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UNIVERSITY OF MAINE
DEPARTMENT OF ELECTRICAL ENGINEERING

FINAL REPORT

LOW FREQUENCY
PARAMETRIC AMPLIFIERS

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ABSTRACT

The study of four-frequency parametric amplifiers described herein was performed in three general areas. A study of four-frequency parametric amplifiers indicates that the terminating admittance on the idler port can be used to modify the amplifier response at the input and output ports. An analytical investigation of methods of designing idler terminations that would result in broadbanding the amplifier was made and resulted in some success. Attempts to develop a low frequency analogue of such an amplifier in order to obtain experimental verification of the analytically derived results pointed up several problems in the idealized model used. In particular, the attempt to absorb the static capacitance of the varactor in the termination at each of the ports resulted in spurious critical frequencies which made the proper separation of the sidebands extremely difficult.

A second area of investigation, which developed as a means of overcoming the filtering problem described above, was the study of hybriding techniques to separate the sidebands by phase cancellation. The results of this study are discussed below.

The final area of study described in this report involves parametric up-converter with a signal frequency of 30 cps and a pump frequency of 100 Kc.
DEFINITIONS

In the report that follows, the following symbolism applies:

A. All subscripts for voltages, currents, or immittances will be chosen in order of ascending frequencies. In the four-frequency reactance amplifier, \( w_1 \) is the signal frequency, \( w_2 \) is the lower sideband idler frequency, \( w_3 \) is the pump frequency, and \( w_4 \) is the upper-sideband idler frequency.

B. Variations of varactor capacitance at the pump and second harmonic frequencies are defined as \( C_1 \) and \( C_2 \) respectively.

C. Normalized admittances are defined as follows:

\[
\gamma_{jk} = \frac{Y_{jk} + j\omega C_{0j}}{\omega j} \quad \left| \frac{C_2}{C_1} \right| = a_j + j\beta_j
\]

\( \gamma_{jk} \) is the normalized transfer admittance from the \( j \)'th to the \( k \)'th port.

D. \( \theta = \) the angle by which \( C_1^2 \) leads \( C_2^2 \). I.e., if \( C_1 = \left| C_1 \right| e^{j\Psi_1} \) and \( C_2 = \left| C_2 \right| e^{j\Psi_2} \), then, \( \theta = 2\Psi_1 - \Psi_2 \).

E. \( K_j = \frac{Q_j}{Q_1} \frac{\omega_1}{\omega j} \)

F. \( \delta = \frac{\omega_1 - \omega_{10}}{\omega_{10}} \)

G. \( x = 2Q_1\delta \)

For a double tuned circuit,

H. \( s = \) relative coupling coefficient = \( k/k_c \), where \( k \) is the coefficient of coupling and \( k_c \) is the critical coefficient of coupling.

I. \( Q_{j1} = \) primary \( Q \) of double tuned circuit at the \( j \)'th port.

J. \( Q_{j2} = \) secondary \( Q \) of double tuned circuit at the \( j \)'th port.

K. \( b = Q_{j1}/Q_{j2} + Q_{j2}/Q_{j1} \)
It has been shown by D.K. Adams that the transducer gain between the signal port and the k'th port in a four-frequency parametric amplifier can be expressed as

\[
G_{t1k} = \frac{4g_s g_k}{w_1 w_2^2} \begin{vmatrix} C_2 & 2 \end{vmatrix} \begin{vmatrix} M_{1k} \end{vmatrix}^2 = \frac{4g_s g_k}{w_1} \begin{vmatrix} C_2 \end{vmatrix}^2 \frac{1}{\gamma_{1k}} \tag{1}
\]

where \( D \) is the normalized network determinant and is given by

\[
D = \begin{vmatrix} \gamma_1 & j \frac{C_2}{C_1} & j \frac{C_2}{C_1} \\ -j \frac{C^*_1}{C_2} & \gamma_2 & j \frac{C^*_2}{C_2} \\ j \frac{C_1}{C_2} & j \frac{C_2}{C_2} & \gamma_4 \end{vmatrix} \tag{2}
\]

and \( M_{1k} \) is the cofactor of the first row and k'th column.

In both the upper and lower sideband up-converter, the termination at the idler sideband can make a significant contribution in the control of the gain and bandwidth of the amplifier. The two cases will be considered separately.

**Upper Sideband Upconverter**

In the four-frequency upper sideband upconverter, the transducer gain from the signal port to the output port is given by

\[
G_{t14} = 4 \frac{w_4}{w_1} \frac{1}{a_4} \left( \frac{\alpha_2}{\alpha_4} + \sin \theta \right)^2 + \left( \beta_2 + \cos \theta \right)^2 \frac{1}{\gamma_1 \gamma_2 \gamma_4 + \gamma_2 - \gamma_1 \gamma_4 + 2j \cos \theta} \tag{3}
\]

A choice of signal and output terminations to produce a match at the input and output ports independent of the idler termination results in

\[ Y_1 = Y_4 = \sin \theta + j \cos \theta \]  \hspace{1cm} (4)

It appears advantageous to choose the relative phase angle between fundamental and second harmonic pumping to be 90°. Among other things, such a choice eliminates the odd-powered frequency dependence in the gain expression giving a symmetrical frequency response curve. With such a choice, the required mid-band terminations are \( Y_1 = Y_4 = 1 + j0 \) and the mid-band transducer gain is given by:

\[ G_{t_{014}} = \frac{\omega_{40}}{\omega_{10}} \left( \frac{a_{20} + 1)^2 + \beta_2^2}{(a_{20} - 1)^2 + \beta_2^2} \right) \]  \hspace{1cm} (5)

It is seen that the mid-band gain can be adjusted by controlling the idler termination.

Two cases were studied with the upper sideband up-converter. First, it was assumed that all three ports were terminated in resonant tuned circuits. Then, to show improved bandwidth, the idler port was terminated in a double tuned circuit. The results of this investigation were reported in detail in an earlier report and is summarized for reference here.

Terminating all three ports in single tuned circuits, with the input and output terminations specified to produce an impedance match at signal and output frequencies, results in normalized admittances which may be expressed as

\[ \begin{align*}
Y_1 &= 1 + jX \\
Y_2 &= \alpha_{20}(1 + jK_2 X) \\
Y_4 &= 1 + jK_4 X
\end{align*} \]  \hspace{1cm} (6)
Substituting these values into equation (3) under the restriction of quadrature fundamental and second harmonic pumping gives, after some manipulation,

\[ G_{t14} = G_{t014} \left[ 1 + \left( \frac{a_{20}}{a_{20} + 1} K_2 \right)^2 \right] \cdot \left\{ 1 + X^2 \left[ \frac{(1+K_4)^2}{4} - \frac{K_4}{1-1/\alpha_{20}} + \frac{K_2^2}{(1-1/\alpha_{20})^2} \right] \right\} \]

\[ + x^4 \left[ \frac{-K_2K_4(1+K_4)}{2(1-\alpha_{20})} + \frac{(K_2+K_4+K_2K_4)^2 - 4K_4K_2^2}{4(1-1/\alpha_{20})^2} \right] \]

\[ + x^6 \left[ \frac{K_4^2K_2^2}{4(1-1/\alpha_{20})^2} \right] \]  

(7)

where \( G_{t014} \) is given by Eq. (5), \( \beta_2 \) being zero for a resonant idler tank. If \( \alpha_{20} \) is chosen to obtain the desired mid-band gain, \( K_2 \) and \( K_4 \) remain as variables usable in adjusting the frequency response. For example, the \( X^2 \) coefficients of the numerator and denominator may be made equal if

\[ K_2 = \frac{(\alpha_{20}-1)^2}{4\alpha_{20}} \left[ 2K_4 - \frac{(1+K_4)^2}{4} \right] \]  

(8)

The higher order terms in \( X \) remain in the denominator but some broadbanding might be achieved by judicious choice of \( K_2 \) and \( K_4 \) to satisfy Eq. (8) while minimizing the effect of the \( X^4 \) and \( X^6 \) coefficients in the denominator.

A greater degree of freedom is possible with a more complex termination at the idler port. For example, using a double tuned circuit at the idler port results in

\[ Y_2 = \frac{\alpha_{20}}{1+S^2} \left[ \frac{1+S^2+K_2^2X^2}{1+K_2^2X^2} + jK_2X \frac{1-S^2+K_2^2X^2}{1+K_2^2X^2} \right] \]  

(9)

where

\[ \alpha_{20} = \frac{1+S^2}{Q_2} \left[ \frac{C_0^2}{C_2} \right] \]  

(10)
Substituting Eq. (9) into Eq. (3), restricting the input and output terminations as before, results in a transducer gain which can be expressed as the ratio of two polynomials. Such an expression for general values of $a_{20}, S, K_2,$ and $K_4$ is extremely cumbersome. It is preferable, at this point, to illustrate the method under somewhat more restrictive conditions.

If the idler circuit is critically coupled and $a_{20}$ is taken to be 2, a value which adequately assures stability in the amplifier, the gain can be written as

$$G_{t14} = G_{t014} \left(1 + \frac{1}{3} K_2^2 X^2 + \frac{1}{9} K_2^4 X^4\right) 1 + X^2 \left[(1-K_4^2)(1+K_4^2)^2 + \frac{1}{4} (1+K_4^2)^2 - K_2^2\right]$$

$$+ X^4 \left[K_4^{2+K_4^4} - \frac{1}{2} (1+K_4^2)^2 K_2^2 + \frac{1}{4} (1+K_4^2)^2 K_2^2\right]$$

$$+ X^6 \left[\frac{1}{4} (1-K_4^2)^2 K_2^4 + \frac{1}{2} K_2^3 (1+K_4) K_4\right]$$

$$+ X^8 \left[\frac{1}{4} K_2^4 K_4^2\right]$$

(11)

where the mid-band gain $G_{t014}$, for the special case being considered can be shown to be 9 times the frequency ratio.

The fact that there are $X^2$ and $X^4$ terms in the numerator and $X^2, X^4, X^6,$ and $X^8$ terms in the denominator, with all coefficients dependant upon $K_2$ and $K_4$, make it apparent that the shaping of the frequency response can be effected by a judicious choice of $K_2$ and $K_4$. Curves illustrating such shaping are shown in Fig. 1. Curves I, II, and III of that figure show the effect of varying $K_2$ while holding $K_4$ constant. Choosing $K_4 = 0.17$ makes the $X^2$ coefficient in the denominator equal to $-K_2^2$. This negative coefficient in the denominator, aided by the frequency dependant terms in the numerator, tend to produce an increase in gain as the frequency is varied from its center value. As the curves indicate, this effect is less pronounced as $K_2$ becomes smaller and the higher order terms in the denominator become increasingly important. Curve IV illustrate the large effect of variations in $K_4$ on the frequency response. For curve V, $K_2$ and $K_4$ were chosen by equating the coefficient in the
numerator and denominator for both the $\chi^2$ and $\chi^4$ terms. In this case, fractional bandwidth is $2.8/Q_1$ with no peaking in the response curve.

The procedure illustrated can be expanded to include families of curves for different values of coupling coefficient, $S$, and idler termination, $a_{20}$. Alternately, one of the $K$'s can be chosen as a constant, and $S$ used as a design variable. Such a procedure might have considerable merit in experimental adjustment.

**Lower Sideband Up-converter**

In the lower sideband up-converter, it is desirable, as was the case for the upper sideband case, to choose the phase angle between the square of the fundamental and second harmonic pumping to be $90^\circ$. If the normalized input and output terminations, $\gamma_1$ and $\gamma_2$ are restricted to be equal, it is possible to separate the gain expression into two factors, one dependant only upon the idler termination, and the other dependant only upon the input and output terminations. Thus, subject to these restrictions, Eq. (1) can be written for the gain between ports 1 and 2 as

\[
G_{12} = 4 \alpha^2 \frac{w_{02}}{w_{10}} \frac{\left|1 + \gamma_4\right|^2}{\left|\gamma_4\right|^2} \cdot \frac{1}{\left|\gamma^2 - 1\right|^2}
\]

where

\[
\gamma_1 = \gamma_2 = \gamma = \alpha + j\beta
\]

Eq. (12) makes it obvious that it is possible to use the idler termination at the upper sideband port to compensate for frequency variations of the input and output terminations. Assuming that the input and output are terminated in single-tuned circuits, the denominator of the last factor of Eq. (12) can be expressed as

\[
\left|\gamma^2 - 1\right|^2 = (\alpha_0^2 - 1)^2 \left[1 + 2 \frac{\alpha_0^2 (\alpha_0^2 + 1)}{(\alpha_0^2 - 1)^2} \chi^2 + \frac{1}{(\alpha_0^2 - 1)^2} \chi^4\right]
\]

Eq. (13) being an increasing function of the frequency variable, $X$, makes it apparent that the idler termination must be arranged such that $\left|1 + \gamma_4\right|^2 / \left|\gamma_4\right|^2$ be also an increasing function of $X$. 

\[
-\ldots- 
\]

\[
-\ldots- 
\]
One circuit which would produce the desired admittance function at the idler port is the series tuned circuit. It should be noted, at the outset, that a simple series tuned circuit at the idler port would violate the original assumption that each termination offers a short-circuit to all frequency except those in the vicinity of its own resonant frequency. Thus the termination must be more complex than a simple R-L-C series circuit, and would actually consist of a filter that provides an admittance pole at the input and output frequency as well as the idler frequency. Assuming a series resonant circuit at the idler port, the gain expression, after some manipulation can be expressed as

\[ G_{t_{12}} = \frac{G_{t_{012}}}{1 + \frac{(a_4)K^2}{(a_4+1)}} \]

where

\[ G_{t_{012}} = \frac{4a_0^2(a_4+1)^2}{(a_0^2-1)^2} \frac{w_{20}}{w_{10}} \]

and

\[ a_4 = \frac{|C_2|}{R_4} \]

Such a configuration can theoretically be used to provide wide-band operation. Very high L to C ratios are necessary, however, in the practical circuitry.

A second possible termination to provide the desired variation with frequency of the term, \( |1+\gamma_4|^2/|\gamma_4|^2 \), is a heavily coupled double tuned circuit. The method of approach to broadbanding with such a circuit consists of determining the maximum peak-to-valley ratio of the term involving \( \gamma_4 \) and adjusting the position of the peak to match the curve of Eq. (13) over as wide a range as possible. Assuming the relative coefficient of coupling, S, to be much larger than unity results in
The maximum value of Eq. (15), occurring for $K_4 X / S = 1$ is

$$\left| \frac{1+\gamma_4}{\gamma_4} \right|^2 = \left( \frac{1-K_4 X}{S} \right)^2 + \left[ \frac{(Q_{41}/Q_{42} a_{40}) + \sqrt{b^2+2}}{S} \right]^2 \left( \frac{K_4 X}{S} \right)^2$$

(15)

For large peak-to-valley ratios, the ratio of primary to secondary $Q$'s must be large and $a_{40}$ should be in the vicinity of unity.
Summary

The design and construction of a physical circuit to verify the results of the preceding section met with only limited success. Attempts were made to separate the sidebands first by filtering, and second by hybriding techniques. Of the two methods, the latter appeared to offer the greater promise. Attempting to absorb the static component of varactor capacity into the tuned circuit at each port results in a series of spurious resonances around the loop. In the frequency range of this project, it was not possible to adequately isolate the individual frequencies in this fashion. Good separation of the sidebands is possible by phase cancellation in the hybrid circuit, but signal currents do not cancel, so that the resultant output voltages had appreciable signal frequency components. In order to approximately realize the conditions assumed in the analysis, it was necessary to separate and control the magnitude and phase of the second harmonic component of varactor capacitance. The necessary control was achieved by a double bridge arrangement of varactors, driving them separately with harmonic pumping signals. The pumping signals had to be kept small to minimize the spurious harmonics generated by the non-linear diode characteristic. But for good amplification, the diodes must be driven hard. As a result, it was necessary to sacrifice gain in the search for experimental verification of our analytical results.

Driver Chassis

In order to obtain as much versatility as possible for input, output, and idler terminations, the varactor diodes and associated pumping circuitry were constructed on a main chassis, with port terminations made on sub-chassis. Banana-plug interconnections simplified the interconnection between driver and sub-chassis and made possible the quick testing of several different pre-constructed amplifier configurations.

A block diagram of the pumping circuitry is shown in Fig. 2. The basic pump signals are derived from a 2.42 megacycle crystal controlled oscillator through a phase-shifting buffer stage. One
output from the buffer is used directly through an amplifier stage as a fundamental driving signal applied to a varactor bridge arrangement. The second output from the buffer is phase shifted through 90°. By means of a dual potentiometer, the two pump signals with a resulting phase difference of 90° are mixed in variable proportions in an adding circuit producing an output whose phase can be adjusted over a 90° range. Doubling the variable phase signal produces a second-harmonic pumping signal with a phase variable over 180°. This double-frequency signal can then be applied to a second diode bridge through an amplifier to produce a second harmonic component of capacitance variation. The output of the two bridges are applied in parallel to pins A and A' of the output terminal strip.

For filtering terminations, where the sidebands are to be separated by filter structures, no additional pumping circuitry is required. However, in hybrid structures, it is necessary to provide additional time-varying capacitances, with the phase shifted by 90°. The bottom half of the block diagram of Fig. 2 illustrates the method of obtaining these components which are then terminated at terminals B and B' of the output terminal strips.

Filtering Terminations

The theoretical model of a four-frequency parametric amplifier, assumed in the theoretical investigation, consisted of the time varying capacitance terminated in a series connection of ideal filters supporting the signal and both sideband components while offering zero impedance to all other frequencies. Such an idealized model is naturally impossible to obtain with physical circuits. There are two main factors which make it difficult to even approximate such a model at low radio frequencies. The first is the necessity of using lumped parameter circuits. The second is the requirement of tuning out the static value of the time varying capacitance at all the supported frequencies.

One means of eliminating the effect of the static component of static varactor capacitance in order to better approximate the idealized model consists of placing across the varactor terminals a network which will resonate the static capacitance at the signal and sideband frequencies. Then, only the fundamental and second
harmonic components of variable capacitance would be observed at the frequencies of interest. It is then possible to terminate the time varying capacitance with a series arrangement of three parallel resonant structure, each of which is tuned to one of the three frequencies of interest, to provide ports for the application of the signal, the extraction of power, and the termination of the idler. A simple analysis shows, however, that such a structure exhibits an effective capacitance many times the static capacitance of the varactor, which necessarily implies a narrow bandwidth and is therefore inappropriate to a study of broadbanding.

Another approach uses a series combination of three filter circuits, each of which would provide an impedance zero at two of the three pertinent frequencies, and in conjunction with the static diode capacitance, provide an impedance pole at the third frequency. Such an approach was attempted and a circuit synthesized and constructed. The considerable number of redundant elements necessary to provide the required zeros results in an equivalent capacitance which is prohibitively large for a broad-band amplifier. Furthermore, the problem of satisfying the filtering problem while at the same time providing the terminating restrictions required by the theoretical analysis made such an approach impractical for the purposes of this investigation.

Hybrid Terminations

In order that the port terminations might be determined entirely from theoretical considerations of optimum performance, it is desirable to separate the sidebands without extensive dependance upon frequency sensitive elements. This suggests the use of phase cancellation in a hybrid configuration.

If a varactor diode is pumped such that its variable component of capacitance can be approximated by

\[ C(t) = C_0 \cos (\omega_3 t + \phi_3) \]  \hspace{1cm} (17)

and a signal voltage

\[ v_1 = V_1 \cos (\omega_1 t + \phi_1) \]  \hspace{1cm} (18)

is impressed across the varactor, the current through the varactor can be expressed as

\[ i = \frac{1}{2} \omega_2 C_0 V_1 \sin (\omega_2 t + \phi_3 - \phi_1) + \frac{1}{2} \omega_4 C_0 V_1 \sin (\omega_4 t + \phi_3 + \phi_1) \]  \hspace{1cm} (19)
where

\[ \omega_2 = \omega_3 - \omega_1 \]
\[ \omega_2 = \omega_2 + \omega_2 \]

By using two identical varactor configurations, but constraining both the pump and signal voltages fed to one to be in quadrature with the corresponding voltages fed to the other, it is possible to derive two output currents, one frequency component of which will be additive, the other being subtractive. For example, if for amplifier A, \( \phi_1 = \phi_3 = 0 \), and for amplifier B, \( \phi_1 = \phi_3 = \pi/2 \), Eq. (19) shows that

\[ i_A = \frac{1}{2} \omega_2 C_0 V_1 \sin \omega_2 t + \frac{1}{2} \omega_4 C_0 V_1 \sin \omega_4 t \]

and

\[ i_B = \frac{1}{2} \omega_2 C_0 V_1 \sin \omega_2 t - \frac{1}{2} \omega_4 C_0 V_1 \sin \omega_4 t \]

Combining the two currents in an adding circuit will eliminate the upper sideband, while a subtractive circuit will eliminate the lower sideband.

The two variable capacitances with the proper phase shift are available from the driver chassis described above. The sub-chassis for the hybrid amplifier must provide; two signal voltages, equal in amplitude but 90° out of phase over the desired band; networks to "add and subtract" the upper and lower sideband outputs to provide separated sidebands at \( \omega_2 \) and \( \omega_4 \); and a means of terminating the pumped diodes such that the terminating admittance may be independently controlled at each port.

A very simple signal frequency phase shifter was used for the initial investigation. Two R-C networks provide phase shifts of +45° and -45° at 500 Kc as shown in Fig. 3. The network is self-compensating because of the inverse relationship between the phase shift in the two branches. The actual phase shift is \( 90° \pm 1.5° \) between 400 and 600 Kc. The resistance networks introduce attenuation and noise and should be replaced by reactive structures in a final amplifier. However, the simplicity of the R-C justifies its use in the initial investigation.

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The hybriding to separate the two sidebands is accomplished by means of two hybrid coils, $T_2$ and $T_4$, shown in Fig. 3. The two primary windings of each transformer must be identical and closely coupled, a condition that was met by the use of bifilar windings on General Ceramic $Q_1$ toroidal cores. The primary of $T_4$ is designed to resonate the static capacitance of the varactor diodes at $\omega_4$ and capacitors $C_{2A}$ and $C_{2B}$ are connected across the primaries of $T_2$ to provide similar termination at $\omega_2$. The circuit $Q$'s are adjustable by means of resistors $R_2$ and $R_4$.

While good sideband separation is obtainable from the existing circuit, the output signal does contain components at signal and pump frequencies. The pump frequency component can be eliminated by careful balancing of the diode bridge structure, but the signal components, in the present configuration, results from the addition of two quadrature components in the two primaries of each transformer, and cannot be eliminated without extensive redesign of the circuitry.

The hybrid configuration offers considerable promise in the study of broadband parametric amplifiers. The use of phase cancellation, rather than frequency discrimination for the separation of output and idler frequencies makes it possible to terminate each port for optimum performance with a minimum of interference with practical circuit problems.
A need was expressed by the prime contractor for a 30 cps low-noise amplifier to be utilized in the detection circuit of sensitive bridge measurements. In an attempt to satisfy that need, a 30 cps parametric amplifier, utilizing lower-sideband up-conversion to 100 Kc was designed and constructed. For the proposed application, a single input frequency was contemplated and the output could be at any frequency since only an analog of the input signal was required. For this purpose, and to obtain the best possible noise performance, a narrow-band design was chosen with a single sideband extracted for the output.

To obtain the relatively large time-varying capacitance required at these low frequencies, the collector-base junctions of two power transistors in a bridge pumping circuit was utilized.

The selection of the lower sideband is accomplished by utilizing the zero of a single crystal transformer coupled to the output coil. The pole of the crystal is positioned at the pump frequency to eliminate pump leakage in the output. The circuit for the amplifier is shown in Fig. 4.

Power gains in excess of 20 db. have been obtained with the present amplifier. The gain has been primarily the result of impedance transformation, the voltage gain being the order of 5 db. The power gains are explained primarily by the very small input admittance of the lower sideband up-converter. The input admittance is given by

\[ y_{in} = G_s (1 - \alpha + j\omega C_s / G_s) \]

where

\[ \alpha = \frac{\omega_1 \omega_2 C_s \omega_1^2}{j_2 G_s} \]

As \( \alpha \) approaches unity, the real part of the input admittance approaches zero resulting in large power gains due to admittance transformation. Transducer power gains have been quite small, in the order of 8 db. with a 10,000 ohm source resistance.

The value of the present amplifier in the application cited above is questionable, since the severe mismatch between the source impedance and the input resistance of the amplifier makes it impossible to achieve the full gain of the amplifier.
\[ \frac{G_{114}}{\omega_2/\omega_1} \]

\[ a_{10} = a_{40} = 1 \]
\[ a_{20} = 2 \]
\[ \phi = 90^\circ \]
\[ s = k/k_c = 1 \]

FIGURE 1. Frequency Response of the Four Frequency Up Converter with Double Tuned Idler Circuit.
FIGURE 3

SIGNAL AND HYBRID CIRCUITS
100 Ke pump

140 pf 1.5 V

100

0.1μfd

500

430 pf 140 pf

140 pf

0.1μfd

140 pf

300

48

140 pf

70 turns

70 turns

100

30 CPS PARAMETRIC AMPLIFIER

FIGURE 4