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# Analog Orbit Computer

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

# ANALOG ORBIT COMPUTER

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*Group 76*

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## ABSTRACT

The problem of pointing an instrument on the earth's surface at an object in orbit about the earth is investigated. Necessary expressions are derived for calculating continuously azimuth, elevation, range, and range rate from inputs of instrument site and orbit parameters. An electro-mechanical orbit computer capable of performing the required mathematical operations is described. Test results are listed for a similar computer used against a typical orbiting body.

Accepted for the Air Force
Franklin C. Hudson, Deputy Chief
Air Force Lincoln Laboratory Office

# GLOSSARY

| | |
|---|---|
| A | elevation angle. The angle between the local horizontal plane and the line of sight measured in the vertical plane through the line of sight. |
| a | semi-major axis. One-half the length of the major axis of the ellipse of orbit. |
| B | azimuth angle. The dihedral angle between the plane of the north meridian at the site and the vertical plane through the line of sight, measured about the site zenith in a clockwise direction from north. |
| b | semi-minor axis. One-half the length of the minor axis of the ellipse of orbit. |
| c | focal distance. The distance between the center of the ellipse of orbit and a focus of the ellipse. |
| E | eccentric anomaly. The angle at the center of the ellipse between the radius to the point of perigee and the radius to the orthogonal projection, from the major axis, of the target on the circumscribed circle, measured from perigee in the direction of orbital motion. |
| e | eccentricity. The eccentricity of the ellipse of orbit, defined as the ratio between the focal distance and the length of the semi-major axis, or $c/a$. |
| f | flatness. The flatness of the earth's surface, defined as the ratio between the difference in semi-major and semi-minor axis lengths and the length of the semi-major axis. Its presently accepted value is 1/297.0. |
| h | slant range. The distance from the site to the target, measured along the line of sight. |
| i | inclination. The dihedral angle between the plane of the celestial equator and the orbital plane. It is measured about the ascending node counterclockwise from the equator to the first intersection with the orbit plane. |

$LST$ local sidereal time. The dihedral angle between the plane of the site meridian and the plane of the meridian through the first point of Aries. It is measured about the north pole from the first point of Aries toward the east.

$M$ mean anomaly. The angle at the center of the earth between the radius to the point of perigee and the radius to a point the target would occupy at the time under consideration if it were in a circular orbit about the earth. This angle is equal to the time since perigee passage multiplied by the average orbital rate, or $M = 2\pi/P\ (LST - T)$. It is measured from perigee in the direction of target motion.

$P$ period of orbit. The time required for the target to complete one orbit.

$R$ radius of the earth. The distance from the center of the earth to the site.

$r$ geocentric range. The distance from the center of the earth to the target.

$S$ orbit area. The area of the ellipse of orbit.

$\gamma$ range angle. The angle at the center of the earth between the radius to the ascending node and the radius to the target, measured from the ascending node in the direction of target motion along the orbit.

$\gamma_o$ range angle of perigee. The angle at the center of the earth between the radius to the ascending node and the radius to the point of perigee, measured from the ascending node in the direction of target motion along the orbit.

$\delta$ latitude correction. The angular difference between geodetic latitude $\lambda_g$ and geocentric latitude $\lambda_o$.

$\lambda_g$ geodetic latitude. The angle between the equatorial plane and the perpendicular to the earth's surface at the site, measured in the plane of the meridian through the site.

$\lambda_o$ geocentric latitude. The angle between the equatorial plane and the radius from the center of the earth to the site, measured in the plane of the site meridian.

$T$ perigee passage. The time at which the target passes perigee.

$\Omega$ longitude of the ascending node. The dihedral angle between the plane of the ascending node meridian and the plane of the first point of Aries meridian, measured about the north pole in the east direction from the first point of Aries.

## I. INTRODUCTION

Location of a satellite in unperturbed orbit about the earth may be described in terms of six parameters. Location of a site on the surface of the earth may be described in terms of three parameters. From these parameters continuous pointing angles, range, and range rate may be calculated. A special effort is made in this report to present the analytic solution to the orbital pointing problem in such a manner that the suggested machine solution is self-evident.

## II. ANALYTIC SOLUTION

When target position is known in its plane of orbit, it is possible by a series of successive rotations and translations, determined by the orbit and site parameters, to arrive at the coordinates of target position with respect to a site of known location. Assume from Fig. 1 that the range angle from perigee ($\gamma - \gamma_o$) is known. Then

$$x_1 = r \sin (\gamma - \gamma_o) \quad ,$$

$$y_1 = r \cos (\gamma - \gamma_o) \quad ,$$

$$z_1 = 0 \quad . \tag{1}$$

The coordinate system is rotated about its $Z_1$-axis through the angle $\gamma_o$ so that the $Y_2$-axis passes through the ascending node. In the new system

$$x_2 = x_1 \cos \gamma_o + y_1 \sin \gamma_o \quad ,$$

$$y_2 = y_1 \cos \gamma_o - x_1 \sin \gamma_o \quad ,$$

$$z_2 = 0 \quad . \tag{2}$$

Next, the system is rotated about the $Y_2$-axis through the inclination angle i, as shown in Fig. 2. The $X_3$-axis is in the equatorial plane and passes through the meridian $90^o$ east of the ascending node; the $Y_3$-axis is in the equatorial plane and passes through the ascending node; the $Z_3$-axis passes through the north pole. Target coordinates are

$$x_3 = x_2 \cos i \quad ,$$

$$y_3 = y_2 \quad ,$$

$$z_3 = x_2 \sin i \quad . \tag{3}$$

Then, a rotation about the $Z_3$-axis through the angle (LST - $\Omega$) places the $Y_4$-axis through the meridian of the site.

$$x_4 = x_3 \cos (\mathrm{LST} - \Omega) - y_3 \sin (\mathrm{LST} - \Omega) \quad ,$$

$$y_4 = y_3 \cos (\mathrm{LST} - \Omega) + x_3 \sin (\mathrm{LST} - \Omega) \quad ,$$

$$z_4 = z_3 \quad . \tag{4}$$

Next, the coordinate system is rotated about the $X_4$-axis through the angle $(90^\circ - \lambda_g)$ to place the $Z_5$-axis perpendicular to the horizontal plane at the site, as shown in Fig. 3.

$$x_5 = x_4 \quad ,$$

$$y_5 = y_4 \sin \lambda_g - z_4 \cos \lambda_g \quad ,$$

$$z_5 = z_4 \sin \lambda_g + y_4 \cos \lambda_g \quad . \tag{5}$$

Translations along the $Y_5$- and $Z_5$-axes are required to position the origin of the coordinate system at the antenna site. From Fig. 3 the angular difference $\delta$ between geodetic latitude $\lambda_g$ and geocentric latitude $\lambda_o$ is calculated. The slope of the perpendicular to the surface at $(y_4, z_4)$ is

$$\frac{1}{(1 - f)^2} \qquad \frac{z_4}{y_4} \quad ,$$

where f is the flatness of the earth's surface. The slope of the radius to the same point is $z_4/y_4$. The difference between the two latitudes

$$\delta = \tan^{-1} \frac{(2f - f^2) \sin \lambda_g \cos \lambda_g}{\cos^2 \lambda_g + (1 - f)^2 \sin^2 \lambda_g}$$

or, since $\delta$ is small and f is much less than unity,

$$\delta \approx 2f \sin \lambda_g \cos \lambda_g = f \sin 2 \lambda_g \quad .$$

The origin of the coordinate system, then, is translated to the site by making

$$x_6 = x_5 \quad ,$$

$$y_6 = y_5 - R \sin \delta \approx y_5 - Rf \sin 2\lambda_g \quad ,$$

$$z_6 = z_5 - R \cos \delta \approx z_5 - R \quad . \qquad (6)$$

The $X_6$-axis is horizontal in the east direction, the $Y_6$-axis is horizontal in the south direction, and the $Z_6$-axis is vertical. From Fig. 4 the pointing angles are for azimuth

$$B = \tan^{-1} \frac{x_6}{-y_6} \quad , \qquad (7)$$

and for elevation

$$A = \tan^{-1} \frac{z_6}{+(x_6^2 + y_6^2)^{1/2}} \quad . \qquad (8)$$

The slant range is

$$h = (x_6^2 + y_6^2 + z_6^2)^{1/2} \quad . \qquad (9)$$

Range rate is the time derivative of Eq. (9).

To calculate pointing information in real time, it is necessary to establish a relationship between time and $\gamma - \gamma_o$, the orbital range angle measured from perigee to the target. Referring again to Fig. 1, other expressions for the components of target position are

$$x_1 = b \sin E \quad ,$$

$$y_1 = a(\cos E - e) \quad . \tag{10}$$

From these expressions, range from the center of the earth to the target is

$$r = (x_1^2 + y_1^2)^{1/2} = a(1 - e \cos E) \quad . \tag{11}$$

Settings Eqs. (10) equal to the corresponding Eqs. (1)

$$r \cos (\gamma - \gamma_o) = a(\cos E - e) \quad ,$$

$$r \sin (\gamma - \gamma_o) = b \sin E = a(1 - e^2)^{1/2} \sin E \quad . \tag{12}$$

From Eqs. (12)

$$d(\gamma - \gamma_o) = \frac{(1 - e^2)^{1/2}}{1 - e \cos E} \, dE \quad . \tag{13}$$

From Fig. 1 the incremental area

$$dS = \frac{r^2}{2} \, d(\gamma - \gamma_o) = \frac{a^2}{2} \, (1 - e^2)^{1/2} (1 - e \cos E) dE \quad . \tag{14}$$

Kepler's second law for orbiting bodies states that the time derivative of the area swept out by the radius (r) from a focus of the ellipse to the orbiting body is a constant. Making use of this principle,

$$\frac{dS}{dt} = K = \frac{\text{area}}{\text{period}} = \frac{\pi ab}{P} = \frac{\pi a^2 (1 - e^2)^{1/2}}{P} \quad , \tag{15}$$

and

$$dS = \frac{\pi a^2 (1 - e^2)^{1/2}}{P} \, dt \quad . \tag{16}$$

Combining Eqs. (14) and (16),

$$\frac{2\pi}{P} \, dt = (1 - e \cos E) dE \quad . \tag{17}$$

Integrating both sides of Eq. (17) over the time elapsed since passage of perigee,

$$\frac{2\pi}{P} \int_{T}^{LST} dt = \int_{o}^{E} (1 - e \cos E) dE \quad , \tag{18}$$

$$\frac{2\pi}{P} (LST - T) = E - e \sin E \quad ,$$

or

$$M = E - e \sin E \quad . \tag{19}$$

From Eq. (19) the eccentric anomaly E may be determined as a function of time. The range angle from perigee $(\gamma - \gamma_o)$ is calculated by Eqs. (12) after E is determined.

## III. MACHINE SOLUTION

The series of operations necessary to proceed from inputs of site and orbit parameters to the desired outputs is accomplished readily by an electromechanical analog computer. A functional diagram of such a machine is shown in Fig. 5.

The basic component used in the computer is the resolver, which is capable of performing almost all the mathematical manipulations required. It accepts two electrical inputs, $u_1$ and $v_1$, and a mechanical angle input $\psi$. Its two electrical outputs are

$$u_2 = u_1 \cos \psi - v_1 \sin \psi \quad ,$$

$$v_2 = v_1 \cos \psi + u_1 \sin \psi \quad , \tag{20}$$

which describe the operations necessary for rotating a rectangular coordinate system. Such an operation is required for Eqs. (1), (2), (3), (4), and (5). If components $u_1$ and $v_1$ are given and the angle $\psi$ is driven until $u_2$ in Eq. (20) is equal to zero, then

$$\psi = \tan^{-1} \frac{u_1}{v_1} \quad ,$$

$$v_2 = (u_1^2 + v_1^2)^{1/2} \quad . \tag{21}$$

This technique is used to solve Eqs. (7), (8), and (9). The only other necessary operation is algebraic summing, which is performed by parallel input operational amplifiers.

From Fig. 5, one of the required inputs to the computer is $\Delta M$, the time integral of the mean anomaly rate. This shaft rotation is produced by the mechanism shown in Fig. 6. A synchronous motor is driven at constant speed by a constant frequency source. The motor drives through reduction gearing the inputs to 23 clutches which are capable of switching 23 different speeds into the output. The output rate is the sum of the output rates of the clutches which are engaged. Gearing is so arranged that the relative speeds of the clutch outputs, starting at the least significant clutch, are 1, 2, 4, 8, 10, 20, 40, 80, etc. Thus, five full decades and one partial decade of discreet rates may be selected by controlling the clutches in 8-4-2-1 binary coded decimal format. Rates from $1 \times 10^{-5}$ degree per minute to 8.0 degrees per minute are available in increments of $1 \times 10^{-5}$ degree per minute. Accuracy of the average output rate is dependent only upon the accuracy of the constant frequency source.

The computer requires a total of nine inputs.

| | | |
|---|---|---|
| 1. | $e$ | eccentricity of the orbit. |
| 2. | $R/a$ | ratio between radius of earth at site and semi-major axis of orbit. |
| 3. | $M_o$ | initial value of mean anomaly. |
| 4. | $dM/dt$ | mean anomaly rate. |
| 5. | $\gamma_o$ | range angle of perigee |
| 6. | $i$ | orbit inclination. |
| 7. | $(LST)_o$ | initial value of local sidereal time. |
| 8. | $\Omega$ | longitude of ascending node. |
| 9. | $\lambda_g$ | geodetic latitude of site. |

Changes to mean anomaly M and local sidereal time LST are generated by the machine to provide continuous solutions in real time.

## IV. IMPLEMENTATION

Accuracy of the computer depends to a great extent upon the accuracy of the resolvers used. The selected winding compensated resolvers have a functional error of less than 0.05 percent of output at maximum coupling and total interaxis error of less than 1.5 minutes of arc. Gain of unity and phase shift of zero between resolver inputs and resolver outputs is maintained by making use of high gain solid state buffer amplifiers. The resolver compensator winding is magnetically coupled to the primary winding so that it sees any changes in transformation ratio between the primary and secondary windings. The input voltage and compensator winding voltage are compared at the input of the buffer amplifier. Any change in the resolver transformation characteristics will be reflected by a change in compensator voltage. This, in turn, will adjust the closed loop gain of the resolver-buffer amplifier combination to restore transformation ratio to its original value. The high input impedance of the buffer amplifier is also useful in providing isolation between interconnected resolvers.

A number of parameters must be introduced into the computer by accurate angle setting of resolver shafts. A separate dial system and associated precision reduction gearing for each resolver is costly and space comsuming. If, instead, a dual speed multipole synchro is coupled directly to the resolver shaft, it will provide indication of shaft angle to within 20 seconds of arc. A single precision dial system with another multipole synchro is used to set consecutively each resolver shaft by servo slaving.

The complete orbit computer package will occupy about 16 cubic feet and weigh about 500 pounds.

## V. TEST RESULTS

Pointing data from a computer similar to that described have been compared to the calculated positions of a satellite in a medium altitude orbit about the earth. Observed errors for a representative group of satellite passes are tabulated below.

| | *Total Pointing Error (Degrees) | Range Error (Nautical Miles) | Range Rate Error (Nautical Miles/Minute) |
|---|---|---|---|
| Average | 0.091 | 9.2 | 1.4 |
| RMS | 0.100 | 11.0 | 1.9 |
| Maximum | 0.230 | 25.0 | 3.5 |

The magnitudes of these errors are in close agreement with those calculated from the known tolerances of the various components used in the computer.

## VI. CONCLUSIONS

The computer described is capable of the simple, straight-forward, and economical solution of the orbital pointing problem. It may be used, as well, against targets which appear fixed in space and against orbiting belts. Well-known techniques and conventional, readily available components are used. Solutions are instantaneous, continuous, and in real time. The accuracy obtained appears to be compatible to that with which the orbit parameters of most earth satellites are known.

*Total pointing error is the great circle distance between the target and the line of sight. It is the resultant error due to the individual errors about the two pointing axes.

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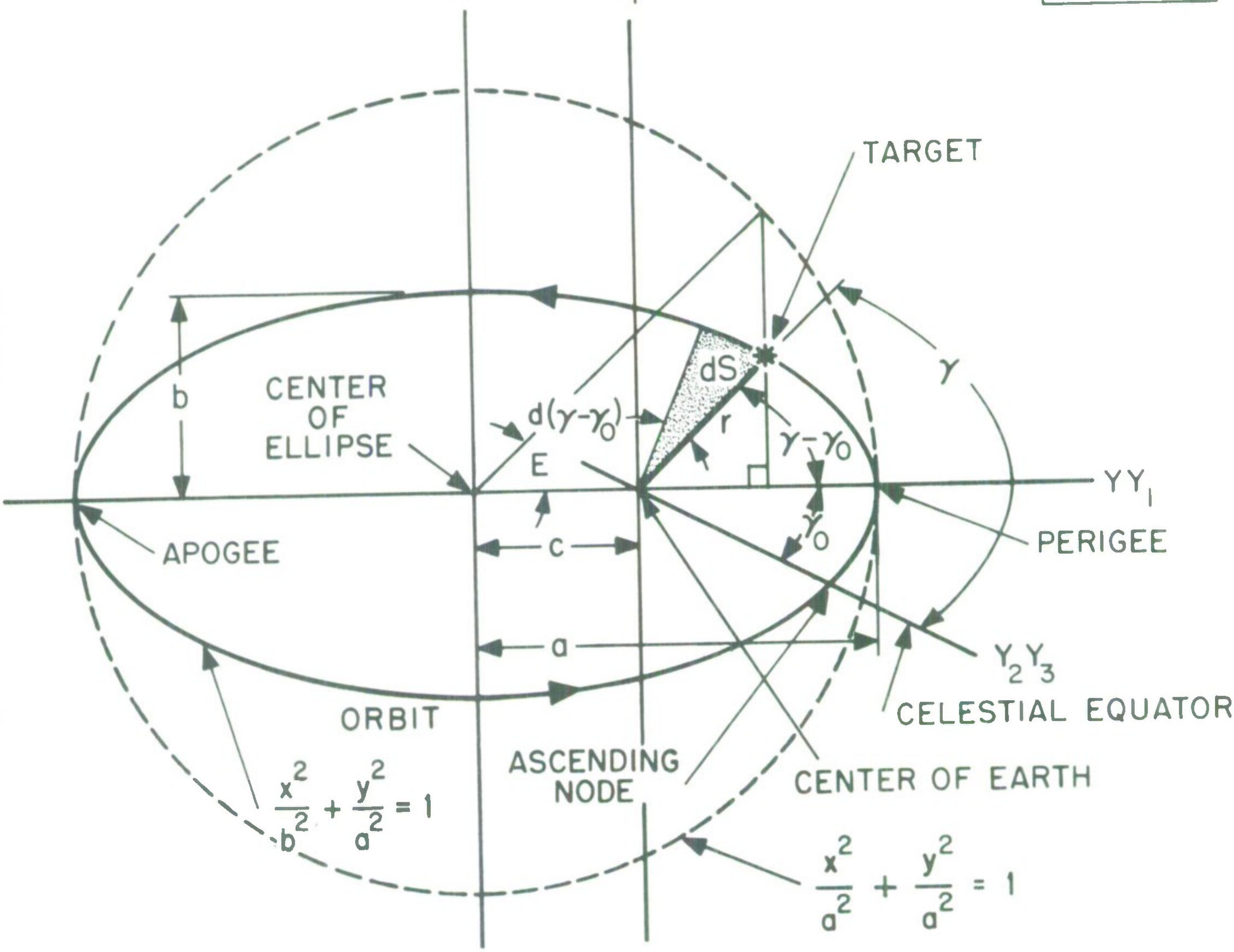


Fig. 1. Plane of orbit.

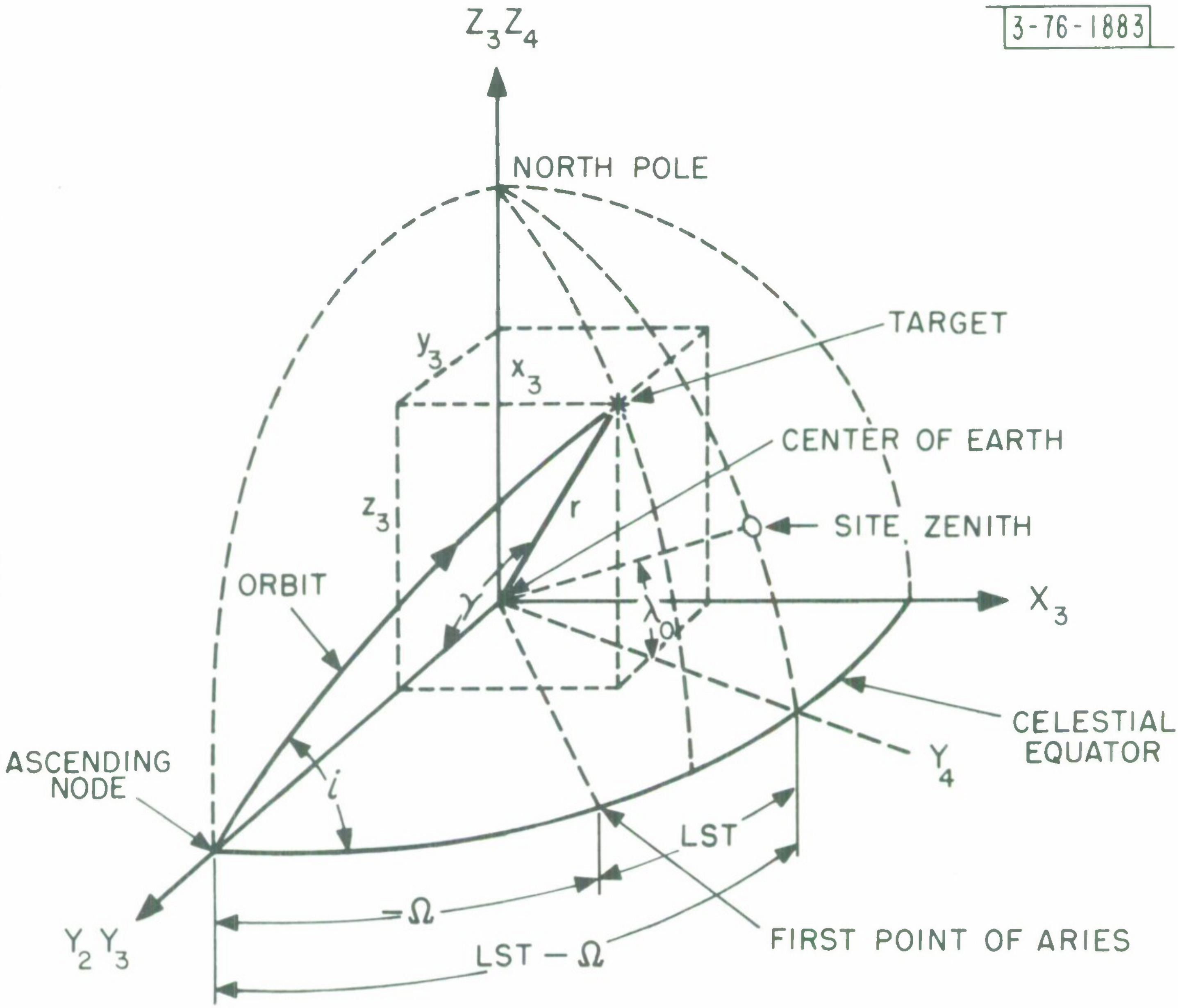


Fig. 2. Location of target and site in $X_3Y_3Z_3$ coordinate system.

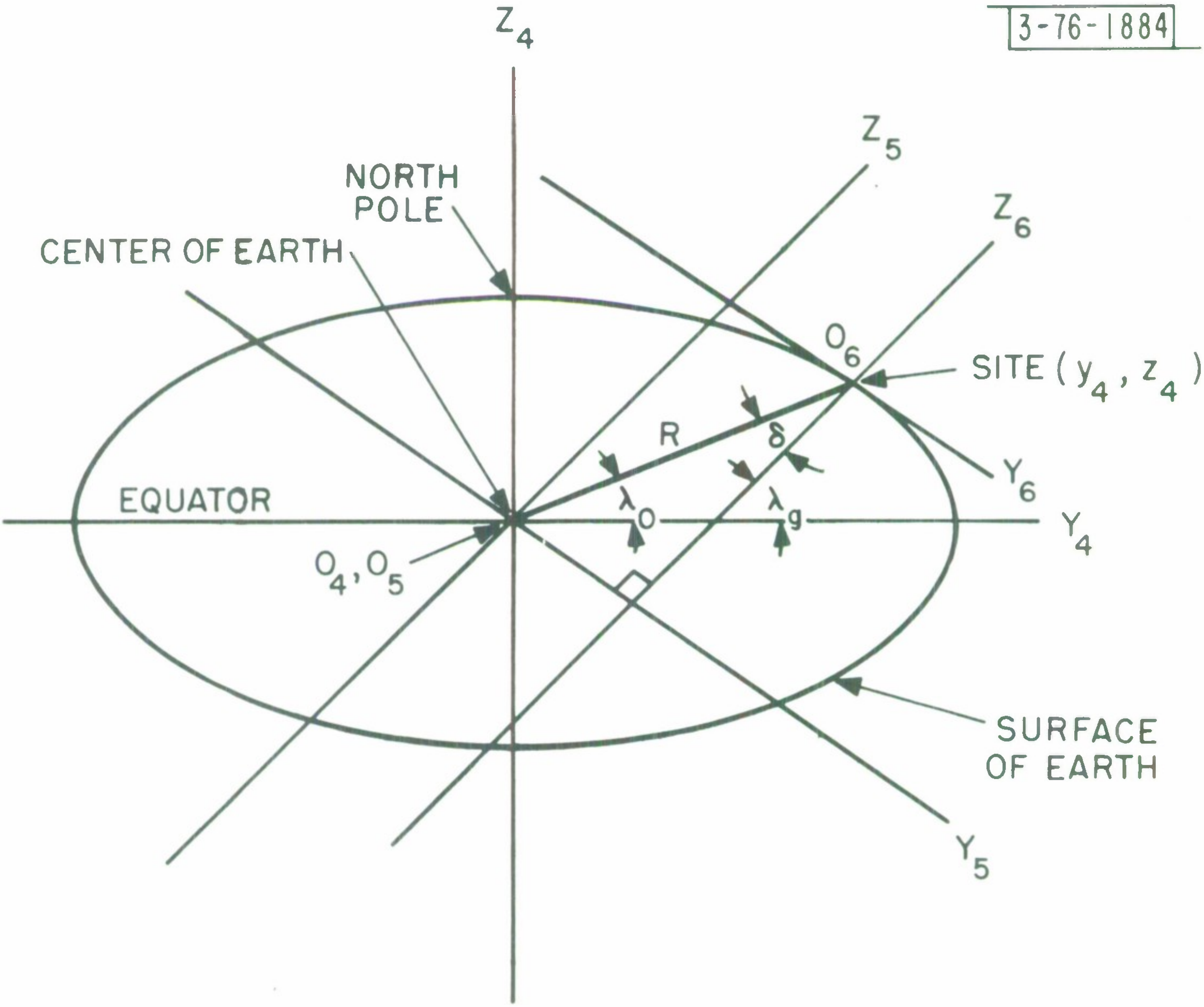


Fig. 3. Plane of meridian through site.

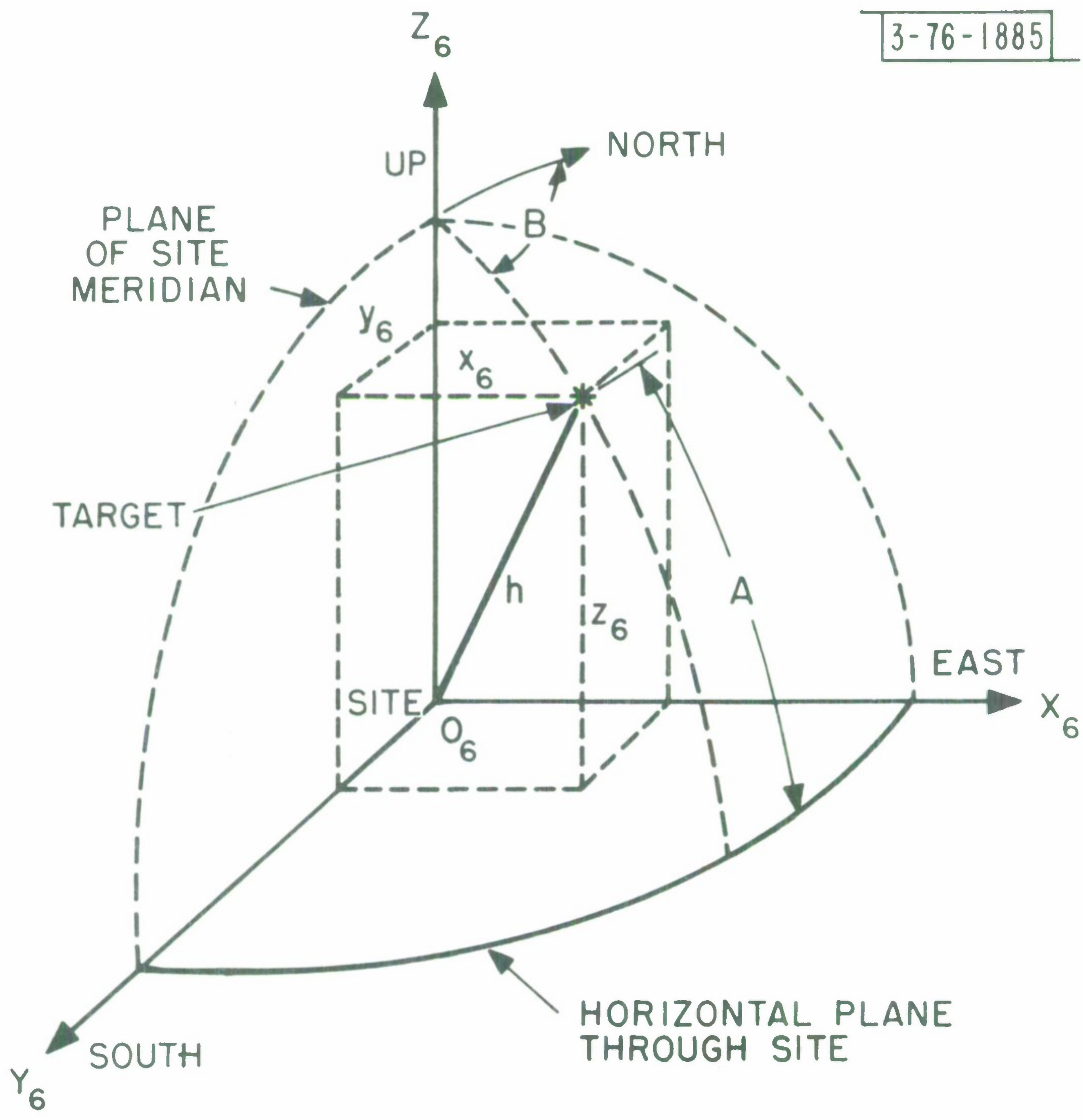


Fig. 4. Location of target in site-centered coordinate system.

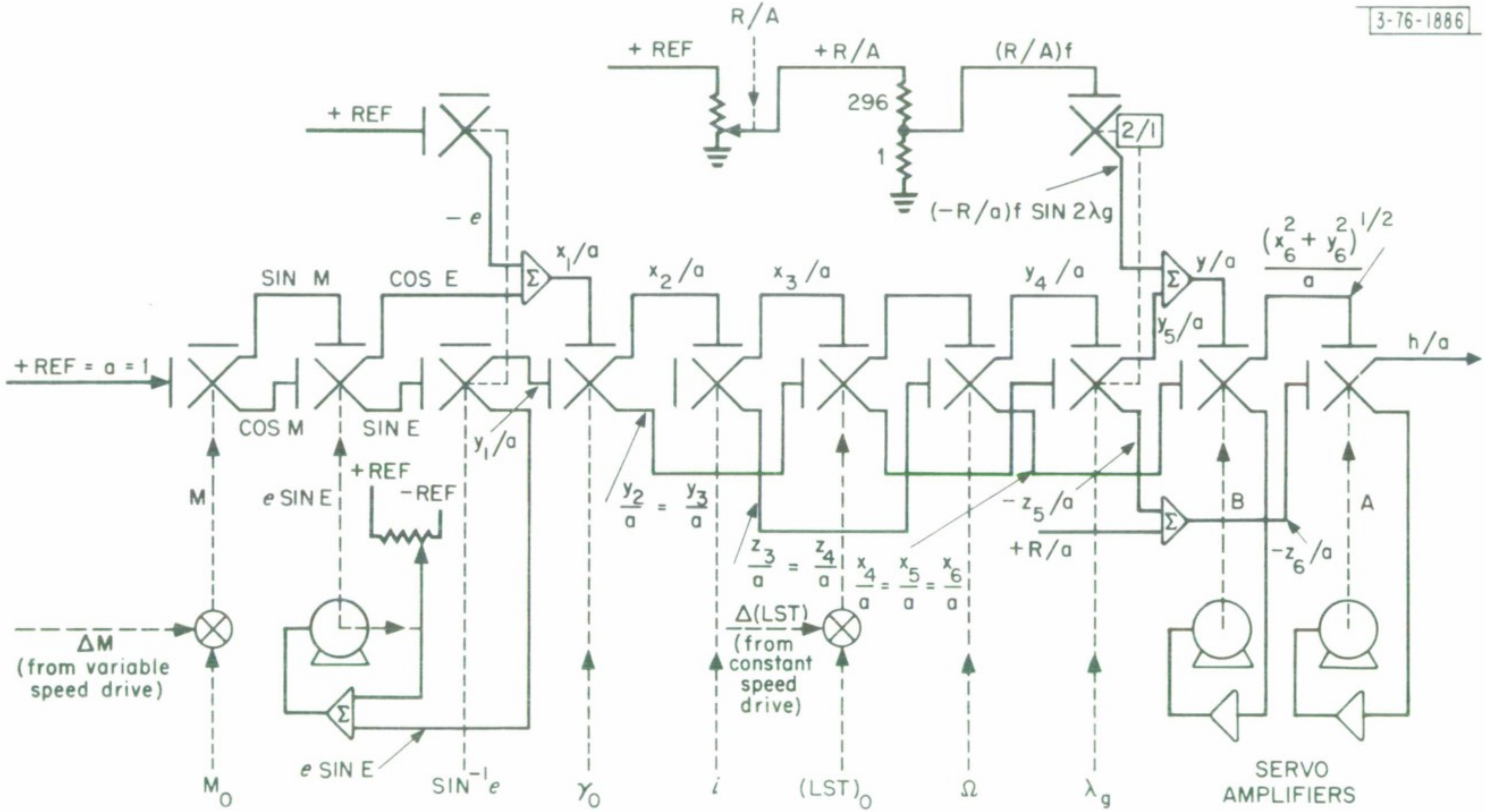


Fig. 5. Functional diagram - analog orbit computer.

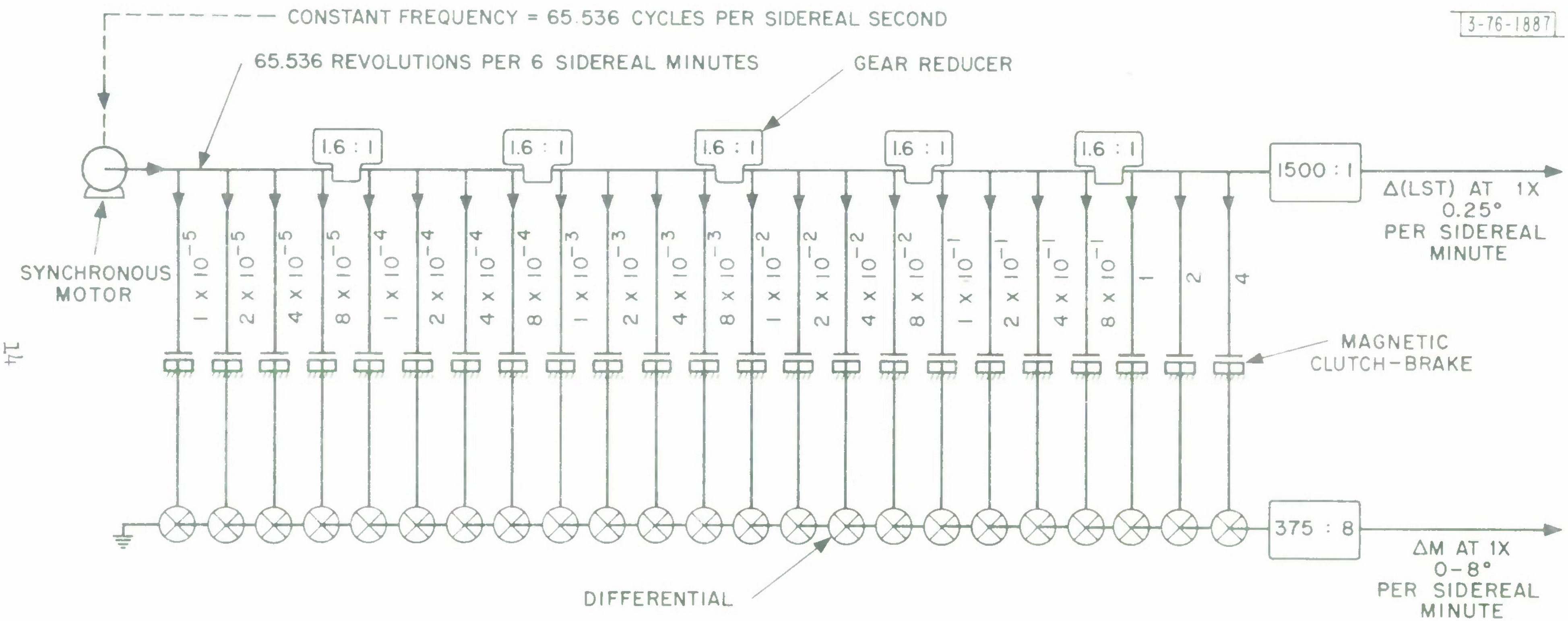


Fig. 6. Mean anomaly and local sidereal time drives.