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ESD-TDR-64-393

ESPOD FUNCTIONAL DESCRIPTION

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-393

JUNE 24, 1964

496L Systems Program Office
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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Line No. 2, AFSC Form 40

Prepared under Contract AF 19(628)-594

STL NO. 8497-6067-RU-000

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FOREWORD

This document is one of three reports which describe ESPOD, a general orbit determination program prepared for the Air Force Electronic Systems Division for use in the SPACETRACK/ SPADATS Center at Ent Air Force Base, Colorado Springs, Colorado.

This report is:

ESD-TDR-64-393 - ESPOD Functional Description (TRW/STL No. 8497-6067-RU000)

The companion reports are:

ESD-TDR-64-394 - ESPOD Operating Instructions and Card Formats (TRW/STL No. 8497-6066-RU000)

ESD-TDR-64-395 - ESPOD Mathematical and Subroutine Description (TRW/STL No. 8497-6065-RU000)

The ESPOD program was prepared by TRW Space Technology Laboratories under Air Force Contract Number AF 19(628)-594.

ABSTRACT

ESPOD is a general orbit determination program prepared for use by the SPACETRACK/SPADATS Center, Ent Air Force Base, Colorado Springs, Colorado. Its primary purpose is to determine satellite orbits. It determines the elements of a satellite orbit and a covariance matrix of uncertainty in the determination, starting from some initial estimate of these elements and correcting it in accordance with observational data. It predicts the future position and velocity of the satellite from the best elements obtained. The program includes a unique collection of mathematical, statistical, and operational techniques to make it operate rapidly and automatically and to produce high precision in the results.

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1. INTRODUCTION

This is a functional description of ESPOD, a precision orbit determination program designed primarily for determining earth satellite orbits. The description is intended to acquaint the reader with the features and capabilities of ESPOD for general information and planning purposes. Analysts, programmers, operators, and those wishing to become familiar with ESPOD and its operation in detail are referred also to the companions of this report:

ESD TDR-64-394 - ESPOD Operating Instructions and Card Formats

ESD TDR-64-395 - ESPOD Mathematical and Subroutine Description

The tables of contents of the above reports will be found in the appendix to this report.

This document reviews the following information concerning ESPOD:

a) the kinds of spacecraft it is applicable to,

b) the class of problems it solves,

c) the nature of the answers it obtains,

d) its structure as a computer program,

e) its general internal processes, and

f) the mathematical models employed in its orbit simulation.

2. APPLICATIONS OF ESPOD

ESPOD has the ability to perform two principal functions — orbit determination and position and velocity prediction. Under orbit determination ESPOD accepts an initial estimate of the orbit elements of an object and a set of observations taken for this object by a set of sensors. It updates the given elements to corrected elements and produces a covariance matrix of uncertainty in the corrected elements, and simultaneously solves for and removes sensor biases in the observations. Under prediction, ESPOD accepts a set of orbit elements and a specification of the uncertainty in those elements. It propagates the position and velocity of the vehicle to a requested time and indicates the uncertainty in position and velocity as implied by the uncertainty in the elements.

Sections immediately following discuss the objects and the sensor network calibration problems to which ESPOD is applicable.

2.1 EARTH SATELLITES

2.1.1 Altitude

ESPOD will determine the orbits of earth satellites at all altitudes. The effects of atmospheric drag on low altitude satellites are accounted for through and including the reentry phase, but ESPOD is not appropriate after drag becomes a function of Mach number and aerodynamic effects dominate.

2.1.2 Inclination

ESPOD will determine the orbits of earth satellites at all inclinations. ESPOD uses as elements the instantaneous position (specified as right ascension, declination, and the distance from earth-center to satellite) and velocity (specified as flight path azimuth, flight path angle from vertical, and the vector magnitude of velocity) of the satellite at some pertinent epoch. Since inclination is not an explicit parameter, ambiguity in determining the ascending node of a low inclination satellite is avoided.

2.1.3 Eccentricity

ESPOD will determine the orbits of earth satellites at all eccentricities, from circular to extremely elongated ellipses. Since eccentricity is not an

explicit parameter, ambiguities in determining the argument of perigee of a near circular satellite or in computing highly eccentric orbits are avoided.

2.2 LUNAR PROBES

ESPOD will determine the orbits of spacecraft which pass so close to the moon that the effect of the moon's gravity is significant to the motion of the spacecraft and possibly the dominant effect. Near-moon orbits to which ESPOD is applicable are:

a) Cases where a spacecraft on a highly eccentric orbit comes near the moon without going around it (cislunar orbits).

b) Cases where a spacecraft escaping from the earth has its orbit strongly perturbed by the moon, either by design or by accident (lunar fly-bys or lunar impact).

c) Cases where a spacecraft passes behind the moon and then returns to earth (circumlunar orbits).

d) Cases where a spacecraft has been maneuvered to go into orbit about the moon, ESPOD does not shift into moon-centered coordinates nor does it consider higher order harmonic terms in the model of the moon's gravitational potential, therefore, high precision tracking of lunar orbiters is not possible.

In all cases, the position and velocity of the spacecraft with respect to the moon are printed.

2.3 INTERPLANETARY PROBES

ESPOD will determine the trajectories of spacecraft which have escaped from the earth-moon system and are in orbit about the sun, but will not account for the effects of the other planets. The position and velocity of the spacecraft with respect to the sun are printed.

2.4 BALLISTIC MISSILES

ESPOD will determine the free-flight trajectories of ballistic missiles. It will track them through the reentry phase until aerodynamic effects become significant, that is, until drag becomes a function of Mach number and aerodynamic effects dominate.

2.5 COMPARISON CALIBRATION

One way in which ESPOD can be used for sensor network calibration is by comparing one network against another as follows: ESPOD can precisely determine the orbit of a satellite using observations from a reference sensor network. Then, using observations from another "unknown" sensor network, it can solve for biases in these observations and in the locations of sensors in the "unknown" network. For any one sensor, any arbitrary subset of the permitted biases may be solved for; multiple sensors may be handled simultaneously. The total number of items which may be simultaneously solved is approximately 50. Solving for biases in the following items is permitted:

a) Range
b) Azimuth
c) Elevation
d) Range Rate
e) Right Ascension
f) Declination
g) Times assigned to a set of observations
h) Sensor latitude
i) Sensor longitude
j) Sensor height above the ellipsoid

By using the Conditional Start feature (see Section 3.13), the above calibration can be performed automatically. In the first orbit determination case, observations from the unknown network can be suppressed by assigning them weight zero. In the second case, which automatically follows, orbit and drag elements can be suppressed as quantities to be solved for and the weights assigned to the observations from the unknown sensor net can be made non-zero.

2.6 INDEPENDENT CALIBRATION

ESPOD can simultaneously determine the orbit of a satellite and solve for approximately 40 of the permitted biases from a single set of observations taken from available sensors. The relative contribution from the various sensors is controlled by assigned weights. The orbit elements and biases are jointly determined to minimize the sum of squares of the residuals.

3. ESPOD FEATURES

ESPOD is a large, comprehensive computer program. Its full mathematical and operating description appears in the companion reports referenced in the foreword. Certain characterizing features of the program are presented below.

3.1 GENERAL ORBIT MODEL

ESPOD utilizes a Cowell method of special perturbations for propagating the satellite position and velocity. In this method, the position and velocity are calculated step-by-step at successive points in time. At each given time the influences of all of the forces acting on the satellite are calculated and summed. These forces are dependent entirely upon the position and velocity of the satellite at the given time, that is, they are special for the moment. The increment between a current position and velocity and the position and velocity at the next point in time is calculated from these specific forces and the current velocity. To find position and velocity at a specific time, it is necessary to advance as many steps from epoch as may be needed to reach the required time. Thus, with a step size of one minute and a requirement to predict 6 hours into the future, 360 steps must be taken.

The precision of the orbit prediction resulting from the above process depends directly only on the quality of three kinds of input information:

a) The position and velocity of the satellite at a given starting time (the orbit elements).

b) The mathematical model of the conservative and dissipative forces acting on the satellite.

c) The numerical method of evaluating the forces and propagating the position and velocity forward.

If the initial position and velocity are wrong, all subsequent positions and velocities will be wrong. If the mathematical representations of the forces are wrong, the step-by-step process will not approximate the true motion well, and the predicted position will be in error. If the numerical process is wrong, the arithmetic is simply giving the wrong answers. In

ESPOD, very particular care has been taken to hold errors due to all of these sources to the practicable minimum.

3.2 COMPUTER AND OPERATING SYSTEM

ESPOD has been prepared for use on the Philco 2000 computer in the Philco 212 system. This system is installed at the SPACETRACK/SPADATS Center, Ent Air Force Base, Colorado Springs, Colorado, and at the SPACETRACK Center Alternate Facility, Hanscom Field, Bedford, Massachusetts. ESPOD operates within the specified ground rules which ensure compatibility with the SPACETRACK B2 System. All outputs are produced through standard output routines. Standard SPADATS record and sensor data are used; designated B2 system tapes are used for input and output; and the JPL planetary ephemeris tape is used for the position of the sun and moon. Standard SPADATS card formats are used for data input and output, however control data applicable only to ESPOD is in a format special to ESPOD. ESPOD is also compatible with special SPACETRACK/SPADATS Center control cards for precision position determination and for comparison with test results.

Two important operating features of ESPOD are that it has no upper limit on the number of observations it can process, and that it operates fully automatically once it is provided with control data, observations, and sensor data.

3.3 INITIAL CONDITIONS

ESPOD accepts SPADATS mean elements from the SEAI tape or from the 7-card format. From these elements (using appropriate general perturbations formulas) it calculates the Cartesian position and velocity of the satellite at the time designated to become epoch for ESPOD. These Cartesian positions and velocities at epoch are then used as initial conditions for the orbit integration process.

In an orbit determination run, the Cartesian position and velocity are further converted to polar-spherical form for use as the starting values of the orbit elements. These elements will be corrected with successive iterations, eventually to converge on the final solution.

ESPOD will also accept as initial conditions orbit elements in polar-spherical or Cartesian form when specified at the appropriate epoch.

3.4 OBSERVATION AND SENSOR DATA

ESPOD accepts SPADATS observational data from the standard observation card format or from the SRADU tape. The following pertinent data is accepted:

a) Observation time
b) SPADATS sensor number
c) Credence number (for FPS-49 radars)
d) Flag indicating Baker-Nunn precision/field reduced data
e) (Radar) Range, R
f) (Radar) Azimuth, A
g) (Radar) Elevation, E
h) (Radar) Range-Rate, $\dot{R}$
i) (Photographic or Radio Telescope) Right Ascension, α
j) (Photographic or Radio Telescope) Declination, δ

ESPOD preprocesses input data for use by the ESPOD differential correction process. If required, apparent elevation angles are corrected for tropospheric refraction. Observations on cards are sorted to time-sequenced order. SPADATS sensor numbers are interpreted to initiate searching for sensor data from the SEAI tape. Weights are assigned to each separate measurement according to the observing sensor and the observation type. The properly formatted observation is recorded on magnetic tape.

ESPOD accepts SPADATS sensor data from the standard sensor card format or from the SEAI tape. The following pertinent data is accepted.

a) SPADATS sensor number
b) Sensor identifying name
c) Latitude
d) Longitude
e) Height above ellipsoid

The SEAI tape is the primary source of sensor data. The tape is searched to recover data on all sensors represented in a given set of observations. This is then augmented or modified by card input.

ESPOD has stored permanently other special data for 120 selected sensors. This data may be modified with input cards. It includes:

a) SPADATS sensor number

b) Gross outlier rejection criterion (multiplier applied to standard deviation)

c) Type of sensor for purpose of assigning weights

d) Mean surface value of refractivity at the site of the sensor

e) Flag indicating refraction correction required/not required

ESPOD has stored permanently a table of 60 different sets of weights catalogued by sensor type. This table may be modified by card input.

3.5 DATA EDITING

ESPOD tests observations by four separate criteria before using them in the differential correction process. Observations are categorically rejected by tests 1 and 2.

Test 1. Is the observation time removed from epoch by more than some maximum interval?

Test 2. Has the observation been specifically selected by the analyst for deletion?

Observations are rejected on the basis of the size of their residuals by Tests 3 and 4. The residuals may change from iteration to iteration because of the successive corrections applied to the initial conditions. Test 3 applies to all iterations; Test 4 applies only to iterations after the first.

Test 3. Gross outlier test: Does the residual exceed some specified (large) multiple of the standard deviation characteristic of the sensor for this type of observation?

Test 4. KRMSWR Test: Does the weighted residual exceed some specified (small) multiple K of the root mean square of the weighted residuals derived from observations which were not rejected by either Test 1, 2, or 3 on the previous iteration?

Typically, Test 1 would remove items which picked up a transmission or punching error in the time identification. Test 4 is more stringent than Test 3. Test 3 is retained for use on the first iteration and to remove transmission and punching errors from the RMSWR used in the KRMSWR test. Tests 1, 3, and 4 are automatic. Test 2 permits the analyst to force the deletion of an observation. Items may be tagged for deletion without reading in the observations an extra time; items on the SRADU tape may be tagged for deletion. It is possible to delete selectively, for example, the range observation at a certain time while retaining the angle and range rate observations.

If it is desired to stabilize the set of observations on which the least squares fit is being made, items noted as being automatically rejected on a previous run by Tests 3 and 4 may be tagged for deletion by Test 2. Tests 3 and 4 may then be "loosened" by specifying less stringent gross outlier criteria and a larger K. With this modification, the problem of defining convergence is simplified, because the RMSWR which is minimized under the least squares process depends only on the corrected initial conditions, and not on the coincidental inclusion or deletion of marginal residuals from iteration to iteration.

3.6 POLAR/SPHERICAL ELEMENTS

ESPOD solves for the polar/spherical orbit elements at epoch. These elements give the position of the satellite in earth-centered coordinates referenced to the true equator and true vernal equinox at 0^h of day of epoch. That is, position is given as the right ascension α, declination δ, and magnitude R of the radius vector to the satellite. Velocity is referenced to the radius vector. The flight path is projected onto a plane normal to the radius vector and the azimuth A of the projection is measured in this plane each from north. The angle β is measured between the radius vector and the flight path. The vector magnitude of the velocity v is given.

It has become customary to arrange these symbols in the order α, δ, β, A, R, v and to use them as an acronym, "adbarv".

In order to account for the interaction of the atmosphere and the particular satellite, ESPOD includes with the orbit elements two additional drag

parameters. The first is a ballistic drag parameter of the satellite, that is, the area-to-mass ratio multiplied by the coefficient of drag: $C_DA/2m$. The second is a variation in the $C_DA/2m$, either the time dependent secular change in $C_DA/2m$ per 24 hours, or else the position dependent periodic change in $C_DA/2m$ as the satellite repeatedly enters and leaves the atmospheric bulge.

3.7 BIAS SOLUTION

Bias errors in the observations and uncertainty in the locations of the sensors may contribute more error to the orbit element determination than both the mathematical model and computational limitations together. Further, accurate estimation of the uncertainty in the orbit elements requires that the errors in observations be unbiased. Correspondingly, ESPOD has provisions to determine and remove biases from any of the following measured inputs:

a) Range
b) Azimuth
c) Elevation
d) Range Rate

} Radar Sensors

e) Right ascension
f) Declination

} Photographic or Radio Telescope

g) Times assigned to a set of observations
h) Sensor Latitute
i) Sensor Longitude
j) Sensor height above ellipsoid

The selection of which measurements from which sensors are to have their bias solved and removed is the responsibility of the analyst.

ESPOD performs explicitly the following functions:

a) Prints out monitoring information indicating the size of apparent biases as implied by current elements.

b) Accepts known biases provided by the analyst and removes them from the observations.

c) Solves for selected biases in the observations and removes them while simultaneously solving for the elements. In total, more than 40 biases may be solved for (and removed). Both biases and orbit elements are corrected to minimize the sum of the squares of the weighted residuals.

3.8 WEIGHTED LEAST SQUARES ORBIT DETERMINATION

Since the observations of a trajectory that are made by a tracking system are imperfect, no trajectory fits these observations exactly. Therefore, only an estimate of the actual trajectory can be obtained from the data. Many methods of forming the estimate are possible, but the weighted least squares method is probably the most common and is the method employed by ESPOD. This method may be formulated as follows:

A set of N observations (radar tracking data, optical observations, etc.) denoted by an N x 1 matrix R is given. These observations are assumed to be a known N x 1 matrix function f of a set of p parameters, denoted by a p x 1 matrix X_A, plus additive random noise denoted by an N x 1 matrix n:

$$R = f(X_A) + n \quad .$$

In the simplest case, X_A denotes six (or less) position and velocity components of the spacecraft at a specified epoch; all other parameters and constants are assumed to be known exactly and are absorbed into the function f. In ESPOD, X_A may include six (or less) orbit elements plus other non-orbital parameters such as drag parameters, tracking station coordinates, and data biases which are not known exactly. The above equation $R = f(X_A) + n$, is called a nonlinear "regression equation". The orbit determination problem is to estimate X_A, given R, the functional form of f, and the statistical properties of n.

ESPOD orbit determination attempts to select an estimate of X_A which minimizes the weighted sum of squares of observation residuals, where residuals are differences between the actual observations and those computed using the orbit model. More precisely, $\delta R^T W \delta R$ is minimized, where

δR = the N x 1 matrix of observation residuals (actual minus computed observations)

W = an N x N diagonal weighting matrix (see Section 3.4)

δR^T = denotes the transpose of δR.

The method of solution is to linearize the nonlinear regression equation by expanding f in a truncated Taylor series about an initial estimate or reference value, X_R, since the actual value of X_A is unknown. Letting $\delta X_A = X_A - X_R$, $\delta R = R = f(X_R)$, and letting A denote the N x p matrix of partial derivatives of f with respect to X (evaluated at $X = X_R$), one has the approximate linear regression equation

$$\delta R = A\,\delta X_A + n \quad .$$

The weighted least squares solution to the linear regression equation is a well known formula in statistics:

$$A^T W A \delta X_E = A^T W \delta R$$

or

$$\delta X_E = (A^T W A)^{-1} A^T W \delta R$$

or

$$X_E = X_R + (A^T W A)^{-1} A^T W \delta R \quad .$$

X_E is an unbiased* estimate of X_A which minimizes the weighted sum of squares of residuals $\delta R^T W\, \delta R$. The first version of the weighted least squares solution is called the "normal equation" and the matrix $A^T WA$, the "normal matrix".

ESPOD achieves the final elements by iterating on this differential correction procedure. It decides that convergence has been obtained by noting that the sum of the squares of the weighted residuals changes by less than 0.1% due to the last computed δX_E.

3.9 DIFFERENTIAL CORRECTION CONTROL

ESPOD provides a unique automatic control which enhances the program's ability to converge to correct elements. The differential correction discussed in the previous section is actually computed subject to the side condition that

*X_E will be unbiased if the noise is unbiased; and even if the noise is biased X_E will be unbiased provided that noise biases are included among the parameters to be solved for.

the sum of the squares of weighted components of the differential correction vector does not exceed unity. The weights assigned to differential corrections are designated "bounds". Since the differential correction process depends upon the appropriateness of a linear approximation to a non-linear function, the linear approximations can fail if the finite corrections are too large. In this event, the computed "corrections" may prove to be diverging rather than converging. It is the function of the bounds to limit the size of the correction so that the linear approximation is valid.

ESPOD automatically adjusts these bounds to compensate either for diverging "corrections" or for too slow convergence. If a "correction" results in new orbit elements which yield a larger sum of squares of weighted residuals than the previous elements, i.e., divergence, then the bounds are halved and a new, more closely constrained correction is computed. If this too yields poorer elements, the bounds are halved again, and thus continuing until one-eighth bounds have been tried. This process typically brings the solution down within the region where the linear approximation applies and obtains a converging step. On the other hand, if a correction yields a new sum of squares of weighted residuals which actually is less than the previous sum, then the bounds are increased, permitting larger corrections. (An exception to this rule occurs when the new sum of squares is significantly different from a theoretically predicted new sum. A difference greater than 10% is considered to be an indication of nonlinearity and in that case the bounds remain unchanged.)

3.10 DERIVATION AND PRESENTATION OF STATISTICAL DATA

During the differential correction process, the p x p normal matrix $A^T WA$ is computed, where p is the number of unknowns being solved for (see Section 3.6). The inverse of this matrix, $(A^T WA)^{-1}$, is the covariance matrix of uncertainty in the solution vector when certain criteria discussed below are met. This means that the diagonal elements of $(A^T WA)^{-1}$ are the variances of the corresponding elements of the solution vector and that the off-diagonal elements are the covariances of the corresponding solution vector elements. The criteria which must be satisfied in order that this be true are:

a) The observational noise is unbiased, or else the non-zero biases are included in the solution vector.

b) The observational noise is uncorrelated* and the variances of the observations are known and are used in the weighting matrix. (The diagonal elements of the matrix W in Section 3.8 are the reciprocals of the variances of the corresponding noise components.)

c) The mathematical model of the orbit and the observations is correct, and all parameters (biases, station locations, physical constants, etc.) which do not appear in the solution vector are known exactly.

When these criteria are met, the weighted least squares solution is also the minimum variance unbiased estimate of the parameters to be solved for. Obviously, these criteria can never be met exactly in any real tracking problem. When they are not met, $(A^TWA)^{-1}$ is, to that extent, an incorrect estimate of the actual covariance matrix.

There is still another more subtle qualification to identifying $(A^TWA)^{-1}$ with the covariance matrix of uncertainty. In a non-linear regression problem such as orbit determination, the true covariance matrix is actually equal to $(A^TWA)^{-1}$ plus terms involving higher order partial derivatives (of observations with respect to parameters to be solved for) and the data sample itself.** These higher order terms are neglected in ESPOD, as in every other orbit determination program, and experience tends to justify this policy.

The ESPOD elements are the position and velocity of the satellite in polar spherical coordinates. The orbit uncertainty is given in these coordinates. ESPOD also transforms the position, velocity, and uncertainty to Cartesian coordinates referenced to the plane of the equator and the direction to the vernal equinox. In addition, ESPOD rotates the uncertainty to Cartesian coordinates referenced to the orbit plane and the radius vector to the satellite. This places the measure of uncertainty in an easily visualized and useful form. Instead of printing covariance matrices, uncertainty is printed out as a matrix

---

*In case the observational errors are correlated, an "equivalent-or-worse" variance may be used in weighting observations, resulting in a value of $(A^TWA)^{-1}$ which is an upper bound on the actual covariance matrix. See T.A. Magness and J.B. McGuire, "Statistics of Orbit Determination - Correlated Observations," Space Technology Laboratories Report 8976-6001-RU-000, 15 December 1961.

**See B.C. Douglas, W.W. Lemmon and T.A. Magness, "Numerical Aspects of Orbit Determination," Space Technology Laboratories Report 8408-6048-RU-000, June 10, 1964.

of standard deviations and correlations. Lastly, ESPOD prints the orientation and dimensions of the three-dimensional position 1σ error ellipsoid.

3.11 PRINTOUT FOR DIFFERENTIAL CORRECTION

ESPOD prints out a comprehensive report of the progress of the differential correction for review and monitoring purposes. The output is available on every iteration.

The following information is printed for each observation on each iteration:

a)	Number of sensor making observation	
b)	Time of observation	Radar observations
c)	Range Residual	
d)	Azimuth Residual	
e)	Elevation Residual	
f)	Range Rate Residual	
g)	Right Ascension Residual	Photographic or Radio Telescope observations
h)	Declination Residual	
i)	Radius Vector Residual	
j)	Horizontal, In-Orbit-Plane Residual	
k)	Normal to Orbit-Plane Residual	
l)	Time interval satellite is early or late	As required to compensate for flight path residual
m)	Argument of latitude of observation	
n)	Angle by which observation is out of orbit plane	
o)	Sensor Latitude Offset	
p)	Sensor Longitude Offset	Alternate to i, j, k above
q)	Sensor Height Offset	
r)	Flight Path Residual	
s)	Normal to Flight Path, In-Orbit-Plane Residual	Alternate to i, j, k above
t)	Normal to Orbit-Plane Residual	

The following is printed after each iteration:

a) Iteration Number

b) Mean residuals by sensor and type of observations

c) Estimated standard deviation about the above mean by sensor and type of observation

d) Number of observations entering each of these statistics

e) Number of candidate observations rejected by gross outlier or KRMSWR editing tests from each of these statistics

f) Number of observations selectively deleted by analyst

g) Orbit Elements from which this trajectory was propagated

h) Differential Corrections computed

i) New orbit elements, i.e., old elements plus computed correction

j) Uncertainty in new orbit elements

k) Bounds applied in computing correction

l) Total root mean square of weighted residuals for this trajectory

m) Best root mean square of the weighted residuals yet achieved

n) Predicted root mean square of the weighted residuals for new orbit elements

o) Normal Matrix

p) Standard deviation and correlation matrix for polar spherical orbital elements

3.12 ESPOD predicts the position, velocity, and uncertainty in position and velocity for arbitrary times specified by the analyst. At each of these times ESPOD prints out the following:

a) Day number plus fraction since 0^h, January 1

b) Minutes plus fraction since epoch

c) Year, month, day, hour, minute, second (to milliseconds), Universal Time

d) Cartesian position and velocity

e) Osculating classical elements

f) Polar-Spherical position and velocity

g) Osculating indeterminacy free elements

h) Latitude, Longitude, and Altitude

i) Apogee altitude

j) Perigee altitude

k) Time until next ascending node

l) Selenocentric position and velocity

m) Heliocentric position and velocity

n) Standard deviations and correlations in equator/vernal equinox Cartesian coordinates

o) Standard deviations and correlations in orbit plane/radius vector Cartesian coordinates

p) Standard deviations and correlations in polar spherical coordinates

q) Normalized eigenvectors of the position covariance matrix

r) Square roots of the eigenvalues of the position covariance matrix, i.e., principal axes of the position error ellipsoid

s) Rotation angles of the principal axes of the position error ellipsoid from the orbit plane/radius vector coordinates

3.13 RESTART FEATURES

When ESPOD completes an interation, it leaves recorded on a magnetic tape all information pertinent to the iteration. Specifically, it records:

a) Satellite identification data

b) Run identification data

c) All observation data

d) All sensor data, as augmented or modified by input data

e) Any special control data

f) Best elements obtained to data (or on option, result of latest iteration)

g) Best root mean square of weighted residuals to date

h) Biases as specified or solved

i) Current bounds

j) Control flags indicating confidence in current elements

This information is available after the completion of a run. A special "Conditioned Start" mode of operation is provided in which is magnetic tape record is referenced for input conditions. This obviates the necessity for re-reading observation data and for setting up all conditions of the solution. It also provides the capability for running predictions to different times automatically, once a correct set of elements is computed. The conditioned start feature is flexible, in that it allows modification of any of the solution conditions.

A special mode of the conditioned start feature is designated "Conditional Start". Conditional start predicates the continuation upon the confidence in the elements determined on the last iteration of the previous run. With this feature, a planned, automatic series of runs processing the same data with increasing levels of sophistication can be set up. If any stage in the process does not converge properly, the series will be truncated and not occupy the computer time. For example, the following series of runs develop predicted position with bias compensation with two set-ups on the computer.

a) Cold Start: Provide observation data and specify solution for orbit elements.

b) Conditioned Start: Augment solution to solve for biases, provide initial estimates.

c) Conditional Start: Augment solution to include drag.

d) Conditional Start: Predict position for specified times.

e) Review output and select biases to solve for. Use mean residuals from best elements to estimate biases.

More and different series set-ups may be developed to approach different problems automatically.

3.14 RESEARCH CAPABILITY

ESPOD is first an operational program. Nominally it runs automatically, using the standard SPADATS files. Certain data, specific to the case at hand, such as dynamic atmosphere parameters, must be included with the input data. Further, if the analyst wishes to refine the solution, he is permitted to enter bias estimates and/or to call for the solution of biases.

ESPOD also permits the analyst to change any constants defining the mathematical models, to change the weights which are applied to residuals, to change other sensor parameters, to weight a priori estimates, to force the integration process to particular step sizes, to change any physical constants, etc. With this convenience, ESPOD can be employed as a research tool. The effects of varying the gravitational potential model, the effects of varying atmosphere models, the solution for multiple biases, the inclusion of all points on intensive tracking, can be studied for their general and particular influence on many classes of satellites.

4. TRAJECTORY PROPAGATION

ESPOD propagates the trajectory of the spacecraft with the Cowell special perturbations method. This is to be compared with general perturbations methods in which the position of the satellite is calculated at any instant of time by using an algebraic formula. In a simplified general perturbations scheme, the fine structure of the orbit is averaged out, and the resulting position predictions can not account for all of the influences in detail.

Many other orbit determination programs besides ESPOD use a Cowell special perturbations propagation scheme. Comparable programs operating at other facilities are "TRACE" at Aerospace Corporation, "KODM" at the Satellite Control Facility, Sunnyvale, and "AT-4 General Tracking System" at TRW Space Technology Laboratories.

ESPOD incorporates the features discussed below to ensure that the calculated trajectory will correspond as closely as possible to a satellite's actual behavior.

4.1 EARTH GRAVITATIONAL POTENTIAL MODEL

ESPOD provides any of four gravitational potential models of graduated accuracy, the particular model to be selected by the analyst according to the case.

a) (Nominal) Utilizing the first three zonal harmonics in a coherent set

b) Utilizing the first nine zonal harmonics in a coherent set

c) Case a or b above augmented with selected low order sectorial and tesseral harmonics

d) Utilizing the first nine zonal harmonics and all sectorial and tesseral harmonics through order and degree four, in a coherent set

ESPOD permits the change of any coefficient of any harmonic and also permits arbitrary inclusion and deletion of individual harmonics as specified with the input data. Thus, any currently published form of the earth's potential model may be conveniently simulated.

4.2 PERTURBATIONS DUE TO THE SUN AND MOON

ESPOD nominally calculates the perturbing effect on the satellite's motion due to the fact that the sun's and moon's position with respect to the satellite are different from their positon with respect to the center of the earth. ESPOD obtains the positions of these bodies with respect to the earth from the SPADATS maintained planetary ephemeris tape developed by the California Institute of Technology Jet Propulsion Laboratory (JPL). These perturbations are nominally present but may be selectively omitted.

4.3 ATMOSPHERIC DRAG MODEL

ESPOD calculates the effect due to atmospheric drag as the product of a "drag parameter" and a tabular atmospheric density.

The drag parameter can take on the following forms. If drag is to be included, the analyst must specify the form.

a) Simple ballistic parameter: $C_C A/2m = D$.

b) Ballistic parameter plus secular variation K in this parameter since epoch: $C_D A/2m = D + K(t - t_o)/1440$ where t_o is Epoch.

c) Ballistic parameter plus periodic effects in this parameter as it's angle ψ with respect to the atmosphere's bulge changes: $C_D A/2m = D + K(1/2 \cos^5 \psi/2 - 1/4)$.

The tabular atmospheric density can take on four forms:

a) (Nominal) The United States Standard Atmosphere (1962, COESA)

b) Same as a, but with special terms added to account for the angle with respect to the atmospheric bulge, effects related to the geomagnetic planetary amplitude (A_p), and effects related to the 10.7 centimeter solar flux (F_{10}^p) in the altitude range between 90 and 1600/km

c) The ARDC 59 standard atmospheric model

d) Same as c up to 120 km, but above that the Paetzold 1962 atmospheric model which accounts for latitude with respect to the atmospheric bulge, season of year, and effects related to A_p and F_{10}

4.4 RADIATION PRESSURE MODEL

ESPOD simulates the radiation pressure forces on a satellite as a function of the effective mass-to-area ratio, which must be supplied by the analyst.

4.5 INTEGRATION PROCESS

ESPOD integrates the equations of motion with a Cowell integration process including certain refinements. The process is initiated with a Runga-Kutta starter which sets up the finite differences from which the Cowell integration proceeds. The velocity is summed from the accelerations with a tenth order Adams-Moulton single sum process. The position is summed from the accelerations with an eleventh order Cowell second sum process. Both of these methods use a predictor-corrector formulation. Interpolations for times intermediate between the time steps of the integration are calculated with a Cowell step.

The time interval between successive steps, step-size, is automatically controlled to keep seventh order differences in acceleration within a certain numerical range. This guarantees a given accuracy but permits the stepsize to be as large as possible.

5. INTERNAL FUNCTIONS OF ESPOD

The entirety of ESPOD operates as three interrelated segments of the total program:

a) A preprocessor segment for reading in the preliminary data and setting up the problem for solution (designated ESPØD)

b) An orbit prediction and observation processing segment for performing the differential correction and removing biases (designated ESPØDDC)

c) An orbit prediction and ephemeris data publishing segment for providing predicted positions and uncertainties (designated ESPØDEPH)

Figure 5-1, "Summary Block Diagram", is an abridged diagram of the flow of process within the program.

The following lists the internal functions within each segment. It has been abridged and simplified from the full program description included in the companion documents referenced in the foreword.

5.1 ESPØD INTERNAL FUNCTIONS

a) Selecting observations, sensor data, and elements data from ESPOD cards, SPADATS cards, or from tape files

b) Sorting and compiling appropriate data into working form

c) Correcting observations for refraction and coordinate rotations

5.2 ESPØDDC INTERNAL FUNCTIONS

a) Special perturbations integration of orbit and calculation of partial derivatives of observed quantities with respect to solution variables

b) Removal of observational biases

c) Calculation of weighted residuals

d) Rejecting residuals which are unjustifiably irregular

e) Solution for the corrections to solution variables by minimizing the sum of squares of weighted residuals

f) Generation of covariance matrix of uncertainty in solution variables

g) Printing, punching, and taping all pertinent results

5.3 ESPØDEPH INTERNAL FUNCTIONS

a) Special perturbations integration of orbit and propagation of position and velocity to specified times

b) Transformation of positions and velocities to all pertinent coordinate systems

c) Propagation of error estimates to specified times

d) Transformation of error estimates to all pertinent coordinate systems

e) Printing, punching, and taping of all pertinent results

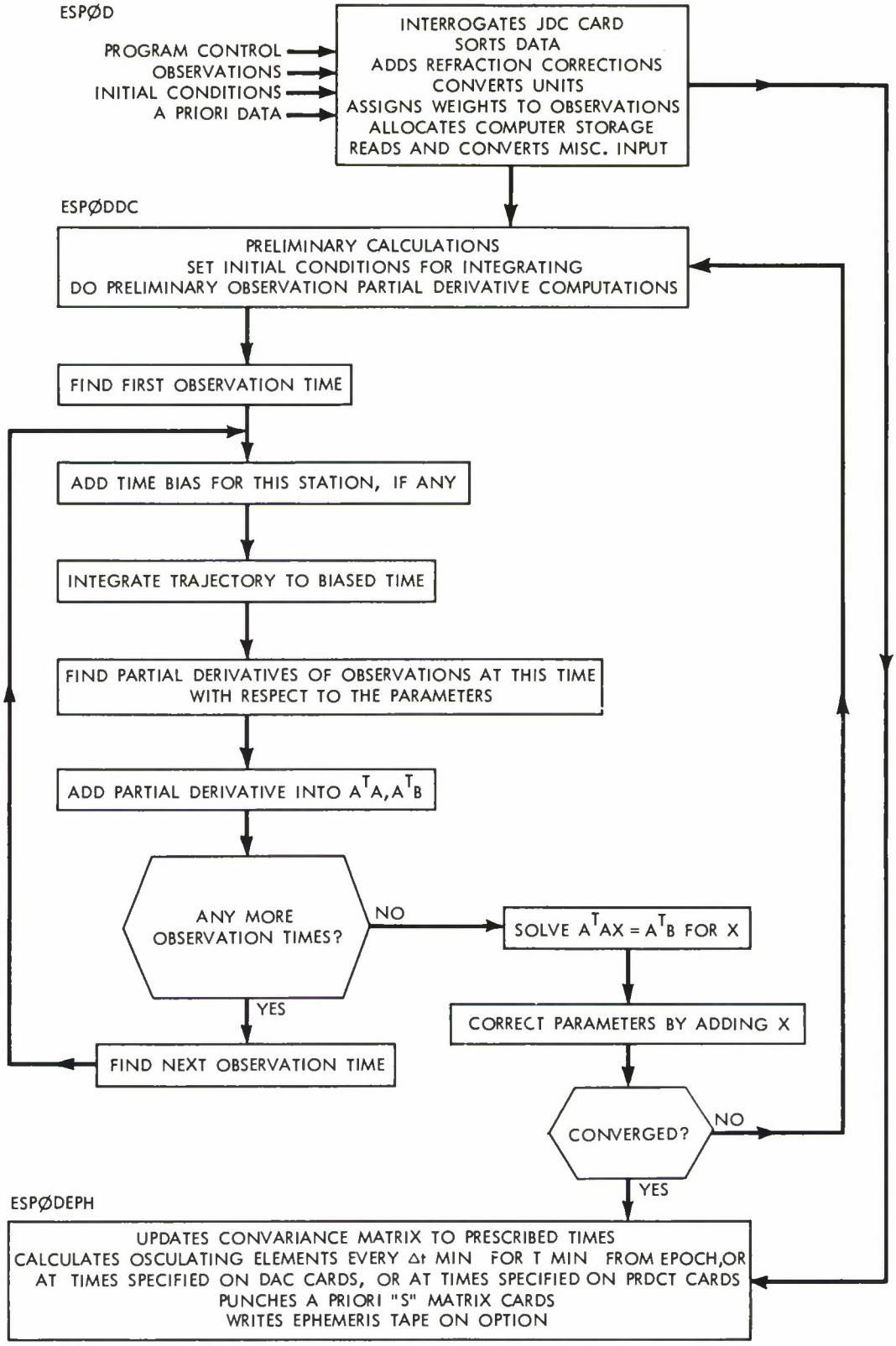

Figure 5-1. Summary Block Diagram

APPENDIX I

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APPENDIX II

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POSTSCRIPT

The word "ESPOD" was coined as a name to identify this computer program within TRW Space Technology Laboratories. The letters were an acronym for "Electronic Systems (Division) Precision Orbit Determination (Program)" and were used as the five letter code for calling the program into operation within TRW/STL's computer utilization system. It appeared in some early documentation and eventually came into general use.