were:

> What strategic allocation of forces $\{\beta_1, \beta_2; x_s, x_a\}$ should each side use against the other during the ensuing engagement?
>
> What is the mathematical structure of the strategic decisions made by both sides (i.e., $\{\beta_1, \beta_2, x_s, x_a\}$) as a function of initial conditions, such as force levels (x,y) available to each side at the beginning of the battle, mobility of the assault and defense units on the battlefield, fire power available to each fighting unit, and the time sequencing or staging of force units into battle?
>
> How does the analysis relate back to the real world in terms of logistics, equipment, force levels, operational plans, etc.?

The above amphibious operation problem was presented mathematically in a game theoretic context which attempts to relate the interplay of the many parameters enumerated above as they affect the outcome of the battle. The payoff of the game was defined in terms of time dependent solutions of equations reflecting the above scenario. The significant outputs of the mathematical model formulated in this study were:

- The number of survivors at the end of the battle.
- The duration of the battle for any threshold of defeat of one side.
- The optimum allocation strategies available to each side. That is, what fraction of the total forces goes across the beach and/or is deployed air mobile as opposed to the defender's allocation when splitting his force in defense against the amphibious operation?
- The natural strategic discontinuity levels. That is, the areas of the strategy domain beyond which both sides must play to obtain strategic optimality.
- Sub-optimal strategies (restricted within discontinuity levels). That is, areas of the strategy domain in which strategic optimality cannot be achieved and sub-optimal strategies become important.

The last two outputs reflect the constraining nature of the real-world situation in that mathematically optimal strategies may not always be possible to achieve because of the physical constraints of the tactical systems employed, e.g., landing craft and helicopter capacity, speed of operation and duty cycle of the logistic support systems, etc. However, it should be noted that the model does not delineate these constraints directly but rather defines the mathematical structure of the decisions made in terms of natural discontinuity levels in the decision domain. These discontinuities in turn define the areas in which optimal strategies shift violently if the real world were for any reason so constrained to operate only in these areas. The resulting strategies derived from the shift from optimality are sub-optimal.

A sensitivity analysis of the mathematical model formulated using the above scenario as a basis was performed by taking the important input parameters and perturbing them away from what was considered a standard model configuration. Because of the multi-dimensional complexity of the model, it was felt that the best way to explore the structure of the problem was to set up a standard case, which would represent a typical amphibious operational exercise. Then, based upon the nature of the results obtained from solving this standard case, other input parameters were perturbed sequentially in order to test for their sensitivity to the overall measure of merit defined by the model.

The results of the standard case indicated that the landing force's optimal strategy is to send all of its forces air mobile, even though such a strategy would represent a severe constraint upon the fire power availab for an air mobile operation. In other words, the value of quick deploym of landing forces via the air mobile mode more than compensates (in terms effectiveness) for the inherent fire power weakness associated with the ai mobile mode.

The disadvantage of the surface assault mode for the deployment of attacking forces lies in the fact that the assault units that contribute most to the heavy fire power characteristics of the assault could not be deployed until the beachhead was first secured by the rifle and close-sup artillery units, which have relatively the same fire power characteristics as the air mobile mode. In other words, to deploy heavy fire power via th surface mobile mode requires the assault units to deploy sequentially in time, thus losing the effectivness derived from the quick deployment as demonstrated by the air mobile mode. Of course, if all fire power of both sides in the standard case were substantially reduced (by a factor of five and kept in the same proportion), the optimal strategy reverts to the sur- face mobile mode, which indicates that time sequencing of assault units relative to quick deployment is no longer detrimental to the overall optim effectiveness of the operation. More significantly, however, the analysis of the standard case also reveals a natural strategic discontinuity in the decision allocation domain, which severely limits the commander's ability make an optimal decision.

The results of this analysis indicate that the amphibious task force commander is constrained to send greater than 90% of his forces via the ai mobile mode if his decision is to remain optimal. If for any reason such percentage of total forces cannot be sent via this mode, then optimality reverts to a decision requiring 100% of the commander's forces going acros the beach in the surface mobile mode.

The natural discontinuity level of 90% is called the decision threshold level for air mobile deployment and represents in the real world the amphibious operation commander's 'flip-flog' decision level in which, based upon long years of experience in command, he determines the mode of attack during the planning stage of the operation. The current vertical envelopment doctrine deploys about 20% to 30% of the landing force via the air mobile mode, which is considerably less than the threshold level resulting from the standard case.

Although our standard case model is not purported to be an accurate representation of the real world, it does yield combat results in the range of values characteristic of the World War II Pacific Island Campaigns. Also, this study does not attempt to develop absolute evaluations of amphibious operations via exercising mathematical models. What this study does attempt to establish functional relationships between various important parameters that are characteristic of the amphibious operation, such as mobility, fire power, time dependent deployment of assault units, and order of battle, within the framework of optimizing an overall measure of merit for the operation. Concerning the amphibious commander's decision threshold of 90%, we can only say that it is high, not only in its range of possible values but also with respect to reality. The primary objective of the sensitivity analysis performed on the standard case was to determine the parameter or set of parameters that will influence this threshold of 90% in such a way as to reduce its value so it conforms to present doctrine.

The results of the sensitivity analysis indicate that the air mobile decision threshold can be reduced. The perturbation of the mobility factor assigned to assault units on the battlefield caused the decision threshold to approach values consistent with reality. That is, it was determined that under conditions of amphibious landing force superiority (force ratios of four-to-one) the decision threshold would drop to zero if the mobility of the assault units on the battlefield was extremely large. The mathematical model indicated that distance between separate and distinct battle engagements on the battlefield had to be traversed by the individual landing force assault units at infinite speed. In other words, if a finite period of time was necessary for the amphibious assault units (completing their local battle engagement) to link up with other amphibious assault units at a different geographical location on the battlefield, then the air mobile decision threshold remained high. Only infinite speed (a physical impossibility) allowed for the air mobile deployment of forces in percentage of total forces consistent with current doctrine.

Next, perturbations of various fire powers associated with assault and defensive units were tried. Increasing all the assault units' fire power relative to the defense did not seem to have any effect upon the air mobile decision threshold level. However, increasing the defender's fire power relative to the amphibious force reduced the amphibious commander's decision threshold to 60% (equivalent to a four-fold increase in defender fire power). Unfortunately, the level of superiority of the amphibious operation (two-to-one order of battle ratio superiority) over the defense was insufficient in the face of heavier fire power to gain the objective of the operation, that is, a successful amphibious landing. Increasing the order of battle ratio superiority of the amphibious operation to four-to-one over the defense achieved victory for the landing force, but it also had the surprising effect of raising the air mobile threshold back to 90%. Finally, only the amphibious operation's air mobile force's fire power was perturbed upward relative to the fire power of all other battlefield units. For two-to-one superiority, it was possible to reduce the air mobile decision threshold to 70% - still not compatible with real-world doctrine. Again, increasing the order of battle ratio in favor of the landing force to four-to-one causes the air mobile decision threshold to return to the 90% level.

Based upon these results, one might ask whether the commander's low air mobile decision threshold level of 20% to 30% is consistent with the optimal achieving of the amphibious operation objective, that objective being the winning of the battle? What the mathematical model indicates is that the air mobile mode of quick deployment of troops is not as effective as the time sequencing of the various surface assault units, which inherently deploy greater fire power, unless: either almost all of the amphibious force can go via the air mobile mode, or the assault units can move about the battlefield with infinite speed, or the margin of victory (overall superiority) of the amphibious operation is low or nonesistent.

Still another possible interpretation of the results is that the mathematical model used in the completed study was too abstract a representation of the amphibious operation and therefore did not adequately reflect all the important parameters affecting the outcome of the battle. This presumes that a more realistic presentation of the amphibious operation, taking into consideration parametric effects omitted from the completed study because the were considered of second order of effect, would yield results more compatibl with reality. Such a study certainly bears consideration in view of the importance of the information potentially available from the mathematical models and techniques demonstrated by the completed study.

II. THE AMPHIBIOUS OPERATION

A. INTRODUCTION

A typical fleet/air/amphibious operation is shown in Figure 2 illustrat the complexity of a MEF landing operation envisioned for the early 1970 tim period. The geographic area of involvement is over a hundred thousand squar miles, both on land and sea. The initial assault includes a beachhead landing which contains the bulk of the total MEF and a vertical-envelopment inland landing designed either to secure the beachhead or to achieve some objective that will aid in successfully getting the total force ashore. Or the beachhead is secure and control of the air accomplished, command is tra ferred ashore and inland objectives are pursued. The area of involvment in establishing the beachhead depends upon the nature of the threat and the geo political environment. The beachhead and its geographic neighborhood (shown by the dashed area) can be as low as 600 square miles in area for a non-nuclear threat, and as high as 2500 square miles for a nuclear threat (or a high probability of such a threat).[4]

Air warfare such as interdiction, air-to-air/ground, surveillance, etc. extend beyond the neighborhood of the beachhead for at least a radius of 300 n. mi. Air-to-air detection, acquisition, and kill extends another 150 Surface-to-air missile systems will be deployed on the beachhead with effective ranges up to 50 n. mi. The MEF vertical envelopment activity ranges are up to 25 n. mi. beyond the beachhead area. This distance can be extended to 50 n. mi. under exceptional circumstances. The Naval ship-to-shore fire support includes guns and missiles at ranges from 10, 20, 40 and even perhaps to 75 n. mi. inland from the beachhead. Figure 3 summarizes the geometry of the battlefield as visualized in reference 4.

B. PROBLEM DEFINITION, FORMULATION, AND SYNPOSIS

Based upon a simple scenario derivable from Figure 2, the following problem has been abstracted:

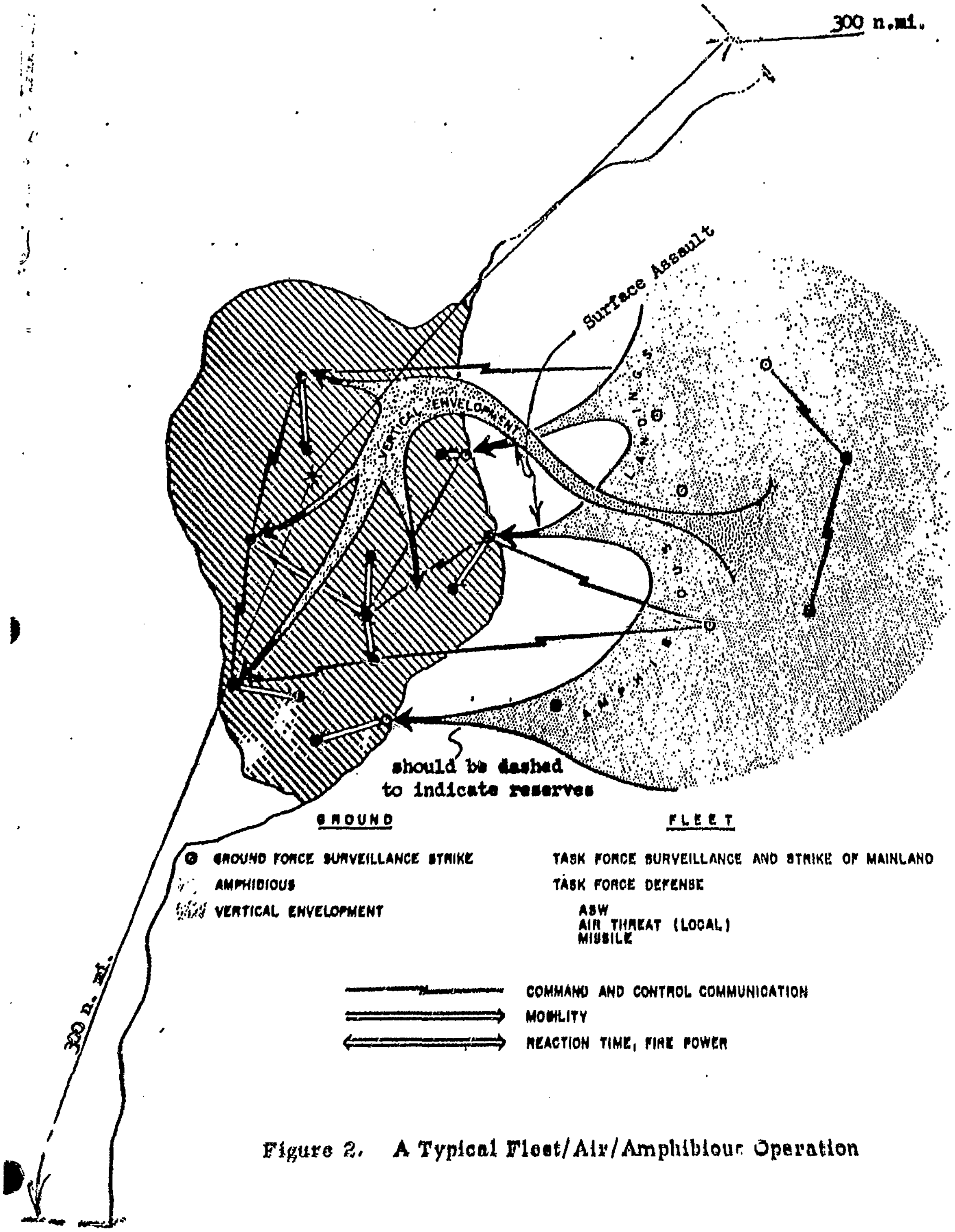


Figure 2. A Typical Fleet/Air/Amphibious Operation

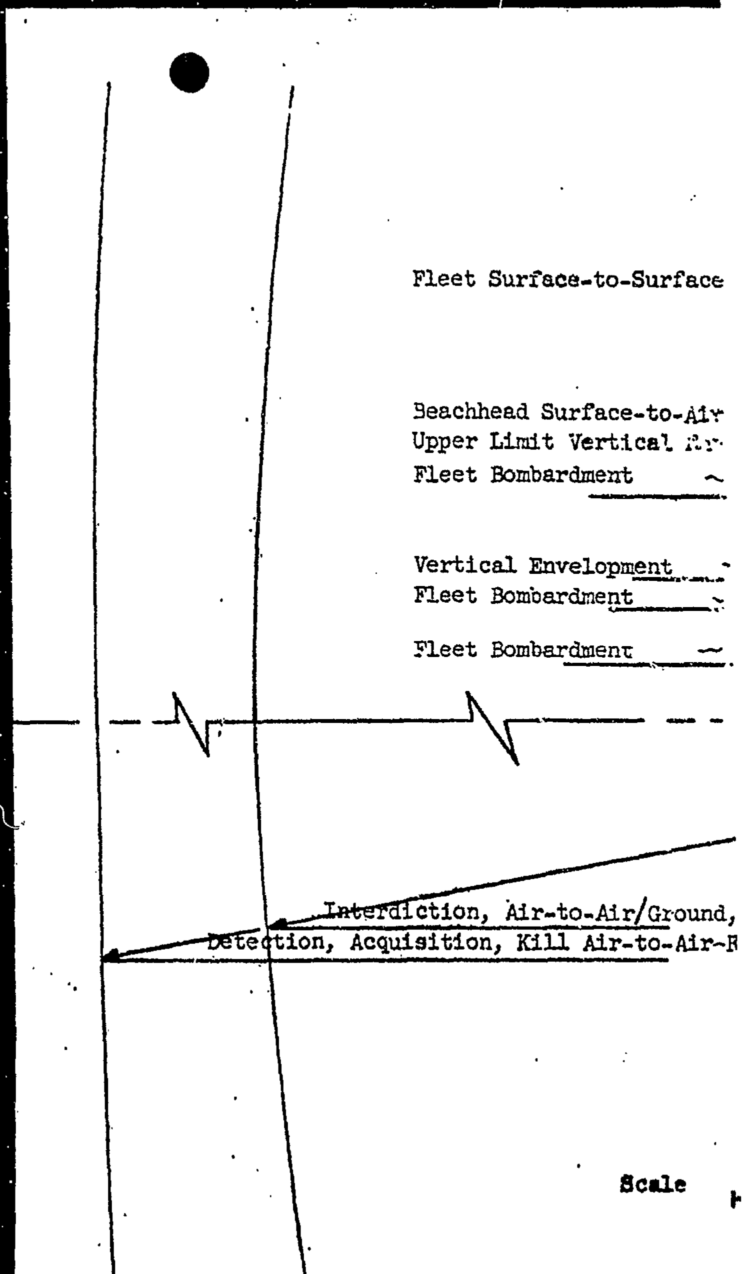
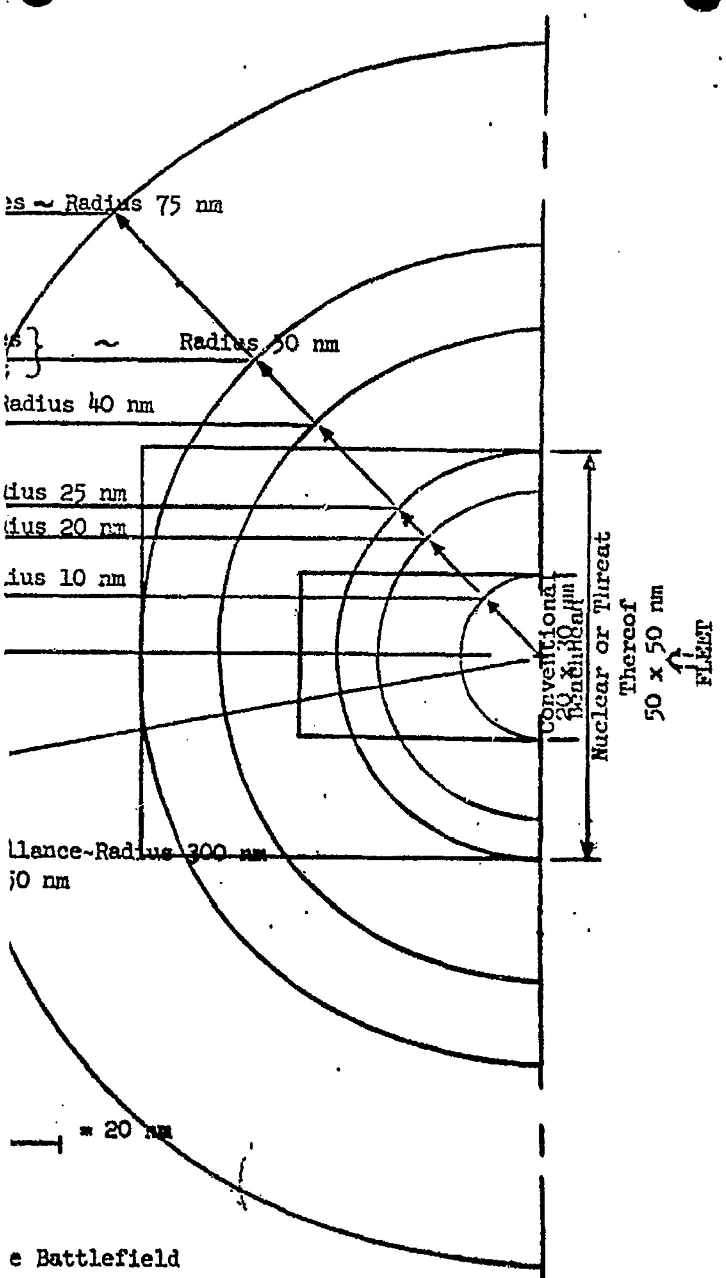


Figure 3. Geomet



e Battlefield

An amphibious landing force, x, at sea, is to assault a limited area defended by a force y. The landing force, x, is to be split up into air mobile, x_a, and a surface mobile, x_s. The defending force y in turn commit its defense by allocating β_i; i = 1 to 2 of its force to each element of x where $\sum_{i=1}^{2} \beta_i = 1$.

The questions to be asked are:

a) What allocation for forces should each side use against the other during the ensuing engagement?

b) What is the mathematical structure of the tactical decisions made by both sides (as defined by a) as a function of initial conditions (e.g., force levels (x, y) at time t = o) and constraints (e.g., spatial and temporal limitations when allocati-g forces)?

c) How does the analysis relate back to the real world in terms of logistics, equipment, force levels, operational plans, etc.?

To abstract the analytic nature of combat during an amphibious operat' from a scenario based upon the above descriptive analysis, one is tempted at first to start simply with a "Lanchester Equation" model approach. This model is by far the oldest analytic approach to land warfare and has great flexibility in its generalized form.[5]

The form of the Lanchester Model which seems applicable is

$$\frac{dx}{dt} = - by$$

$$\frac{dy}{dt} = - ax$$

$$x(o) = x_o$$

$$y(o) = y_o$$

where x_o, y_o represent the force levels of both sides at time $t = o$ and a, b reflects each side's normalized rate of attrition of the opposing side. Lanches Square Law can be deduced from (1) by taking the ratio of the two differential equations and integrating.

$$\frac{dx}{dy} = \frac{by}{ax} \tag{2}$$

$$\int_{x_o}^{x} ax\, dx = \int_{y_o}^{y} by\, dy \tag{3}$$

$$a(x_o^2 - x^2) = b(y_o^2 - y^2) \tag{4}$$

At this point it might be useful to test the above square law relationship with some of the available data on past battles. Weiss[7] has indicated that reasonable agreement with Lanchester's Square Law exists for the Pacific Island Campaign of World War II. In this paper eleven island campaigns were analyzed in terms of United States and enemy strengths before the operation and total losses after the islands were successfully taken. In each campaign the United States initiated the action with superior forces and if Lanchester's Square Law were applicable certain characteristics of the outcome of the battle can be deduced from equation (4). If one rewrites equation (4) as follows:

$$\frac{(y_o - y)\,(y_o + y)}{(x_o - x)\,(x_o + x)} = \frac{a}{b} \tag{5}$$

for the factors on the left hand side can be definable in real world terms. For example,

$\left(\dfrac{y_o - y}{x_o - x}\right)$ is the casualty ratio at time t

$\left(\frac{y_o + y}{x_o + x}\right)$ is the average force ratio from the start of battle to time

$\left(\frac{a}{b}\right)$ is a constant representing the average fire power ratio between sides.

If one converts the battle statistics available in Table I of reference 7 to the above ratios and inserts these ratios into the following logarithmic version of equation (5)

$$\log\left(\frac{y_o - y}{x_o - x}\right) + \log\left(\frac{y_o + y}{x_o + x}\right) = \text{constant} \qquad (6)$$

the plot shown in figure 4 results. The dispersion of all the battle points does reasonably confirm the minus-one slope of the line defined in equation (6). Assuming the validity of the square law in this plot, an estimate of the superiority of the attacking forces fire power over the defending forces fire power $\log\left(\frac{a}{b}\right)$ can be made by noting the ordinate-intercept of the band about the dispersed battle points. This intercept varies from 3 to 18 indicating the extreme variance in fire power superiority over the enemy for different amphibious operations of the last war. This begs the question as to why the effective fire power varies so much relative to the defender. Does the answer lie in the differences in tactical systems employed or operational doctrine used, or the enemy capabilities encountered etc? Certainly this wide dispersion of relative fire power found in real world situations suggests to the analyst the importance of being able to functionally relate the specific amphibious operational characteristics of the campaign such as systems employed, military doctrine used, and enemy capability encountered to an overall measure of effectiveness of the campaign. This type of analysis is one of the objective of this study.

In another plot of these same statistics[7], United States loss ratios have been plotted (see figure 5) against Order of Battle Ratios $(B/R) \sim \left(\frac{x_o + x}{y_o + y}\right)$ indicating the effect of superior forces in keeping the loss ratios low. In fact for a constant defending force, the attackers absolute loss goes down when

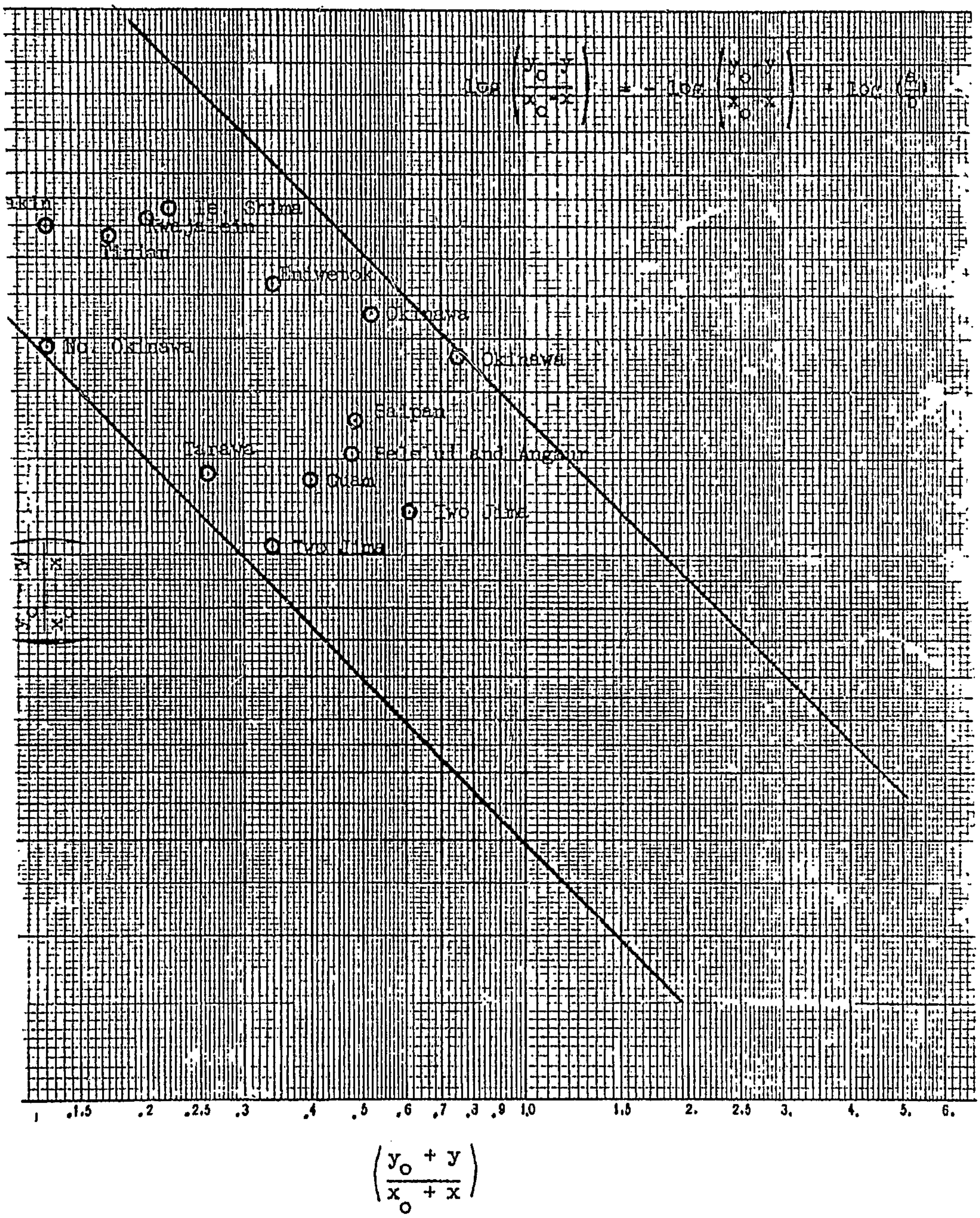


Figure 4. CASUALTY VS. AVERAGE FORCE DURING BATTLE (ON BOTH SIDES) RATIOS FOR WORLD WAR II PACIFIC ISLAND CAMPAIGNS

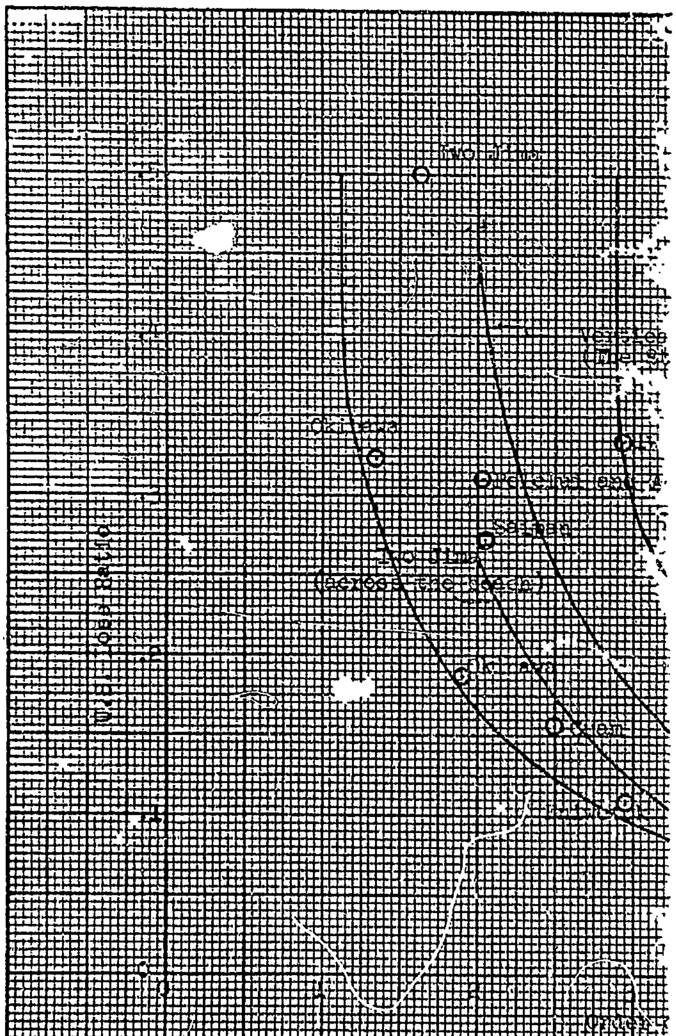
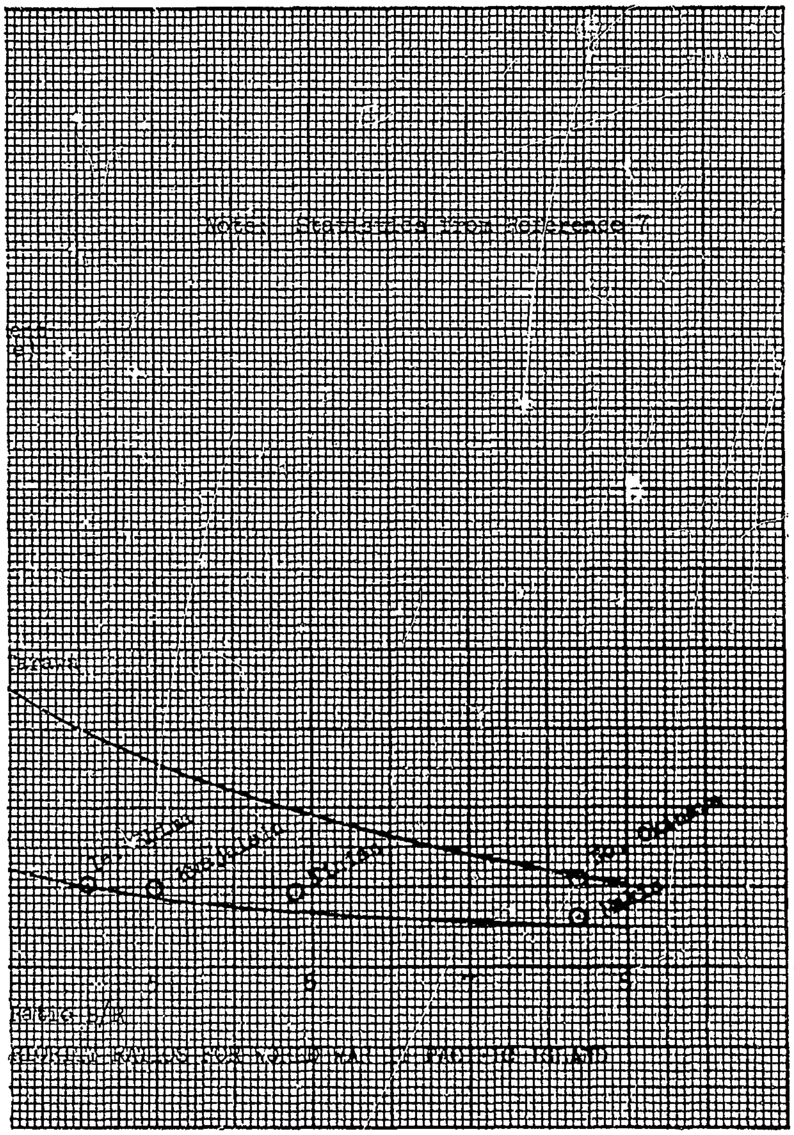


Figure 5. U.S. Loss vs. [illegible]
CAMPAIGN


Note: Statistics from Reference 7

order of battle ratio $\left(\frac{B}{R}\right)$ goes up if the square law is applicable to the situ Also shown in figure 5 as solid lines are some theoretical battles taken from o study computer runs. These battles were defined analytically in terms of Lanch Square Law and the results show an amazing similarity to the actual island camp The reason for the difference between the two runs is due to the fact that each curve is based upon a different scenario defining the amphibious operation batt Also the individual campaigns plotted from reference 3 represent different scenarios, however, their scatter on this particular chart does conform to a characteristic horn-shaped channel outlined by the envelope lines enclosing the battles shown in figure 5.

Let us now proceed with the development of the Lanchester Model. Equation 4 indicates that the normalized attrition rate varies inversely as the square of the force level. This suggests the following transformation whi will allow one to consider a force level and its attrition rate as an effectiv force level only.

$$x \longrightarrow \frac{x}{\sqrt{a}}$$

$$y \longrightarrow \frac{y}{\sqrt{b}}$$

$$t \longrightarrow \frac{t}{\sqrt{ab}} \qquad (7)$$

This reduces equation (1) to

$$\frac{dx}{dt} = -y$$

$$\frac{dy}{dt} = -x$$

$$x(o) = x_o$$

$$y(o) = y_o \qquad (8)$$

We are now in a position to relate this model formulation directly to the scenario of the amphibious operation described above. However, before we describe the mathematical model actually programmed, it might be useful to give a brief mathematical synopsis of this model utilizing the above notation in order to indicate the overall direction of the analysis.

Using the dot notation for time derivatives, the amphibious operation can be described mathematically by the following equations:

$$\dot{x}_a = -\beta y$$

$$\dot{x}_s = -(1-\beta)y$$

$$\left.\begin{aligned} \beta\dot{y} &= -x_a \\ (1-\beta)\,\dot{y} &= -x_s \end{aligned}\right\} \quad x = x_a + x_s$$

$$\left.\begin{aligned} &x(o) \\ &y(o) \end{aligned}\right\} \quad \text{given at } t = o \tag{9}$$

where β; $o \le \beta \le 1$ is the allocation of that part of y's force to x_a and $(1-\beta)$ to x_s, and $\{x_s, x_a\}$ represents an allocation of all of the amphibious landing force x to either the surface or air mobile mode of attack.

If any of the units on either side are reduced to zero during the battle, the opposing side's remaining allocation against the vanishing unit will be assumed to be transferable for combat against the other side's non-vanishing unit. The winner will be the side that survives the battle. To relate force levels and normalized attrition rates, the following scaling laws apply:

$$x_a \longrightarrow \sqrt{a_a} \quad x_a$$

$$x_s \longrightarrow \sqrt{a_s} \quad x_s$$

$$y \longrightarrow \sqrt{b} \quad y$$

$$t \longrightarrow \sqrt{a_i b} \quad t; \quad i = a, s$$

The model actually programmed can be extended to include x's reserve for x_r which can be thrown into the action at some later time. The computerized mo also takes into consideration the differences in fire power available to x when projected through a vertical envelopment assault or via a surface beachhead lan That is, x's Regimental Landing Teams, RLT's, are projected ashore as quickly possible with the heavy ordnance coming after. In the above model it is assume x has many units, each with its own characteristic fire power. The greater nor attrition rate a_s as associated with the surface force can only be projected at a given rate. The vertical envelopment with the lower attrition rate, a_a, assumed to be projected ashore instantaneously. In this way x is faced with an allocation choice of projecting into battle low fire power forces x_a at a high (instantaneously) or a high fire power force x_s at a much lower rate.

It is felt that these limiting assumptions are justified in that we are cerned only with the essence of the problem at this stage of the study; that is the gross outcome of the battle as a function of input parameters $\{ x(o); y(o) \}$ decision parameters $\{ x_a(o), x_s(o); \beta \}$ and constraints $\{ a_s, a_a;$ b; time to ini battle $\}$. Hopefully, this initial analysis will develop useful insight into the structure of amphibious operations such that more significant models can be deve later on.

Summarizing, equation (9) represents a simple model of an amphibious ope in a game theoretic context which attempts to relate the interplay of force lev allocation decisions, and constraints as they affect the outcome of the battle. payoff of such a game will be defined by the solutions of the above equations as function of time at the specific time when one side's force level is either redu to zero or any finite level. The remaining side's force level represents the va the game, positive to blue if the winner and negative to blue if the loser. This called a zero sum two person game.

Another possible zero sum two person game based upon the above model could be constructed by defining the payoff function or value of the game as the time t* it takes to complete the amphibious assault, either successfully or successfully. That is, the time it takes to reduce the opposing side either to zero or any predetermined level. This time would be minimized by the attacker (the amphibious landing force) while the defender would attempt to maximize the same time. Strategies for both sides defined by this type of payoff function could have a realistic interpretation in that one of the primary purposes of an amphibious operation is to secure a beachhead as quickly as possible such that the main body of attacking forces can be placed ashore safely. The defending forces will attempt to hinder the amphibious landing force as long as possible, such that superior reserves deployed elsewhere can be brought to bear upon the attacker while he is in the vulnerable position of attempting to establish a beachhead. While it is true that the attacking amphibious force would never knowingly go into an area under an adverse force posture, it is also true that if the defender knew the location of the beachhead in advance and martialled all his forces against this beachhead, the attacker would be under an extreme disadvantage in concluding the landing successfully. Since the defending force is unaware of the landing point because it is kept secret by the attacker and the attacking force further hinders the ability of the defender to obtain this secret by employing deception tactics, such as false landing starts, etc., the defending force is forced to keep troops in reserves. This makes it possible for the amphibious landing force to gain local superiority at a beachhead provided, the time between committing the attacking forces and securing of the beachhead is less than the time necessary for the defender to effectively deploy his reserves against the known landing point determined by the actual landing. The outcome of such a battle using the above model can then define the effectiveness of the deception techniques employed by the attacker. The model could also determine the relationship and value of fire power, mobility, dispersion, reaction time, and surveillance to various proposed deception techniques.

Up to now we have been describing the mathematics and possible use of the above model in extreme generality; however, before we get involved with the actual use of the amphibious landing model, we should describe the model as it was actually programmed. The next section describes the actual model from the mathematical point of view, and Appendix A gives the detailed computer program used to implement the mathematics.

C. THE AMPHIBIOUS OPERATION MODEL

1. Introduction

In the construction of a model of an amphibious operation the entire operation is first formulated as a complicated abstract process in the physical world. Certain items, such as number of battle units, force levels, firepower, etc., may be considered as inputs to the physical process while other items such as casualties, duration of the battle, etc., may be viewed as outputs. In mathematical terms, the amphibious operation is an "operator" with the physical input variables as its domain and the physical output variables as its range. In view of the complexity of a general amphibious operation and the extremely large number of contingent possibilities that can arise during the execution of any individual operation, it would be exceedingly difficult if not impossible to completely represent the operator mathematically. Our task then is to construct a mathematical operator that is an approximation to the physical process. The nature of the approximation is determined by the uses to which the model will be put and the resultant simplfications that can be tolerated without materially affecting the significant results.

As the purpose of this study is to qualitatively determine the nature of optimal battle strategies under very general conditions, no attempt will be made to abstract the analytic nature of the amphibious operation via an all-inclusive model. The basic unit of force for the protogonists, Blue and Red, will be taken as a battle unit and the battle as a whole will be viewed as an aggregation of local conflicts among individual battle units. Thus, at any one instant of time, we need only consider a series of relatively simple local conflicts to determine the state of the battle as a whole. Since we will assume that the dynamics of any local conflict are governed by Lanchester's Equations, the only pertinent information is the composition of the local conflicts and the force levels and attrition constants for the individual battle units. This information may be summarized in a list of the following form:

	Unit	Force Level	Attrition rate
	R1	-	-
	R2	-	-
	.		
	.		
	.		
	Ri	-	-
Local Conflict No. 1			
	B1	-	-
	B2	-	-
	.		
	.		
	.		
	Bj	-	-
	Ri+1	-	-
	Ri+2	-	-
	.		
	.		
	.		
	Rk	-	-
Local Conflict No. 2			
	Bj+1	-	-
	Bj+2	-	-
	.		
	.		
	.		
	Bm	-	-
etc			

The composition of the local conflicts will be determined by giving an Opponent Priority List (OPL) for each battle unit. In order to reduce the complexity of the computer program that implements this model, the decision was made t limit the OPL to two levels, the first priority opponents and the second priority opponents. The following behavior is then postulated for the individual battle uni

RULE 1: A battle unit will seek to engage his first priority opponents if they have not been already eliminated from the battle. If a unit is engaged with an opponent, that opponent is termed a direct opponent of the given unit.

RULE 2: If a unit's first priority opponents have been eliminated, he will, after a specified time delay, seek to engage his second priority opponents.

RULE 3: Any two units that share a common direct opponent are considered to be battle allies.

RULE 4: All the direct opponents of a given unit's battle allies are taken to be additional direct opponents of the unit itself.

These rules offer a great flexibility in the determination of the conflic since a unit can be drawn into a given conflict in many ways - by attacking an opponent who is involved in that conflict, by being attacked by an opponent who is involved in that conflict, or by sharing a common direct opponent with an ally wh is involved in that conflict.

In general, the application of Rules 1 and 2 and the repeated application of Rules 3 and 4 will completely determine the composition of all the local conflic (A formal proof can be constructed that shows that Rules 3 and 4 need only be appli a limited number of times and that a unique splitting up of the units into local conflicts results. The proof, which will be omitted here, rests on the fact that t above rules define an equivalence relation on the set of all units and this relatio completely partitions the set into equivalence classes.)

Within each local conflict, the progress of the battle may be measured by means of Lanchester's Equations. These differential equations can be solved analytically and evaluated to determine the force levels of any unit in a given loc conflict at any time in the future, as long as the basic composition of the conflict remains unchanged. Once the basic composition changes the coeffi of the differential equations change and the original analytic solution is valid. At this point we must redetermine the composition of the local conf calculate the new coefficients for the differential equations, and begin a of analytic solutions to continue where the old solutions left off.

The composition of a local conflict can be changed one of two ways:

1. a unit is added to the battle
2. a unit is eliminated

The specification of a time of arrival for each unit determines the time at unit is added to the battle. A single time of arrival is given since it is that once a force is committed to the battle it will not be withdrawn an at another time. Once a unit is added, it of course seeks out its opponen ing to Rules 1 and 2.

The elimination of a unit occurs when its force level drops below a specified minimum force level. The time at which this takes place can be de by inverting the analytic solutions of the differential equations and solvi time as a function of force levels. This inverse solution has several possi depending on the coefficients and initial values of the differential equatio These solutions are given in detail in the next section.

After all the first priority opponents of a given unit are eliminate specifies a time delay before that unit may engage its second priority oppone (The rule also applies if the first priority opponents have been eliminated the unit's time of arrival.) This delay factor is intended to reflect the ge graphical location of the units and their relative mobilities.

In summary, the following items are the necessary input parameters f unit:

1. initial force level
2. minimum force level
3. attrition constants
4. time of arrival
5. time delay factor
6. first priority opponents li
7. second priority opponents l

The application of Rules 1, 2, 3 and 4, in conjunction with the preceding items, determines the composition of the local conflicts while Lanchester's equations give the force level of each of the units as a function of time and indicates the time at which a unit is eliminated.

As analytic methods are used throughout and no time-step simulation is utilized, this furnishes us with an extremely rapid means of determining the expected outcome (or "payoff") of the battle determined by the given battle plans and initial force levels.

If various elements of the battle plans and/or initial force levels are considered as parameters we thus are in a position to generate trade-off tables showing the effect of a variation in one or more of the parameters, for example, the degradation of payoff due to increased time spacing of the various landing groups. Furthermore, game theoretic techniques may be used to simultaneously optimize the choice of parameters for both the attacker and defender.

2. The Application of Lanchester's Equations to Local Conflicts

As outlined in the previous section, Lanchester's Equations are u to measure the force levels of any unit at any given time. It should again noted that the application of these equations depends upon the various coefficients of the differential equations remaining constants To insure this, we have specifically assumed that the composition of the local conflict remains unchanged during the time period to which we are limiting ourselves.

For the sake of simplicity in the following discussion we will begin by assuming that the local conflict occurs between two individu battle units, A and B, on opposing sides and then generalize to a larger number of units. The force level of each unit will be termed n_A and n_B respectively and the attrition rate by a_A and a_B. The familiar Lanchester equations governing this simplified combat are:

$$\dot{n}_A = - a_B\, n_B$$

$$\dot{n}_B = - a_A\, n_A$$

when the dot notation is used to denote differentiation with respect to time. These simultaneous equations may be solved to give n_A and n_B as functions of time:

$$n_A(t) = n_A(t_o) \cosh\left(\sqrt{a_A\, a_B}\,[t-t_o]\right) - \frac{\sqrt{a_B}}{\sqrt{a_A}}\, n_B(t_o) \sinh\left(\sqrt{a_A a_B}\,[t-t_o]\right)$$

$$n_B(t) = n_B(t_o) \cosh\left(\sqrt{a_A\, a_B}\,[t-t_o]\right) - \frac{\sqrt{a_A}}{\sqrt{a_B}}\, n_A(t_o) \sinh\left(\sqrt{a_A a_B}\,[t-t_o]\right)$$

In general, however, a local conflict may have more than a single unit each side. If side A has k battle units involved and side b has ℓ battle units volved we may identify the forces levels as n_{A1}, n_{A2}, ..., n_{Ak}, n_{B1}, n_{B2}, ..., $n_{B\ell}$ the attrition rates as a_{A1}, a_{A2}, ..., a_{Ak}, a_{B1}, a_{B2}, ..., $a_{B\ell}$. For simplicity sake, may assume without any loss of generality, that the subscripts are assigned in a manner that

$$a_{A1} \geq a_{A2} \geq \cdots \geq a_{Ak}$$

and

$$a_{B1} \geq a_{B2} \geq \cdots \geq a_{B\ell}$$

Under these circumstances it can be shown that optimum behavior on each side demands that fire be concentrated on units A1 and B1. (It is recognized th this type of behavior does not occur in practice. It is felt, however, that the optimum behavior postulated here offers a sufficiently close approximation to r for our purposes. If subsequent investigation should prove this not to be corre then an alternate scheme of behavior could easily be postulated.)

The differential equations, in this case, become:

$$\dot{n}_{A1} = - a_{B1} n_{B1} - \sum_{i=2}^{\ell} a_{Bi} b_{Bi} \qquad ($$

$$\dot{n}_{aj} = 0 \qquad (\text{for } j = 2, \ell) \qquad ($$

$$\dot{n}_{B1} = - a_{A1} n_{A1} - \sum_{\alpha=2}^{k} a_{A\alpha} n_{A\alpha} \qquad ($$

$$\dot{n}_{B\beta} = 0 \qquad (\text{for } \beta = 2, k) \qquad (1$$

Equations (8) and (10) indicate that:

$$n_{Aj}(t) = n_{Aj}(t_o) \qquad (\text{for } j = 2, \ell)$$

$$n_{Bs}(t) = n_{Bs}(t_o) \qquad (\text{for } s = 2, k)$$

and so equations (7) and (9) can be represented in the form:

$$\dot{n}_{A1} = - a_{B1}\, n_{B1} + r$$

$$\dot{n}_{N1} = - a_{A1}\, n_{A1} + s$$

where r and s are both constants:

$$r = - \sum_{i=2}^{\ell} a_{Bi}\, n_{Bi}$$

$$s = - \sum_{\alpha=2}^{k} a_{A\alpha}\, n_{A\alpha}$$

The general time solutions for the force levels become:

1) If $a_{A1} > 0$ and $a_{B1} > 0$

$$m_{A1}(t) = \left[m_{A1}(t_o) - \frac{s}{a_{A1}}\right] \cosh\left(\sqrt{a_{A1}a_{B1}}\,(t-t_o)\right)$$

$$\frac{\sqrt{a_{B1}}}{\sqrt{a_{A1}}}\left[m_{B1}(t_o) - \frac{r}{a_{B1}}\right] \sinh\left(\sqrt{a_{A1}a_{B1}}\,(t-t_o)\right)$$

$$+ \frac{s}{a_{A1}}$$

2) If a_{A1} or $a_{B1} = 0$

$$n_{A1}(t) = -\frac{a_{B1}}{2} s(t-t_o) - a_{B1}\, n_{B1}(t_o) - r \quad (t-t_o) + n_{A1}(t_o) \qquad (14$$

The equations for $n_{B1}(t)$ are identical in form to equations (13) and (14) above, with the subscripts, of course, being reversed.

For the purpose of determining the "time to elimination", the inverse solution of the above equations are needed. These are:

1) If $a_{B1} = 0$ and $r < 0$:

$$t = t_o + \frac{n_{A1} - n_{A1}(t_o)}{r} \qquad (15$$

2) If $a_{B1} = 0$ and $r \geq 0$:

no solution exists; i.e., $t = \infty$

3) If $a_{A1} = 0$ and $s = 0$ and $r < a_{B1}\, n_{B1}(t_o)$:

$$t = t_o + \frac{n_{A1} - n_{A1}(t_o)}{(a_{B1} n_{B1}(t_o) - r} \qquad (16$$

4) If $a_{A1} = 0$ and $s = 0$ and $r \geq a_{B1} n_{B1}(t_o)$:

no solution exists; i.e. $t = \infty$

5) If $a_{A1} = 0$ and $s \geq 0$:

$$t = t_o + \frac{[r - a_{B1} n_{B1}(t_o)] + \sqrt{(a_{B1} n_{B1}(t_o) - r)^2 + 2 a_{B1} s (n_{A1}(t_o) - n_{s1})}}{a_{B1} s} \qquad (1$$

6) If $a_{A1} = 0$ and $s < 0$ and $r > a_{B1}n_{B1}(t_o) - \sqrt{-2a_{B1}s[n_{A1}(t_o) - n_{s1}]}$;

no solution exists; i.e., $t = \infty$

7) If $a_{A1} = 0$ and $s < 0$ and $r \leq a_{B1}n_{B1}(t_o) - \sqrt{-2a_{B1}s(n_{A1}(t_o) - n_{A1})}$;

$$t = t_o + \frac{[r - a_{B1}n_{B1}(t_o)] + \sqrt{[a_{B1}n_{B1}(t_o) - r]^2 + 2a_{B1}s[n_{s1}(t_o) - n_{A1}]}}{n_{B1}d}$$

8) If $\sqrt{a_{B1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] + \sqrt{a_{A1}}\left[n_{A1}(t_o) - \frac{s}{a_{A1}}\right] = 0$ and $\frac{s}{a_{A1}} < n_{A1}$:

$$t = t_o + \frac{1}{\sqrt{a_{A1}a_{B1}}} \ln \left| \frac{n_{A1}^{(t_o)} - \frac{s}{a_{A1}}}{\frac{s}{a_{A1}} - n_{A1}} \right| \quad ($$

9) If $\sqrt{a_{B1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] + \sqrt{a_{A1}}\left[n_{A1}(t_o) - \frac{s}{a_{A1}}\right] < 0$:

$$t = t_o + \frac{1}{\sqrt{a_{A1}a_{B1}}} \ln \left\{ \frac{\left(n_{A1} - \frac{s}{a_{A1}}\right) - \sqrt{\left(n_{A1} - \frac{s}{a_{A1}}\right)^2 + \frac{a_{B1}}{a_{A1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right]^2 - \left[n_{A1}(\right.}}{\frac{\sqrt{a_{B1}}}{\sqrt{a_{A1}}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] + \left[n_{A1} - \frac{s}{a_{A1}}\right]} \right. \quad ($$

10) If $\sqrt{a_{B1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] + \sqrt{a_{A1}}\left[n_{A1}(t_o) - \frac{s}{a_{A1}}\right] > 0$ and $\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] \geq$

no solution exists; i.e., $t = \infty$

11) If $\sqrt{a_{B1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] + \sqrt{a_{A1}}\left[n_{A1}(t_o) - \frac{s}{a_{A1}}\right] > 0$ and $\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] < 0$

and $\sqrt{\left[n_{A1}(t_o) - \frac{s}{a_{A1}}\right]^2 - \frac{a_{B1}}{a_{A1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right]^2} > n_{A1} - \frac{s}{a_{A1}}$:

no solution exists; i.e., $t = \infty$

12) If $\sqrt{a_{B1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] + \sqrt{a_{A1}}\left[n_{A1}(t_o) - \frac{s}{a_{A1}}\right] > 0$ and $\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] <$

and $\sqrt{\left[n_{s1}({}_o) - \frac{s}{a_{A1}}\right]^2 - \frac{a_{B1}}{a_{A1}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right]^2} \leq n_{A1} - \frac{s}{a_{A1}}$:

$$t = t_o + \frac{1}{\sqrt{a_{A1}a_{B1}}} \ln \left\{ \frac{\left(n_{A1} - \frac{s}{a_{A1}}\right) - \sqrt{\left[n_{A1} - n_{A1}(t_o)\right]\left[n_{A1} + n_{A1}(t_o) - \frac{2s}{a_{A1}}\right] + \frac{a_{B1}}{a_{A1}}\left[\cdots\right.}}{\frac{\sqrt{a_{B1}}}{\sqrt{a_{A1}}}\left[\frac{r}{a_{B1}} - n_{B1}(t_o)\right] + \left[n_{A1}(t_o) - \frac{s}{a_{A1}}\right]} \right. \tag{21}$$

D. PROBLEM I

1. Introduction

To illustrate the methodological techniques for handling an amphibious ing situation, a specific problem will be investigated (see Figure 6). An ampl landing force B, at sea, is to assault a limited area defended by a force R. ing force is to be split up into three surface mobile elements B_1, B_2, B_3 and mobile element B_4, each of which is to initiate action sequentially in time. elements are projected ashore sequentially in time with the first wave consist infantry battalion type units, designated B_1, then a short time δt later more i plus close support artillery type units, designated B_2, and finally after anot increment δt some more infantry plus a tank section designated B_3. The air mo elements consisting of a vertical envelopment team of infantry and close supp artillery units, designated as B_4, which are projected instantaneously inland tance d from the beach (see Figure 6) at the same time B_1 arrives across the b The relative fire power of these four elements of blue are assumed to be in the lowing ratio: $B_1 : B_2 : B_3 : B_4 :$ 1.0: 2.0: 4.0: 1.5. The red defending force R turn, commits its forces either to the beach (designated R_1), to the airmobil (designated R_2), or to both at the same time B_1 and B_4 are deployed. The relat firepower of these two elements of red are assumed to be equal to blue's maxim $B_3 : R_1 : R_2$: 1.0: 1.0: 1.0. Note (from Figure 6) that each element of blue and has a predetermined battle commitment time which reflects the amphibious land force logistic constraints (Reference 4). This time of commitment is symbolize the clock next to each element in Figure 6. Also, take note that the ensuing b takes place at two different locations, at the beach and at some inland point. model developed for this problem reflects this spatial characteristic of the bat by defining a set of time delays $\{t_{d_1}\}$ which applies to each force element c battle indicating the amount of time required for each element to traverse this tance if required to do so during the course of the battle. Whether an element verses this distance depends upon whether the force element has an opponent to f during the initial allocation of forces and successfully destroys the opponent s that any new fighting opponents can only be reached by traversing the distance d shown in Figure 6. If a force element does not have an opponent initially at it particular point on the battlefield then the time delay t_d will denote the time necessary to meet an opponent located at the other point on the battlefield. Na this time delay reflects the mobility characteristics of the u ts involved and sents an important trade-off parameter.

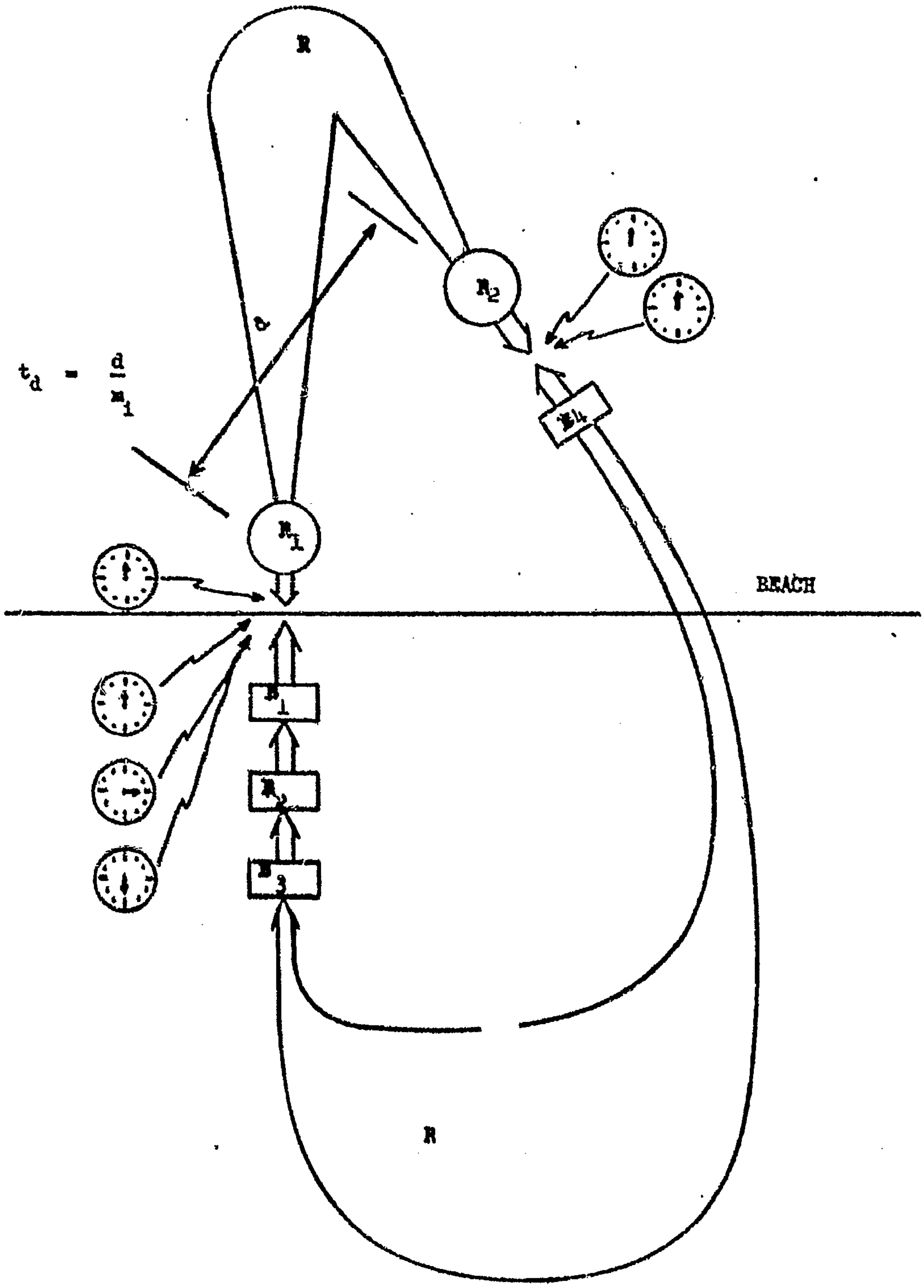


Figure 6. THE AMPHIBIOUS LANDING MODEL

Another aspect of problem I not indicated in Figure [illegible] is the fact that fighting element has a priority list of opponents. This means that every fight element knows its first priority opponent and when the action is successfully [illegible] pleted, it can then attack a predefined second priority opponent. In some case fighting elements are placed in position to protect the flanks of an operation, no priority is utilized in which case fighting initiates only when the opposing priority initiates the battle action against the flank.

The following questions are pertinent to the above problem description:

a) What is the mathematical structure of the tactical decisions made by both sides in selecting the action variables such as number of forces, time delays, battletime commitments, priorities, etc.?

b) How does this type of an analysis relate back to the real world?

c) What allocation of forces should each side use in initiating the battle? e.g. Blue's allocation is that of deploying its total units to the beach and/or inland locations and red's total unit allocation is that of defending the beach and/or staying in the rear to engage the vertical envelopment attack. Both sides must make such allocations quantitatively.

d) What time sequence should the amphibious landing force adhere to in initiating its elements into combat?

The significant input variables to the model based upon the above [illegible]

a) Order of battle values - the total number of troops each side has available for the operation.

b) Firepower - each fighting element's rate of kill per man per hour.

c) The sequencing of units into the battle

d) Battle priorities

e) Unit mobility factors

The significant output variables are:

a) The number of survivors at the end of the battle.

b) The duration of the battle for any threshold of defeat of one side.

c) The optimum allocation strategies available to each side, e.g., what fraction of the total forces goes across the beach? Employed air mobile as opposed to the defender's allocation of what fraction of his forces is defending the beach as opposed to the forces kept in reserve to defend against the air mobile attack.

criteria that will be used to determine the optimality of the strategies will
min max payoff for red and max min payoff for blue. When min max equals max m
pure solution exists and the payoff corresponding to this pure solution is cal
value of the game. If no such equality exists, then it can be shown that min
min and the only way to get equality is to redefine the payoff function as an
value of survivors with the players picking their strategic variables (i.e.,
total forces allocated to air mobile and defending against air mobile) accordi
probability distribution. Decisions made in this manner are called mixed stra
usually represent marginal strategies for the side having to employ them. Fr
tactical system design or requirements point of view, one would never knowing
an amphibious operation against a defending force which depended upon a mixed
to gain the objective of the operation. This would be tantamount to having
order to achieve success. Whereas the logical plan for an amphibious operatio
to land with overwhelming superiority and allow any advantage accrued by the m
of secrecy in initiating the operation to compensate for faulty threat intell
estimates, acts of God (e.g. bad weather that is unpredictable), etc. To det
just what constitutes overwhelming superiority in planning an amphibious oper
just how much can one degrade this superiority and still achieve objectives w
certainty is one of the objectives of this study. The techniques employed t
these types of questions will be the subject matter of this section.

2. Discussion

To obtain the game-theoretic solution to the amphibious operation desc
above (see Figure 6) we first compute the mathematical surface representing th
of all possible allocations for both sides (Figure 7). Then we tag the minim
row and the maximum of each column shown respectively as rectangles and elips
Figure 8. Blue, the attacker, will select the maximum of the minimums tagged
rows while red, the defender, will select the minimum of the maximums tagged
columns. The point on the grid (see Figure 9) in which the maximum of the mi
off and the minimum of the maximum payoff occurs is the same point and the va
game is defined as the number of survivors of blue/red (positive for blue, ne
for red) located at this point on the surface. The strategies associated with
point are pure. In most cases the min max = max min solution occurs at corne
the matrix.

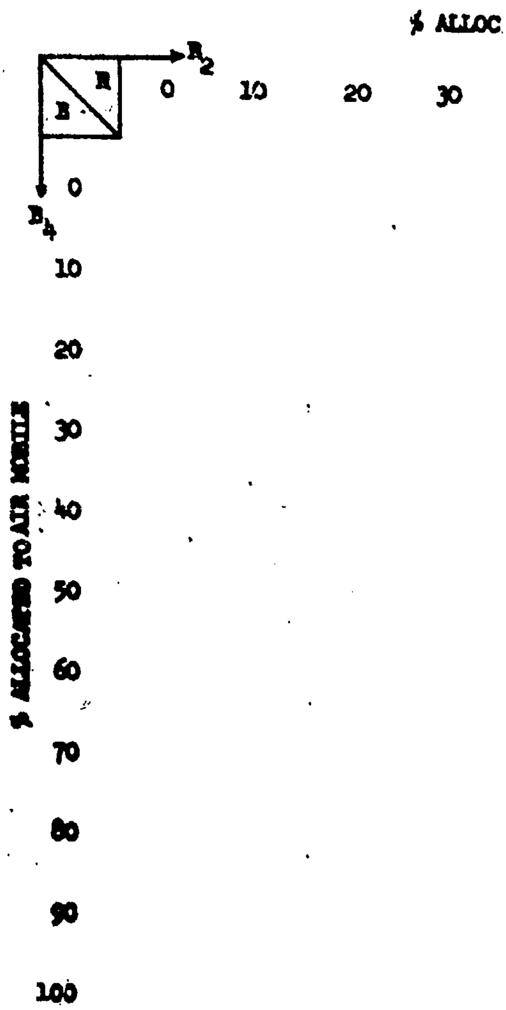
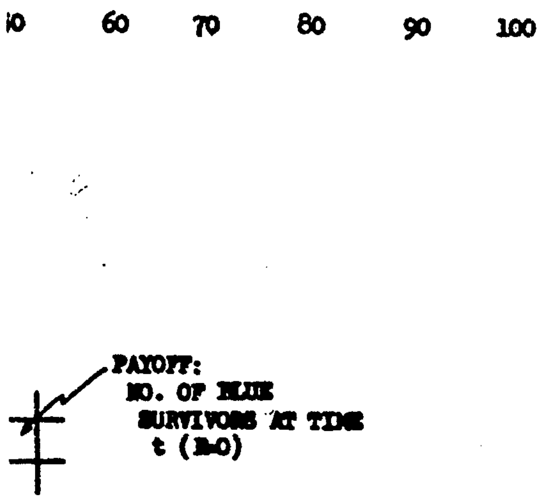

% ALLOC
R_2
R
B
B_4
0
10
20
30
0
10
20
30
40
50
60
70
80
90
100
% ALLOCATED TO AIR MOBILE

Figure 7.

F MATRIX

FEND AGAINST AIR MOBILE



. Decision Surface

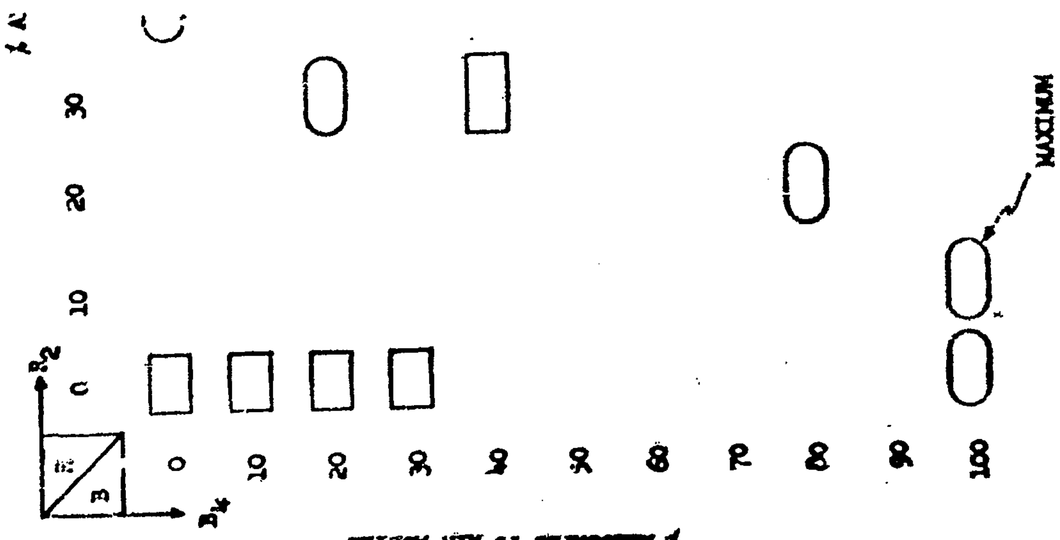
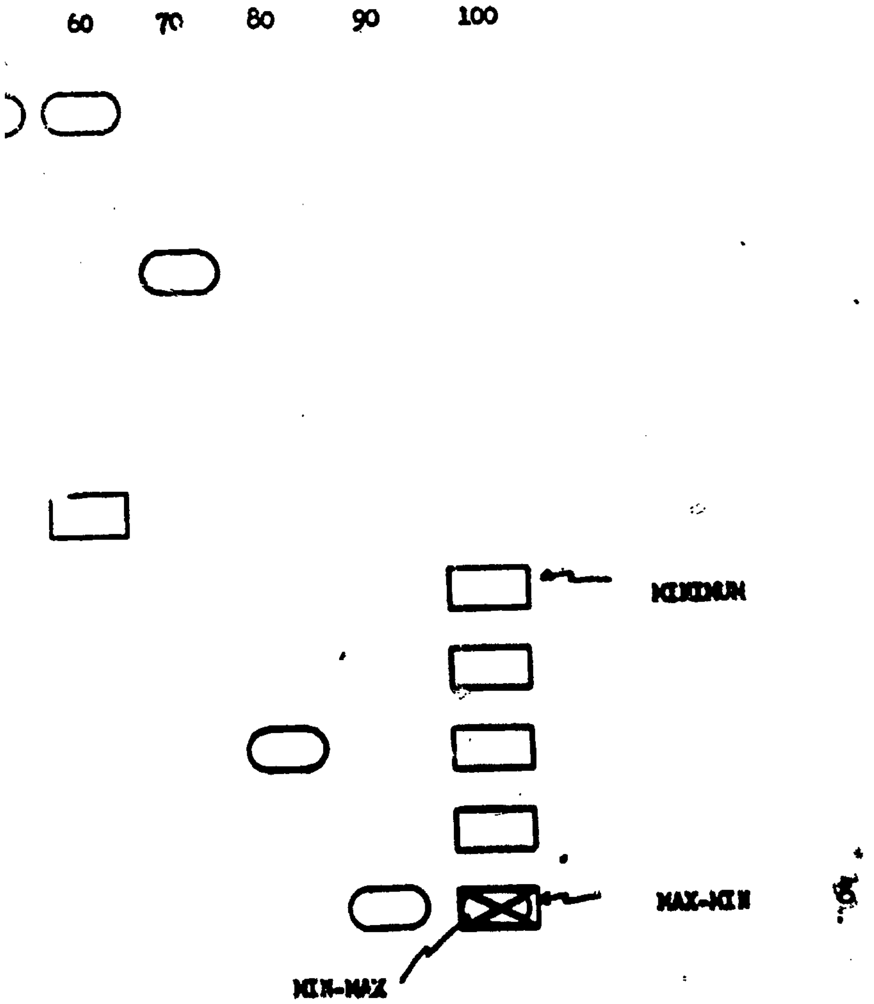


Figure 8. Game Theoretic Solution - Unrestricted Case

O DEFEND AGAINST AIR MOBILE



Once the value of the game and associated decision variables are known wit constraints, the solutions determined from natural discontinuities of the surface generated. This is accomplished by placing a constraint raster first on blue's st line starting from the 100% allocation decision level and moving the raster toward the 0% allocation. For each placement of the raster the matrix is solved for the min max = max min solution in the same manner as outlined above on for the parti matrix (see Figure 9) consisting of blue's strategy line from 0% allocation to the raster and red's unrestricted strategy line. This sub-optimum solution to the gam is then related to the placement of the raster by noting when the solution change abruptly as the raster moves from 100% to 0% allocation. For example, if blue's u restricted strategy is to allocate 100% of the fighting elements to the air mobile or vertical envelopment decision the raster is then placed under the 90% level re ing blue's strategy from 0% to 90% and the solution to this restricted game is not If the optimum solution is the 90% allocation of the fighting elements via the ai mobile mode decision, then this is considered to be no change in the basic strat. That is, the restricted game still demands that blue send all his fighting units a mobile even though blue is restricted by the 90% allocation level. After this det mination, the raster is then moved to the 80% allocation level and the restricted game again solved. If the solution yields the 80% allocation level decision the is moved to the 70% level, etc. In most cases during this process of methodically constraining blue's decision level the game theoretic solution will abruptly chang to yield an optimum decision other than the maximum possible fighting units going air mobile. The position of blue's constraint raster when such an abrupt decisio occurs is then recorded and the associated sub-optimum strategy restricted within this partitioned matrix is called a natural strategic discontinuity level. The physical meaning of such sub-optimum strategies is that if for any physical reason blue cannot send 100% of his forces air mobile (the optimal policy), at what level of decision constraint must blue change his tactics completely concerning a given mode of attack. Therefore, the natural strategic discontinuity level represents a threshold in blue's strategic thinking, above which blue will attack in the vertic envelopment mode with all the fighting elements he physically can get air mobile, and below which he will use the sub-optimum strategy based upon the solution of the partitioned matrix. An example of this threshold from the real world would be the decision of a commander not to send an air mobile strike in support of an across-[illegible] operation if he felt that the air mobile forces would not be able to act a fighting unit during the time necessary for the main body forces from

SOLUTION W

% ALLOCA

R_2 0 10 20 30 40

R

B

B_4

0

10

20

30

40

50

60

80

90

100

% ALLOCATED TO AIR MOBILE

PAYOFF:
NO. OF BLUE
SURVIVORS
% (R=0)

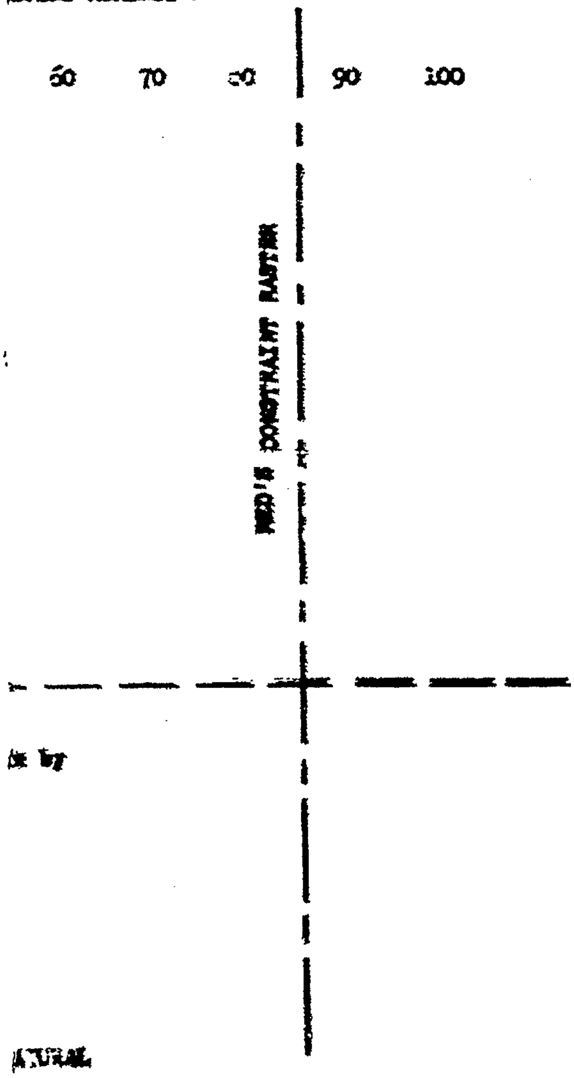
BLUE'S CONSTRAINT R

Note: Solution to the constra
solving the partitioned

Figure 9. GAME THEORETIC SOLUTIO
DISCONTINUITY LEVELS

NAL DISCONTINUITY LEVELS

EFEND AGAINST AIR MOBILE



a by

ACTURAL

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

	Perturbed Parameter	Order of Battle Ratio (B/R)
Standard Case	~	$1 \le B/R \le 4$
Time Delay	$0 \le t_d \le 4$	2, 4
Amphibious Operation Firepower	$.1 \le B_1 \le .3$ $.2 \le B_2 \le .6$ $.4 \le B_3 \le 1.2$ $.15 \le B_4 \le .45$	2,4
Defender's Firepower	$.4 \le R_1 \le 1.6$ $.4 \le R_2 \le 1.6$	2,4
Vertical Envelopment Firepower	$.15 \le B_4 \le .45$	2,4

FIGURE 10. SENSITIVITY ANAI

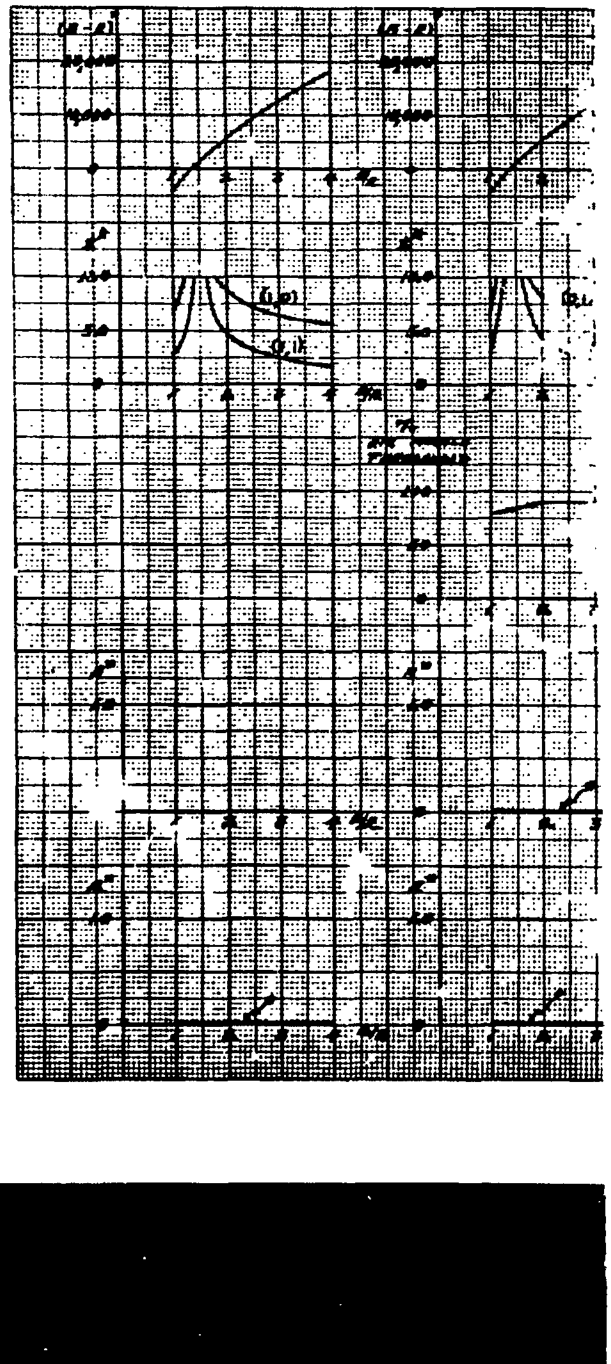
(hours) Time Delay Required units to traverse the distance between the beach and reserves (t_d)	(kill/hr/man) Blue's Fire Power Surface Mobile (B_1, B_2, B_3)	(kill/hr/man) Blue's Fire Power Air Mobile (B_a)	(kill/hr/man) Red's Fire Power Beach and Reserve (R_1, R_2)
4	.1, .2, .4	.15	.4, .4
~	.1, .2, .4	.15	.4, .4
4	~	~	.4, .4
4	.1, .2, .4	.15	~
4	.1, .2, .4	~	.4, .4

The major inputs to the model are represented by the columns. Als in this analysis is the fact that each fighting element has a second level opp means that the surviving element after an engagement on the battlefield is cor seek out a new opponent as opposed to staying put waiting for an enemy element from another section of the battlefield. In this way mixed (or asymmetric) ob will n t be an influencing factor. In other words the payoff function, i.e., of survivors, will reflect active combat on both sides.

b. The Standard Case

The standard case with inputs shown in Figure 10 was analyzed by Computer Program No. AM104B titled, "Assault Tactics." A complete descriptio computer program is contained in Appendix A of this report. The results of th are displayed as a matrix of survivors for blue, red and the time length of e as defined by the particular allocation for each side. Examples of these outr shown in Appendix A. The game-theoretic analyses of these outputs were done b and displayed in the form of a series of graphs placed in columns, each colum senting a different natural constraint of the problem.

Figure 11 gives the game-theoretic analysis of the standard case. first column represents the unrestricted play of the game in that the strategic available to both sides encompasses the full payoff matrix or decision surface. independent variable is the order of battle ratio (B/R) which reflects the degr superiority with which the attacking force initiates the amphibious operation. In the fir in the upper left hand corner the attacker's optimum payoff (B-R)* in terms of n survivors (positive for blue or attacking survivors, negative for red or defend survivors) is plotted. The defending (red) force is kept constant at 5000 men attacking (blue) force is allowed to vary from 5000 to 20,000 men. Note that a both sides are reduced to zero at the same time for the optimal allocation (min max min criteria) of forces on the battlefield. This order of battle ratio is attacker's threshold for successfully completing the operation, i.e., the attack ducing the defender to no survivors. However, an inspection of the actual decis surfaces (Figures 12 to 16) reveals that this threshold has no meaning as a p factor because many of the non-optimum choices for both sides will result in the attacker not achieving his objectives. In other words, the optimum allocation d point is an unconservative criteria for use in determining the level of superior



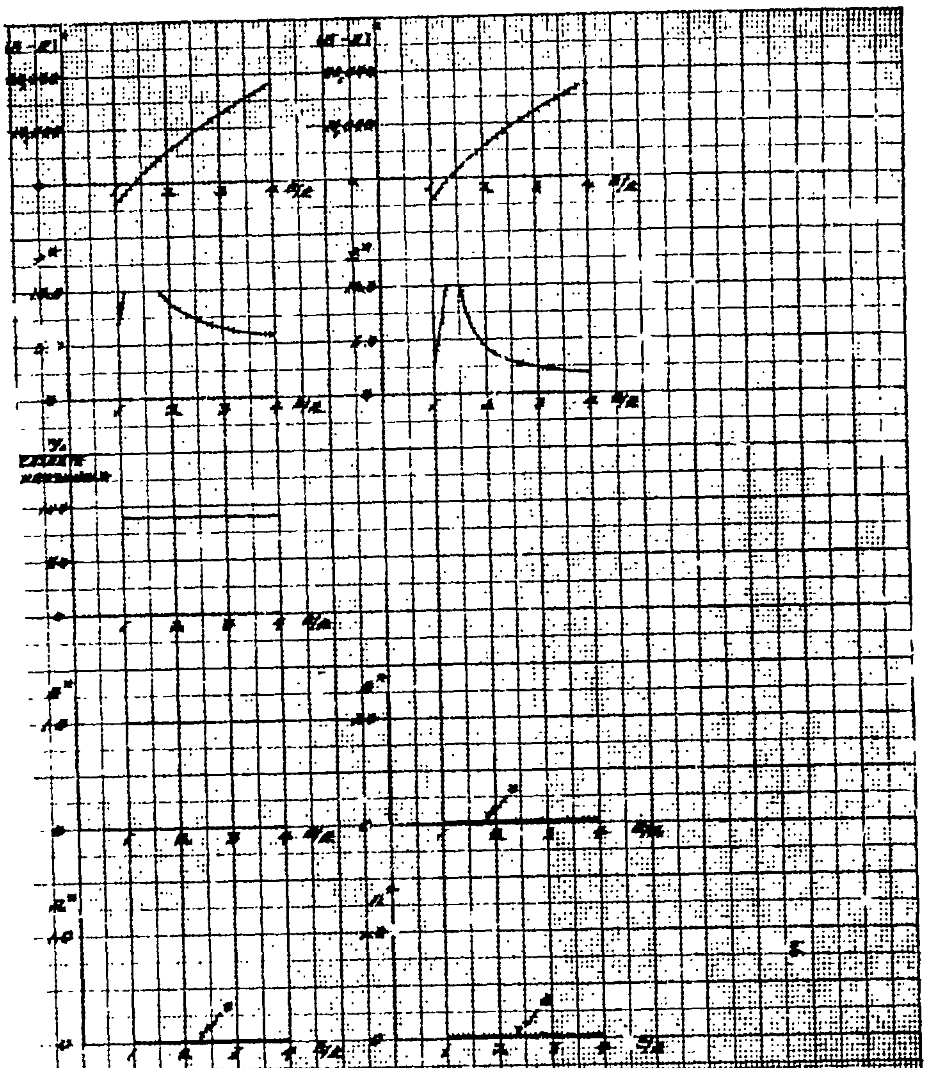


Figure 11 STANDARD CASE m 5, t_1 4.0, { a; b } = { .1, .2, .4, .15, .4, 4 }

		0	.1	.2	.3
	0	-4226	-4074	-3861	-3530
	.1	-4407	-4199	-4100	-3889
	.2	-4538	-3978	-4171	[illegible]
	.3	-4622	-4078	-3964	-4141
	.4	-4662	-4127	-3651	[illegible]
B	.5	-4630	-4116	-3657	-355[illegible]
	.6	-4566	-4043	-3584	-3217
	.7	-4469	-3933	-3460	-307[illegible]
	.8	-4337	-3782	-3287	-2882
	.9	-4166	-3586	-3059	-2619
	1.0	-3953	-3335	-2761	-2264

Figure

R

.4	.5	.6	.7	.8	.9	1.0
031	-2124	-1960	-2238	-2678	-3215	-4286
510	-2835	-2369	-2609	-3001	-3493	-4407
377	-3410	-2642	-2866	-3231	-3678	-4536
071	-3835	-3200	-3036	-3388	-3840	-4618
117	-4044	-3752	-3134	-3482	-3927	-4656
004	-4089	-4011	-3572	-3517	-3962	-4630
552	-3985	-4062	-3966	-3495	-3947	-4566
098	-3697	-3959	-4035	-3884	-3881	-4469
609	-3172	-3683	-3925	-4008	-3761	-4337
315	-2508	-3208	-3642	-3875	-3980	-4166
904	-2145	-2801	-3251	-3574	-3798	-3953

ANDARD CASE B/R = 1.0

		0	.1	.2	.3
	0	5492	6440	7094	7566
	.1	4400	5520	5810	6367
	.2	1674	4464	5548	5209
	.3	-2128	1317	4437	5576
	.4	-2812	-1734	1546	4317
B	.5	-2913	-1910	709	2840
	.6	-2733	-1643	1690	3200
	.7	-2205	- 398	3074	4081
	.8	- 471	3378	4554	5235
	.9	3854	5182	6015	6546
	1.0	5774	6733	7394	7832

R

.5	.6	.7	.8	.9	1.0
8112	8212	8196	8054	7754	5492
6968	7060	7013	6806	6378	3413
5886	5968	5878	5571	4859	-1352
4916	4984	4837	4371	3293	-2328
4601	4194	3987	3315	933	-2719
5633	4761	3476	2703	- 980	-2789
4124	5661	4870	3167	- 740	-2569
4802	4719	5689	4927	3330	-1252
5775	5690	5361	5717	4927	3397
6942	6845	6551	6036	5745	4860
8165	8083	7832	7394	6733	5774

STANDARD CASE B/R = 2.0

		0	.1	.2	.3
	0	9026	9518	10087	10453
	.1	8640	9070	8692	9096
	.2	7998	8840	9114	8444
	.3	6798	8148	8936	9158
	.4	4764	6582	8123	8997
<u>B</u>	.5	2225	4329	6217	7929
	.6	2937	4639	5635	6270
	.7	4351	5622	6450	6596
	.8	6048	6970	7621	8075
	.9	7791	8527	9058	9418
	1.0	9465	10079	10532	10844

Figure I

R

.5	.6	.7	.8	.9	1.0
10922	11033	11062	11006	10845	9026
9593	9699	9712	9624	9415	7485
8364	8465	8458	8332	7896	6007
7425	7382	7353	7185	6757	4642
8889	7790	6469	6266	5852	3560
9246	8967	8066	6767	5297	3434
9038	9289	9121	8246	7039	5708
7458	8949	9333	9055	8320	7204
8452	8399	8540	9377	9063	8281
9697	9631	9433	9085	9421	8989
11087	11026	10844	10532	10079	9465

RD CASE B/R = 2.5

		0	.1	.2
	0	12004	12477	12868
	.1	11750	12062	11324
	.2	11390	11931	12120
	.3	10803	11583	12031
	.4	9680	10837	11685
B	.5	7905	9369	10739
	.6	6626	7549	8865
	.7	7629	8427	9006
	.8	9058	9725	10220
	.9	10805	11347	11751
	1.0	12583	13051	13404

F

R

.4	.5	.6	.7	.8	.9	1.0
13431	13613	13730	13783	13769	13667	12004
11881	12068	12182	12223	12188	11762	10349
10633	10650	10760	10788	10729	10332	8859
11978	11191	9930	9533	9449	9097	7619
12236	12072	11534	10445	9210	8132	6764
12167	12293	12146	11697	10828	9677	8469
11682	12216	12351	12207	11791	11053	9995
9803	11332	12243	12409	12256	11835	11103
10779	10866	10829	12199	12467	12286	11794
12196	12251	12208	12054	11786	12525	12251
13796	13844	13796	13650	13404	13051	12583

STANDARD CASE B/R = 3.0

		0	.1	.2
	0	17515	17881	18196
	.1	17351	17589	[illegible]
	.2	17138	17513	17663
	.3	16843	17317	[illegible]
	.4	16390	17003	17435
B	.5	15505	16444	17087
	.6	13869	15089	16282
	.7	12606	13114	[illegible]
	.8	14145	14588	14928
	.9	16106	16474	16755
	1.0	18257	18583	18833

R

.4	.5	.6	.7	.8	.9	1.0
18678	18848	18972	19050	19076	19026	17515
16672	16845	16967	17038	16964	16625	15480
17034	15916	15135	15197	15178	14831	13718
17666	17360	16720	15637	14486	13317	12345
17812	17757	17523	17074	16333	15272	14171
17781	17886	17839	17635	17258	16676	15796
17577	17853	17960	17914	17717	17365	16794
16909	17587	17916	18035	17982	17773	17372
15328	16115	17428	17959	18109	18038	17760
17078	17128	17099	16992	17871	18183	18042
19114	19149	19114	19009	18833	18583	18257

16 - STANDARD CASE B/R = 4.0

the attacker must plan for in order to guarantee his objective, i.e., a posit off value for the attacker. This unconservatism is due to the fact that in t world situation intelligence for both sides would be less than perfect and op decisions would have to be judged from a probabilistic basis. The model used study assumes both sides have perfect intelligence which is certainly not the world case. However, it is possible to use the results of this run to determ proper order of battle ratio for the attacker which is independent of the al decision made by either side, thus removing the intelligence aspect of the pr (i.e., attacker's advance knowledge as to the defenders allocation of forces) can be done by determining the order of battle ratio above which the payoff decision surface always stays positive. This occurs approximately at (B/R) - represents a measure of the amount of intelligence indeterminancy the part bious operation scenario (in this case the model of Figure 6) contains. The can be defined in this case by the increment of troops necessary to guarante success of the operation independent of intelligence, i.e., from an order of ratio of (B/R) = 1.4 to 2.3 representing the increment of forces necessary perfect intelligence to guarantee success of the operation with imperfect in gence. One example of imperfect intelligence would be for the commander of amphibious operation to err, based upon poor surveillance information in his of the defense's allocation of forces to the beach and/or reserves.

The next graph in the left hand column of Figure 11 represents the ti duration of the battle t^* based upon both sides optimizing their decision var in the sense defined in the graph above, e.g. $(B/R)^* = f(B/R)$. Two time solu exist because red, the defender, has two different force allocations which yi the same optimum payoff. The following two charts in this column represent b blue's and red's optimum strategies as a function of (B/R) for the unrestrict of the game. Blue, the attacker, sends all his assault elements in the air m mode ($R^* = 1.0$) and red, the defender, either defends the beach with all his at the time the battle is initiated, or stays in the rear with all his force attacks all of blue's air mobile units deployed. If no blue air mobile units deployed (a non-optimum play on blue's part), red traverses the distance to th in time t_d after the battle is initiated and attacks blue's units landing on beach. If red defends the beach and blue goes 100% air mobile (optimum) then battle is initiated t_d hours after vertical envelopment touchdown. Depending red's optimum choice of strategy, the length of time of the battle will refle not only the rate of attrition of both sides, but also the possible transport time between both points of the battlefield, i.e., between the beach and the Note that when both sides reduce each other to zero force strength at the same the time of the t^* goes to infinity (see figure 11). This is a consequence Lanchester's equations which represents a mathematical discontinuity for this cular solution. It can be avoided by having a finite number or percentage of survivors determine the pay off surface.

Summarizing the above, we can say the left-hand column of Figure 11 re the results of the unrestricted optimal play of the game between blue, the attacke the defender (defined in Figure 6) as a function of the order of battle ratio para (B/R). The payoff function $(B-R)^* = f(B/R)$ indicates that an order of battle ra. at least $\{(B/R) > 1.4\}$ is necessary for blue to achieve its objectives (reducing re zero survivors while blue survives) under optimal decisions and perfect intellige both sides. It was also implied (Figures 12 to 16) that a $(B/R) > 2.3$ was necessa guarantee the objective independent of the strategic decisions made by either sid perfect intelligence). It should be noted that both sides had complete freedom decision choice. That is, blue could send any portion of its forces air mobile to the fire power constraints of the air mobile mode. While red could defend th and/or inland area without constraint. This is called the unrestricted play of t We are now in a position to discuss the natural discontinuity levels of the st decision surface for both sides. These are indicated in the next three columns in Figure 11.

c. The Natural Strategic Discontinuity Levels

Based upon the above analysis, we would like to pose the following question:

> To what extent does possible real world constraints affect the outpu recorded by the standard case solution for the unrestricted play of the game?

From the mathematical model point of view, real world constraints such as logist of the amphibious operation have not been directly programmed. However, inheren payoff matrices of Figures 12 to 16 lies mathematical constraints which indicate sensitivity of optimum strategic decisions made by both sides to the value of th function. For example, in the second column of Figure 11, blue's (the attacker strategic discontinuity levels are recorded as a function of the order of battle (B/R).

The third graph in this second column of Figure 11 illustrates the t mathematical constraint we are talking about. This chart indicates blue's perce mobile allocation threshold above which blue must deploy to the air mobile mode for the air mobile mode to remain the optimum force allocation or decision defi unrestricted play of the game (the first column of Figure 11). If blue in atte play optimally, cannot deploy a greater percent of his forces to the air mobile this threshold percent indicates, then a radically different strategy is called this case, blue, the attacker, is forced to go 100% across the beach if his to remain optimum vis-a-vis red, the defender. This abrupt change in strat threshold level indicated is caused by a discontinuity of the optimum decis when solving for the min max = max min payoff point on the decision surfac limiting blue's capacity to fully allocate forces in the air mobile mode.

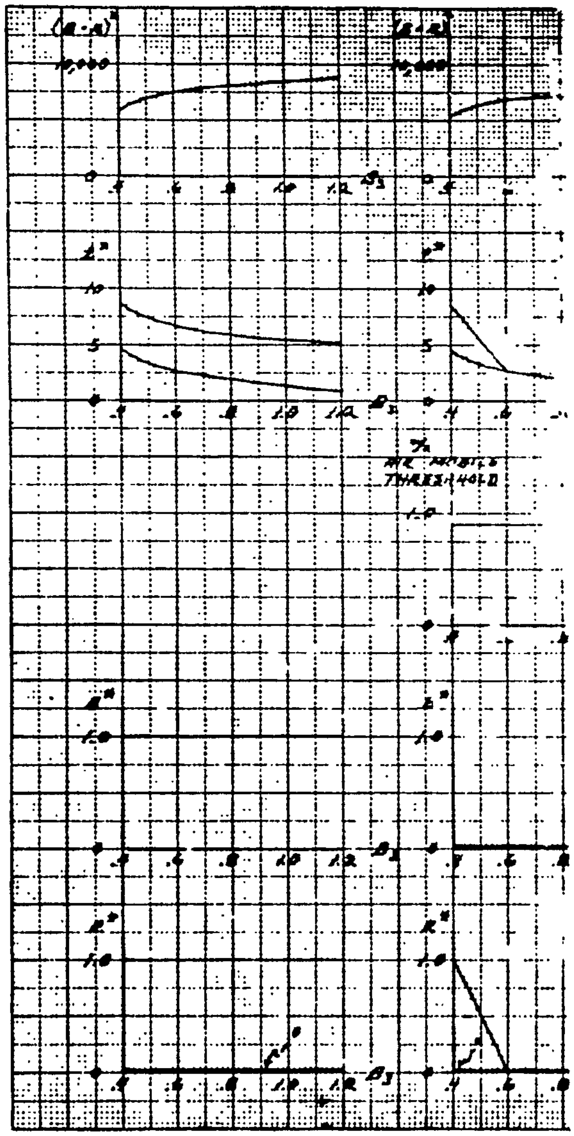
This threshold parameter is a significant output reflecting the of the mathematical model exercised which was not obvious to the analyst du construction phase of this study. Also, the air mobile threshold represent fication of a command decision made many times in the past by commanding o planning and tactical phase of great battles. The question as to whether to the rear via the air mobile mode and in what percentage of the total for deploy in this mode is usually answered at the highest levels of command whe judgment and experience are without peer. For the amphibious model to be this parameter quantitatively is a significant test of the model in that th deploy air mobile troops is a relatively new concept. Also to relate such the other inputs of the amphibious operation model such as fire power, mobi size, priority levels, etc. of both the attacker and defender should cert the knowledge and experience necessary to develop requirements, plan, and c amphibious environment. Before continuing the analysis in this direction, t columns of Figure 11 will be defined.

The third column represents red's, the defenders, threshold in s forces to the beach and/or remain inland to attack the air mobile force if d This same type of decision was made in the Battle of Okinawa by the Japanese they elected to dig in inland and let the United States amphibious landing p opposed. Based upon the results of this battle, one can say the Japanese c played optimally if not successfully. Columns two and three of Figure 11 a the new payoff values $(B-R)^*$ and the time t^* based upon the constrained str sulting from the natural discontinuity effect inherent in the strategic surf ly, column four of Figure 11 illustrates the results of the play of the game sides are forced into strategies constrained within their thresholds.

Summarizing we can say that the unrestricted play of the game re allocating completely to the air mobile mode while red defends completely ei beach or inland reserve (Figure 11, first column). If blue is physically co hand B/R=4 is called the superiority case in that it represents the greate relative to the defender, of blue forces used in the standard case. The t parameter t_d is allowed to run from $0 \le t_d \le 4$ hours and there does not see any visible effect on blue's air mobile threshold for the parity case (se third graph, second column). However, in the superiority case shown in Fi for zero time delay the air mobile threshold drops suddenly to zero. In o if each fighting element traverses the two points on the battlefield at i (a physical impossibility) no air mobile threshold exists for the superior and blue will send as many of his force elements via the air mobile mode a while still keeping within the optimality criteria for both the restrict restricted play of the game. It should be noted that only discrete t_d val computed and the curves shown in all these game runs are only valid at the where computations were made. In other words t_d for zero and one were it is not known when the threshold value jumps from zero to one. However, sake of simplicity, straight lines were drawn between computed points on t

The next variable that was perturbed was the attacker's (blue) Figure 10 indicates the range of values given to all four of blue's figh The fire power of each element was increased by a factor of half in both and superiority case (see Figures 19 and 20). There was no effect on blue decision threshold level which remained constant compared to the standard c Apparently increasing the relative strength of blue over red using additio power indicates that the surface attack mode becomes more desirable vis-a- mobile mode, thus keeping the commander's decision threshold relatively hig scale.

The next perturbation would be to weaken blue relative to red use of fire power. This was done for the parity and superiority cases in and 22 by raising red's fire power relative to blue (see Figure 10). Red's was raised in increments of 50% and a marked decrease in the commander's de hold is observed in the parity case (Figure 21, third graph, second column) Unfortunately blue, the attacker, does not achieve the objective of the ope cause just before the threshold starts to decrease the payoff $(B-R)^*$ to blu negative (that is, red wins). This is demonstrated by the first graph in t column of Figure 21. This parity case could have represented too marginal order of battle ratio superiority for blue. Possibly if blue used a B/R=4 Case, Figure 22), this would not only reduce the commander's decision thres


10,000
10
5
1.0
.4
.6
.8
1.0
1.2

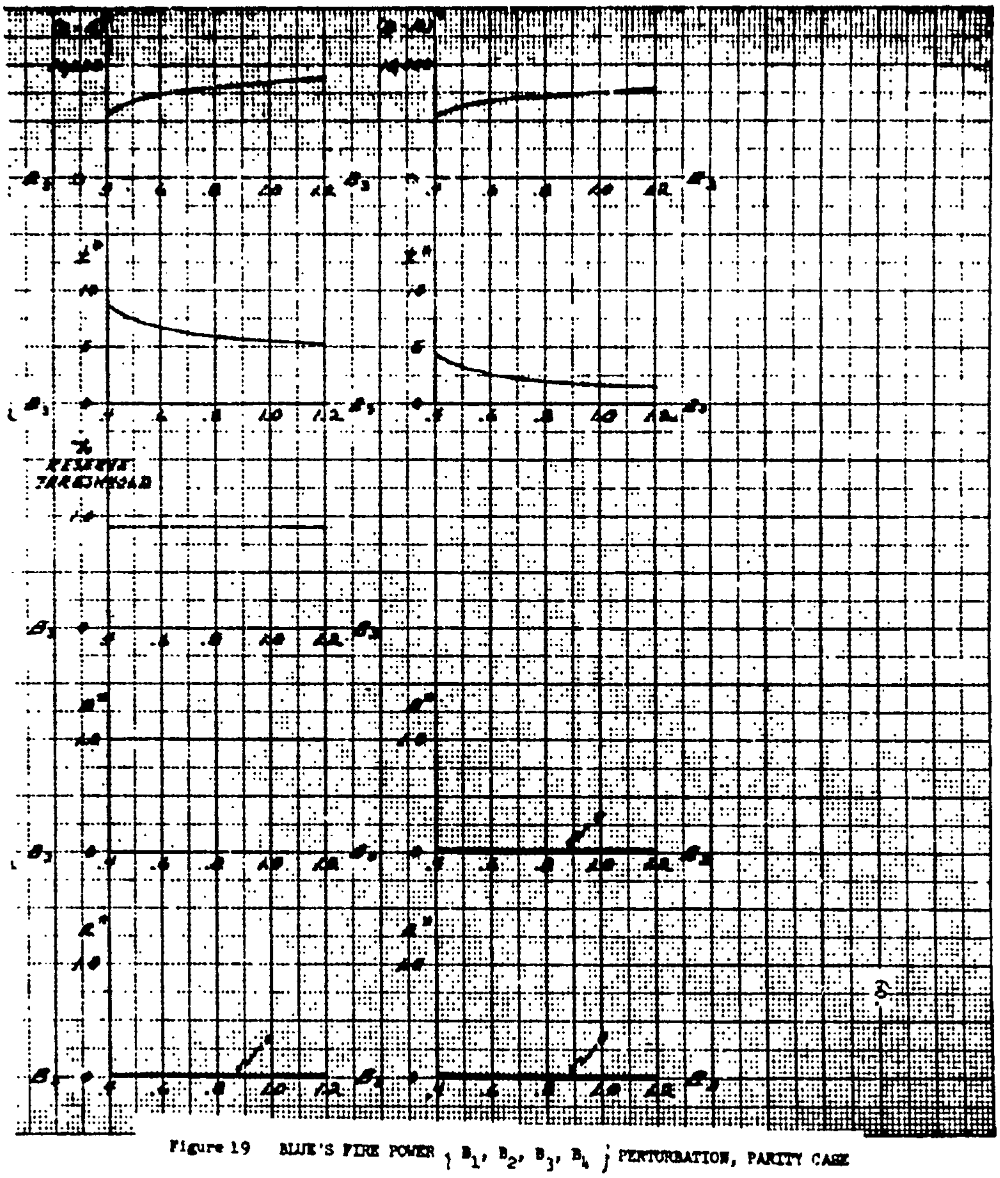
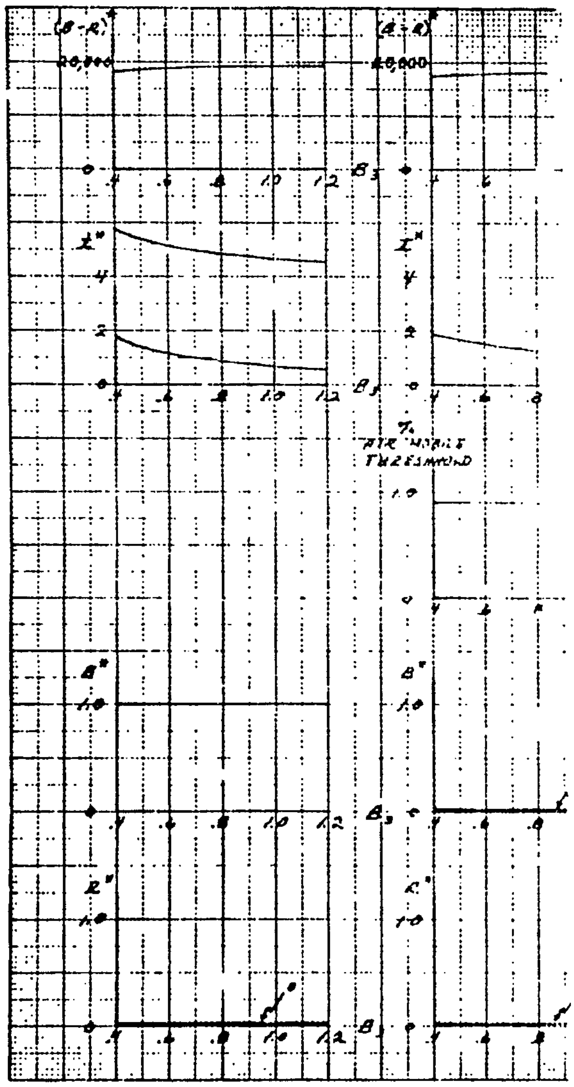


Figure 19 BLUE'S FIRE POWER { B_1, B_2, B_3, B_4 } PERTURBATION, PARITY CASE



(B-R)
20,000
.4
.6
.8
1.0
1.2

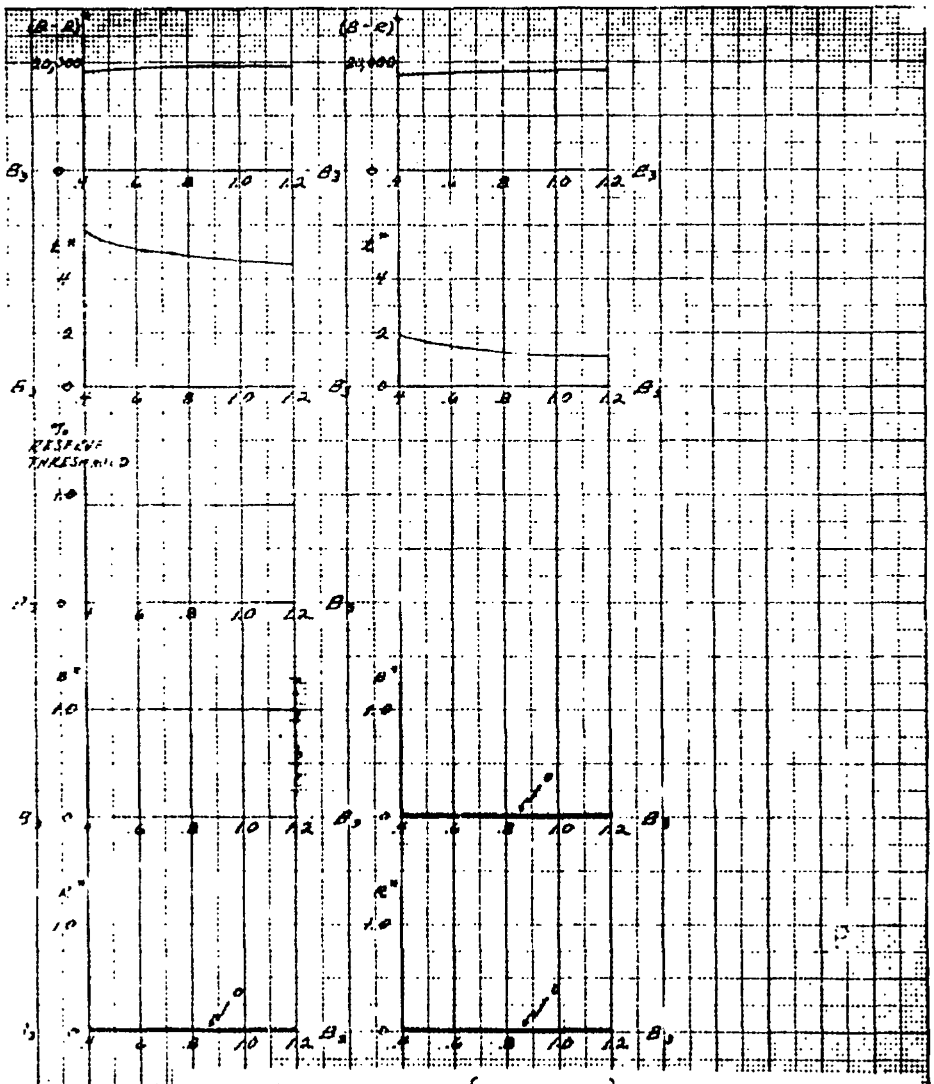
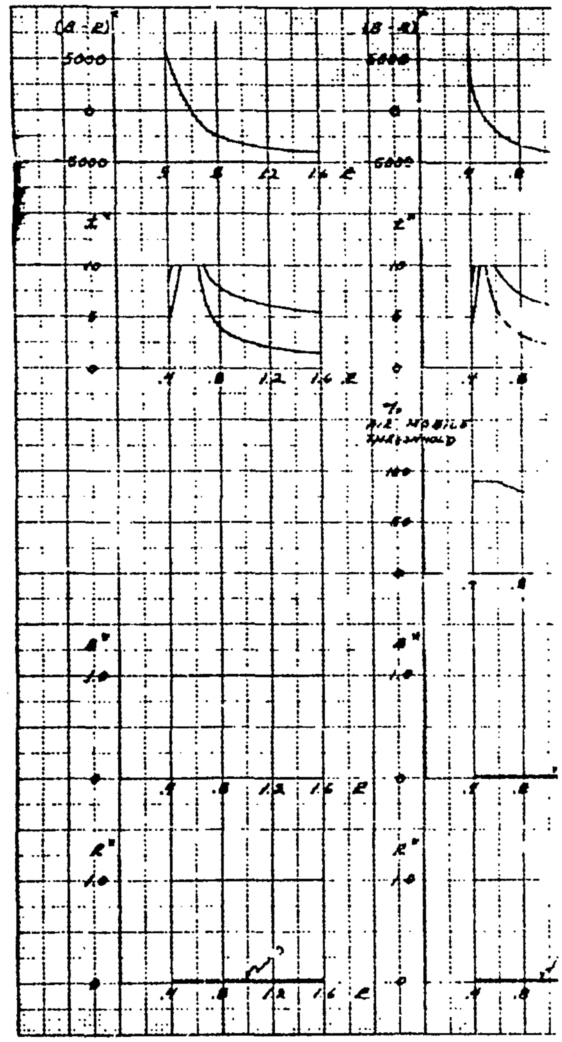


Figure 20 BLUE'S FIRE POWER $\{B_1, B_2, B_3\ B_4\}$ PERTURBATION, SUPERIORITY CASE



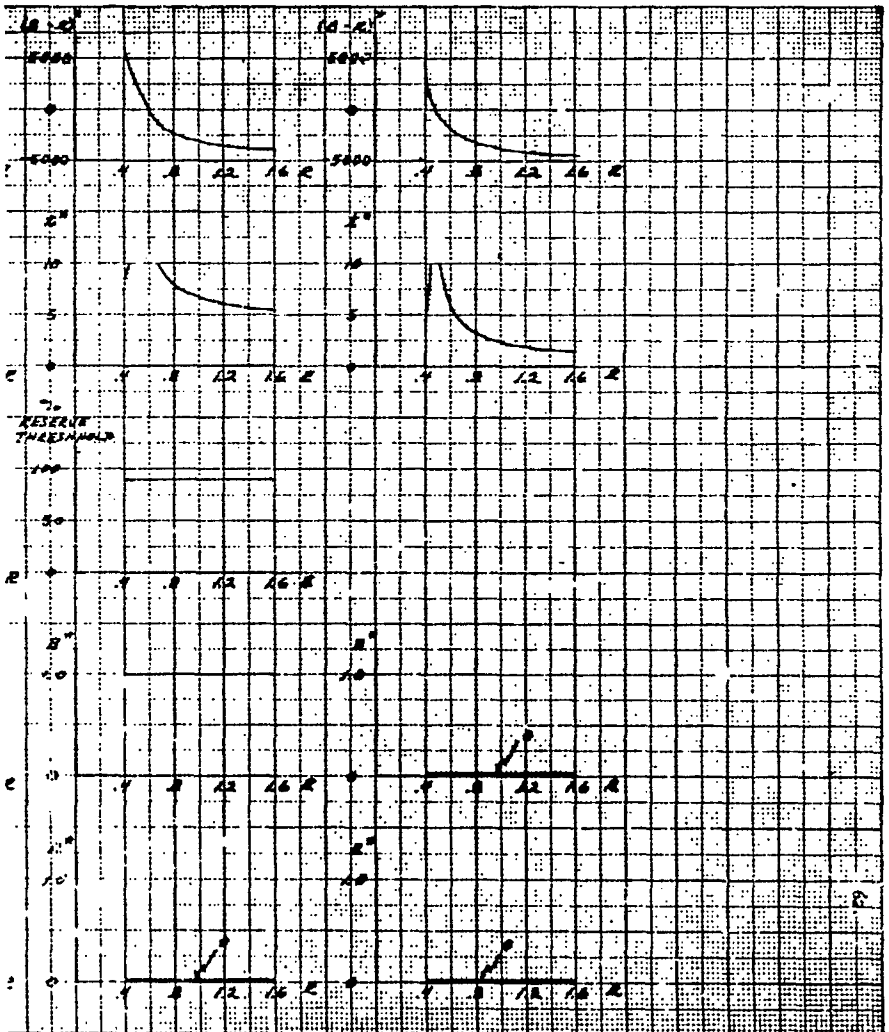
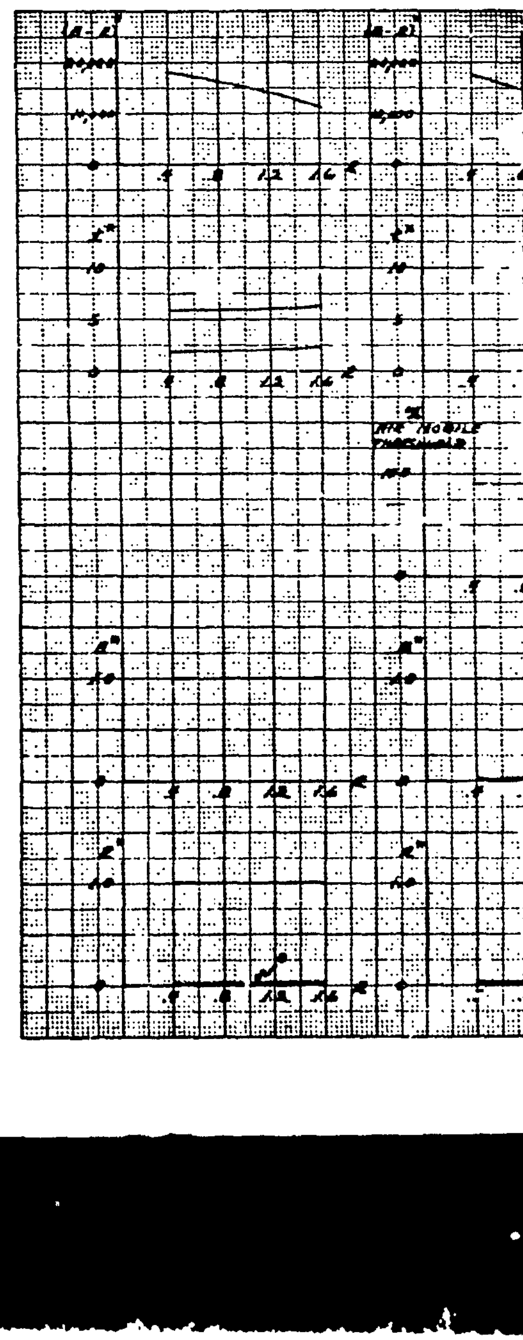


Figure 21. RED'S FIRE POWER $\{R_1, R_2\}$ PERTURBATION, PARITY CASE


AIR MOBILE THRESHOLD

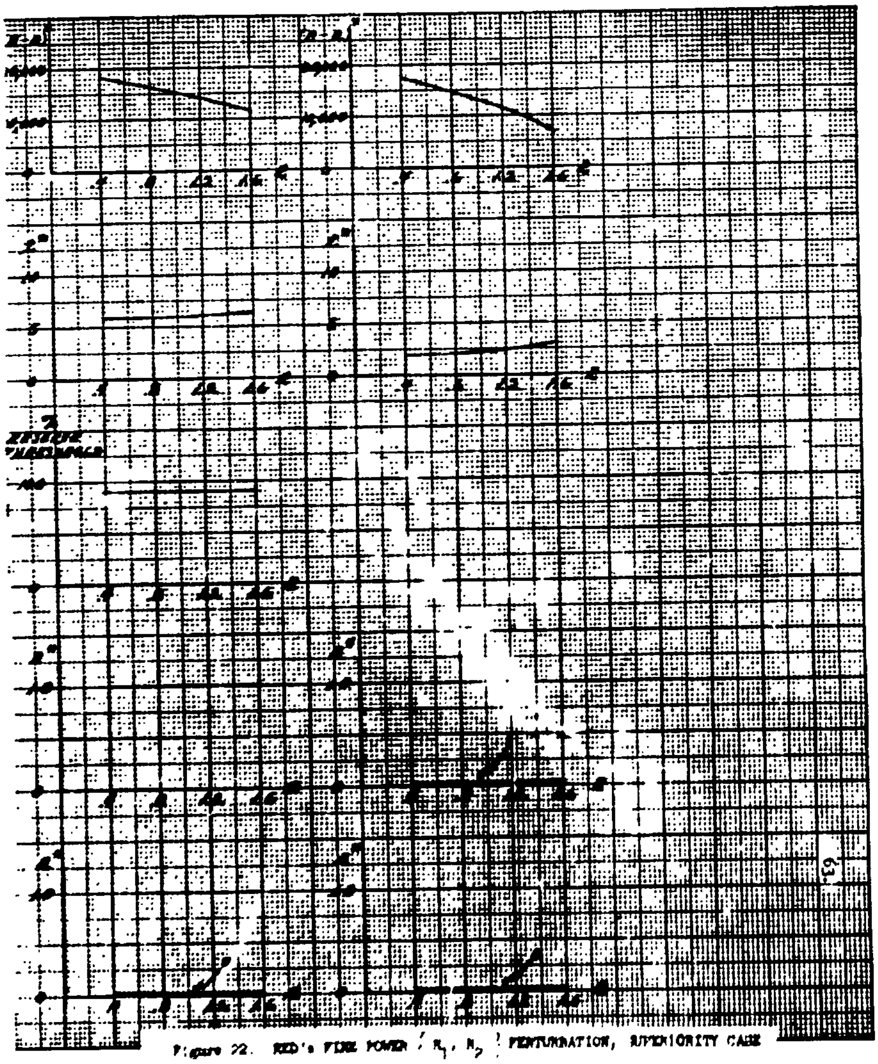


Figure 22. RED's FIRE POWER (R_1, R_2) PERTURBATION, SUPERIORITY CASE

but also achieve for blue the objective of the operation success. Figur us by achieving only half the conjecture of the last sentence. That is, operation objective is truly achieved, but the commander's decision thre right back to the standard case level. It is only when red's fire powe causing blue to lose the battle $R > 1.2$) that the commander's decision t turns down. At this point one might ask whether the commander's low thr consistent with optimally gaining the objective of the amphibious opera threshold level is not consistent with the attacker's achieving a succes landing operation in an optimum sense, why does the equivalent real wor this amphibious operation contain an air mobile mode of troop deploym magnitude relative to the surface landing represented by the abstract m model seems to be saying is that the air mobile mode of deployment of tr in this case represents instantaneous deployment is not as effective ing (that is slower deployment) of the surface elements $\{B_1, B_2, B_3\}$ have greater fire power. If this be the case, then let us increase the air mobile elements only and see what happens to the commander's decisi

Figure 23 gives blue's air mobile element fire power pert parity case. As expected for $B_4 \geq .3$ the commander's decision threshol reduced to 70% which is still in the upper range of possible values and current doctrine. It is interesting to note that for $B_4 \geq .225$ not only optimal strategy reduce to 100% allocation across the beach for the rest the game, but also red, the defender, optimal strategy reduces to defend only as opposed to the multiple choice available to red for the unrestri (Figure 23, fourth graph, column one). This mild reduction in threshold to be caused by increasing the desirability of the air mobile deployment creasing its fire power. What happens if the order of battle ratio (B/ increased?

Figure 24 gives these results for the superiority case and the commander's decision threshold level goes back to the 90% level of t case. This again illustrates that the commander's air mobile decision t but also achieve for blue the objective of the operation success. Figur us by achieving only half the conjecture of the last sentence. That is, operation objective is truly achieved, but the commander's decision thre right back to the standard case level. It is only when red's fire powe causing blue to lose the battle $R > 1.2$) that the commander's decision t turns down. At this point one might ask whether the commander's low thr consistent with optimally gaining the objective of the amphibious opera threshold level is not consistent with the attacker's achieving a succes landing operation in an optimum sense, why does the equivalent real wor this amphibious operation contain an air mobile mode of troop deploym magnitude relative to the surface landing represented by the abstract m model seems to be saying is that the air mobile mode of deployment of tr in this case represents instantaneous deployment is not as effective ing (that is slower deployment) of the surface elements $\{B_1, B_2, B_3\}$ have greater fire power. If this be the case, then let us increase the air mobile elements only and see what happens to the commander's decisi

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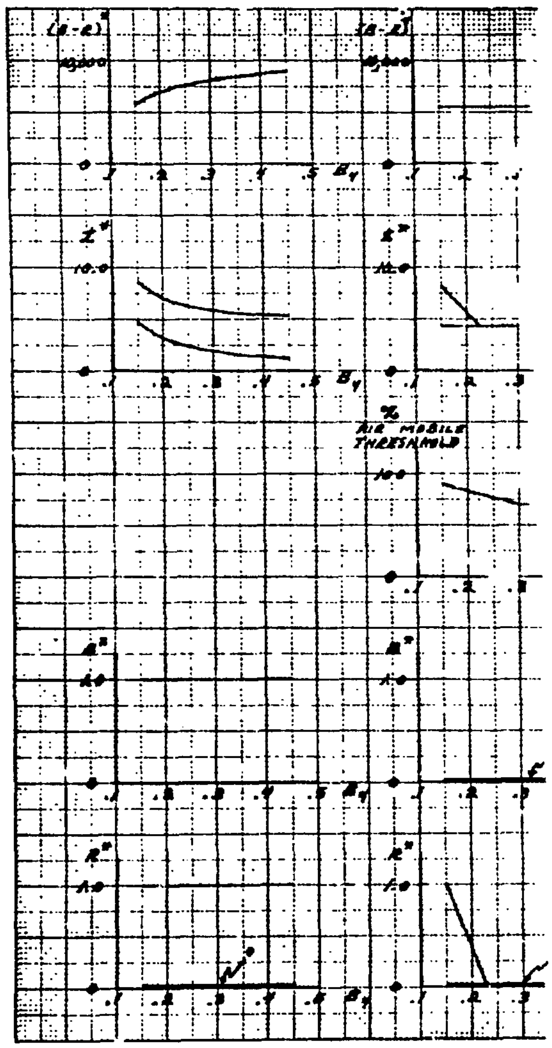
Figure 24 gives these results for the superiority case and the commander's decision threshold level goes back to the 90% level of t case. This again illustrates that the commander's air mobile decision t


%
AIR MOBILE
THRESHOLD

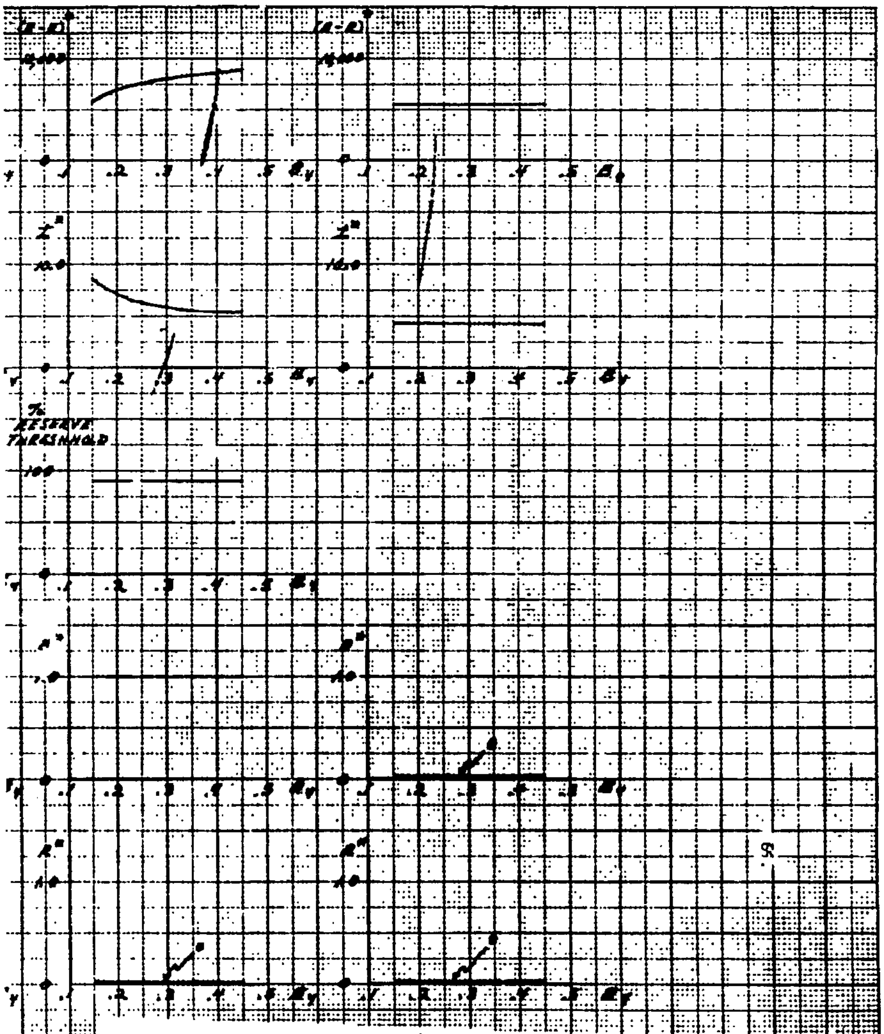
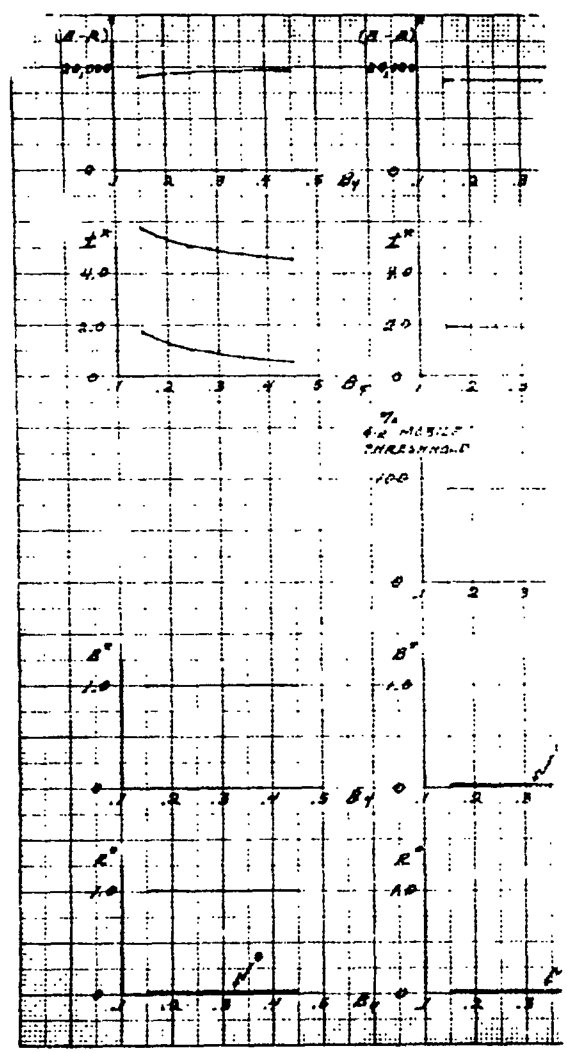


Figure 23. AIR MOBILE FIRE POWER { B_4 } PERTURBATION, PARITY CASE



4.0
2.0
.2
.3
.4
100
1.0

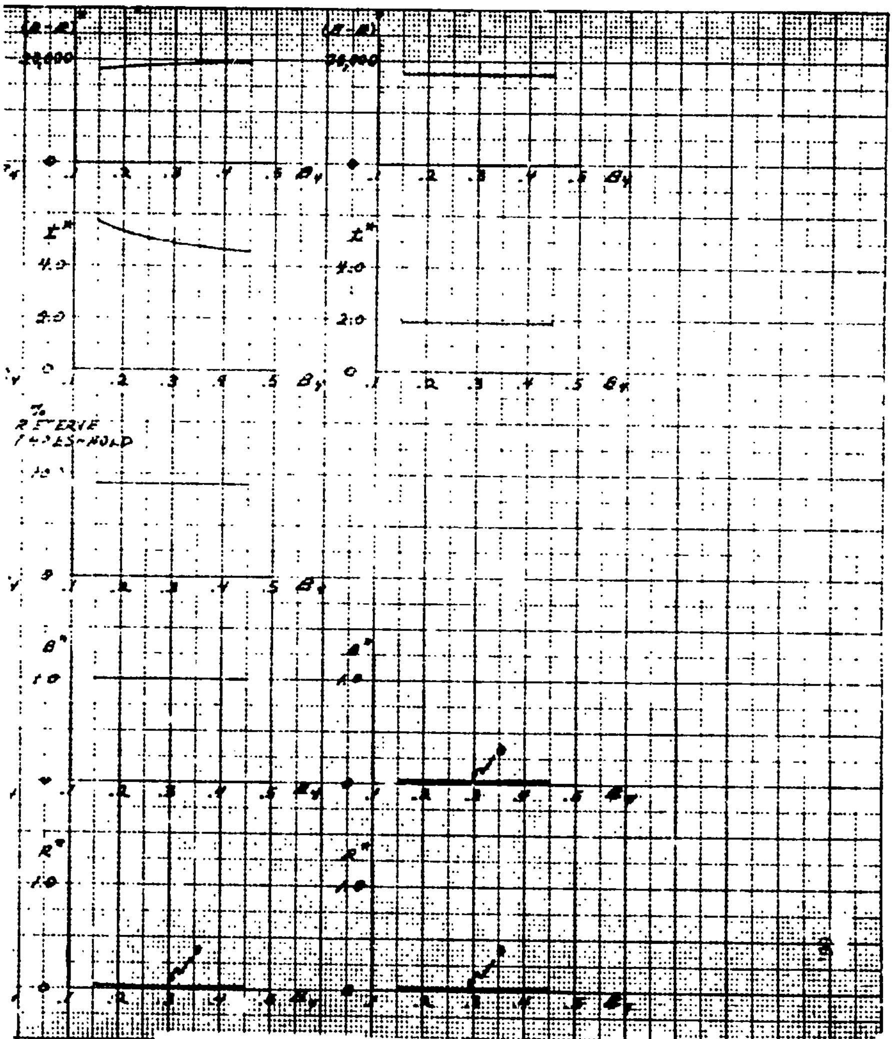


Figure 24 AIR MOBILE FIRE POWER (B_4) PERTURBATION, SUPERIORITY CASE

is directly related to the success of the amphibious operation. That is, :
commander has overwhelming superiority of forces and fire power, optimum d
in the air mobile mode infers most (greater than 90%) of his forces land v
ode. If the magnitude of such deployment is for some reason (technically
.conomically) not feasible, then the optimum allocation of forces is made l
all forces across the beach.

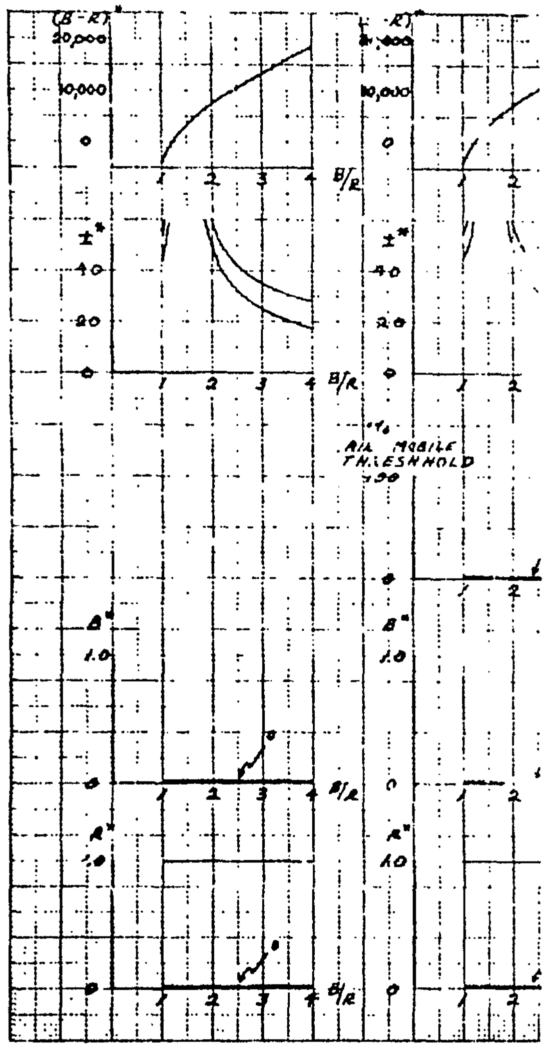
e. Summary

Justifying these model results to current doctrine is impossib amphibious operation as presently conceived is designed to overwhelm the ing force such that a toe-hold can be achieved on the defender's territor size to support future major objectives. The planning of an amphibious op definition presumes the fact that it represents an operation based upon vis the defender. Then why does current doctrine also require a vertical or air mobile deployment mode of forces of insufficient magnitude to assu results? Even if one assumes that the payoff is insensitive to the opt for the unrestricted or restricted play of the game (which may be true i the economic and/or technical feasibility of the air mobile deployment mo more severe then the surface mode of deployment of forces. Thus if one p amphibious operation using the cost-effectiveness criteria instead of a p military payoff criteria, the justification for the air mobile deploymen even more difficult.

Still another possible interpretation of the results is that t matical model used in the completed study was too abstract a representatio amphibious operation and therefore did not adequately reflect all the impo parameters affecting the outcome of the battle. This presumes that a more presentation of the amphibious operation, taking into consideration parame omitted from the completed study because they were considered of second or would yield results more compatible with reality. Such a study certainly sideration in view of the importance of the information potentially avail the mathematical models and techniques demonstrated by this study.

4. Iwo Jima

J. H. Engel in his paper titled, "A Verification of Lanchester's L the applicability of the Square Law in an actual combat situation where Un forces captured the island of Iwo Jima. In this analysis of the capture o the Lanchester's equations, as defined by the Square Law, were found to be The fire powers acredited to both sides as measured by the author were one the magnitude of the standard case used in this study. This factor of one



AIR MOBILE
THRESHOLD

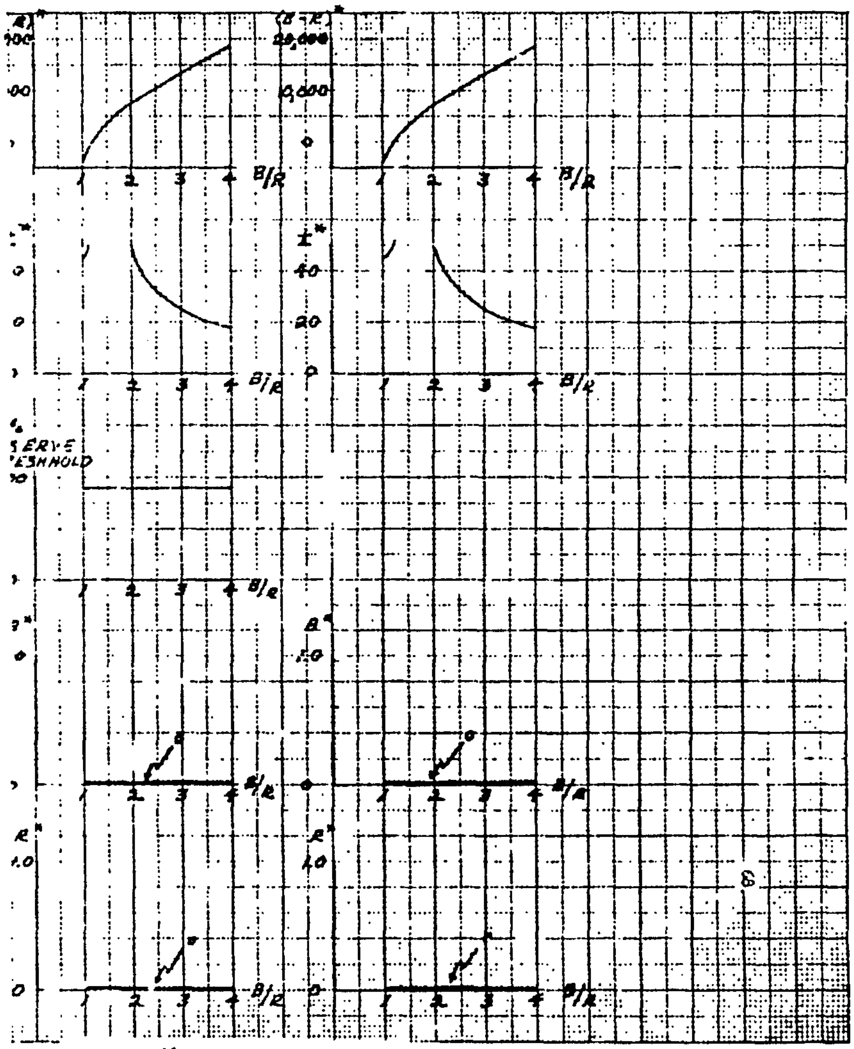
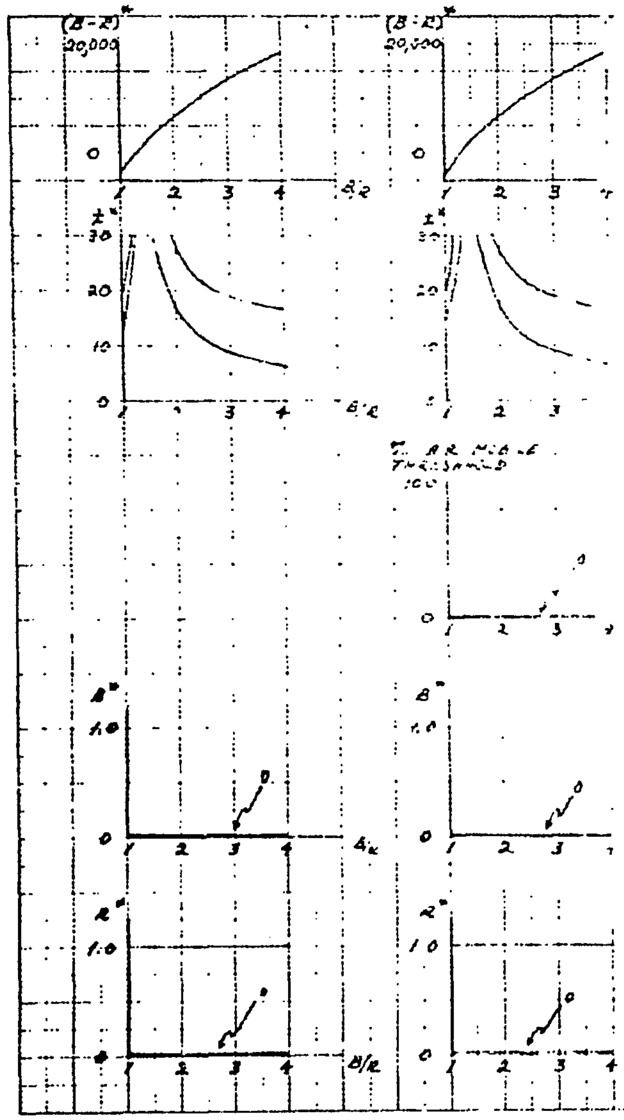


Figure [illegible] IWO JIMA, ONE-FIFTEENTH THE STANDARD CASE FIRE POWER assumes fighting occurred predominately in the day time. This fire p as an input to the amphibious model of this study to see if any usefu would result. In Figure 5, the U.S. loss ratios as a function of ord ratio (B/R) is plotted against actual World War II data. The mathem results for this case fell well within actual data. Figure 25 gives the computer run in the same form as the previous sensitivity analysi delay t_d used was ten hours. The first column, third graph, indicat optimum strategy of allocating all blue's forces across the beach. I fire power strength available during the Iwo Jima Campaign does not upon the assumptions of the model) the sophistication of air mobile mode. Of interest to the analyst would be the strategy crossover po 26 gives the results of the standard case for one-fifth the fire powe unrestricted blue strategy remains across the beach. Figure 27 ind strategy crossover for blue only at B/R=1 and 4. Column two, third the commander's decision threshold of 90%, the same as the standard


(B-R)*
20,000
B/R
1.0

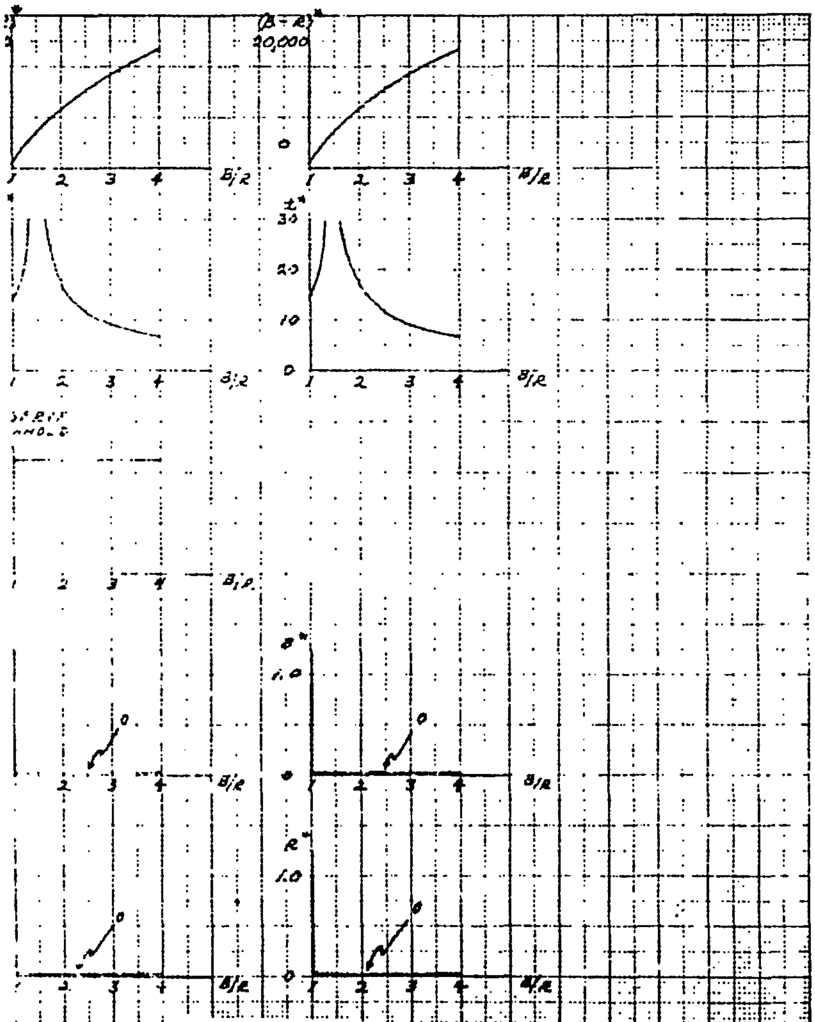
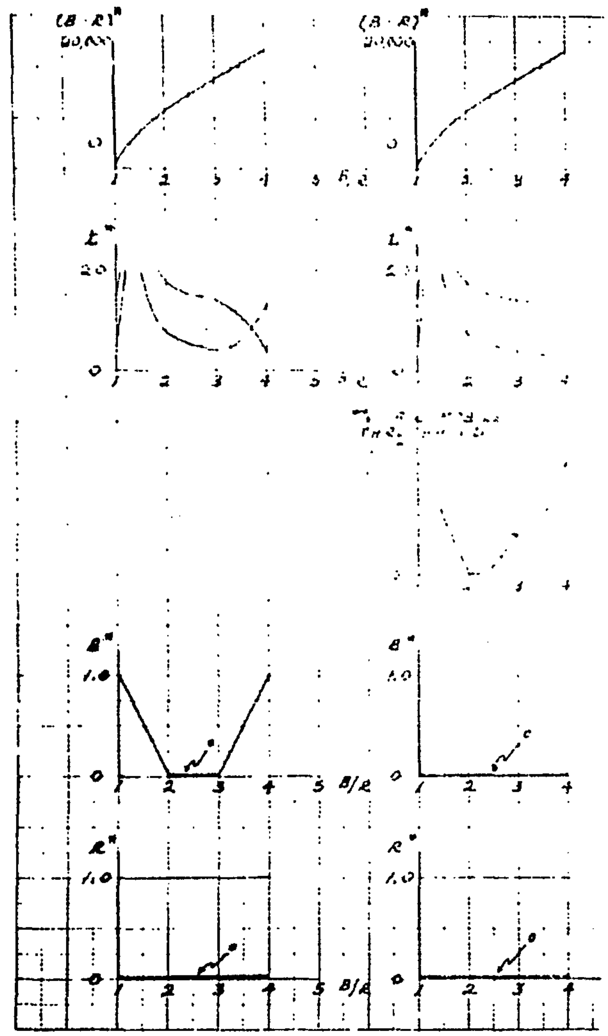
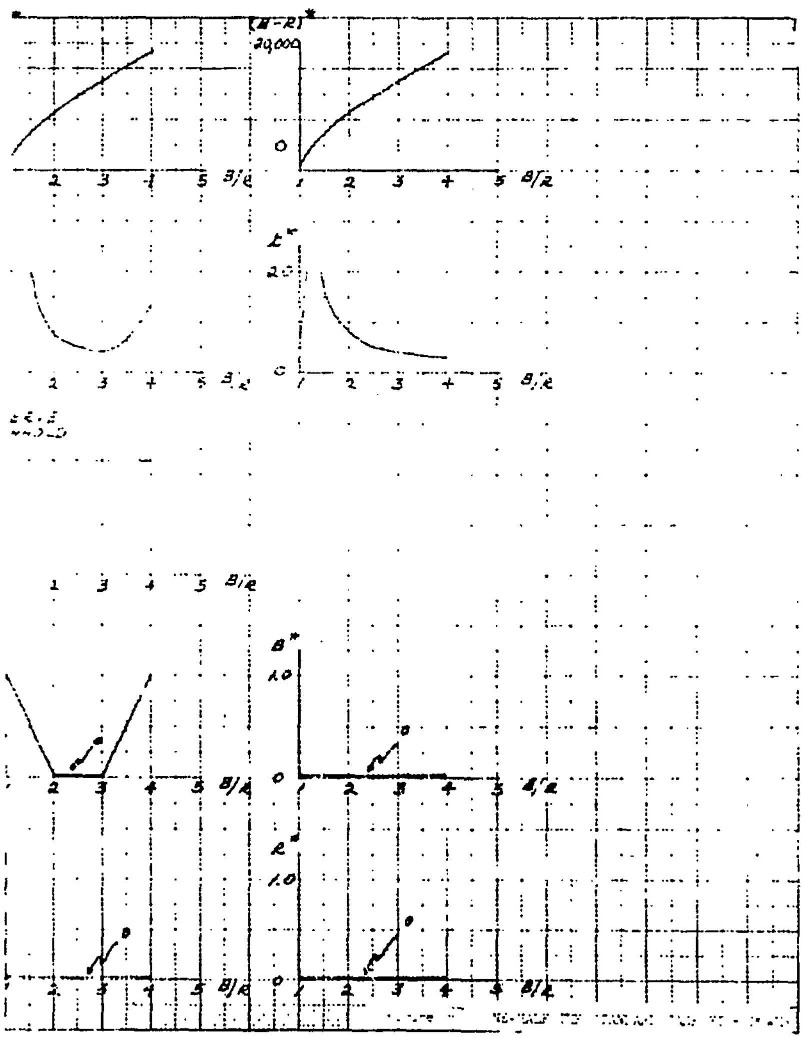


Figure [illegible] ONE-FIFTH THE STANDARD CASE FIREPOWER



(B · R)*
L*
B*
1.0
B/R
R*
1.0
B/R



I. THE VALUE OF MOBILITY, DISPERSION AND SURVEILLANCE IN ACHIEVING EFFECTIVE UTILIZATION OF FIRE POWER

The requirements consistent with limited and tactical warfare have pl stress on the characteristics of mobility and dispersion of units on the The Lanchester Equations Model used in the previous section does not des characteristics in a natural manner. This is due to the fact that the po surveillance characteristics of units on the battlefield are very importa mathematical description of such action is not easily accomplished using functional relationships. No longer is warfare necessarily characterized attrition of the opposing sides until one side predominates. The high l modern weapons allows great advantage to the side pre-empting the attac the force size.

In no way is it possible to determine a tactical force's effectivenes enemy as a function of mobility, dispersion, surveillance, fire power ca deception, etc. by utilizing the Lanchester approach only! This is due the Lanchester Model is only sensitive to the resultant attrition rate (e and its effect over time and is not sensitive to how this attrition rate the battlefield characteristics of the units involved. For example, one firepower tactical unit such as a tank or artillery battalion against a and if circumstances are such that the high-firepower is never in positio enemy because of lack of surveillance, and/or mobility, then the choice of rates $\left\{ a_a, a_s; b \right\}$ cannot be made on the basis of weapons capability alc fore this device becomes arbitrary if one resorts only to the Lanchester M is needed is a mathematical description of the battlefield taking into con mobility, dispersion, surveillance, deception, etc. such that true tactica ness can be derived subject to or constrained by weapon systems and/or org capability. Such a model will now be developed.

1. Battlefield Surveillance and Mobility

a. Introduction

The detection of enemy targets on the battlefield represents t problem area in the successful employment of tactical units and associated systems against such targets. In order for such units to be effective, a tegrated tactical force comprised of aircraft (both reconnaissance and log supporting personnel, and equipment must be deployed. Since the enemy presum also attempting to use its own tactical units and weapon systems in an effecti the situation reverts to a 'red' and 'blue' type analysis because each opposir unit represents a potential target to each other. The question then arises as type of mobility and dispersion characteristics each side (red relative to blu versa) would need to perform effectively. If either side was immobile, then only be a question of time when the opposing sides would detect and destroy ea The net effect of redversus blue would be a function of the relative number of aircraft, weapon systems and supporting equipment opposing each other and how dispersed. Such a situation could effectively be described by the Lanchester To increase survivability, each side could either disperse into smaller unit periodically move about the battle area, or incorporate into the tactical sys active defense. The latter is objectionable because it makes the tactical fo cumbersome and there is serious technological doubt as to whether a defense sy ever be effective against the all out use of offensive force in a battlefiel ment (e.g. "The best defense is a good offense."). It seems clear that peric bility and unit dispersion are the onlyalternatives when attempting to susta tions on the battlefield. This is true for a wide variety of combat situatio guerrilla warfare to tactical nuclear war. The key factor determining the and importance of mobility and dispersion to insure survivability is the effe of the fire power utilized against the tactical system. When the attrition ca the threat of overwhelming fire power is present, then mobility/dispersion cor tions are much more important that fire power/reaction time considerations only when the latter predominates do we have a case for Lanchester's Model. are interested in the successful employment of tactical systems under all cond including the threat of a nuclear environment, we can assume that the importar mobility and dispersion supersede fire power considerations and the model des that follows will have these assumptions in mind. Also please note that Appe contains a different approach to the same model which will be developed below

b. The Effect of Intermittent Mobility on Detection

Let us suppose two opposing tactical forces (referred to as the re blues) operate in a battle area. Let capital 'R' be the total area red contr searches to acquire the blue tactical force and small 'r' the rate of search area per unit time red searches oking for blue. Define capital 'B' and sma in a similar manner for the blue tactical force. If both sides search for eac systematically covering their controlled battle areas, and if the probability being in an area of the battlefield of size 'r' equals r/R the question arises is the probability $P_{r,b}$ red detects blue first, assuming red moves periodicall time units and blue moves periodically every m time units, each move being in random. This question can be similarly phrased for $P_{b,r}$. The probability red blue first is important because this indicates which side has the benefit of a action against the other. Also if one assumes that the threat of fire power is sufficient to overwhelm the tactical force unit, then the side that is abl empt his attack suffers negligible attrition. For this reason the determina detects first is all important with firepower attrition factors secondary an a probability of kill conditional on the probability of detecting first. Fig indicates in a time sequence one cycle of the periodic mobility considered f blue. $P_{r,b}$ will be computed for one cycle, i.e., the time period m that blue stationary.

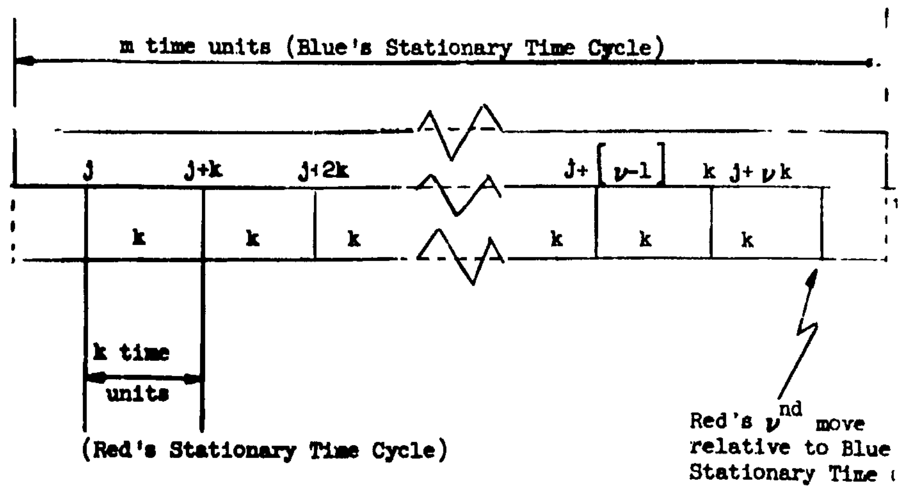


Figure 28. RED'S AND BLUE'S PERIODIC MOBILITY AS A FUNCTION OF ONE OF B TIME CYCLE

Using Blue's move as the location of the origin of the time scale (Figure 29) and the random move of Red starting at time = j, we compute the probability of red detecting blue first at time i, $P_{r,b}(i)$.

Let time run from $1 \le i \le m$, then $P_{r,b}(i)$ for $m > k$ can be developed inductively:

$$P_{r,b}\left\{1 \le i \le j\right\} = \left(1 - \frac{b}{B}\right)\frac{r}{R} \text{ for } i = 1 \quad \left(1 - \frac{b}{B}\right)\left(1 - \frac{r}{R}\right)\frac{r}{R-r} \text{ for } i =$$

$$\left(1 - \frac{b}{B}\right)\left(1 - \frac{b}{B-b}\right) \cdots \left(1 - \frac{b}{B-[i-1]b}\right)\left(1 - \frac{r}{R}\right)\left(1 - \frac{r}{R-r}\right) \cdot$$

$$\left(1 - \frac{r}{R-[i-2]r}\right) \quad \frac{r}{R-[i-1]r} \quad \text{for } i$$

The i^{th} expression can be reduced to

$$P_{r,b}\left\{1 \le i \le j\right\} = \left(\frac{B-ib}{B}\right)\frac{r}{R}$$

The first term on the right indicates the probability bl does not detect red during the (0, i) time period and the second indicates probability that red does not detect blue during the (0, i - 1) time perio but does detect at the i^{th} time period. Another way of looking at this probability model would be to assume that red and blue, each sampling sequentially in time and in unison, an urn placed before each, containing white balls except one which is black. The number of white balls in each is (R/r) - 1 for red and (B/b) - 1 for blue. Therefore the above equation probability red pulls the black ball out of the urn before blue does so o of his urn at the i^{th} time period conditional to all previous balls pulle from the urn not being replaced (i.e., sampling without replacement).