HELICOPTER SHAFT FORCE MEASUREMENT SYSTEM

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prepared by:
CORNELL AERONAUTICAL LABORATORY, INC.
Buffalo, New York

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HELICOPTER SHAFT FORCE MEASUREMENT SYSTEM

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Prepared by:
Cornell Aeronautical Laboratory, Inc.
Buffalo, New York

for
U.S. ARMY TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA
Prepared by:
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William R. Deasley
CAL Project Engineer
FOREWORD

The work described in this report has been performed by the Flight Research Department of the Cornell Aeronautical Laboratory, Inc. The program was sponsored by the U. S. Army Transportation Research Command under Contract No. DA 44-177-TC-700. Its purpose was to instrument a HU-1A helicopter in order to measure main rotor shaft horizontal and vertical forces separate from fuselage forces in simulated forward flight in the NASA-Ames Research Center 40 feet by 80 feet wind tunnel. This data is required to determine rotor-pylon-fuselage interference drag effects and to determine rotor and helicopter performance at high tip speeds and high advance rotors.

The project was administered under the direction of the U. S. Army Transportation Research Command, Port Eustis, Virginia. The report has been reviewed in detail by USATRECOM and the data set forth are concurred with.

FOR THE COMMANDER:

[Signatures]

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LIST OF SYMBOLS

\( x, y, z \) axes in fuselage frame of reference, \( z \) axis parallel to rotor shaft axis

\( H_x \) longitudinal shaft shear

\( H_y \) lateral shaft shear

\( T \) shaft tension

\( \Theta \) instantaneous shaft angle

\( \phi \) angle of applied force

\( N(t) \) force

\( S_t \) stress due to tension

\( \varepsilon \) strain

\( I \) moment of inertia

\( M \) moments resulting from horizontal shear

\( C \) distance from neutral axis to strained surface

Subscripts

\( \Delta \) differential
INTRODUCTION

The development of instrumentation to measure shaft shear and tension forces was undertaken by the Cornell Aeronautical Laboratory, Inc., as a result of a request by the Transportation Research Command (TRECOM) of the Army to aid in the measurement of the rotor shaft forces on an HU-1 Helicopter to be tested in the 40 foot by 80 foot wind tunnel at the NASA Ames Research Center. Similar instrumentation was developed by CAL for use in a helicopter stability derivative measurement program discussed in References 1 through 5.

Since it is not possible to measure the rotor-fuselage interaction forces on the wind tunnel balances, it is necessary to measure the main rotor shaft forces. These forces are the only direct measurement of the interaction between the fuselage and rotor system. Strain gages have been mounted on the rotor shaft to measure directly the strains resulting from shear and tension forces. The three forces which can be measured are the \( \lambda_l \), \( \lambda_y \), and \( \lambda_z \) components of the rotor-fuselage interaction.

These forces are called the Longitudinal Shaft Shear (\( \lambda_l \)), Lateral Shaft Shear (\( \lambda_y \)), and Shaft Tension (\( \lambda_z \)). All measurements are necessarily referenced to the axis of the main rotor shaft.

For ease of computation and data evaluation, it is desirable to obtain an indication of these shaft forces in the fuselage frame of reference rather than the rotating axis system of the rotor. Since the only forces available for direct measurement exist in the rotating shaft, a coordinate transformation is necessary.

To obtain a measurement of shaft forces in the non-rotating axis system, the signals representing shaft forces in the rotating system must be summed in a sine-cosine resolver rotating at shaft speed. This transforms the rotating system data into data in the non-rotating coordinate system. The shaft tension measurement is independent of shaft angle, so no axis transformation is necessary.
DISCUSSION OF $H_x$, $H_y$ MEASUREMENT SYSTEM

The following is the mathematical basis for the system operation. Scale factors and conversions between forces, strains, voltages, and trace deflections are omitted since they only tend to obscure the simple trigonometric operations.

Let $\phi$ be the instantaneous shaft angle ($\phi = 0$ when the reference fixed in the shaft is aligned along the longitudinal axis of the aircraft and to the rear) and $\theta$ the angle of the applied force $H(t)$ as shown in the diagram.
Then the shear forces which would be observed in the two planes at right angles to each other, parallel to the axis of the shaft and fixed with respect to the rotating shaft (i.e., the planes coincident with the strain gage locations) are

\[ H_x = H(t) \cos(\Theta - \Phi) \]
\[ H_y = H(t) \sin(\Theta - \Phi) \]

These signals are introduced into a sine-cosine resolver which is connected to form the products and sums

\[ H_a = H_x \sin \Phi + H_y \cos \Phi \]
\[ H_b = H_x \cos \Phi - H_y \sin \Phi \]

Substituting (1) into (2)

\[ H_a = H(t) \sin \Phi \cos(\Theta - \Phi) + H(t) \cos \Phi \sin(\Theta - \Phi) \]
\[ H_b = H(t) \cos \Phi \cos(\Theta - \Phi) - H(t) \sin \Phi \sin(\Theta - \Phi) \]

This reduces to

\[ H_a = H(t) [\sin(\Phi + \Theta - \Phi)] = H(t) \sin \Theta = H_y \]
\[ H_b = H(t) [\cos(\Theta - \Phi + \Phi)] = H(t) \cos \Theta = -H_x \]

These equations are independent of \( \Theta \). \( H_x \) and \( H_y \) are the components of shaft shear along the longitudinal and lateral directions.

The selection of signs and proper summation is accomplished in practice by reversing the sign of \( H_x \) or \( H_y \) until an input appears to be independent of shaft angle. The proper \( x \) and \( y \) axes are obtained by rotating the resolver case relative to the shaft until the output of \( H_y \) is zero when only a \( H_x \) input is applied.
Figure 1 is a block diagram of the $H_x$, $H_y$ measurement system:

Data in Rotating System

$H_1$ Bridge

Preamp

Gain Equalization and Power Amplification

Slip Rings

$H_2$ Bridge

Preamp

Gain Equalization and Power Amplification

Data in Non-Rotating System

Demodulator and Filter

$H_x$ CEC 7-235

Galvanometers

Demodulator and Filter

$H_y$ CEC 7-235

FIGURE 1  BLOCK DIAGRAM OF $H_x$, $H_y$ MEASUREMENT SYSTEM
ANALYSIS OF ROTOR SHAFT STRAIN AND SIGNAL LEVELS

Tension Measurement

The shaft strain resulting from axial tension is computed, assuming the tensile stresses are uniform around the shaft.

Shaft Cross-Section Area = \( \pi \left( \frac{3.500^2 - 3.125^2}{4} \right) = 1.95 \text{ in}^2 \)

Assume a vertical load of 1000 lb, then the stress due to tension is:

\[
S_T = \frac{1000 \text{ lb}}{1.95 \text{ in}^2} = 512.8 \text{ lb/in}^2
\]

This is equivalent to 25640 lb/in\(^2\) for a 5000 lb load.

The strain \( \varepsilon \) = \( \frac{512.8 \text{ lb/in}^2}{30 \times 10^6 \text{ lb/in}^2} = 17.1 \times 10^{-6} /1000 \text{ lb} \)

1 microinch/inch strain is equivalent to 58.5 lb of applied load. A 5000 lb load produces 85.5 microinches/inch strain.

Since the strain gage bridge is a four arm bridge employing tension and Poisson's Ratio compression arms, the effective bridge output is (for gage factor of 2.0):

\[
1.29 \times 10^{-6} \text{ volt/volt supply per microinch/inch strain}
\]

This becomes

\[
1.29 \times 10^{-6} \times 26 \text{ volts} \times 17.1 \text{ volts/1000 lb load} = 575 \times 10^{-6} \text{ volts per 1000 lb load.}
\]

Figure 2 is a signal flow diagram which shows the various forms of the signal through the tension data channel.
From calibration results the hysteresis and repeatability of the data was in the range of 5 to 10 lb for a 5000 lb range of input. This is equivalent to a 2.5 to 5 microvolt uncertainty in the bridge balance.

Full bridge output voltage was $2.87 \times 10^{-3}$ volts. This was transformed by a factor of 1.55 and preamplified at the rotor by a solid state feedback stabilized amplifier having a gain of 500. The amplifier exhibited a constant gain into the load for output levels up to 7 volts. Since the output for 5000 lb tension was 2.2 volts, no non-linearity or saturation was to be expected.

Because of the necessary decoupling and phase correcting networks, this reduced to approximately 1.5 volts RMS at the demodulator. The resultant current to the recording galvanometer was approximately $36 \times 10^{-6}$ amperes. The galvanometer sensitivity of $7.9 \times 10^{-6}$ amperes per inch deflection resulted in a deflection sensitivity of approximately 1110 lb/in. This calculation agrees reasonably well with the 1200 lb/inch calibration factor obtained experimentally.

---

**FIGURE 2 TENSION SIGNAL FLOW DIAGRAM**
Longitudinal and Lateral Shaft Shear

Strain gages were located at two stations on the main rotor shaft in such a manner that they would sense differential bending moments in the shaft. This differential is the result of shear forces at right angles to the shaft axis.

These gages were oriented in two planes intersecting at right angles along the axis of the shaft. Thus, the instantaneous orthogonal components of shaft shear were obtained in a frame of reference fixed with the shaft.

The strain levels and resultant signal levels were computed in the following manner.

For a hollow cylinder the moment of inertia along one of its diameters is:

\[ I = \frac{\pi}{64} D^4 - d^4 \]

the stress \[ S_d = \frac{M_d C}{I} \]

where

\( M_d \) = differential moments resulting from horizontal shear

\[ M_d = 500 \text{ lb shear} \times 10 \text{ inch separation of gage locations} \]

\[ C = 1.750 \text{ inch} \]

the differential stress

\[ S_d = \frac{500 \times 1.750}{2.685} = 3260 \text{ lb/in}^2 \text{ per 500 lb} \]

and the differential strain

\[ \epsilon_d = \frac{3260}{30 \times 10^6} = 1.08 \times 10^{-6} \text{ in/in} \]

for a 500 lb shear load.

The resolution and hysteresis limit obtained during calibrations was approximately 1 lb. This is equivalent to 0.2 x 10^{-6} in/in strain.

---

* See Figure 3, which is a pictorial drawing of the gage installation and a schematic of the bridge interconnections, and Figure 4 is a photograph of a portion of the installation.

** 500 lb is the maximum expected shear load.
ALL THE H₁, H₂ & T GAGES USED ARE WIRED WITH GREEN & BLACK WIRES

A SPARE SET OF (NOT SHOWN) H₁, H₂ & T GAGES ARE MOUNTED ON THE SHAFT AND ARE CONNECTED TO THE UNEVENTED RED & WHITE WIRES INCLUDED IN THE SAME SHIELDED CABLES

FIGURE 3 PICTORIAL VIEW OF GAGE INSTALLATION
FIGURE 4  PORTION OF GAGE INSTALLATION
Figure 5 is a signal flow diagram showing the various voltage levels through one of the two shear channels.

Assuming a gage supply voltage of 26 VAC and a gage factor of 2,

\[ e_h = 26 \times 2 \times 108.3 \times 10^{-6} = 5.64 \times 10^{-3} \text{ volts/500 lb.} \]

A 1 lb shear load is equivalent to about \(12 \times 10^{-6}\) volts. This is an easily detectable signal.

A 1.55 step-up in an isolation transformer and an amplifier gain of 150 resulted in a full-scale output of the rotor head preamplifiers of \(1.55 \times 150 \times 5.64 \times 10^{-3}\) volts = 1.31 volts RMS.

The resultant output voltage of the resolver amplifier is approximately 5.5 VAC. This reduces to approximately 2.15 VAC at the demodulator. The demodulator output is 0.65 VDC which deflects the 7-325 galvanometer about 3.5 inches on the high sensitivity range and 1.65 on the low sensitivity range.

\[ 0.65 \text{ VDC} \]

\[ \text{Approximate galvanometer} \]

\[ \text{deflection 3.5 inches or} \]

\[ 1.65 \text{ inches} \]
The mechanization of the measurement system can be separated into four parts:

1. Rotor Installation (strain gages and preamplifiers)
2. Remote Test Unit
3. Resolver and Gearbox Installation
4. Recording System

Figure 6 is a block diagram of the complete system and Figures 7, 8, and 9 show in detail the interconnections and circuits of the four sections.

The strain gages used for the rotor installation are Tatnal Type CB-141-350. These are 350 ohm metal film strain gages having a gage factor of 2.03 ±1/2%. These gages are bonded to the rotor shaft with Eastman 910 cement and waterproofed and protected from injury by a synthetic rubber overcoat and fiberglass tape.

A duplicate set of strain gages for shear and tension measurements was also installed in the event of a failure of the first set. Wiring has been provided so that switching from one set to the other is not difficult. Realignment of the resolver would be necessary for the shear channels if this change is made.

The strain gage outputs are balanced and amplified by the Rotor Preamplifier and Balance Unit which is mounted on the damper mounting assembly Bell Part No. 204 010 417 1. The resulting signals are transmitted through the slip ring assembly (provided by Bell) and approximately 120 feet of shielded cable to the Remote Test Unit. Figure 10 is a photograph of this assembly.

The Remote Test Unit consists of two chassis mounted in a cabinet rack. The lower unit is a regulated 400 cps supply for the instrumentation and the upper is the chassis which provides the gain controls, test or calibration voltage, power amplification, demodulation and filtering necessary for obtaining $H_x$, $H_y$, and $T$. 
FIGURE 6 BLOCK DIAGRAM OF MEASUREMENT SYSTEM
FIGURE 7 SCHEMATIC OF ROTOR HEAD AND RESOLVER UNITS
FIGURE 8 SCHEMATIC OF REMOTE TEST UNIT
FIGURE 9 INTERCONNECTING CABLES SCHEMATICS
Panel mounted 50-0-50 micro ampere meters are provided for easy visual monitoring of the galvanometer signals and for balancing.

The resolver system is mounted on the tachometer generator pad at the lower right side of the main gearbox. Due to structural member interference, it was necessary to rotate the hydraulic pump (and the necessary drain fittings, etc.) 90° to accommodate the gearbox without additional machining. Appendix I is a description of the gearbox.
CALIBRATION PROCEDURES AND RESULTS

Tension

The tension calibration was accomplished by lifting the helicopter by the main rotor and cap which has a lifting eye. A NASA 10,000 lb range proving ring having a calibration factor of 22.8 lb/division was used as a measure of the tension input. The shaft axis was inclined forward about 3 degrees. This would result in a 0.275% reduction in the tension along the axis.

Calibrations were made for every 90° position of the shaft. A noticeable difference in the deflection sensitivity was obtained for the various azimuth positions. Since this was discovered to be mainly due to bending moment cross talk in the extra pair of gages used for nulling the torsion cross talk, a similar pair of gages was mounted at an azimuth position of 180° from the original pair. These gages will reduce the effects of bending moments to a negligible amount.

The calibrations presented here were made with only the two original torsion nulling gages. This shows up as an increase and decrease in deflection sensitivity with shaft azimuth position. It was not possible to lift the helicopter with the slip rings installed since the blade assembly was removed. These calibrations were made without the possible small effect of slip rings. The 3000 ohm series resistor in the preamplifier output should make the effect of the rings negligible.

The average of the four calibrations is very near the deflection sensitivity of the system with no bending moment inputs. This average was obtained by taking the average of the trace deflections for each value of tension load. Since a few of the tension loads did not agree exactly with the nominal value, small correction factors (of the order of .02 inches) were introduced to simplify the averaging procedure.
Table I presents the input tension and resultant trace position for the four azimuth positions. Table II tabulates the corrected deflections for the nominal tension inputs listed and the resultant linear average of the four calibrations. One point for the 180° azimuth position was off-scale. This point was obtained by extrapolation since it was important for a consistent method of averaging. The resulting calibration is shown in Figure 9. The non-linearity between zero and 1000 lb resulted from the rest position of the galvanometer being offset beyond its linear range. This is not important since the data will all be obtained in the 4000 to 6000 range of loads.

The hysteresis of the averaged calibration was not reduced appreciably by the averaging technique. The resulting maximum hysteresis is about .03 in. This is equivalent to about 34 lb hysteresis or $\frac{34}{5000} \times 100\% = 0.7\%$ of full scale.

The data obtained from the tension channel during wind tunnel tests will not be as accurate as was obtained during the calibrations because of the temperature drifts, torque cross talk, vibration and noise resulting from the captive flights. Filtering is provided which will aid greatly in averaging out the noise, but it will be necessary to establish a reference value of tension after a warm-up period. During flight, the rotor shaft temperature rises and it is desirable to establish the 5000 lb point at test operating temperature. The calibration factor will also be changed due to the variation of Young's Modulus with temperature. This is of the order of -1% for each 30° C rise. This correction can be applied to the test data by checking the shaft temperature just after shutdown.

Shaft Shear

The calibration of the shaft shear channels was preceded by a sequence of adjustments necessary for the initial setting up of the measurement system.

In Equation 2 an incorrect sign in either $H_1$, $H_2$, or in the method of summing these signals in the resolver will result in
TABLE I

INPUT TENSION AND RESULTANT TRACE POSITION

<table>
<thead>
<tr>
<th>Azimuth 0°</th>
<th>Azimuth 90°</th>
<th>Azimuth 180°</th>
<th>Azimuth 270°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lb Trace Load Deflection</td>
<td>Lb Trace Load Deflection</td>
<td>Lb Trace Load Deflection</td>
<td>Lb Trace Load Deflection</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1140</td>
<td>1133</td>
<td>1140</td>
<td>1140</td>
</tr>
<tr>
<td>2280</td>
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</tr>
<tr>
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<td>0</td>
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</table>

* extrapolated from 180° calibration curve
TABLE II
CORRECTED DEFLECTION FOR INPUT TENSIONS AND RESULTANT AVERAGE DEFLECTION

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<th>NOMINAL WEIGHT</th>
<th>0° Azimuth</th>
<th>90° Azimuth</th>
<th>180° Azimuth</th>
<th>270° Azimuth</th>
<th>AVERAGE DEFLECTION</th>
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</thead>
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<td>Actual Weight</td>
<td>Required Correction</td>
<td>Corrected Deflection</td>
<td>Actual Weight</td>
<td>Required Correction</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
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<td>-0.10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>1140</td>
<td>0</td>
<td>+1.03</td>
<td>1133</td>
<td>+7</td>
</tr>
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<td>2.03</td>
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<td>-5</td>
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<td>3.00</td>
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<td>0</td>
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<td>+1.05</td>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>-0.12</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

correction of .00087 in/lb
Axial Tension vs. Average Deflection

Slope = 1130 lb/in

Channel #8
Supply Voltage 26 VAC
Galvanometer Type 7-325
Serial No. c154

FIGURE 11 SHAFT TENSION CALIBRATION
\[ H_1 = H(t) \sin \phi - H_2 \cos \phi \]
\[ H_2 = H(t) \cos \phi + H_2 \sin \phi \]

expanding,
\[ H_1 = H(t) \sin \phi \cos(\Theta - \phi) - H(t) \cos \phi \sin(\Theta - \phi) \]
\[ H_2 = H(t) \cos \phi \cos(\Theta - \phi) + H(t) \sin \phi \sin(\Theta - \phi) \]

This reduces to
\[ H_1 = H(t) [\sin(\phi + \Theta + \phi)] = H(t) \sin(2\phi + \Theta) \]
\[ H_2 = H(t) [\cos(\Theta - \phi - \phi)] = H(t) \cos(\Theta - 2\phi) \]

These equations are dependent on \( \phi \) and the output is a second harmonic of shaft RPM for a steady input.

By a reversal of either the sign of \( H_1 \) or \( H_2 \), this second harmonic disappears and the output is a constant.

The output is a constant if the \( H_1 \) and \( H_2 \) channel gains are equal. If they are not, Equation 1 is changed to:
\[ H_1 = H(t) (1 + \Delta) \cos(\Theta - \phi) \]
\[ H_2 = H(t) (1 - \Delta) \sin(\Theta - \phi) \]

Resolving \( H_1 \) and \( H_2 \) into the \( H_A \) component:
\[ H_A = H(t) (1 + \Delta) \sin \phi \cos(\Theta - \phi) + H(t) (1 - \Delta) \cos \phi \sin(\Theta - \phi) \]
\[ = H(t) \sin \Theta + \Delta H(t) \sin(2\phi + \Theta) \]

The second term is a 2/rev ripple superimposed on the correct output which results from unequal channel gain. The average gain is unchanged. In the present system, the ripple frequency is approximately 10 cps. The filter has a cut-off at 1 cps so the ripple due to unequal gain is negligible. If there is a static unbalance in \( H_1 \) or \( H_2 \) due to either bridge unbalance, blade unbalance or tracking errors, this term will appear at the output of the resolver as a function of \( \sin \phi \) or \( \cos \phi \) (i.e., a 1/rev ripple).

The Remote Balance Unit has provisions for detecting the steady state
unbalance at the input to the resolvers so that this unwanted 1/rev ripple can be reduced to negligible amounts. There is no inherent way to balance out higher order errors without rotating resolvers at multiples of blade RPM.

The orientation of the resolved output is determined by the relative position of the resolver stator and its mounting pad, providing the resolver rotor is rigidly connected to the main rotor shaft or through a 1:1 gearing arrangement.*

The most significant feature of this measurement system is that the steady state output of the $H_x$ or $H_y$ channel can only result from a corresponding steady state input shear on the rotor shaft. Electrical unbalance, mechanical or aerodynamic unbalances will not materially change the steady state deflection. The zero shear balance point can be determined quite accurately by switching off the resolver output or by reversing its sign and noting the average. This zero can be obtained when the rotor is spinning or when it is not turning. The zero current position of the galvanometer could be a false zero if any unbalance exists in the demodulator and, for this reason, should not be used.

The $H_x$ and $H_y$ channels were calibrated for each 45° azimuth position of the rotor shaft. This provided one with sufficient data to establish the average sensitivity of the particular channel.

Some variation in sensitivity for the various shaft positions was observed. The sensitivities in the plane of the strain gage bridges were adjusted to be nearly equal, but the stiffness of the shaft was not constant with azimuth position. This is most likely the result of slight errors in the concentricity of the inner and outer surfaces of the shaft.

* See Appendix I for details of the Resolver Gearbox.
FIGURE 12  SHEAR LOADING FIXTURE
FIGURE 13  CALIBRATION SET-UP FOR $H_y$ CHANNEL
This unequal sensitivity with azimuth position will result in a 2/rev ripple superimposed on the desired deflection. The filtering of the recording channels is sufficient to reduce this to negligible amounts.

Calibrations were performed by applying loads at right angles to the rotor shaft axis in the longitudinal and lateral directions. A steel ring mounted on the shaft and a U-shaped collar with two radial ball bearings bolted to the collar were required as calibration accessories. These were constructed in such a fashion that loads applied from a cable attached to the U-shaped collar were transferred to the radial bearings, which in turn contacted the steel ring without interfering with electrical cables to the Slip Ring Assembly. Thus, it was possible to rotate the rotor shaft without removing the calibration fixture or any load that was applied.

Figure 12 is a photograph of this fixture and Figure 13 is a photograph of the calibration technique used for the $H_y$ calibration.

Loads were applied in 50 lb increments and the channel gains were switched from high to low for each load value. The resultant trace deflections are presented in Tables III and IV.

The average for each load value at both high and low sensitivity settings is presented in Table V.

Figures 14 and 15 are plots of the resulting calibrations.

The linearity and hysteresis errors are less than 1/2% of full-scale.

Figures 16 and 17 are polar plots of the calibration factors for $H_x$ and $H_y$ versus azimuth position. This series of plots is included to show graphically the variation of system sensitivity with angular position.
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*Highs and lows in inches of deflection
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*Omitted on plot because of insufficient data

**Lows and highs in inches of deflection
$H_x$ vs. High Average
$H_x$ vs. Low Average

$H_x$/ High Average = 113.8 lb/in
$H_x$/ Low Average = 222.5 lb/in

Channel No. 6
Supply Voltage 26 VAC
Galvanometer Type 7-325
Serial No. 5960

FIGURE 14 $H_x$ CALIBRATION
$H_y$ vs. High Average
$H_y$ vs. Low Average

$H_y$ / High Average = 115.0 lb/in
$H_y$ / Low Average = 227.5 lb/in

Channel No. 7
Supply Voltage 26 VAC
Galvanometer Type 7-325
Serial No. 5867

FIGURE 15 $H_y$ CALIBRATION

平均—英寸的偏转

$H_y$ — lb

平均—英寸的偏转
Figure 17 \( H_y \) CALIBRATION FACTORS AS A FUNCTION OF AZIMUTH

- High \( H_y / \text{in} \) vs. Azimuth Angle
- Low \( H_y / \text{in} \) vs. Azimuth Angle
FIGURE 14  Calibration Factors As a Function of Azimuth

FIGURE 16  $H_z$ Calibration Factors As a Function of Azimuth

TG-1476-F-1
SUMMARY AND CONCLUSIONS

The design, construction, and calibration of a shaft force measuring system has been accomplished. The resulting calibrations indicate that the combined errors due to hysteresis, linearity and resolution are less than 1% of full scale for all three force components - $H_x$, $H_y$, and $T$.

The forces measured at the rotor shaft comprise the major portion of the fuselage-rotor interaction forces. The control rods from the swash plate to the blade assembly introduce additional forces which must be added to the shaft tension to obtain the complete axial or rotor lift. The cyclic pitch control input introduces bending moments which are not sensed by the shear bridge arrangement. Since the rotor is a teetering type, it introduces no appreciable bending moment.

A shear component, $H_x$, will be observed statically because of the 3° tilt of the rotor shaft. For a 5000 lb vertical load, the apparent shear will be approximately 260 lb.

The vector sum of the $H$ and $T$ forces will yield the total rotor-fuselage interaction force and its direction relative to the axis of the rotor shaft.

Provision has been made for switching the filter on and off the tension channel with no gain change. This makes it possible to measure the amplitude of the vertical load fluctuations as well as the more accurate filtered value of the average vertical load.

Careful attention to warm-up and balancing of the signals during operation as well as adjustment of the torque nulling circuit will yield data on the $H_x$, $H_y$, and $T$ forces within 1% to 2% accuracy. The system is capable of higher accuracy, but it is doubtful whether the measurements can be made to that accuracy because of the mechanical vibration and other factors which will introduce random errors and fluctuations in the measured data.
BIBLIOGRAPHY


APPENDIX I

The resolver gearbox designed and constructed at Cornell Aeronautical Laboratory, Inc., was used to convert the shaft rpm of the Tachometer Generator into a shaft output having the exact rpm of the main rotor shaft. This output shaft must also be locked to the main rotor shaft position so that it will never slip out of synchronization with the main rotor shaft. This is essential for the correct resolution of the \( H_x \) and \( H_y \) forces.

The gear ratio between the Tachometer Generator shaft and the main rotor is:

\[
\frac{18}{19} \times \frac{26}{27} \times \frac{41}{55} \times \frac{57}{176} \times \frac{57}{176}
\]

This can be simplified to

\[
\frac{57}{176} \times \frac{41}{176} \times \frac{52}{55}
\]

which is the basis for the gearing employed.

Figure 18 is the assembly drawing of the gearbox.

The actual gearing employed is:

\[
\begin{array}{c}
\text{Tach RPM} \\
\hline
104 \quad 0 \\
\hline
57 \quad 176 \\
\hline
41 \quad 176 \\
\end{array}
\]

= Main Rotor RPM

To Resolver Rotor

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FIGURE 18 ASSEMBLY DRAWING OF THE GEARBOX