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The Radio Direction Finding
Research Laboratory
Department of Electrical Engineering
University of Illinois

# A COMPARISON OF RADIO DIRECTION FINDING SYSTEMS

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# A COMPARISON OF RADIO DIRECTION FINDING SYSTEMS.

(14 TR-1) (9) Technical Report [redacted]. (12) 55p.

1 Dec [redacted] 49 (11)

(15) N6ori-71 (15)

N6-ori-71 Task XV
ONR Project No. 076 161

(16) NR-076-161 TR-9


*Supervisors*

A. D. Bailey E. C. Jordan H. D. Webb

*Research Associates and Assistants*

J. M. Anderson
R. W. Annis
R. H. Brunner
R. A. Campbell
R. H. DuHamel
V. J. Fowler
E. C. Hayden
C. T. A. Johnk
B. D. Kirkwood
R. E. Lepic
T. R. O'Meara
D. E. Royal
N. Yaru


[redacted]

Approved by: E. C. Jordan
E. C. Jordan

[redacted]


~~RADIO DIRECTION FINDING RESEARCH LABORATORY~~
DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS


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# 1. ABSTRACT

Small and large aperture rdf systems (both existing and proposed), and limited space distributions of narrow aperture rdf systems are compared in terms of the bearing error which arises when a desired and an undesired signal of the same frequency but having arbitrary relative amplitude, arbitrary time phase difference, and arbitrary relative directions of arrival are incident upon each of the several systems. The effect of increasing array diameter upon the bearing error of large aperture systems and distributions of small aperture systems is also considered. The comparative results are shown in the form of bearing error curves, photographs of simulated system performance and as a tabulation of average absolute bearing error for each system.

A discussion of the practical problems involved in the realization of some of the proposed systems is included and significant conclusions are stated.

## II. INTRODUCTION

One of the objectives of the radio direction finding research program at the University of Illinois has been the comparison of various possible methods of radio direction finding. This report attempts to summarize some of the more important conclusions which have been reached concerning several classes of systems. The basis which has been used for comparison is the ability of the system to give a correct bearing under the common practical field conditions of a wave interference pattern produced by the presence of several rather than one arriving wave. Under such conditions the first question to be answered is that concerning which of the arriving waves is the "desired" wave, or which wave gives the "correct" bearing under different sets of circumstances. Two cases are of considerable practical importance. When the interfering waves are due to reflections or reradiations at or near the site, giving rise to what is usually termed "site error", it is reasonable to assume that under most circumstances the direct waves will be the strongest, and, therefore, that the direction of the strongest wave will give the correct bearing. In this case the interfering waves can have any direction with respect to the direct or desired wave. When the several waves are due to multipath sky wave transmission, it is no longer so probable that the correct direction will be given by that of the strongest wave. Other criteria, such as the direction of the wave which arrives first or at the lowest vertical angle, or the average direction of all the arriving waves might prove more reliable. However, except for back-scatter and off-path transmission, it seems probable that all the sky waves should arrive from nearly the same horizontal angle, so that the direction of any individual wave should be not too greatly in error, and use of the strongest wave is again a reasonable procedure.

The systems compared in this report include both well-known types which have been built and operated (some for many years) and other types which exist only on paper. Among the latter are two systems which have been proposed by members of this laboratory. The sequence-comparator system devised by R. W. Annis is a small-aperture[1] system which extracts and utilizes more information from a wave interference pattern than does a conventional small-aperture system. The wide-aperture vector-phase system proposed by Thomas O'Meara provides a method for obtaining an instantaneous indication (without switching) which has the same order of accuracy as the wide-aperture switched Doppler-Effect type of radio direction finder. While both of these systems are probably too complicated in their present forms to be useful as practical direction finders, they do serve to indicate how the required information can be obtained and displayed.

The distributions of narrow-aperture systems discussed in Section V,C should be distinguished, on the one hand, from wide-base systems which have been used in the past to obtain a position fix, and on the other hand, from systems of closely-grouped rdf sets which have been used to obtain improved bearings either by a process of averaging individual bearings, or by diversity reception in which the most reliable bearing is selected. The theoretical bearing error curves shown

1. The "aperture" of an array is the length or diameter of the array in wavelengths. The dividing line between small- and large-aperture systems may be taken as about $\lambda/4$.

have been calculated and drawn for an instantaneous vector addition of the individual system responses, such as could be obtained by the combining schemes suggested in Section V,C. Under certain conditions, which will be indicated, such an addition results in a bearing improvement similar to that which can be obtained with wide-aperture arrays.

In this report only sample or "typical" curves for the various systems have been shown. This is because to show all the curves available for each system would have resulted in such a mass of information as to make the report unreadable. Additional information on the various systems will be found in individual reports now in preparation.[2] In addition, information on some systems is still incomplete. To the extent that some of the conclusions of this report are based upon a limited amount of data, these conclusions must be considered tentative until further substantiated.

It is recognized that the ability to discriminate against interfering waves is only one of many considerations involved in the selection of a radio direction finding system. Depending upon the application, other factors, such as sensitivity, relative simplicity, space requirements, ruggedness, portability, etc., are often deciding. It is obviously impossible to consider all such factors in making a general comparison, especially since the deciding factors will be different in different applications. However, to make the comparisons given here more meaningful, the Section VII of this report includes a brief discussion of some of the practical circuitry considerations involved with each of the various systems.

---

2. Among the reports now in preparation are the following:
   1. An Analysis of Radio Direction Finding Systems---Part III.
   2. Direction Finding System Analyzer and Doppler Effect.
   3. Vector-Phase RDF System.

## III. STATEMENT OF THE PROBLEM

The systems which are compared seem to divide naturally into three groups as follows:

1. Narrow Aperture Radio Direction Finding Systems
   - i. Systems Retaining Amplitude Information
     - a. Adcock
     - b. Sequence Comparator
     - c. Spaced-loop
   - ii. Systems With Amplitude Limiting
     - a. Vector-phase
     - b. Narrow-Aperture Doppler-Effect
2. Wide Aperture Radio Direction Finding Systems
   - i. Doppler-Effect
   - ii. Vector-Phase
   - iii. Wullenweber
3. Limited Space Distribution of Narrow-Aperture Radio Direction Finding Systems

Equations have been developed which give the bearing error of each of the above systems for the case of interference between an undesired and a desired signal of the same frequency. This bearing error is a function of the ratio of the magnitudes of the two signals, their time phase difference, their azimuthal difference and the aperture of the system. The vertical angle of the arrival of the signals also affects the error; however, the equations do not explicitly consider this variable. In succeding sections of this report, the equation for the bearing error of each system as a function of the variables will be given. Families of curves have been included to show graphically the functional relationship between the bearing error and the several independent variables. These are assembled for convenient, rapid intrasystem comparison. Photographs of simulated system performance are also included for comparison of small aperture systems. Finally the average absolute bearing errors of the several systems are tabulated for overall or intersystem comparison.

## IV. DEFINITION OF SYMBOLS

| | |
|---|---|
| $h$ | relative magnitude. |
| $\Gamma_{2,1}$ | time phase difference between the strongest and the next strongest wave at the center of a complete system (lead angle weaker minus lead angle stronger). |
| $\Gamma$ | time phase difference between the strongest and the next strongest wave at the center of a complete system (lead angle weaker minus lead angle stronger). $\Gamma = \Gamma_{2,1}$ when only two waves are present which is the case in this report. |
| $\Psi_{a+1,\ L}$ | time phase difference between the lead angle of the resultant waves (or resultant induced voltages) at antenna (a + L) and at antenna a. (Subtract the lead angle of the strongest from that of the next strongest.) |
| $\Phi_n$ | azimuthal angle of the nth strongest wave measured from the reference counterclockwise[3] (spherical coordinates). |
| $\Phi_I$ | indicated direction of arrival measured counterclockwise from $\Phi = 0$ (sense is away from reception point). |
| $\Phi_T$ | true direction of transmitter measured counterclockwise from $\Phi = 0$. |
| $\Phi$ | angular separation measured counterclockwise – from weaker to stronger or angular separation measured clockwise from stronger to weaker. $\Phi = \Phi_{2,1}$ when only two waves are present which is the case in this report. |
| $\varphi$ | azimuthal angle as a variable. |
| $\theta$ | polar angle as a variable. |
| $\theta_n$ | polar angle of arrival of the nth strongest wave, measured downward from the vertical axis (spherical coordinates). |
| $B_I$ | indicated bearing measured clockwise from true north. |
| $B_T$ | true bearing of transmitter measured clockwise from true north. |
| $B_a$ | antenna bearing measured clockwise with respect to true north. |
| $\xi$ | bearing error = -"indication" error = negative of indicated direction minus true direction = indicated bearing minus true bearing. |
| $N$ | total number of incoming waves. |
| $n$ | nth wave |
| $A$ | total number of antennas |
| $a$ | ath antenna (running variable) |
| $S$ | total number of systems |
| $s$ | sth system |
| $E_n$ | field strength of the nth wave |
| $M$ | modulus of a vector |
| $M_s$ | resultant field strength at the center of the sth system |

3. Clockwise and counterclockwise is determined by looking down on a compass from above or looking towards the (-z) axis on a right-handed spherical coordinate system.

$\Gamma(\psi) = \arctan\left(\frac{h \sin \psi}{1 + h \cos \psi}\right)$

$\Gamma_{n\pm} = \Gamma(-n\Phi \pm \Gamma)$

D — system diameter in wavelengths

d — cylinder diameter in wavelengths

R — system radius in degrees

r — cylinder radius in degrees

t — time

C — spacing between antennas

$J_n$ — Bessel function (nth order)

$Y_n$ — Bessel function of second kind (nth order)

$M_0 = (1 + h^2 + 2h \cos \Gamma_{2,1})^{1/2}$

$M_{1+} = [1 + h^2 + 2h \cos(\Phi_{2,1} + \Gamma_{2,1})]^{1/2}$

$M_{2+} = [1 + h^2 + 2h \cos(2\Phi_{2,1} + \Gamma_{2,1})]^{1/2}$

$M_{1-} = [1 + h^2 + 2h \cos(\Phi_{2,1} - \Gamma_{2,1})]^{1/2}$

$M_{2-} = [1 + h^2 + 2h \cos(2\Phi_{2,1} - \Gamma_{2,1})]^{1/2}$

## V. COMPARISON OF INDIVIDUAL SYSTEMS

### A. Narrow-Aperture Radio Direction Finding Systems

#### 1. Systems Retaining the Amplitude Information in the Arriving Waves

Systems incorporating the Adcock array are subdivided for purposes of comparison into the following classes:

a. Adcock array with an automatic bearing indicator (ABI).
b. Adcock array with Watson-Watt type indicator.
c. Adcock array with a matched-line type of indicator (MLI).

It should be noted that the hand-rotated goniometer type may be considered as of the first class above while the null-seeking type such as the SCR-269 may be considered as of the third class.

The bearing error for the first two classes above may be obtained by letting n = 1 in the following general expression for the bearing error of harmonic antenna array [4]:

$$\varepsilon = \frac{\Gamma_{n+} + \Gamma_{n-}}{2n} \qquad 5.1$$

The solid-line curves of Plate I show graphically the bearing error for the first two classes of systems as a function of azimuthal separation between a desired and an undesired signal of the same frequency for selected values of h and $\Gamma$. It is evident that the maximum bearing error is $\pm 90^0$ for the Adcock systems.

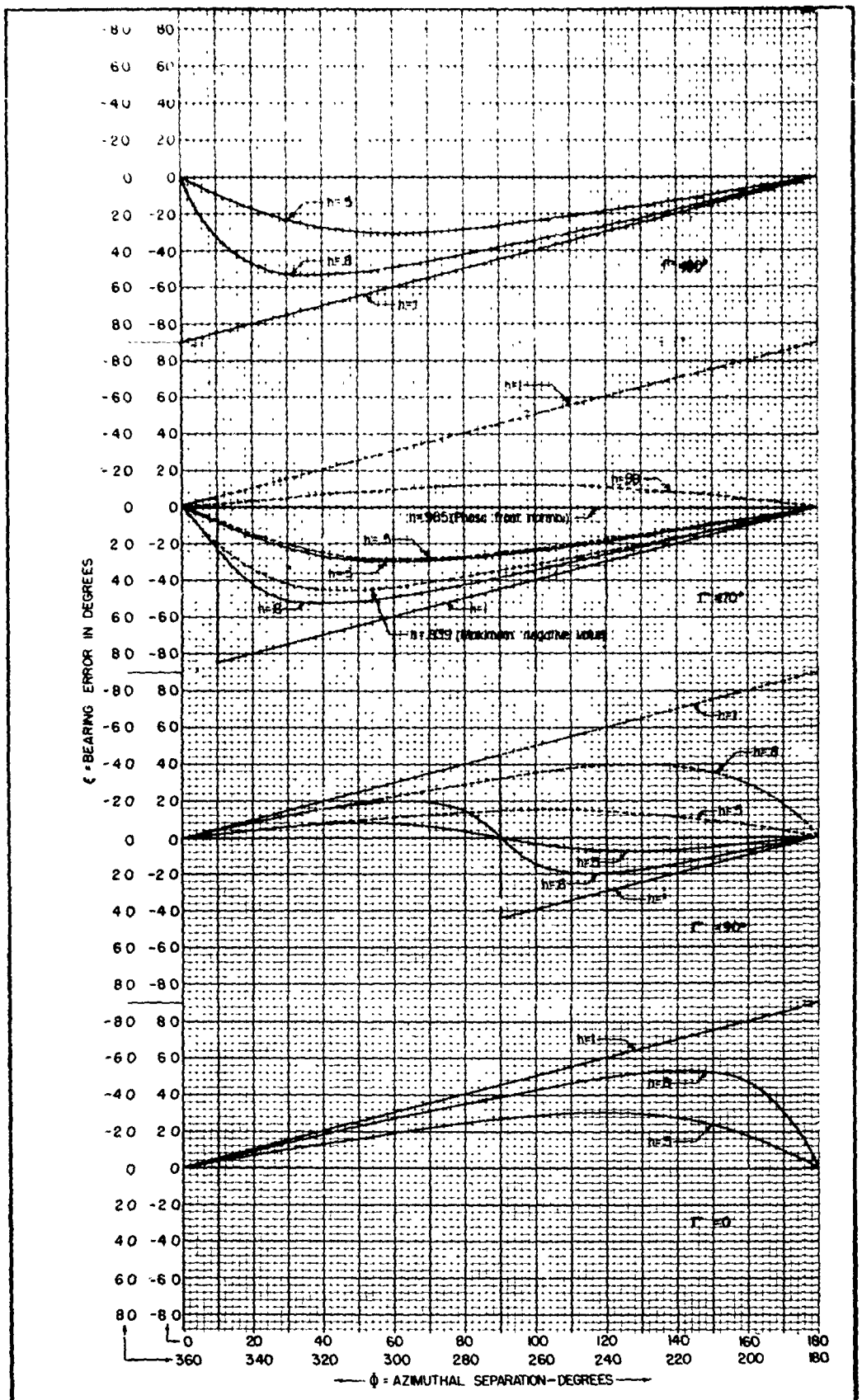
The broken-line curves of Plate I are termed Phase-Front Normal Bearing Error Curves. A word of explanation is probably in order here. Recalling that the comparisons of this report are based upon rdf performance in a wave interference field and that all interference fields have some sort of a standing wave field which is uniquely determined by the values of h, $\Phi$, and $\Gamma$ corresponding to any point, then a direction finder which determines the normal to the equiphase front at any point will in all such cases be subject to the phase-front normal bearing error. The application of these latter curves will be discussed below.

A second basis for the comparison of Adcock systems is the collection of photographs comprising Plates II to VI. The photographs of simulated radio direction finding system operation were obtained from the Antenna Simulator. Each plate is reasonably self-explanatory; however, it should be particularly noted that h = 0.6 for all plates, $\Gamma_{2,1}$ assumes values of 0, 30, 60, 90, 120, 150, and 180 degrees respectively for each plate, and the several plates differ as to the value of $\Phi_{2,1}$ – values of 10, 30, 90, 135, 170 degrees being used. The direction of arrival of the stronger (desired) signal is from above in all cases. The direction of arrival of the interfering signal bears to the left in each case by the particular magnitude of $\Phi_{2,1}$.

In order to evaluate the merit of an indicated bearing, a device termed the Bearing Quality Indicator (BQI) has been developed. The visual output of the device appears on a separate oscilloscope as several vertical-line traces. The relative lengths of the lines give an

---

4. The general expression for the bearing error of a harmonic antenna array was developed as equation 8.10 in Technical Report No. 7 entitled "An Analysis of Radio Direction Finding Systems-Part I". Equation 5.1 above has been written to agree with the symbols as defined earlier in this report.




ADCOCK AND PHASE FRONT NORMAL (DOTTED)
RDF BEARING ERROR

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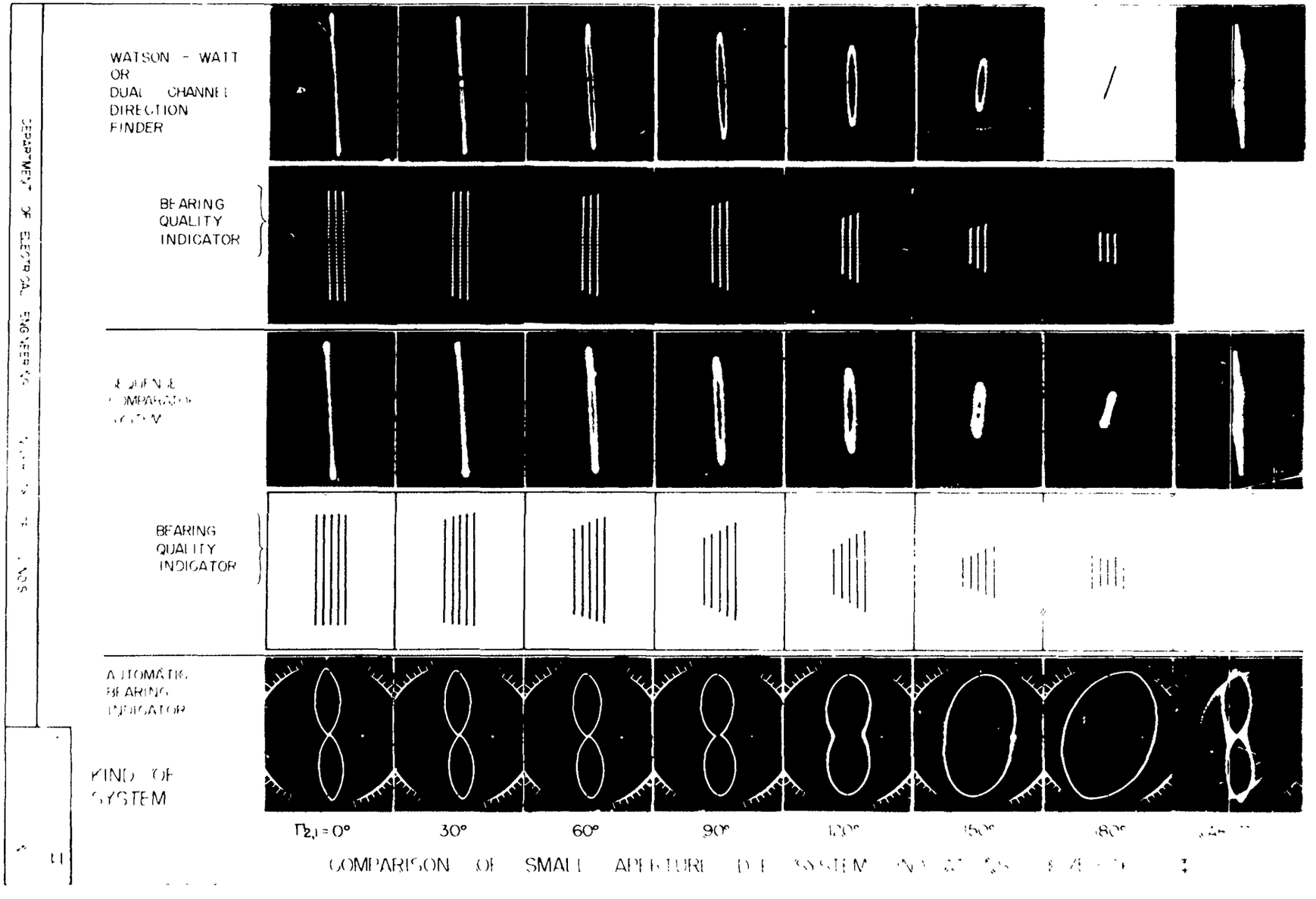
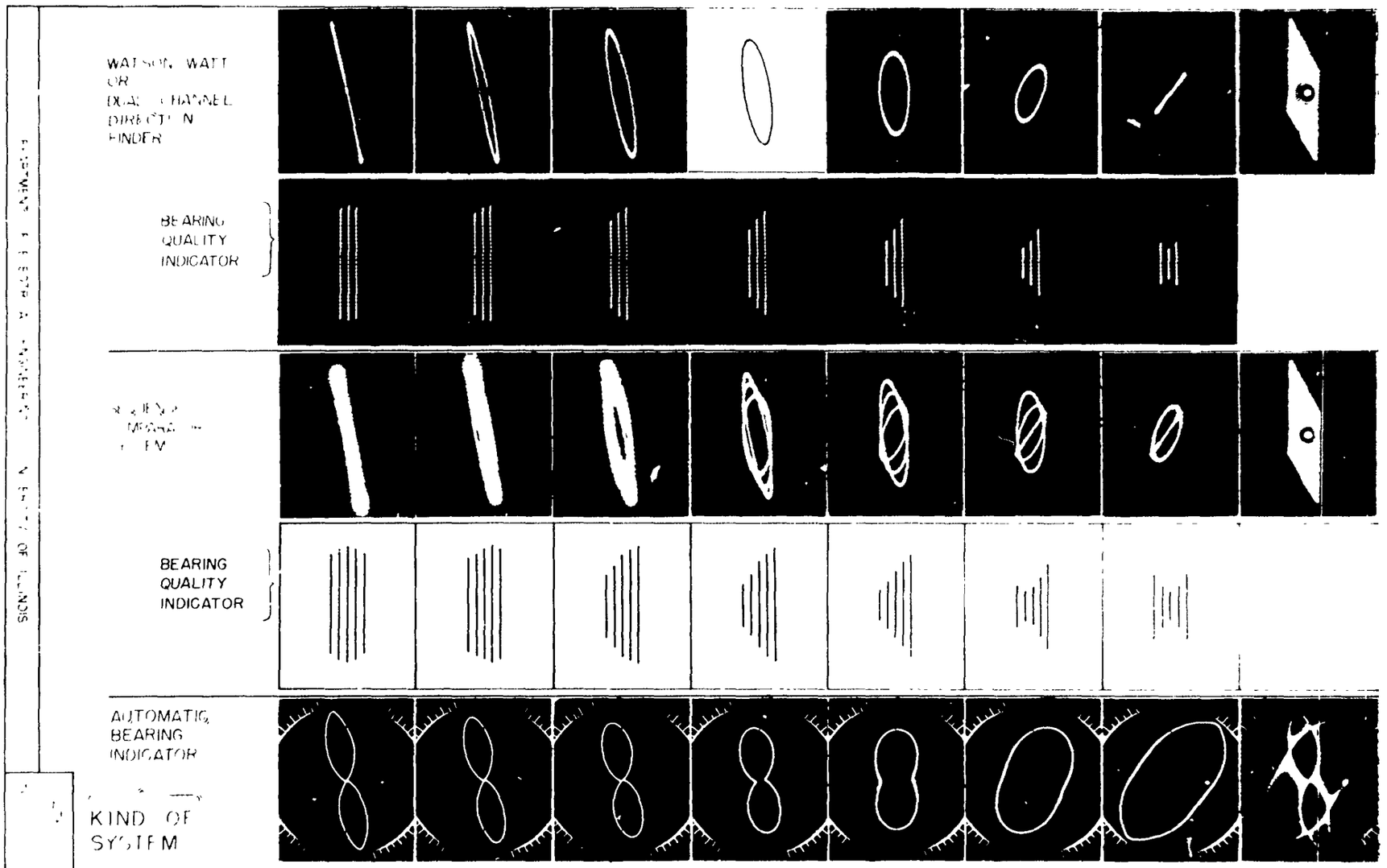
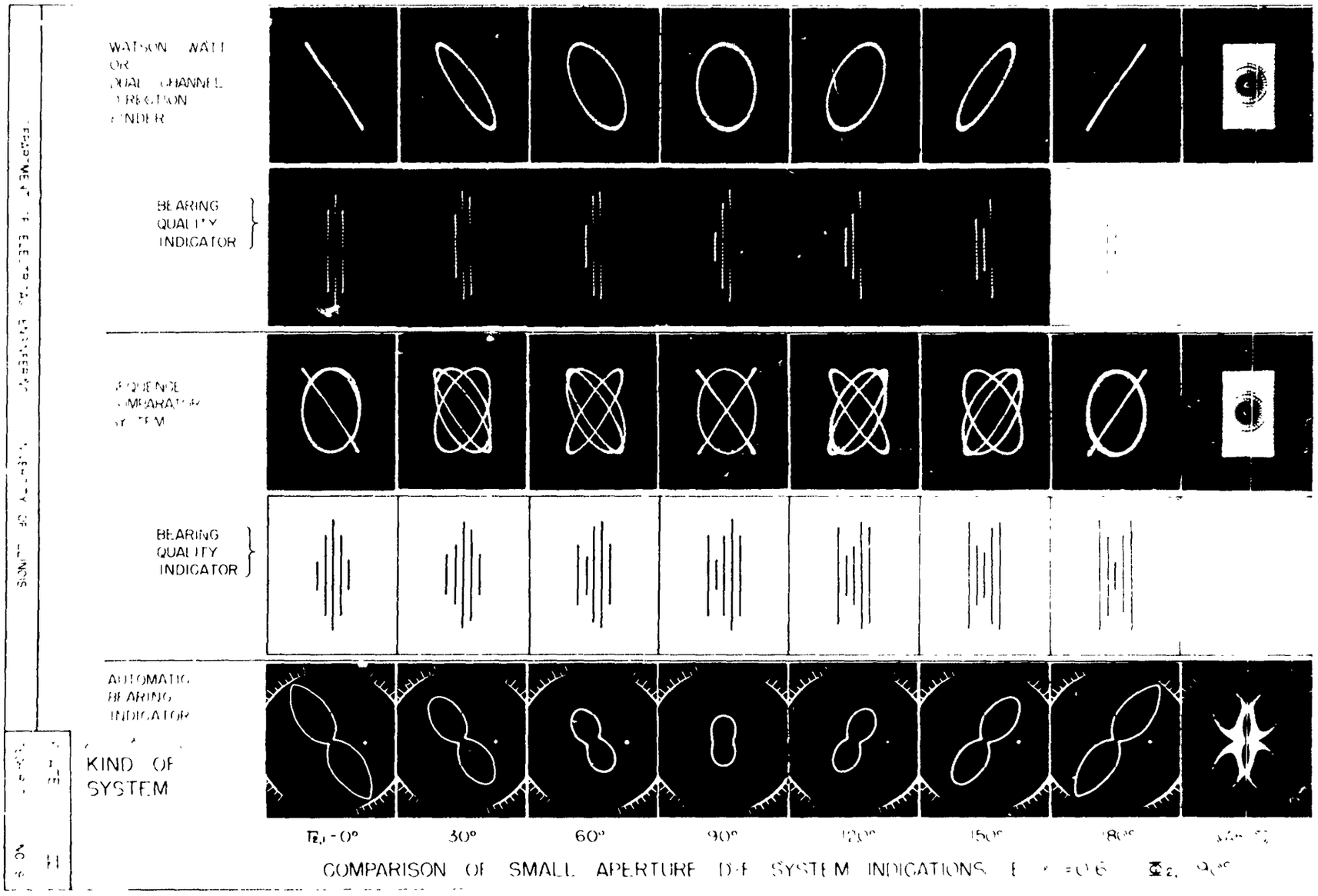
PLATE I
TECH RPT NO 9



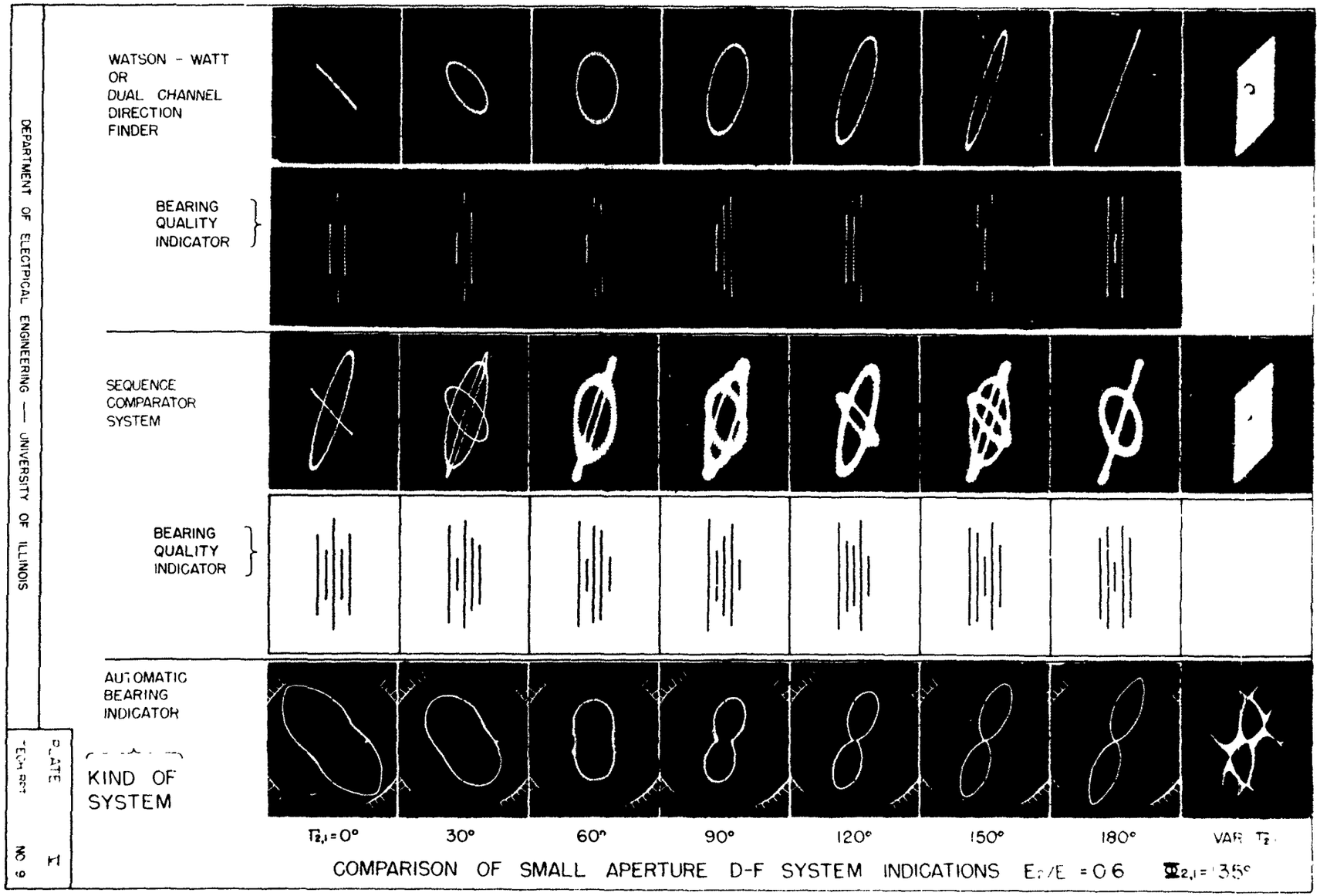

COMPARISON OF SMALL APERTURE DF SYSTEM

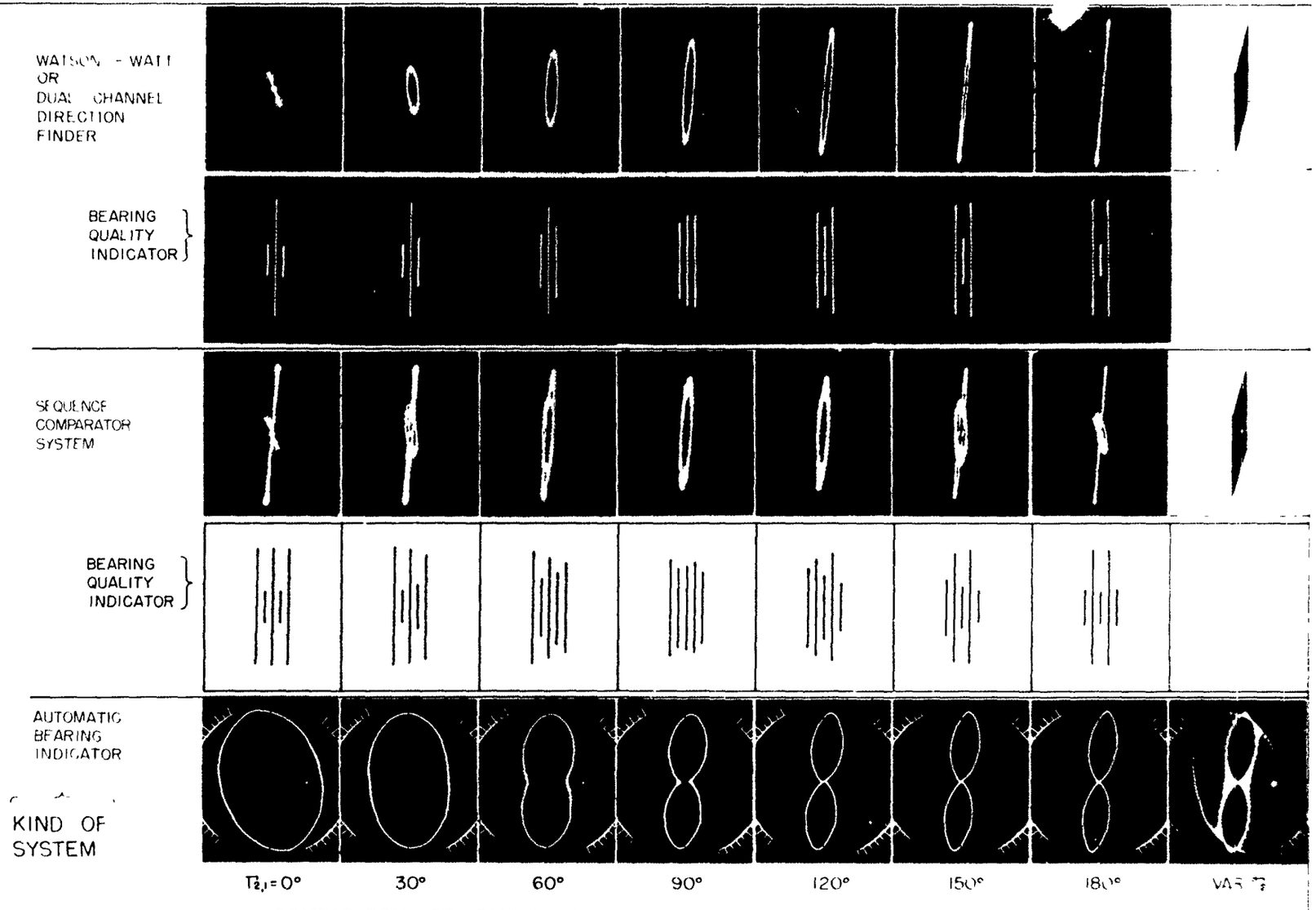



COMPARISON OF SMALL APERTURE D-F SYSTEM INDICATIONS




COMPARISON OF SMALL APERTURE D-F SYSTEM INDICATIONS


DEPARTMENT OF ELECTRICAL ENGINEERING — UNIVERSITY OF ILLINOIS
PLATE
TECH REP
NO 9
KIND OF SYSTEM
WATSON - WATT OR DUAL CHANNEL DIRECTION FINDER
BEARING QUALITY INDICATOR
SEQUENCE COMPARATOR SYSTEM
BEARING QUALITY INDICATOR
AUTOMATIC BEARING INDICATOR
$\Psi_{2,1}$ = 0°
30°
60°
90°
120°
150°
180°
VAR $\Psi_2$
COMPARISON OF SMALL APERTURE D-F SYSTEM INDICATIONS $E_2/E$ = 0.6 $\Psi_{2,1}$ = 135°




COMPARISON OF SMALL APERTURE D-F SYSTEM INDICATIONS E/E =0.6 $\Phi_{2,1}$=170°

DEPARTMENT OF ELECTRICAL ENGINEERING — UNIVERSITY OF ILLINOIS

PLATE II

TECH REP NO 9

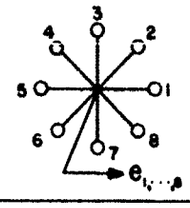
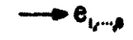
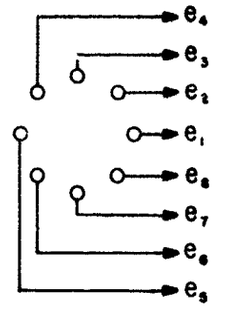
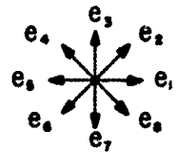
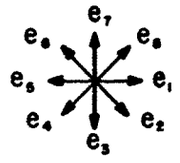
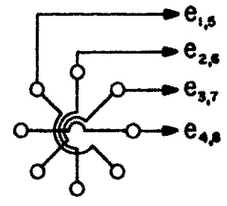
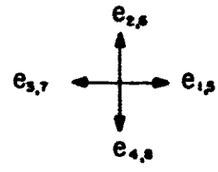
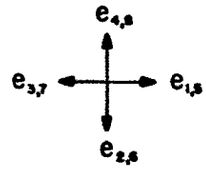
indication of the quality of the indicated bearing on the rdf system with which the BQI is being used. For purposes of comparison, the output of the BQI when used in conjunction with a simulated Watson-Watt system is shown as the second row of photographs on each plate. The rules for the interpretation of the BQI patterns are as follows: When the middle line is longer than the two outside lines, the bearing indication is between the two incoming signals. When the lines increase or decrease in sequence, the bearing is near the direction of the stronger signal. When the center line is shorter than the two outside lines, the bearing lies outside of the direction of arrival of the two signals. If the lines are very nearly equal, the bearing is very close to the bearing of the stronger signal. If the lines vary greatly in magnitude, then the bearing indication varies over wide limits and should not be used. The technical details of the BQI will be covered in Part III of the series of reports on "An Analysis of Radio Direction Finding Systems". It should be stated that the lengths of the lines are proportional to the magnitudes of the quantities $M_{1+}$, $M_0$, and $M_{1-}$.

The third row of photographs appearing on each plate show the simulated performance of the Sequence Comparator, so termed because its principle of operation is to resolve the received complex signal into components which are analogous to the components one obtains in the symmetrical component method of unbalanced power system analysis. The Sequence Comparator rdf system (SCRDF) uses several bearing indications simultaneously on one cathode ray tube. The correct directions of arrival of the two waves are given by the sides of the parallelogram which encloses the composite figure. The bearing information is derived from one antenna array. One possible system is illustrated in Plates VII and VIII, using 8 antennas, but other numbers could have been chosen. This system could be set up to give unilateral bearing in place of the bilateral indication shown. The system shown when used in conjunction with a bearing quality indicator would reduce the bearing error shown by a factor of two or three and in many cases would give complete resolution of the direction of arrival of the individual signals. Since the amount of improvement would partially be a function of the operator, it is difficult to say exactly how great the improvement would be.

The fourth row of photographs on each plate shows the simulated performance of a BQI used in conjunction with the SCRDF system. It is seen by inspection that the latter is a valuable adjunct for interpreting bearings when the angular separation is small. However the type of indication that would be obtained when more than one interfering wave is present is not known.

The fifth row of photographs on each plate shows the simulated performance of an ABI type of rdf system under the same assumed conditions.

A further interesting feature of the several systems is shown in the last column of photographs on each plate. This case entitled VAR $\Gamma_{2,1}$ shows the simulated performance of each system when two signals of slightly different frequency are received. One may readily note that it is now possible by the Watson-Watt method to resolve the individual directions of arrival of the two signals. The same thing can be done with the ABI indicator if the observer takes the direction of arrival to be indicated by the cross-over points of beat frequency interference. In the case of three signals, the ABI method breaks down; however, the

| DESCRIPTION OF CONNECTIONS | FORM OF CONNECTIONS * | DESCRIPTION OF PHASE SHIFTS IMPOSED ON OUTPUTS | REPRESENTING IMPOSED PHASE SHIFTS ** | SYMBOL FOR THE *SUM* OF PHASE-SHIFTED VOLTAGES |
|---|---|---|---|---|
| ANTENNAS CONNECTED IN PARALLEL |   | NO PHASING IMPOSED |   | $e_0$ |
| ANTENNAS FED SEPARATELY INTO SEPARATE AMPLIFIERS |   | $e_1$ : SHIFT 0° <br> $e_2$ : " 45° <br> $e_3$ : " 90° <br> etc. |   | $e_{1+}$ |
| | | $e_1$ : SHIFT 0° <br> $e_2$ : " -45° <br> $e_3$ : " -90° <br> etc. |   | $e_{1-}$ |
| OPPOSED ELEMENTS CONNECTED IN PARALLEL |   | $e_{1,5}$ : SHIFT 0° <br> $e_{2,6}$ : " 90° <br> etc. |   | $e_{2+}$ |
| | | $e_{1,5}$ : SHIFT 0° <br> $e_{2,6}$ : " -90° <br> etc. |   | $e_{2-}$ |

*NOTES:*

* *"Parallel connections" implies that isolation stages will be employed between antenna terminals and parallel-connection points, in order to permit one array to furnish all outputs.*

** *Observe that these vector diagrams represent only the phase shifting to be imposed by circuit stages.*

COMBINATIONS OF ANTENNA ELEMENT CONNECTIONS AND ANTENNA OUTPUT PHASE SHIFTS REQUIRED IN A *SEQUENCE - COMPARATOR SYSTEM* USING AN EIGHT ELEMENT CIRCULAR ARRAY

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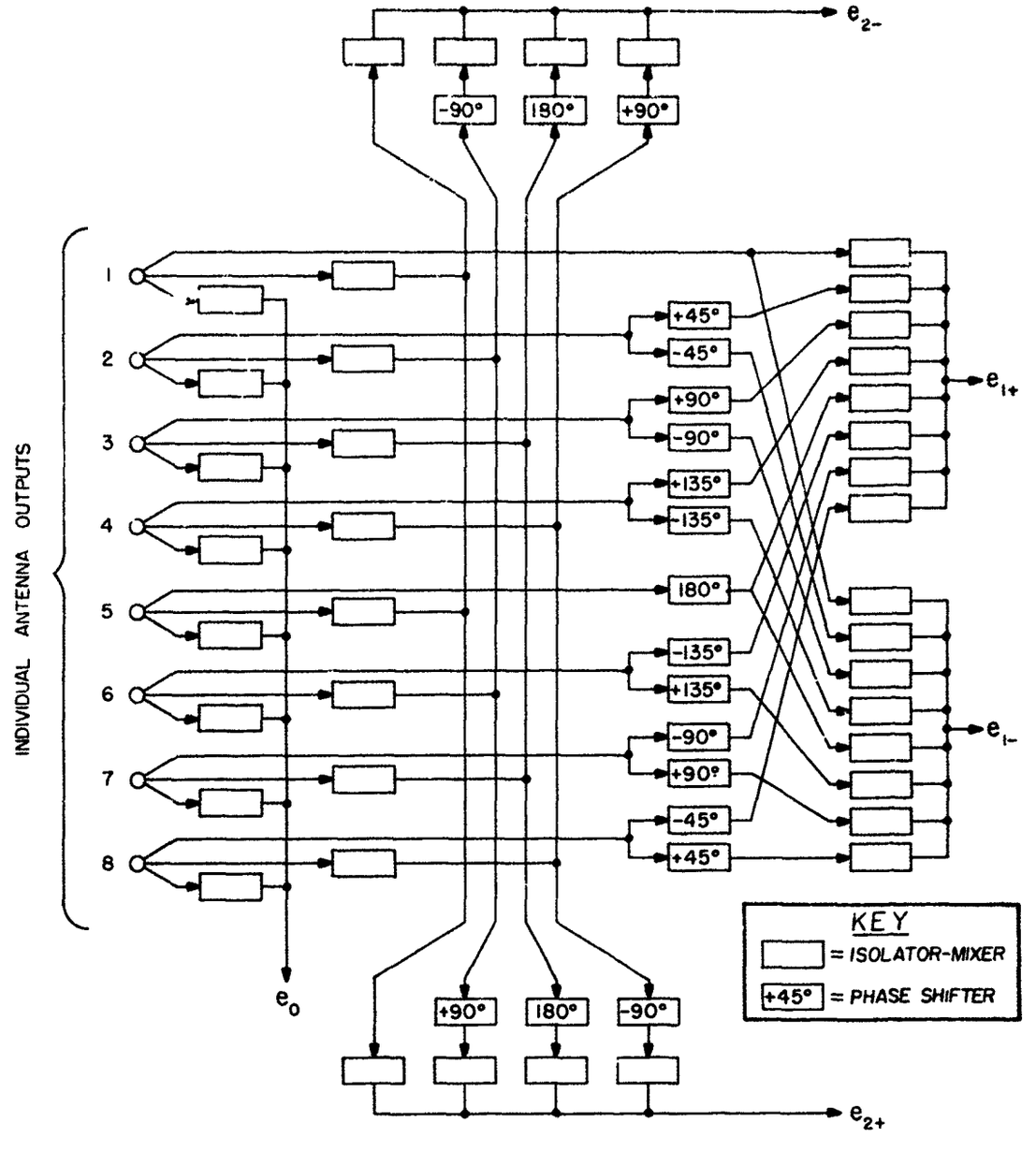
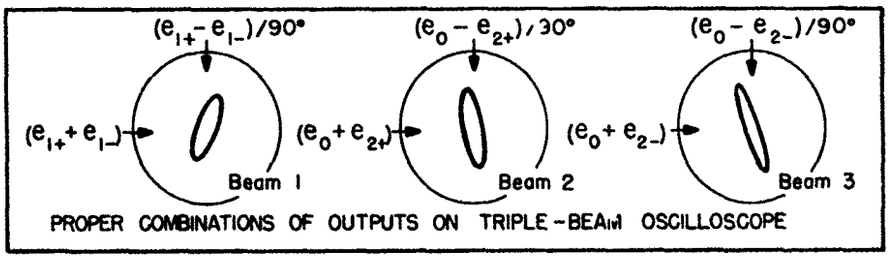
PLATE VII

TECH. RPT. NO.9

# FUNCTIONAL CIRCUIT OF THE SEQUENTIAL PHASING SYSTEM SHOWING THE REQUIRED VOLTAGE-MIXER AND VOLTAGE-PHASE-SHIFTING OPERATIONS




PROPER COMBINATIONS OF OUTPUTS ON TRIPLE-BEAM OSCILLOSCOPE

PLATE VIII

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TECH. RPT NO. 9

Watson-Watt is still able to resolve the individual directions of arrival. The true direction of arrival of each signal in the case of the Watson-Watt is indicated by the sides of the parallelogram.

The matched-line indicator (MLI) has its calculated bearing error compared with the ABI bearing error in Plate IX. The bearing error of the MLI is very nearly equal to the bearing error of the ABI when small values of blur are encountered. For large values of blur the bearing deviates considerably from the ABI bearing. Also, for high values of blur the phase shift between the sense voltage and the goniometer voltage varies the bearing drastically. This effect is an instrumental error and can be used to reduce the bearing error in the case of reradiated signals due to local field obstructions. But in the case of propagation interference, this may have the opposite effect and, hence, give a rather unpredictable performance. This system is particularly noted for its ability to give a very accurate setting of the indicator dial but this does not mean low bearing error. It should be noted that the primary reason for the difference between the bearing error of the MLI system and the ABI system is that the former includes the effect of an added sense antenna along with the Adcock array which may be common to the two systems.

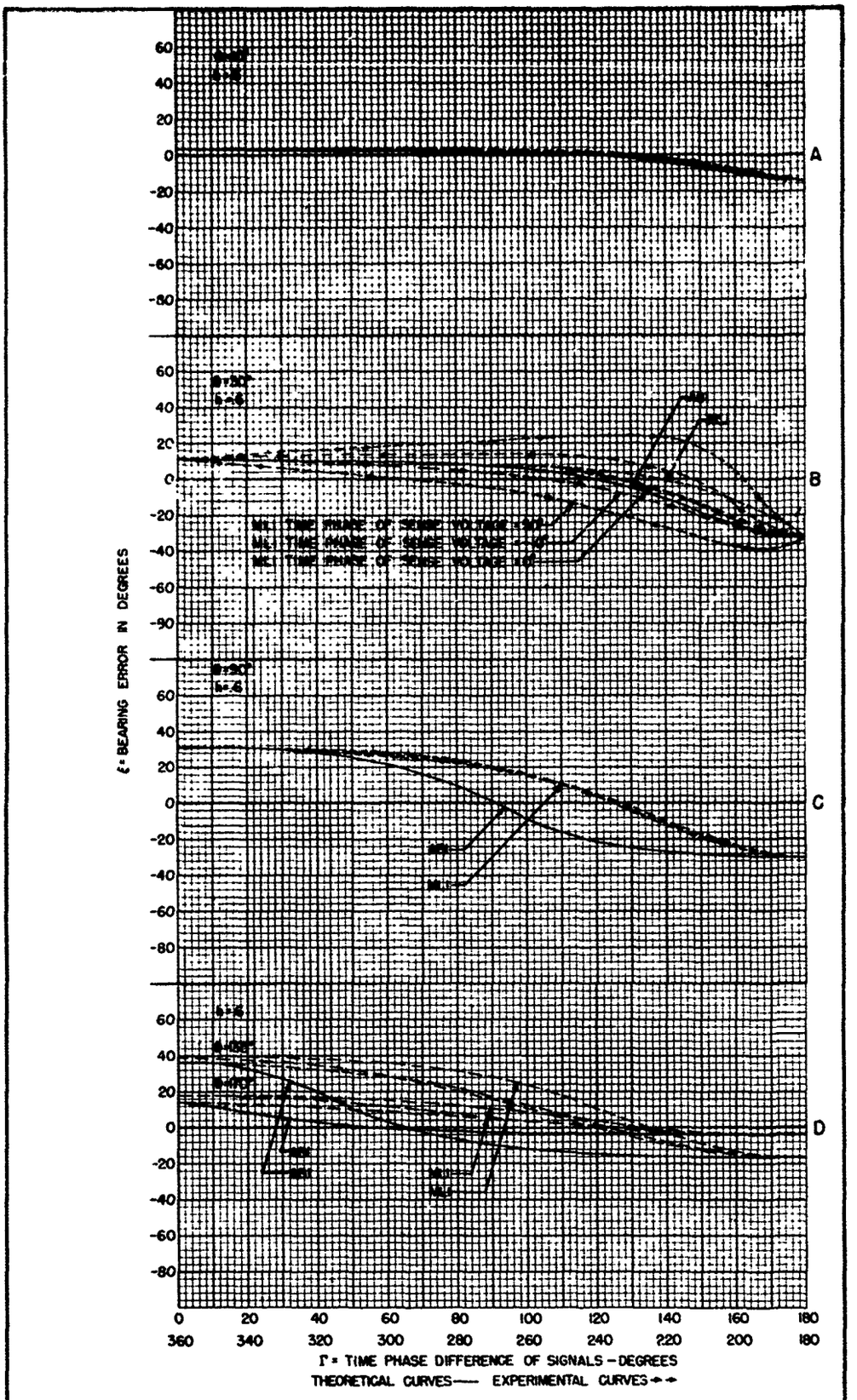
The vertical spaced-loop radio direction finding system consists of two parallel vertical loops coaxially oriented and separated by a distance considerably less than a wavelength. The electrical output of the system is obtained from the difference of the electrical outputs of each of the two component loops. The particular merit of this system is its reliable performance under conditions of polarization which void the bearing indication of a conventional loop or Adcock system. However, the considerably reduced effective height of this system from that of a loop or Adcock and the increased number of ambiguities are disadvantages.

The bearing error for the spaced-loop radio direction finder may be calculated from equation (5.1) above by letting $n = 2$. Plate X shows the bearing error of such a system as a function of azimuthal separation between a desired and an undesired signal for selected values of h and $\Gamma$. The maximum bearing error in this system is one-half that of a conventional loop or Adcock system subject, of course, to the disadvantages mentioned above.

2. Systems with Amplitude Limiting

It is possible to build many small diameter direction finding systems which tend to give a bearing in the direction of the normal to the curve of constant phase through their center. Such systems would be phase comparison systems using only the phase information from the voltages induced in the antennas and, hence, would incorporate amplitude limiting of the antenna signal voltages before any combination of these voltages took place. A direction finder, whose indicated direction as the system diameter approaches zero is in the direction of the normal to the phase front, will be called a phase-front direction finding system. The vector-phase systems (defined in V B (2)) are of this type.

A particular type of phase-front system (also a vector-phase system) which seems well suited as a small-diameter system is shown in Plate XI. The phase of the induced voltages in diagonally opposite




MATCHED LINE AND AUTOMATIC BEARING INDICATOR BEARING ERROR

| | |
|---|---|
| PLATE | IX |
| TECH. RPT. | NO. 9 |

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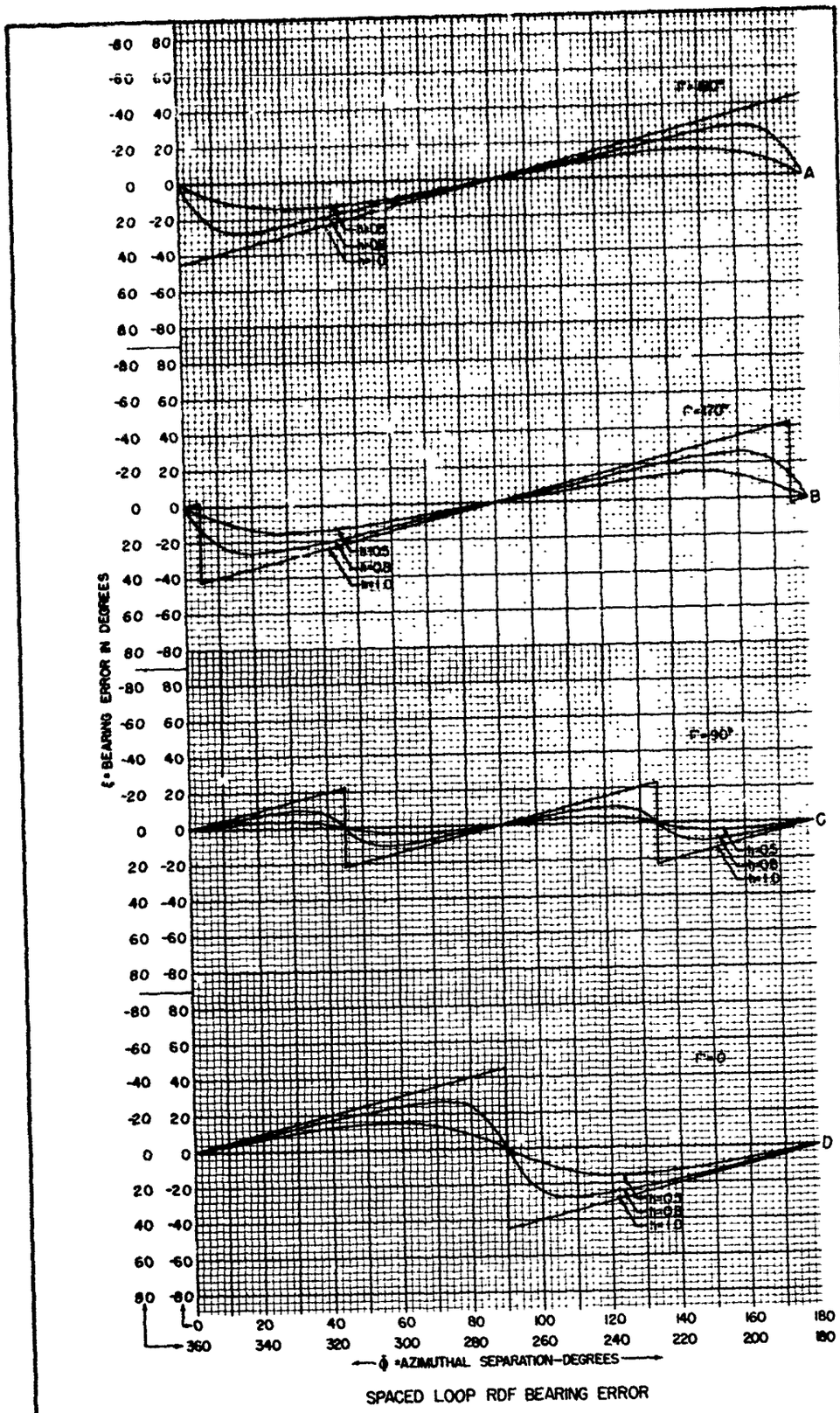
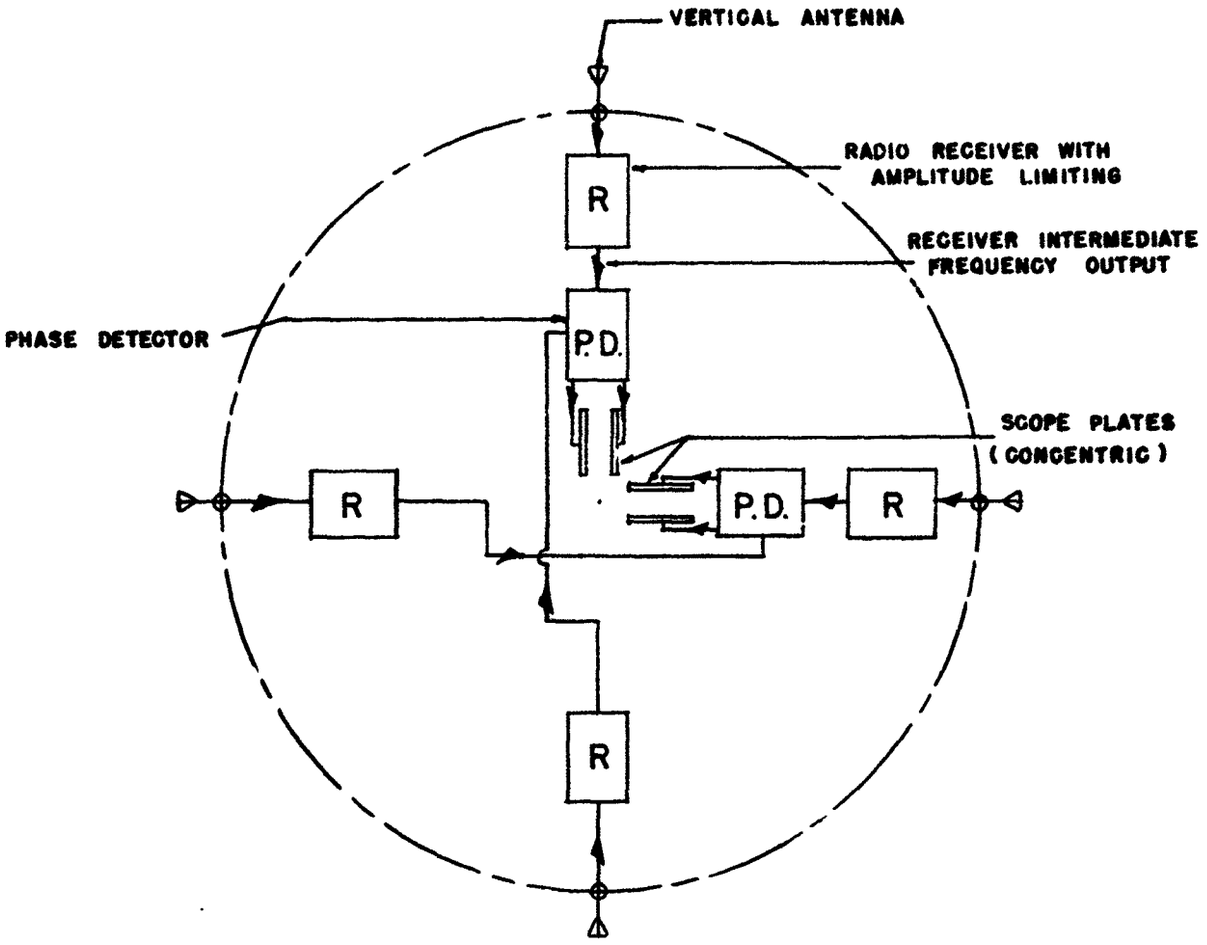



SPACED LOOP RDF BEARING ERROR

| | |
|---|---|
| PLATE | X |
| TECH RPT | NO 9 |

DEPARTMENT OF ELECTRICAL ENGINEERING — UNIVERSITY OF ILLINOIS




4 ELEMENT PHASE COMPARISON SYSTEM OF SMALL DIAMETER ( IN WAVELENGTHS )

PLATE XI

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TECH. RPT. NO.9

antennas is compared in a phase detector connected to these antennas by receivers incorporating amplitude limiting. The two dc output voltages from the two phase detectors are applied to the two sets of orthogonal deflection plates of a conventional oscilloscope to give an indicated bearing. The dot presentation could be changed to a line with suitable circuits (the bearing angle would be unchanged). For phase-front systems of small diameter (under $\lambda/4$) there is little to be gained in the way of error suppression by a more complicated phase-front system (such as a Doppler or 8-element vector-phase system).

A comparison of the error curve of an actual system as described in the previous paragraph (dotted curve of Plate XII) to the deviation of the phase front normal from the direction of arrival of the stronger signal (solid curve of Plate XII) shows the close agreement for systems with diameters less than $\lambda/16$. Because of the similarity of these curves and other similar curves, it is felt that the curves showing deviation of the phase front normal in Plate I give a very good indication of the performance of all phase-front systems with small diameters. The dotted curves are for the Phase-Front Normal Deviation, while the solid curves show Adcock Error. In the upper and lower graphs only solid curves are shown as the Adcock bearing error and phase-front normal are coincident here.

The formula for the normal to the phase front (from which the curves of Plate I are plotted) can be expressed as:

$$\Phi_I = \arctan \left( \frac{h(h + \cos \Gamma_{2,1}) \sin \Phi_{2,1}}{1 + h \cos \Gamma_{2,1} + h(h + \cos \Gamma_{2,1}) \cos \Phi_{2,1}} \right)$$

## B. Wide-Aperture Systems

### 1. Doppler-Effect Radio Direction Finding System

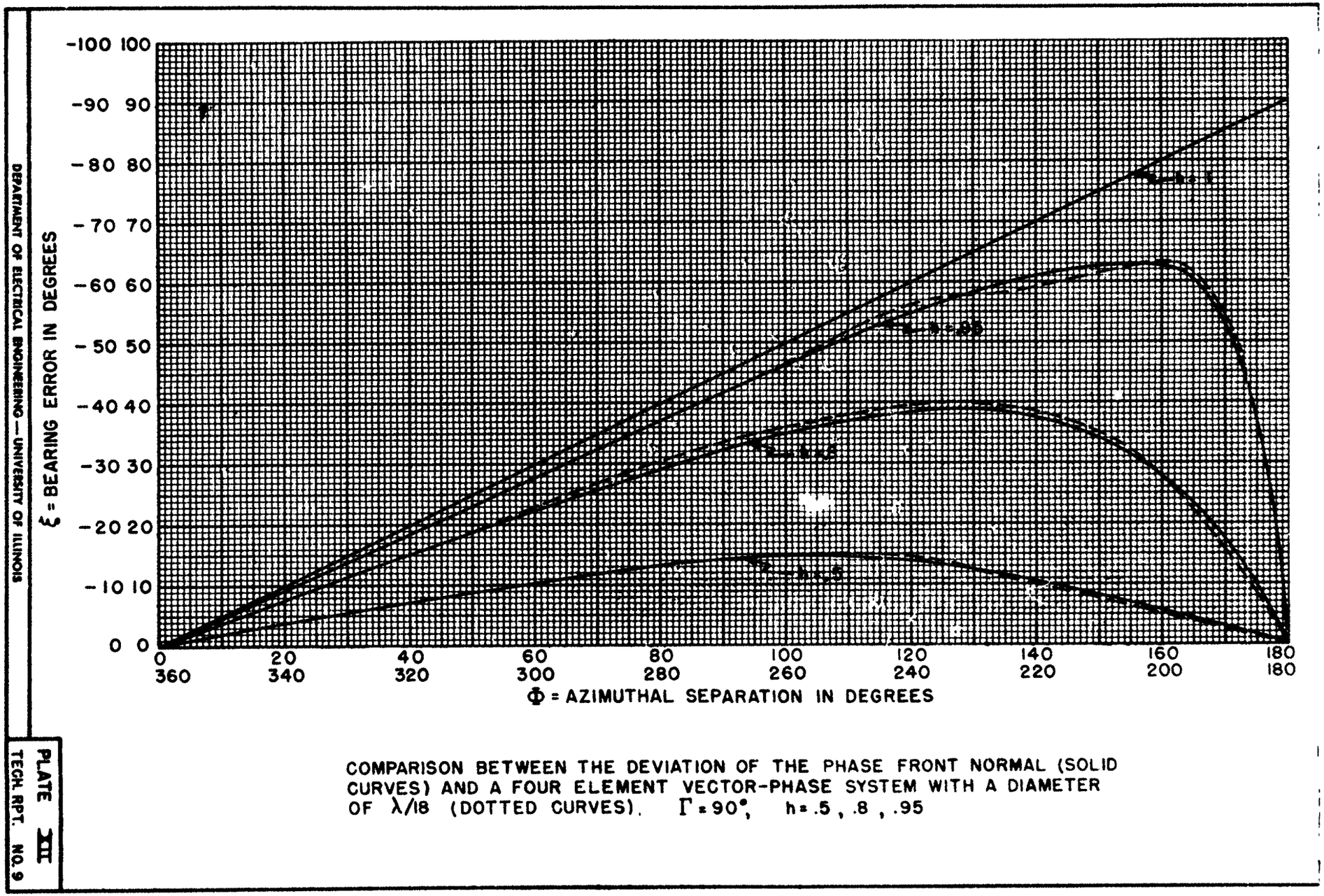
The theory of a doppler-effect radio direction finding system was developed in technical report No. 8. Practical systems would use a fixed antenna array with abrupt or gradual switching between elements. A laboratory model system using a linear coupling law will be described in a later report.

The manner in which the switching is accomplished may take many forms, but the following three are especially important.

a. A single electronic contact may be used with a frequency discriminator at the output of the receiver. See Plate XIII - Fig. 1. The detected voltage in this case is a series of pulses.

b. A single rotating switch may be used, with the phase of the radio frequency output of the receiver compared to the phase of an antenna at the center of the array by a phase detector. (See Plate XIII - Fig. 2.) Whenever a phase detector is used, its output is generally a sinusoidal function of phase input and for this reason cannot handle signals of more phase deviation than $180^\circ$ with good results.

c. To limit the deviation of phase, the commutation may be changed to compare phase between adjacent antenna receivers. See the system schematic of Plate XIV - Fig. 1.




COMPARISON BETWEEN THE DEVIATION OF THE PHASE FRONT NORMAL (SOLID CURVES) AND A FOUR ELEMENT VECTOR-PHASE SYSTEM WITH A DIAMETER OF $\lambda/18$ (DOTTED CURVES). $\Gamma = 90°$, $h = .5, .8, .95$

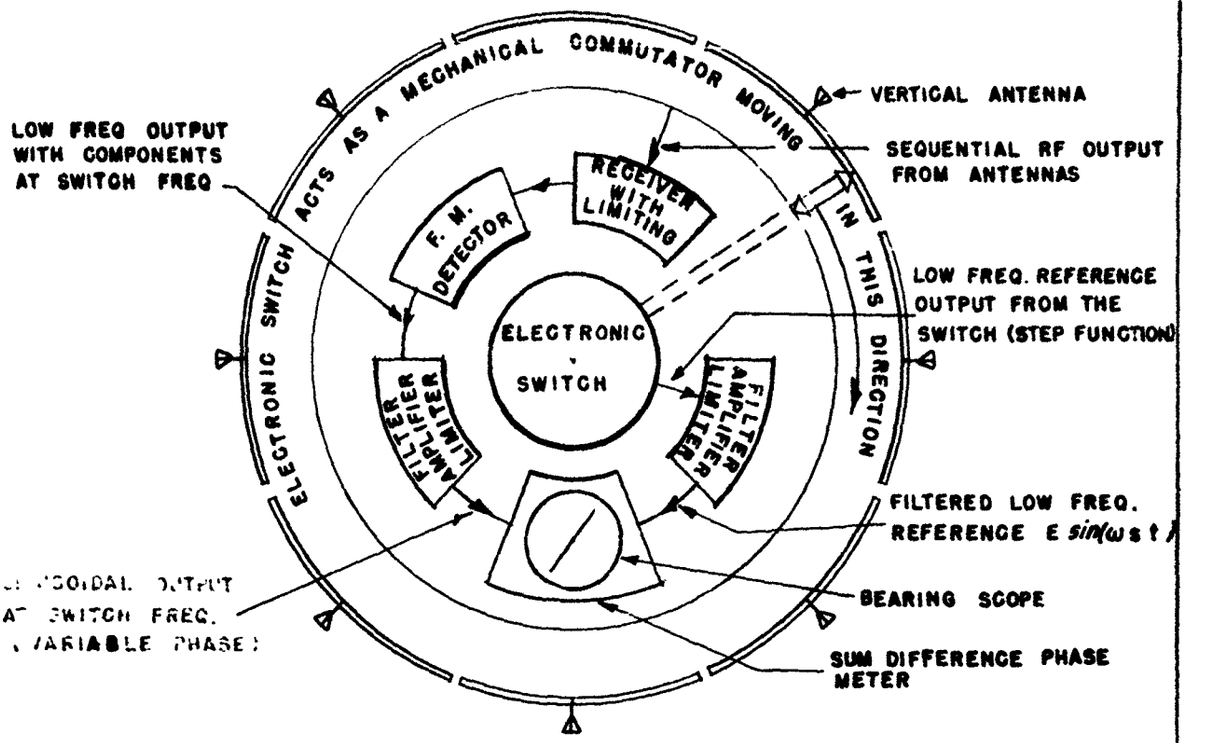



FIG 1

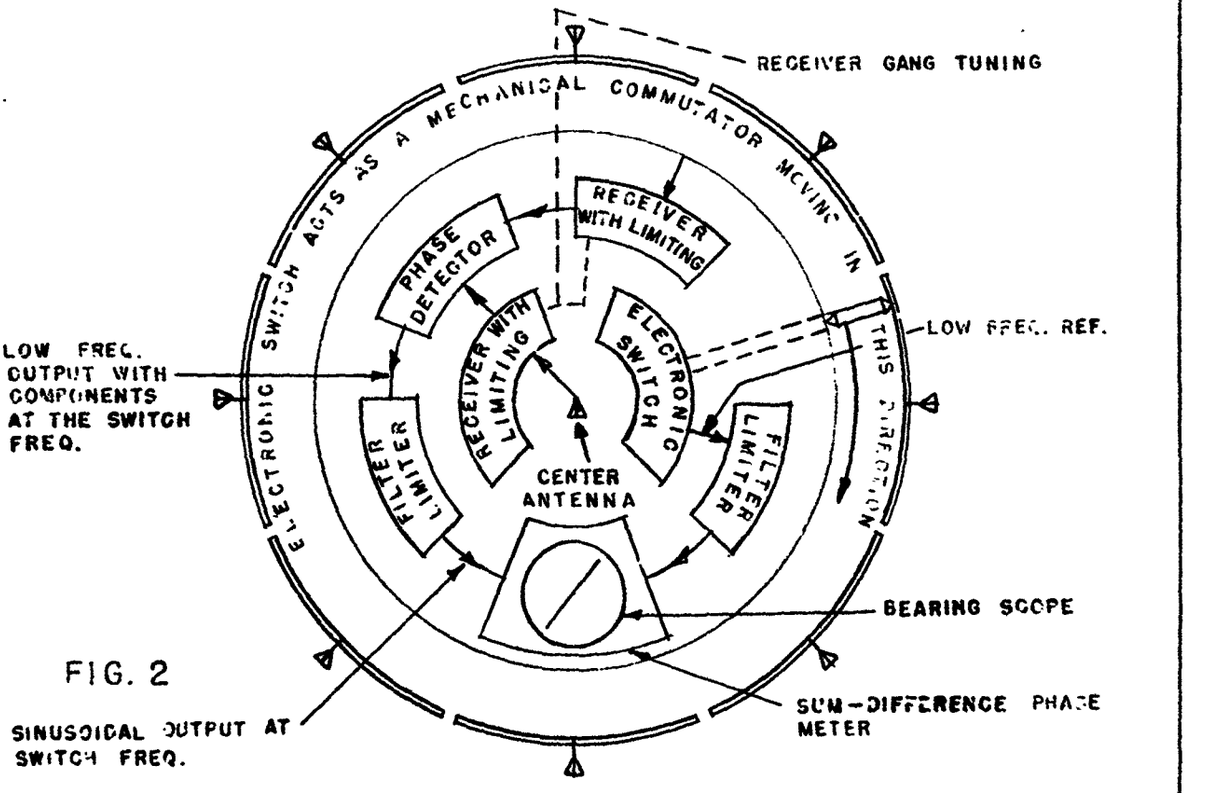



FIG. 2

DOPPLER RDF COMMUTATION SYSTEMS

PLATE XIII

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TECH. RPT. NO. 9

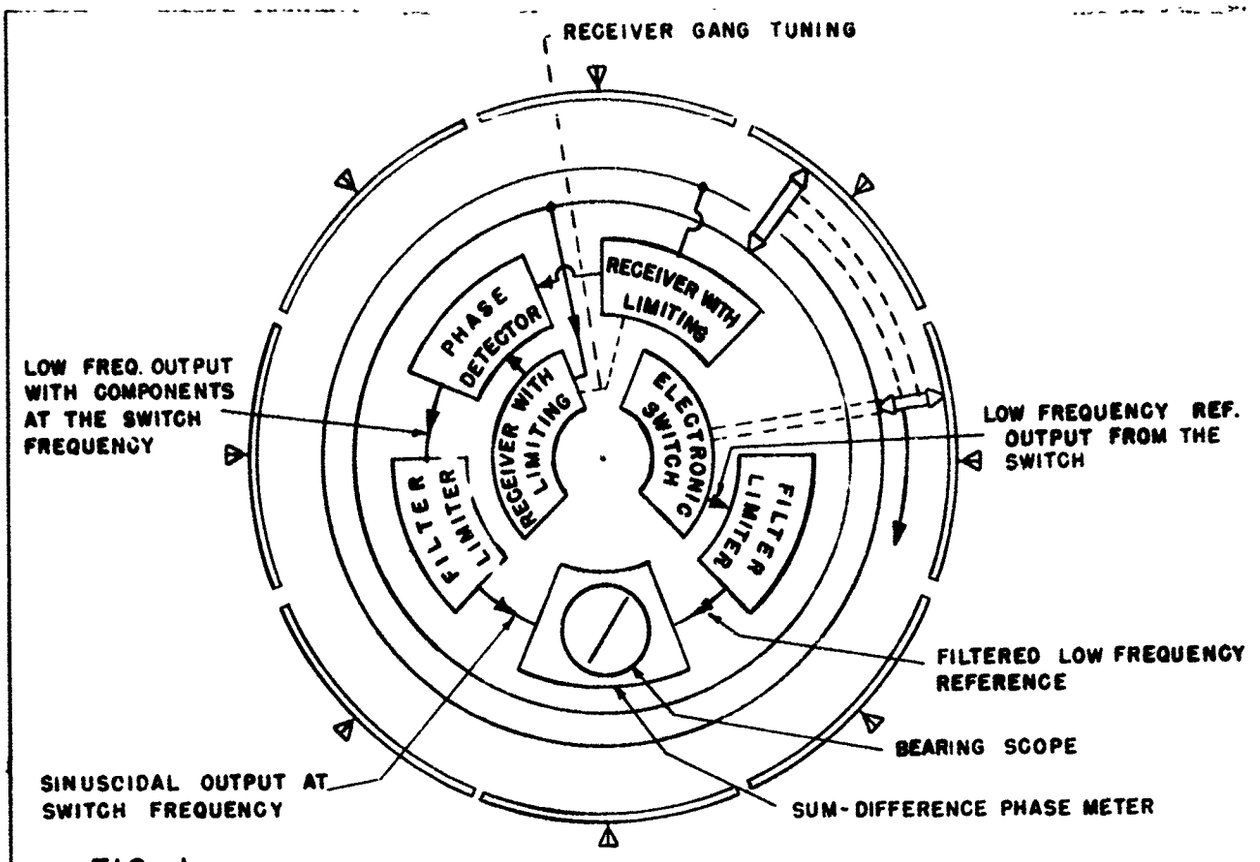


FIG. 1

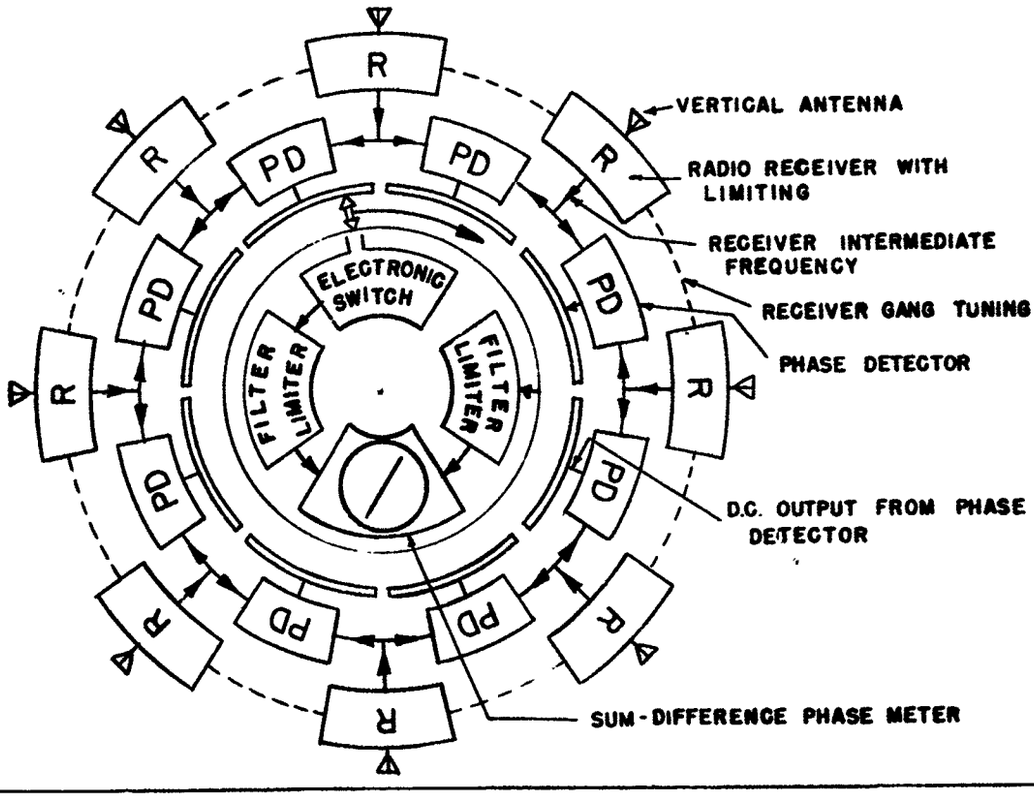


FIG. 2

DOPPLER RDF COMMUTATION SYSTEMS

PLATE XIV

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TECH. RPT. NO. 9

The abrupt switching from antenna to antenna produces transients which are troublesome in the preceding systems. To alleviate this, some form of gradual coupling in and out of the antennas is required. A linear coupling law gives, to a close approximation, a sinusoidal phase variation which is the variation for a single rotating antenna.

If the phase differences between adjacent elements were individually and simultaneously detected, the switching could be shifted to the dc output of each phase detector (see Plate XIV - Fig. 2). The information may be stored by storage devices and later scanned for presentation, making this system essentially instantaneous.

The doppler bearing error is dependent upon the magnitude of a voltage at the switching frequency which adds in quadrature to the desired signal voltage and produces a phase shift and, hence, a bearing error. The magnitude of this quadrature voltage is expressed exactly as an infinite series of Bessel functions. However, for small values of relative magnitude (h), this series converges rapidly and, in this report, the magnitude of the quadrature voltage has been approximated by using only the first few terms. The details of obtaining the desired signal and quadrature voltages are explained in Technical Report No. 8.

The bearing error equation for theoretical Doppler systems (using a single rotating element or an infinite number of fixed elements) is:

$$\xi = - \arctan \left( \frac{D \cos G}{\beta \sin \theta_1 + D \sin G} \right),$$

where

$$D = \sum_{k=1}^{\infty} 2 \left( \frac{n \sin \theta_2}{\sin \theta_1} \right)^2 (-1)^{k+1} \cos k \Gamma_{2,1} + J_1(k g)$$

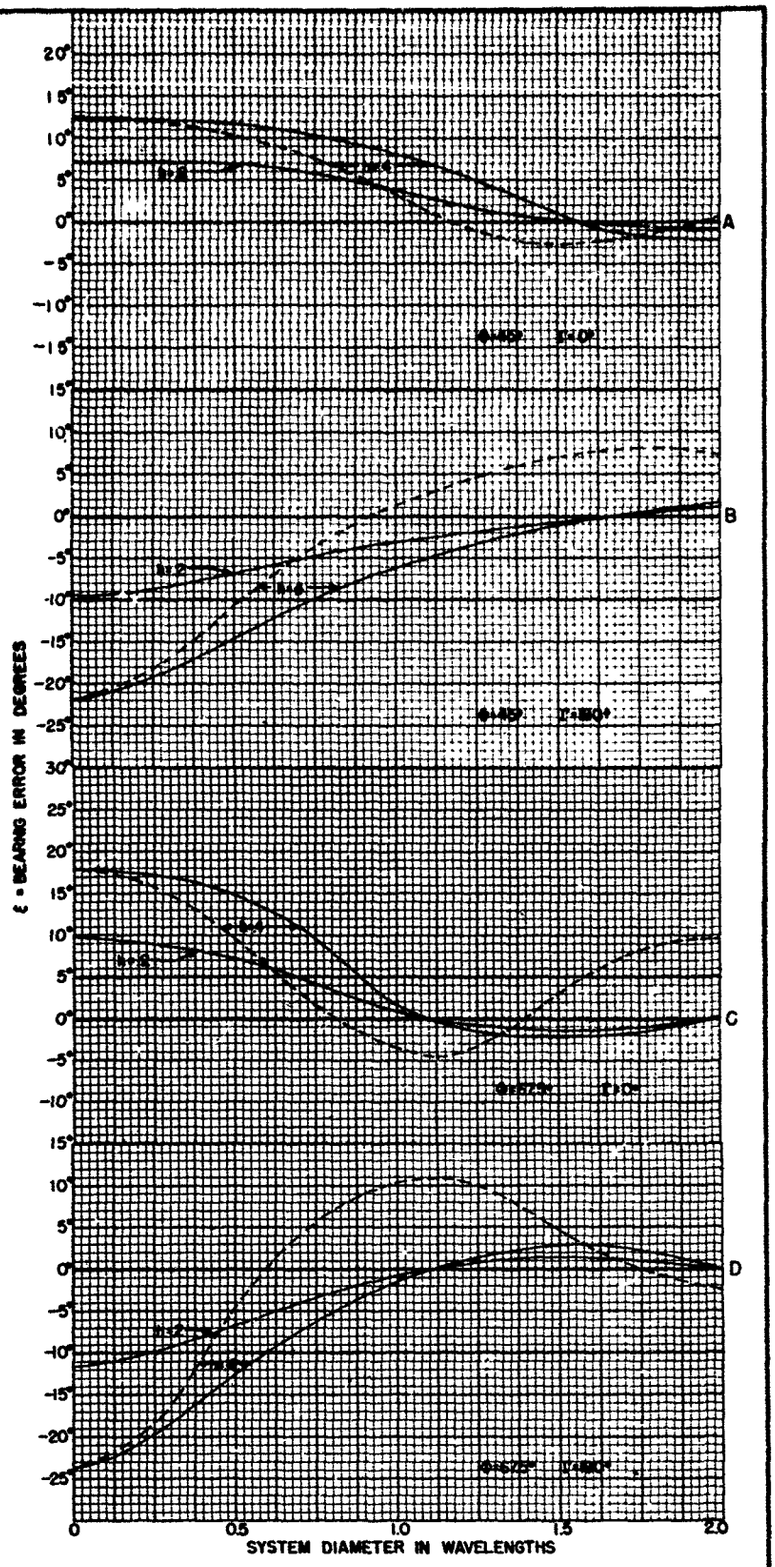
$$g = \beta (\sin^2 \theta_2 + \sin^2 \theta_1 - 2 \sin \theta_2 \theta_1 \cos \Phi_{2,1})^{1/2}$$

$$G = \arctan \left( \frac{\cos \Phi_{2,1} - \dfrac{\sin \theta_1}{\sin \theta_2}}{\sin \Phi_{2,1}} \right)$$

$$\beta = \frac{\pi D}{\lambda} .$$

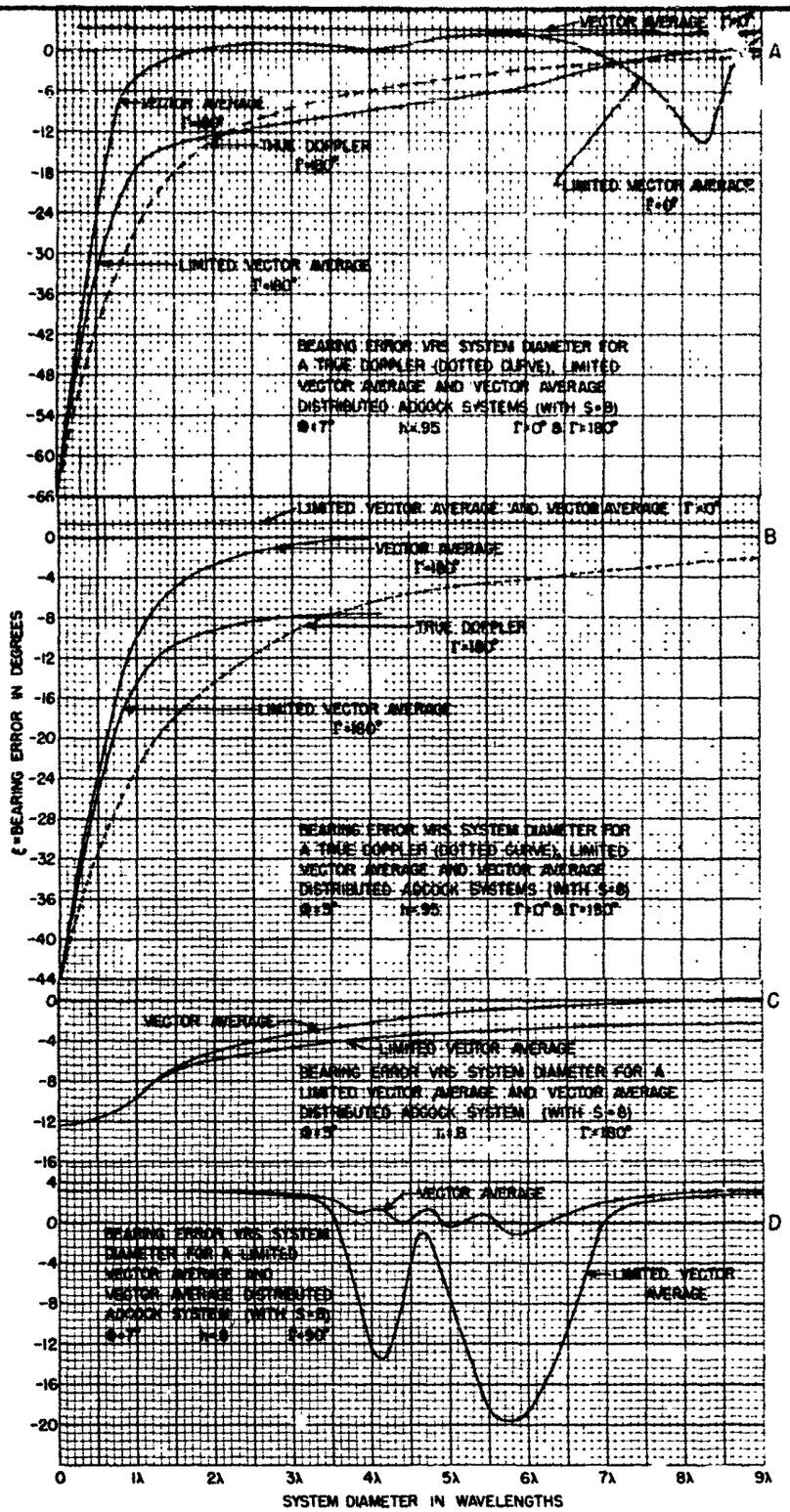
Plates XV and XVI show theoretical bearing error as a function of system diameter. On the same plates are shown for comparison the error vs diameter curves for distributed Adcock systems (see Section V,C) with the same field conditions. Plate XV shows two field conditions where the error is relatively small, while Plate XVI shows a common field condition with small angles of separation where large errors can occur.

Referring to Plates XV and XVI, it is seen that for a large angular separation between desired and undesired waves, the theoretical Doppler curves appear much better than those for the distributed narrow aperture systems. However, for interference effects between waves arriving from nearly the same direction (such as might be expected with multipath skywave transmission) the distributed system appears to give better results than the Doppler.



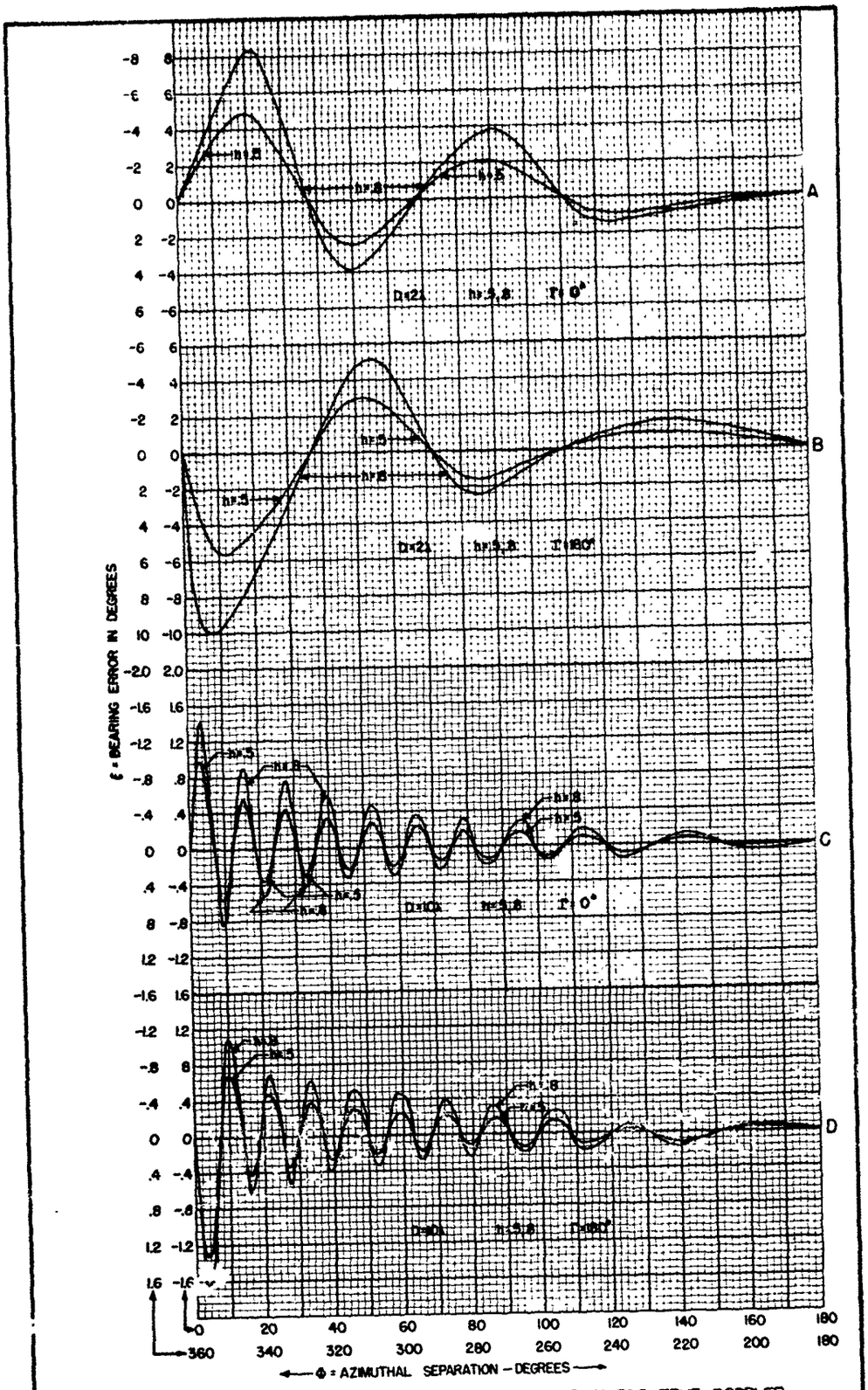

ERROR VRS SYSTEM DIAMETER FOR TRUE DOPPLER SYSTEMS (SOLID CURVES) AND FOR A CIRCULAR DISTRIBUTION OF ADCOCK SYSTEMS - UNLIMITED - S = 8 - (DOTTED CURVES)

| | |
|---|---|
| DEPARTMENT OF ELECTRICAL ENGINEERING - UNIVERSITY OF ILLINOIS | PLATE XV |
| | TECH. RPT NO. 9 |




BEARING ERROR OF DOPPLER AND DISTRIBUTED SYSTEMS FOR LARGE RELATIVE MAGNITUDES AND SMALL ANGLES OF SEPARATION

| | |
|---|---|
| | PLATE XVI |
| DEPARTMENT OF ELECTRICAL ENGINEERING — UNIVERSITY OF ILLINOIS | TECH. RPT. NO. 9 |




BEARING ERROR VS ANGULAR SEPARATION FOR TRUE DOPPLER SYSTEMS WITH DIAMETERS OF 2 AND 10 WAVELENGTHS

| | |
|---|---|
| PLATE | XVII |
| TECH. RPT | NO. 9 |

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Plate XVII shows the variation of theoretical bearing error with angular separation for a Doppler system of diameters of 2 and 10 wavelengths.

The effect of arbitrary vertical angle of arrival and azimuthal angle of both desired and undesired signals has been investigated. For a circular Doppler system, the immediate observation is the apparent reduction of aperture as the vertical angle of arrival is reduced; that is, as the signal arrives from greater angles of elevation. The apparent aperture for each signal is the true aperture times the sine of the vertical angle. If the vertical angles of the desired and undesired signals are the same, then the bearing is the same as a Doppler Bearing with reduced aperture.

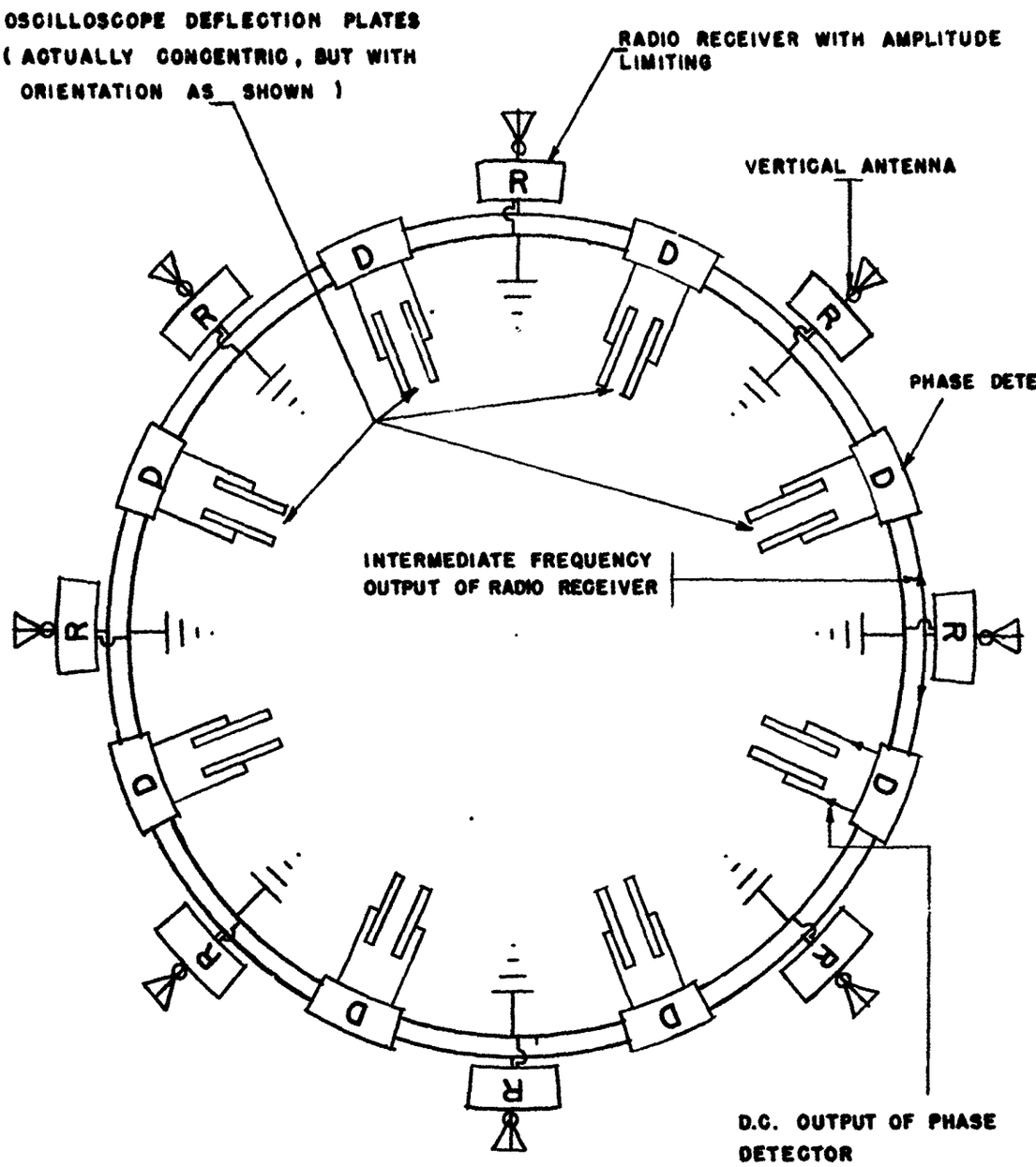
If the antenna is a simple dipole with the characteristic "donut" space pattern then the pickup factor contains a sine factor of vertical angle. Hence, waves from higher elevation are discriminated against first on an amplitude basis and secondly from aperture reduction.

2. Wide-Aperture Vector-Phase Systems

A vector-phase direction finder will be defined as a phase front direction finder which gives the direction of arrival as a vector operated indication which is the vector sum of two or more component vectors. The component vector magnitudes are proportional to the phase differences between any two antennas of an array; the component vector directions are parallel to a line joining these antennas, and their senses are in the direction-sense of the antennas whose induced voltages lead in phase. The vector-phase system which is proposed as being the most practical form would utilize a circular antenna array and would obtain the magnitude and sense of the vectors by means of phase detectors (with sinusoidal characteristics), measuring the phase differences between the intermediate frequency outputs of adjacent radio receivers connected to adjacent antennas. An example system us shown in Plate XVIII. This system has 8 antennas, 8 radio receivers, 8 phase detectors and a vector summing oscilloscope with 8 sets of concentric deflection plates connected to the outputs of the phase detectors. The receivers must incorporate amplitude limiting as only the phase information is desired to obtain a bearing. The 8 sets of deflection plates are oriented so they are perpendicular to the line joining the two antennas whose phase difference is originating the dc voltages presented to the plates.

An alternate type of vector summing device is shown in Plate XIX. Here the function of all but two sets of vector summing oscilloscope plates is replaced by a suitable resistor ring network, hence, a conventional type oscilloscope tube can be thought of as an alternate (and instantaneous) method of presenting the same information as presented by the pseudo-Doppler type radio direction finder shown in Plate XIV - Fig. 2.

It can be shown that a circular vector-phase system of 8 antennas presents the same bearing as an equivalent (one with the same number of antennas) Doppler system of the type shown in Plate XIV, and, hence, has the same bearing error under all electric field conditions. Due to the near equivalence of bearing indication between a

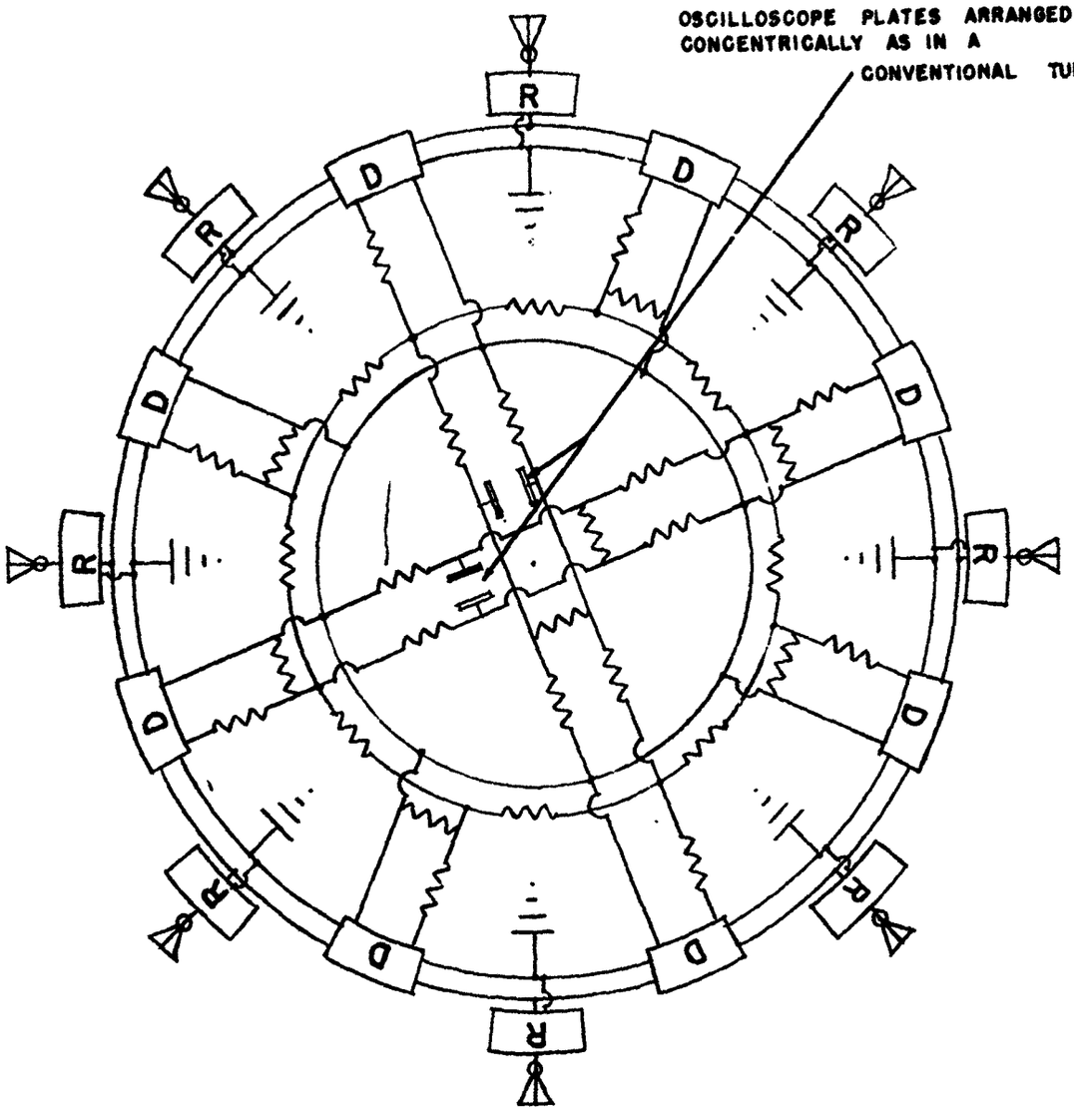



VECTOR PHASE R D F COMBINING CIRCUIT

PLATE XVIII

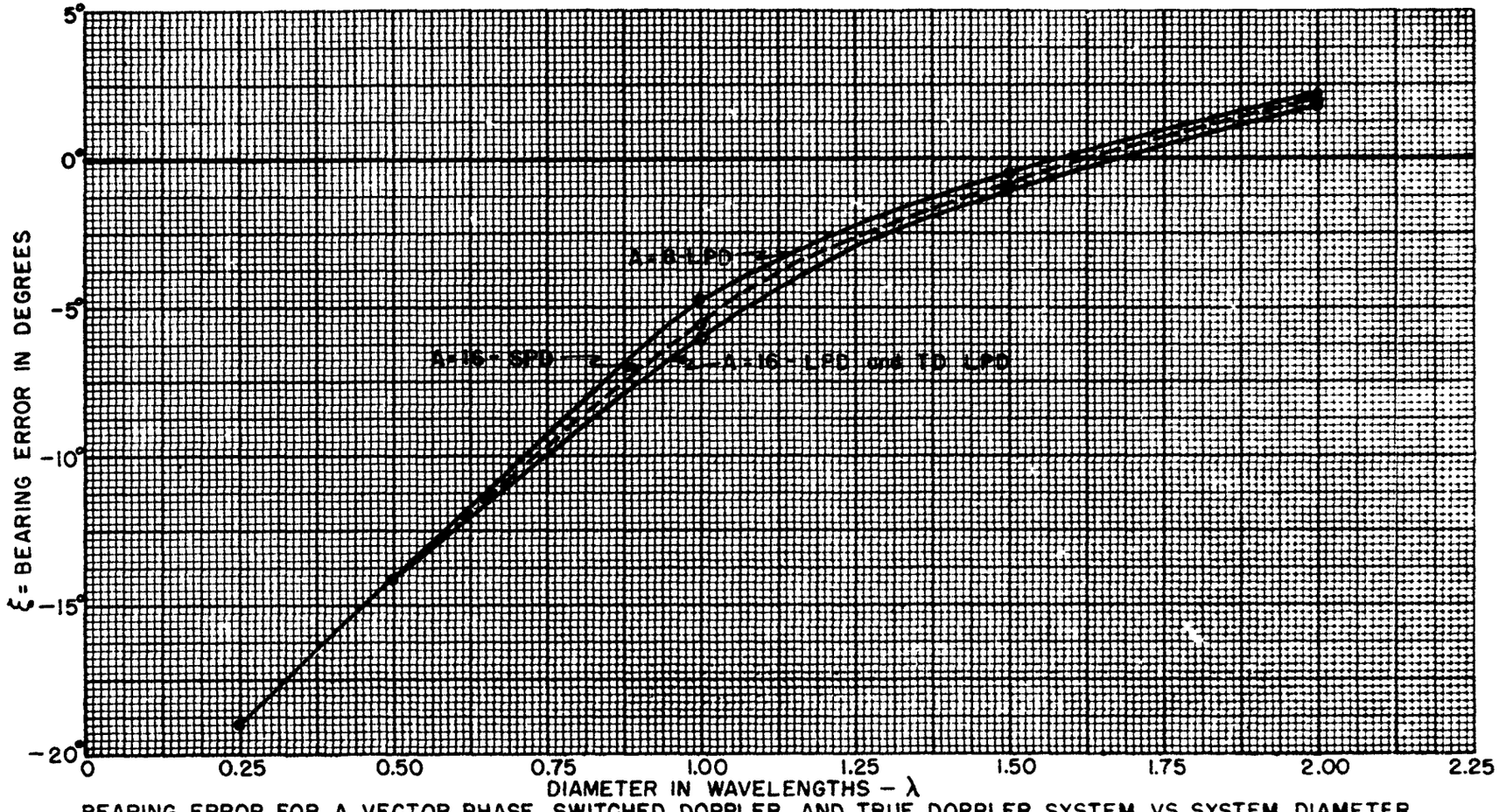
DEPARTMENT OF ELECTRICAL ENGINEERING - UNIVERSITY OF ILLINOIS

TECH. RPT. NO. 9




VECTOR PHASE RDF COMBINING CIRCUIT

PLATE XIX




BEARING ERROR FOR A VECTOR PHASE, SWITCHED DOPPLER, AND TRUE DOPPLER SYSTEM VS SYSTEM DIAMETER

$\Phi = 45°$, $\Gamma = 180°$, $h = .4$

A = 8 LPD – 8 ELEMENT SWITCHED DOPPLER OR VECTOR PHASE SYSTEM WITH LINEAR PHASE DETECTION
A = 16 LPD – 16 " " " " " " " " " " "
A = 16 SPD – 16 " " " " " " " " SINUSOIDAL PHASE DETECTION
TD LPD – TRUE DOPPLER SYSTEM WITH LINEAR PHASE DETECTION

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PLATE XX
TECH. RPT. NO. 9

circular vector-phase system and all types of circular Doppler systems, extensive error calculations have not been made for vector-phase systems. The bearing of a vector-phase system (under the influence of a desired and an undesired signal) can be expressed as:

$$B = \arctan\left[\frac{\sum_{a=1}^{A} K \sin(\psi_{a+1,a})\sin\frac{a2\pi}{A}}{\sum_{a=1}^{A} K \sin(\psi_{a+1,a})\cos\frac{a2\pi}{A}}\right],$$

where

$$\psi_{a+1,a} = \frac{\pi D}{X}\left[\sin\frac{\pi}{A}\cos\left(\frac{2\pi}{A}a - \Phi_1\right)\right] + \zeta(\tau_a + 1) - \zeta(\tau_a)$$

$$\zeta(\tau_a) = \arctan\left(\frac{h \sin\tau_a}{1 + h\cos\tau_a}\right)$$

$$\tau_n = \frac{2\pi D}{X}\cos\left[\frac{\pi}{A}(2a-1) - \frac{1}{2}(\Phi_{2,1} + 2\Phi_1)\right]\sin\frac{\Phi_{2,1}}{2} + \Gamma_{2,1}.$$

The bearing error is given by

$$\xi = B - \Phi_1.$$

Several curves of vector-phase bearing error are shown in Plate XX in comparison with corresponding Doppler Curves.

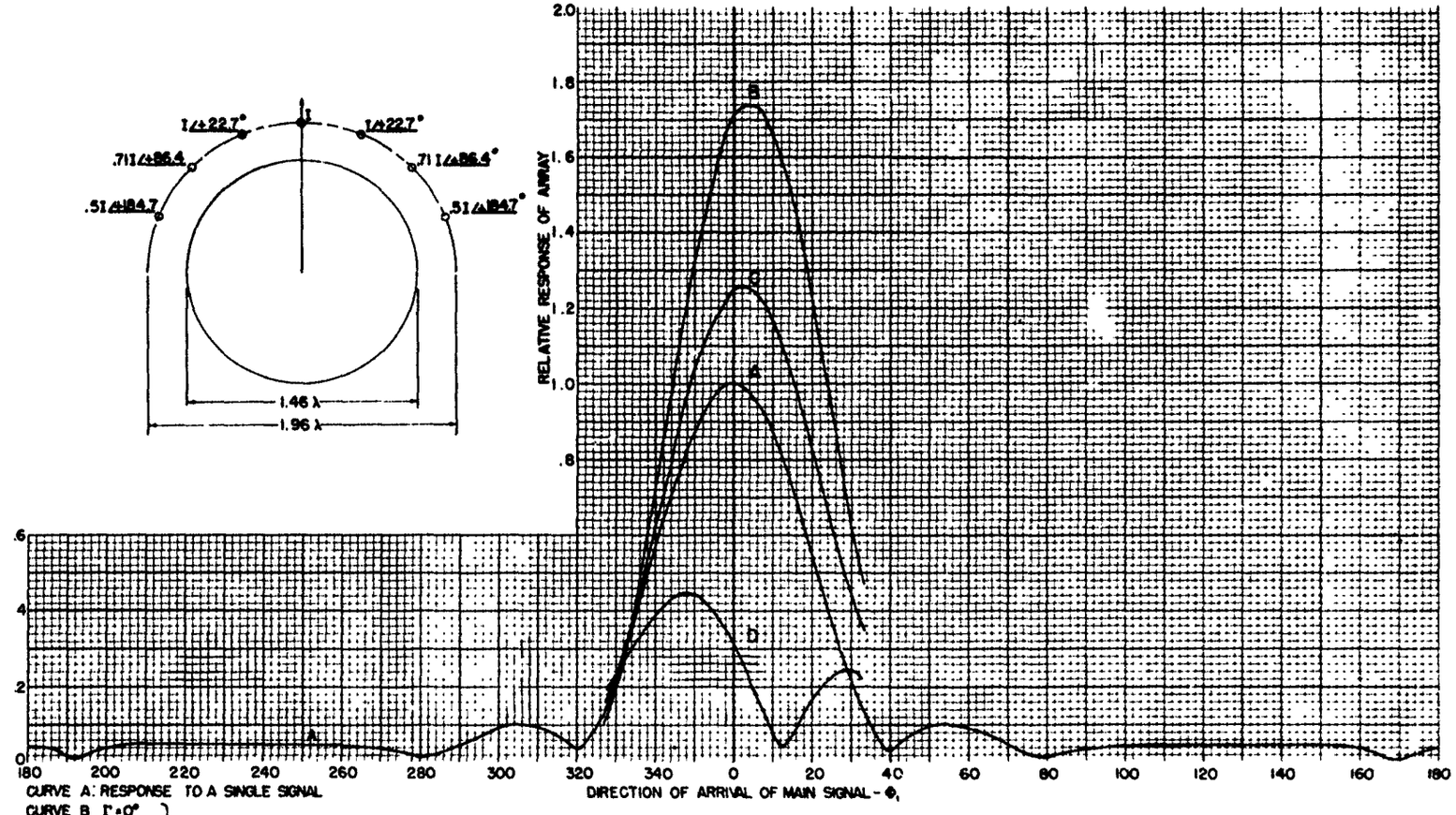
The effect of vertical angle of arrival would be generally the same as with equivalent Doppler systems. The optimum number of antenna elements again is the same as for an equivalent Doppler system. Fifteen or sixteen antennas seem to be a good compromise for system diameters between two and three wavelengths. In general, an odd number of antennas will give less repetitive errors[5].

3. Wullenweber Systems

No extensive investigations of the properties of the Wullenweber type of direction finding system have been carried out at this laboratory. However, it seemed desirable to have enough data on at least one such system to allow comparisons to be made with some of the other types being studied. In particular, since the Wullenweber is a wide-aperture system, it seemed desirable to be able to make some comparisons between a Wullenweber system and a wide-aperture Doppler system of similar size. The wullenweber type system for which calculations were made is shown diagramatically in Plate XXI. The phasings of the elements were so chosen that at a distant point on the radial line $\Phi = 0$, if the antenna were used for transmitting, the signals from each of the radiating elements would have the same phase.

Curve A on Plate XXI shows the response of the array to a single signal arriving in the horizontal plane, along the $\Phi = 0$ radial. The array is assumed to be rotated about a vertical axis. Curves B, C, and D show the response obtained when a second signal, or ray, of the same frequency as the first, and having a field strength eight tenths

5. "Radio Direction Finding by the Cylical Differential Measurement of Phase," C. W. Earp and R. M. Godfrey, Jour. I.E.E., V. 94, Part III A, No. 15 March - April 1947.




CURVE A: RESPONSE TO A SINGLE SIGNAL

CURVE B Γ = 0°
CURVE C Γ = 90°
CURVE D: Γ = 180°
} h = 8 ANGULAR SEPARATION BETWEEN RAYS = 10°

PERFORMANCE OF A WULLENWEBER ARRAY WITH TWO ARRIVING RAYS

DEPARTMENT OF ELECTRICAL ENGINEERING — UNIVERSITY OF ILLINOIS

PLATE XXI
TECH. RPT. NO. 9

that of the first, arrives along the $\Phi = 10^{\circ}$ radial. The phase difference, $\Gamma$, between the arriving signals, measured at the center of the array is $0^{\circ}$, $90^{\circ}$, and $180^{\circ}$ for Curves B, C, and D, respectively. For $\Gamma = 0^{\circ}$ and $\Gamma = 90^{\circ}$, the bearing shift due to the presence of the second ray is not large, but for $\Gamma = 180^{\circ}$, the bearing is split, and the error is considerably larger than for the other two cases. Thus, as do most of the other systems, this system has a tendency to "blow up" when $\Gamma = 180^{\circ}$. Under some conditions, however, as will become apparent, this tendency is much less in the Wullenweber type system than in other types of systems.

If the side-lobe level were zero, then when the difference in angle of arrival of the rays exceeded the beam width[6], the two rays would give separate, correct indications. If the difference in angle of arrival of the rays were less than the beam width, then the indicated bearing (in some cases a split bearing would be obtained) would be in error, the size and nature of the error depending on the relative phase, the angular difference in direction of arrival, and the relative magnitude of the rays. However, the error could in no case exceed twice the beam width.

If the side-lobe level is not zero, but is such that twice the ratio of side-lobe to main-lobe level is less than one minus the ratio of the weaker to the stronger signal, then the conclusions are similar to the previous case. Roughly, if the difference in angle of arrival of the rays exceeds the beam width, two separate bearings will be indicated, one for each wave. Neither, however, may be correct, but their errors will be less than the beam width.

For the same conditions as the last previous case, except that the difference in angle of arrival of the rays is less than the beam width, the indicated bearing (under some conditions a split bearing will be obtained) will be in error by an amount less than twice the beam width.

If the ratio of side-lobe to main-lobe level exceeds the previously specified limit, the error in the indicated bearing may be quite large, especially if $\Gamma$ is near $180^{\circ}$.

The preceding statements indicate why it is important for the side-lobe level to be low as well as for the beam to be sharp when multiray reception occurs. Even if the main lobe is sharp, large errors can be obtained for some conditions if the side-lobe level is too high.

Nothing has been said yet about the variation of the Wullenweber pattern as the vertical angle of arrival is changed. This is a serious design problem. Many arrays that would give satisfactory results for waves arriving in the horizontal plane would be entirely unsatisfactory for waves arriving at an appreciable vertical angle because of breakup of the Wullenweber pattern. The array must be so designed that this breakup is not objectionable.

## C. Limited Space Distribution of Small-Diameter Systems

This section will consider various techniques of instantaneously and automatically combining the bearings of a collection of

6. The term "beam-width" as used here means the angular distances between the minimum adjacent to the main lobe.

small-diameter unit direction finding systems. These systems are spread over a region large in comparison to the diameter of the unit systems, but small in comparison to the distance from the signal source to the system center.

The bearing information from the unit system could conceivably be combined in many ways to form a composite bearing, each combining technique resulting in its own mathematical law of combination. It is believed that three of these combination laws would be the most probable outcome of the majority of combination circuit techniques.

The first and most probable of these laws will be called the vector-average law. If $B_{r_s}$ is the indicated bearing of the sth system and $M_s$ is the field strength at the center of the sth system, then $B_{I_R}$, the resultant bearing for a composite system with a vector-average law can be expressed as:

$$B_{I_R} = \arctan \left[ \frac{\sum_{s=1}^{S} M_s \sin BI_s}{\sum_{s=1}^{S} M_s \cos BI_s} \right]$$

where S = number of systems. This law is the one generally resulting from distributed systems not incorporating amplitude limiting and which combine bearing information either as dc voltages proportional to the sine and cosine components of bearing, or as audio-frequency voltages containing bearing information in their relative phase.

The second combination law will be called an amplitude-limited vector-average law. With the same symbol definitions as used in the previous paragraph, this law is:

$$B_{I_r} = \arctan \left[ \frac{\sum_{s=1}^{S} \sin B_{I_s}}{\sum_{s=1}^{S} \cos B_{I_s}} \right]$$

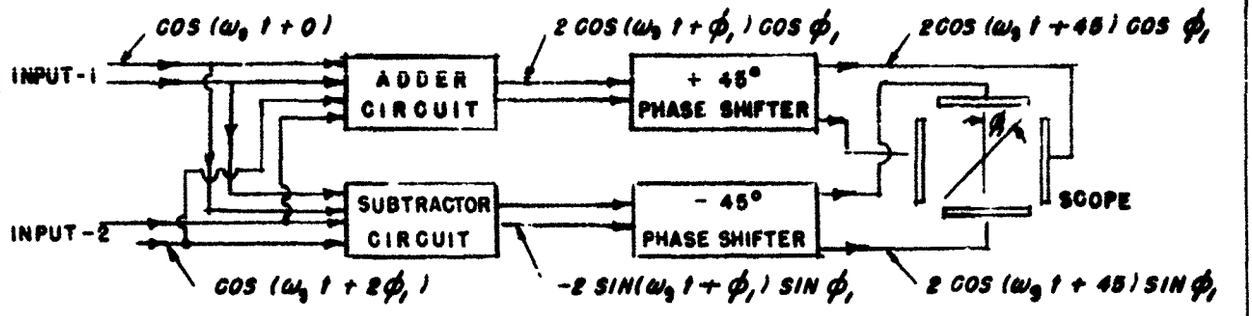
The amplitude-limited vector-average law is the law generally resulting from distributed systems incorporating amplitude limiting (such as the 4 element vector-phase system described in Section V,A2).

The third law of combination to be considered is an arithmetic mean law. In the same symbols as used in the last two paragraphs:

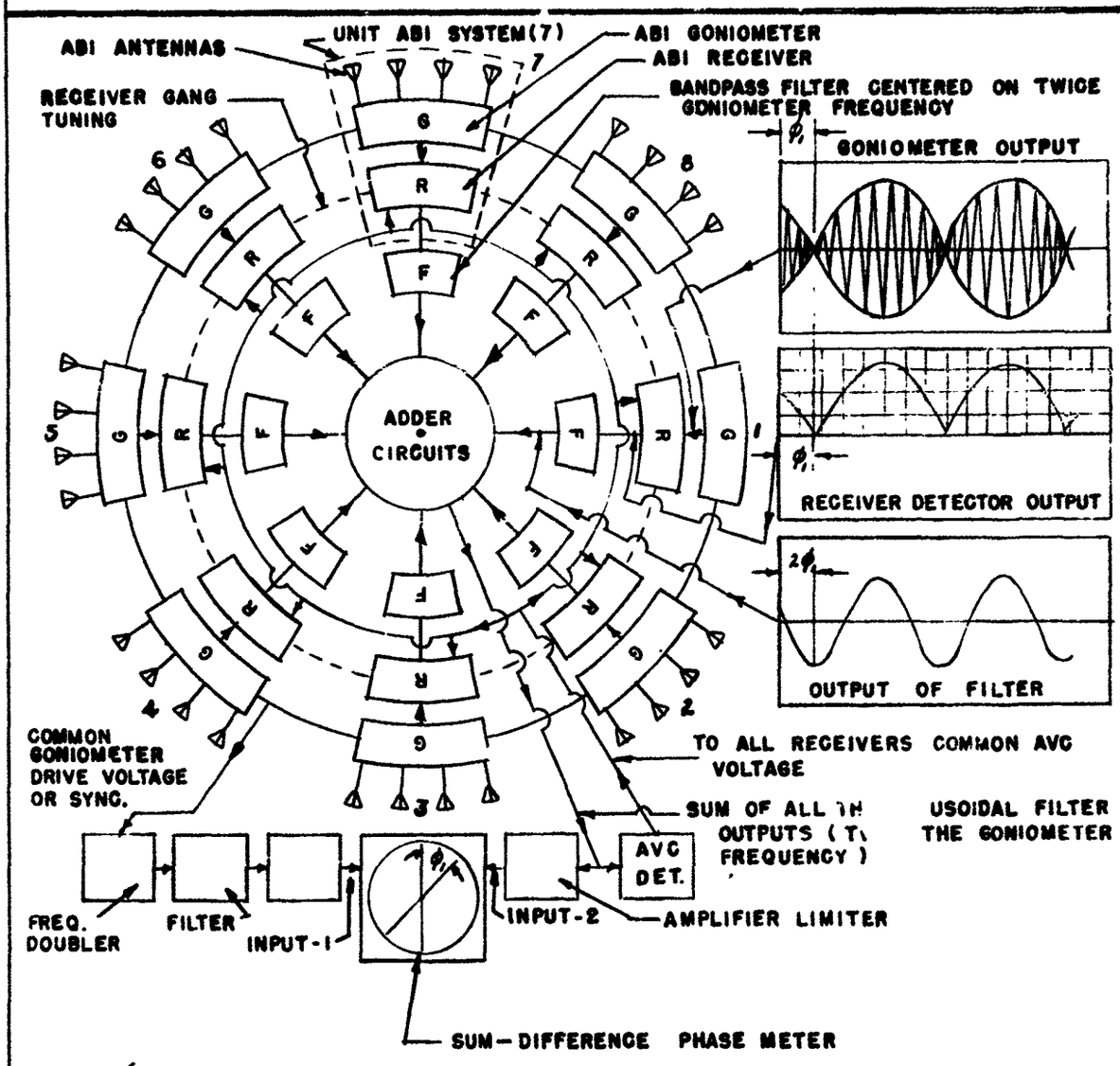
$$B_{I_r} = \frac{\sum_{s=1}^{S} B_{I_s}}{S} .$$

If the variation in $B_{I_s}$ from system to system is not too great (less than $40^\circ$), then the difference between the resultant bearings of an arithmetic mean law and an amplitude-limited vector-average law will generally be very small (less than $1^\circ$).

As an illustration of a circuit combining technique for distributed systems with a resultant bearing given by the vector-average law see Plate XXII. This plate demonstrates a potential method




DETAILS OF SUM-DIFFERENCE PHASE METER




( $\phi_1$ IS PRESENTED ON THE PHASE METER IF ALL UNIT SYSTEMS OTHER THAN SYSTEM (1) ARE INOPERATIVE )

CIRCULARLY DISTRIBUTED COMPOSITE SYSTEM OF 8 ABI UNIT SYSTEMS

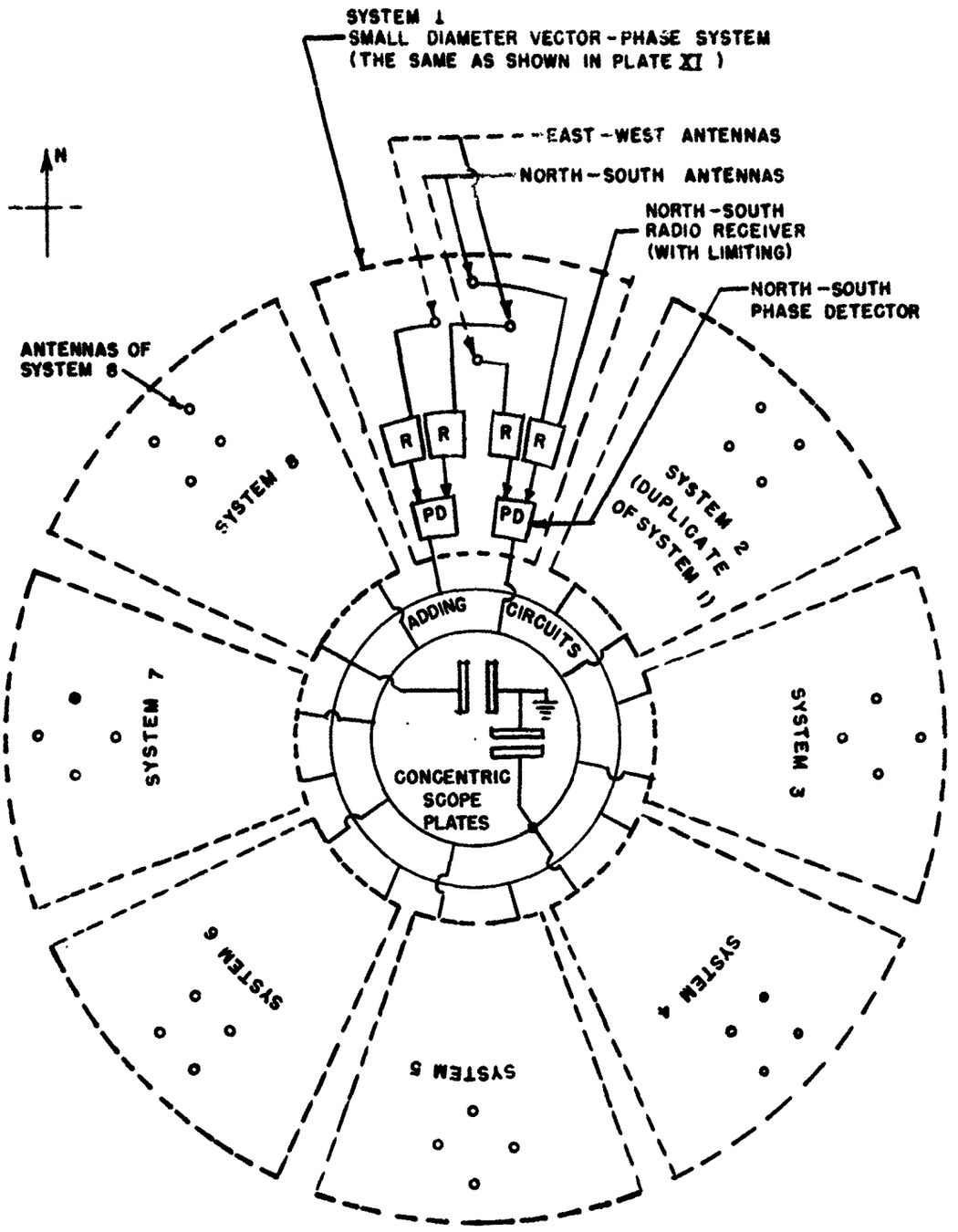
PLATE XXII

of using existing ABI direction finders in a circular space distribution of unit systems. The goniometers of all the unit ABI systems are activated by a common rotation voltage source and the second harmonic of this rotation frequency is compared in phase (by a sum-difference phase meter) to the phase of the sum of all of the filtered outputs from the ABI second detectors. It is not to be expected that this scheme would work well on an incoming signal containing some types of pulse modulation or substantial modulation components at the goniometer rotation frequency. Nor is it proposed that this scheme is the most practical one for combining the bearings of distributed ABI systems. However, it serves to illustrate the general techniques and some of the problems to be faced.

Plate XXIII illustrates a system which gives a bearing expressed by the amplitude-limited vector-average law. This distributed system is a composite of small diameter vector-phase systems of the type discussed in Section V,A,2. The direct current outputs of all the north-south discriminators are added linearly, and the same is done for the outputs of the east-west discriminators. The two sets of resultant dc voltages are applied to the two sets of orthogonal deflection plates in a conventional oscilloscope tube, to obtain the desired resultant bearing. Again there are many practical problems to be overcome, although the majority of these would probably be in the unit systems rather than in the combination techniques. This system would be expected to work reasonably well on code signals.

Assuming that circuit techniques are available for successfully combining bearings (under most modulation conditions) and presenting the resultant bearing on some type of indicator, a distributed system spread out over a region of several wavelengths is seen to have excellent possibilities in discriminating against an interfering signal. A resultant bearing can be no worse than the worst bearing of any of the unit systems and will generally be much better.

The Distributed System bearing error is an average of the bearing errors of the individual unit systems within the composite system. The bearing error of each of the unit systems is a known function of the total time phase difference between the desired and undesired signals. The total time phase difference between the two signals at each unit system is a function of the time phase difference at the center of the composite system plus a time phase difference which is a function of the spacing of unit systems in the composite system, as well as the electrical diameter (in the circularly disposed case) of the composite system. The composite system bearing error is obtained by combining the bearing error of the unit systems by (1) vector-averaging, (2) limited vector-averaging, (3) averaging each unit system bearing error. The vector-average utilizes the magnitude of the vectorial sum of the desired and undesired signals at each unit system. To show the order of improvement under general error conditions, curves of error vs system diameter (and length) have been drawn in Plate XXIV. Curves A and B show the error vs system diameter for circular and linear distributions, respectively, of ABI systems (or other systems with the same bearing error as an Adcock system). One curve of bearing error vs system diameter for a circular distribution of small-diameter vector-phase systems is shown in curve C. In all cases, curves for combination




CIRCULARLY DISTRIBUTED COMPOSITE SYSTEM OF
8 SMALL DIAMETER VECTOR-PHASE SYSTEMS

PLATE XXIII

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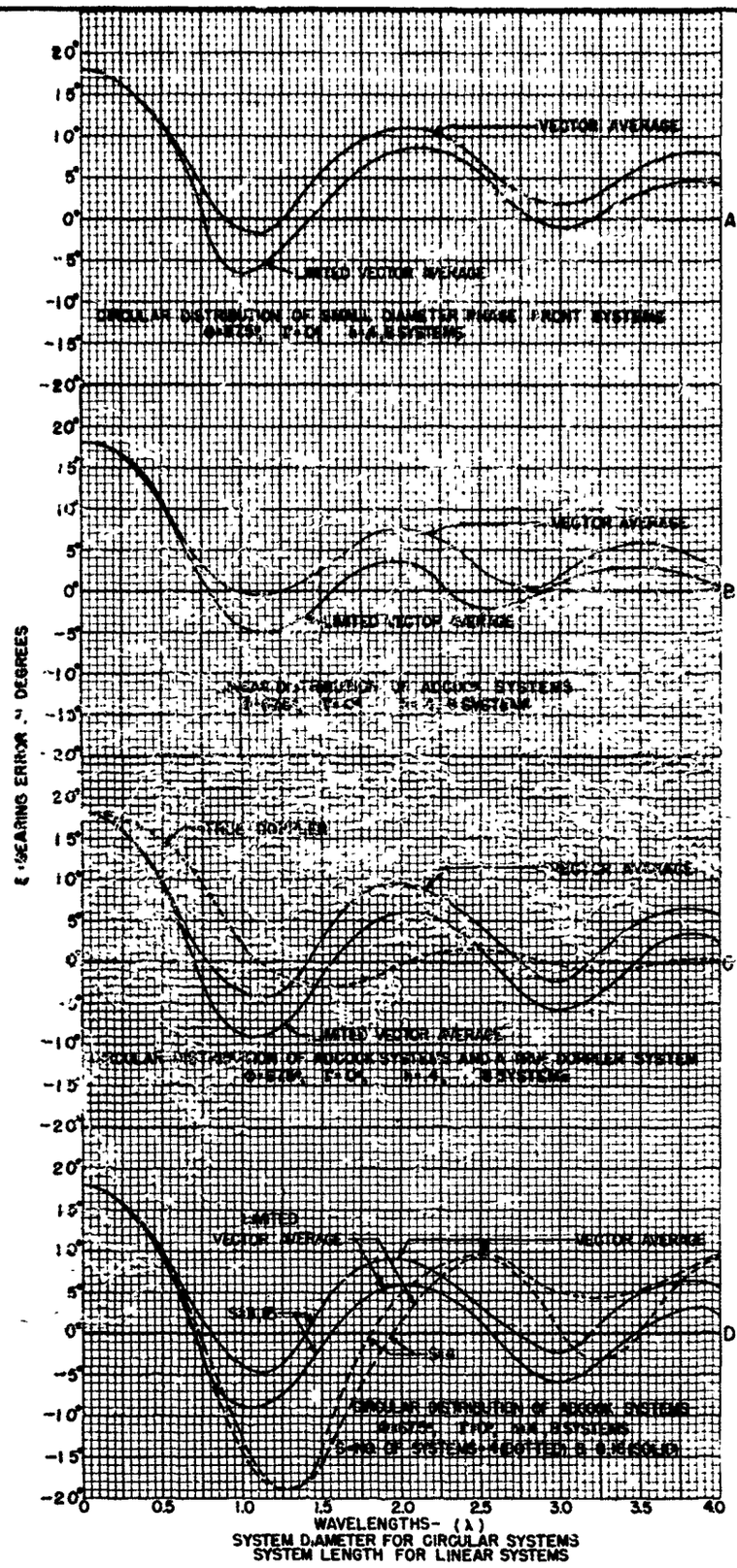
TECH. RPT. NO. 9




BEARING ERROR VS SYSTEM DIAMETER (OR LENGTH)
IN WAVELENGTHS FOR LARGE ANGLES OF SEPARATION

| DEPARTMENT OF ELECTRICAL ENGINEERING -- UNIVERSITY OF ILLINOIS | PLATE XXIV |
|---|---|
| | TECH. RPT. NO. 9 |

circuits following both vector-average and amplitude-limited vector-average laws are shown. A curve of true Doppler bearing error is also shown in Plate XXIV for comparison purposes.

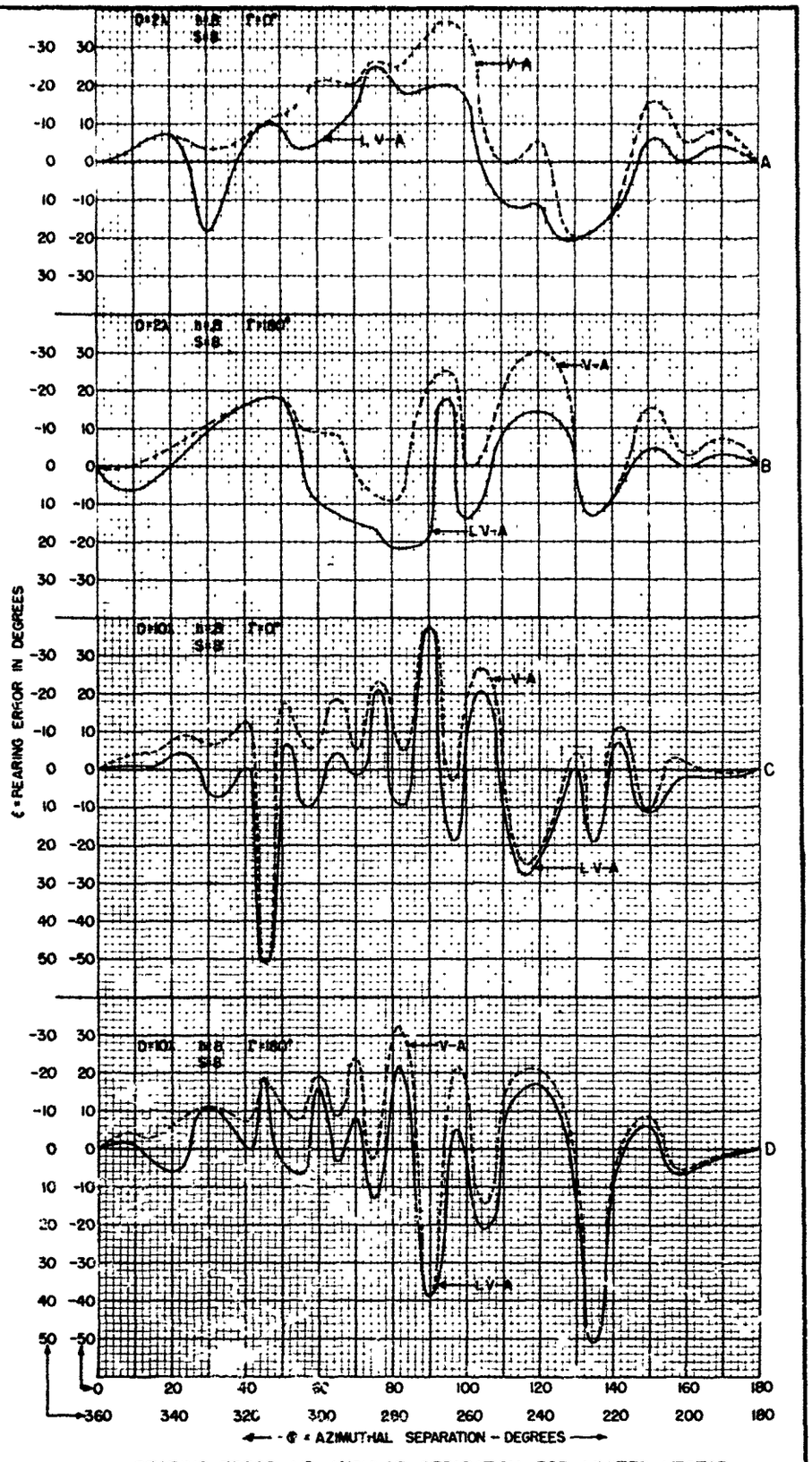
The variation of error with the number of unit systems is illustrated by curve D. This curve shows that for some interfering signal conditions an increase in the number of unit systems need not give a decrease in bearing error, although a reduction is generally the case. An increase in the number of systems gives very little improvement after a certain number is reached. This optimum number of systems is approximately eight for composite system diameters less than three wavelengths.

If the general direction of arrival (within $\pm 30^{\circ}$) could be judged sufficiently far in advance, a linear dispostion of antennas at right angles to the direction of arrival would give better results than a circular composite system with the same number of unit systems and the same maximum dimensions. This is illustrated by the solid curves of B and C or D.

At present, little can be said quantitatively about random distributions of systems. In general, if a random distributed system were fairly well dispersed and contained the same number of unit systems as a circularly disposed composite system (with uniform spacing), in addition to having roughly the same maximum dimensions, then the order of error reduction would be about the same for these two types of composite systems.

Interfering signal conditions where some of the worst errors occur for the unit systems are shown in Plate XVI. Error is plotted as a function of system diameter and it is seen that the improvement is more rapid with vector-average than amplitude-limited vector-average combination laws. In general, the unlimited system will make the average bearing tend more towards the direction half-way between the arrival directions of the desired and undesired signals. If the angle of separation between signals is small, as in Plate XVI, this tendency is a desirable characteristic, but if the angle of separation is large, as in Plate XXIV, this tendency is not necessarily desirable.

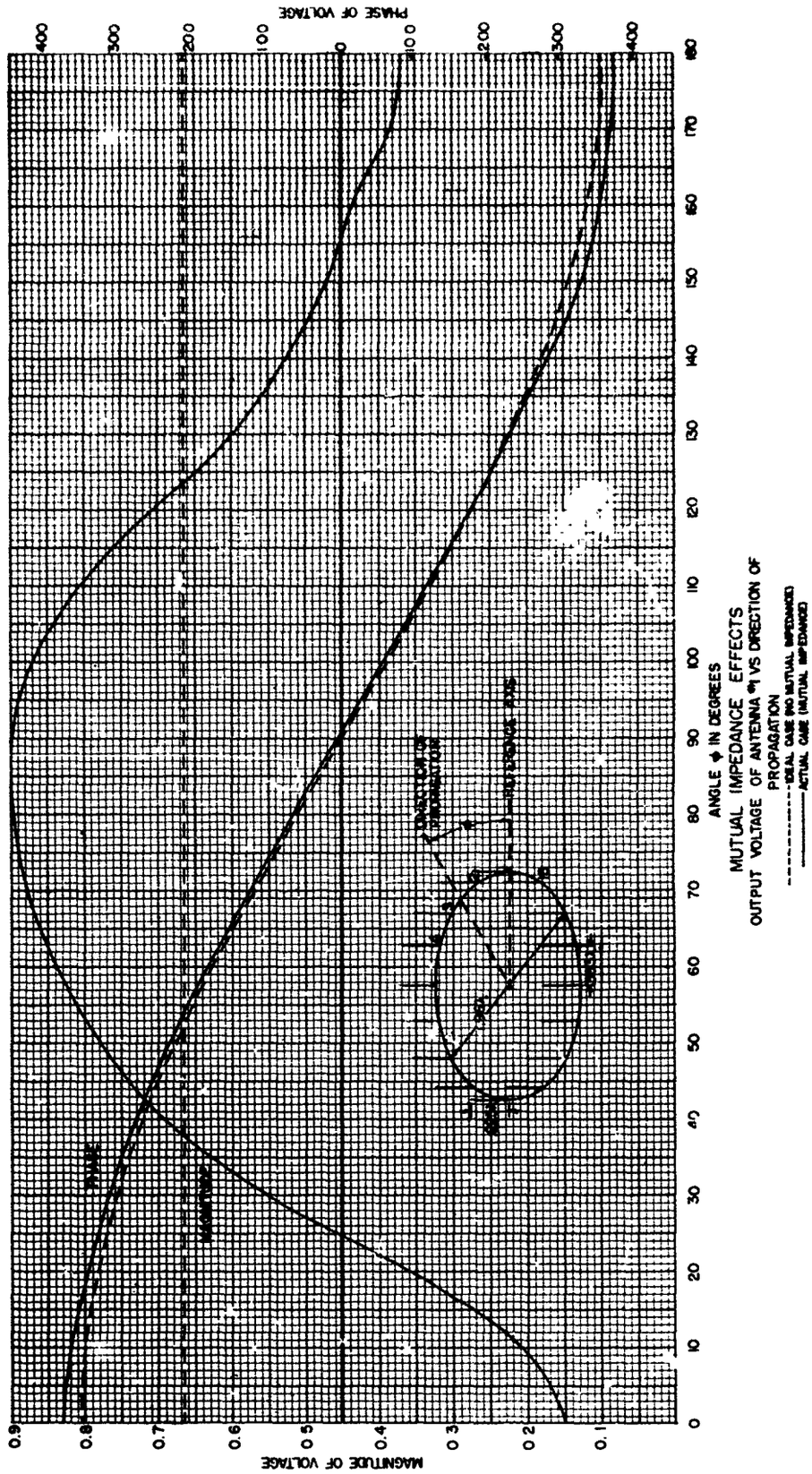
Bearing error as a function of angular separation is shown plotted in Plates XXV for a circular distribution of 8 ABI systems with diameters of two and ten wavelengths. Eight unit systems is an insufficient number for a distributed system with 10 wavelengths diameter. This is illustrated by the $10\lambda$ curves. For these curves with some conditions of angular separation ($\Phi_{2,1} = 45^{\circ}$ and $135^{\circ}$), the errors of all the unit systems are identical and no error suppression is obtained.




BEARING ERROR VS ANGULAR SEPARATION FOR LIMITED VECTOR AVERAGE (SOLID CURVES) AND VECTOR AVERAGE (DOTTED CURVES) DISTRIBUTED ADCOCK SYSTEMS (S=8, CIRCULARLY DISTRIBUTED)

| | |
|---|---|
| PLATE | XXV |
| TECH. RPT. | NO. 9 |

DEPARTMENT OF ELECTRICAL ENGINEERING — UNIVERSITY OF ILLINOIS




MUTUAL IMPEDANCE EFFECTS
OUTPUT VOLTAGE OF ANTENNA #1 VS DIRECTION OF PROPAGATION
- - - - - IDEAL CASE (NO MUTUAL IMPEDANCE)
——— ACTUAL CASE (MUTUAL IMPEDANCE)

| | |
|---|---|
| PLATE | XXVI |
| TECH. RPT. | NO. 9 |

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# VI. PRACTICAL CONSIDERATIONS

## A. Practical Circuitry

### 1. Identical Receiving Channels

Several of the rdf systems previously described, namely the Watson-Watt, Vector-phase, sequence comparator, and Doppler in which switching is done after some amplification, require the use of two or more receiving channels that are alike in phase and gain responses; or, at least, that are alike within certain tolerances which may be dictated by the particular system. The simplest of these systems, the Watson-Watt, requires two such receiving channels, or three if the $180^{\circ}$ ambiguity is eliminated.

Briefly, the requirements of the receiving channels are as follows: they must have (1) identical gain vs frequency response curves, (2) identical phase vs frequency response curves, (3) identical time delays in each channel. Because the gain vs frequency and phase vs frequency curves are interdependent, two channels can be made exactly alike if their gain vs frequency curves are made to have equal amplitudes at all frequencies, provided that the time delays of the two channels are identical. Methods for attaining these results, including some of the practical difficulties involved, will be described in a later report.

Some of the indicating system problems have been previously discussed. If the usual type of cathode ray tube is used, which has two pairs of deflection plates, the pairs of plates must be at right angles electrically. If this is not true a correction can be made by applying voltages to the plates through a resistance bridge system.

Cross-feed is a problem in all multichannel receiving systems. It must be kept to a minimum by using suitable chassis layouts, adequate shielding, and suitable ground connections. A considerable amount of cross-feed can be eliminated in a dual-channel system by using a bridge circuit in the output leads. If bridges were required for each two receiving channels in an n channel system, the number of bridges required would be $n(n-1)/2$.

### 2. Gain Control

It is necessary that the overall gain control give the same attenuation for all channels at all points. This control must not, therefore, depend upon the characteristics of electron tubes. It can take the form of matched attenuators used at some low impedance level in the receiving systems, such as in cable lines between receiver components.

### 3. AVC

In order to control all receiving channels by the same amount of strong signals, it is necessary that the AVC voltage comes from a common source. Dual-channel receiving systems may derive this voltage from the sense channel. For the vector-phase and distributed narrow aperture systems the AVC voltage may be derived from a pilot signal that is made common to all receiving channels by means of connecting cables. It is required that the AVC voltage for systems having

receivers with identical phase and gain characteristics be applied in such a manner that the gain vs frequency response is not affected. For the distributed narrow aperture systems the AVC must be applied in such a manner that the gains of all the receivers relative to each other are known.

4. Amplitude Limiter

The vector-phase and Doppler systems require amplitude limiters in each of the receiving channels. These limiters must have the same threshold of saturation. The effective bandwidth will be larger on strong signals than on weak signals, since the required amplitude threshold will be farther removed from band center in the case of strong signals. It is possible to get around this difficulty by placing a tuned circuit or filter after the limiter. Since the purpose of the limiters in each channel is to get equal amplitude responses in each channel for the signal components at the same frequency entering the channels, the filters following the limiters will do no harm, but the filters in all of the channels must be identical in phase and gain characteristics. The filters are also required for use in eliminating undesired frequency components in the limited inputs to the resistance-coupled phase discriminators used in the vector-phase and Doppler systems.

5. Cables

In order to provide inputs of proper relative magnitudes to all receiving channels in a multichannel receiving system, it is necessary that the cables from the antennas to all of the channels have the same electrical characteristics, and have the same electrical length so that the total attenuation and the time delays will be the same for all of the cables. If it should not be practical to use cables of the same length, attenuation and delay equalizing networks must be used at the receiver ends of the cables. In the case of the Wullenweber system the required attenuations and delays will not be equal, but must maintain prescribed relations over the entire frequency band in order to obtain the desired antenna pattern.

6. Polyphase Oscillators

The sequence comparator system requires that the local oscillator voltages fed to the mixers on the various receiving channels have definite phase relations between each other. This may be accomplished by using a tunable polyphase oscillator to furnish the required voltages, or by using a single-phase oscillator operating through a tunable phase shifter to each channel. The tunable polyphase oscillator seems to be the more feasible of the two possible methods.

7. Switching

The Doppler and Wullenweber systems with stationary antennas require switching. Any switching produces a transient problem. The transients may be reduced if there is some amplification, before switching, but this means that identical amplifiers would be required in all of the channels. If the switching can be done so that the transients are not objectionable or cancelled out in the system, then the switching problem is largely solved. Results obtained with square-law (abrupt) and triangular (linear) switching will be discussed in a later report.

## B. Practical Considerations

### 1. Mutual Impedance Effects

#### a. Wide-Aperture Systems

In theoretical analysis on systems such as the switched Doppler, vector-phase or Wullenweber, it is customary to neglect the effect of mutual impedance between elements of the antenna array. This is done because to attempt to include these effects directly would make the analysis exceedingly difficult. Therefore the assumption is usually made that (a) mutual impedance effects are indeed negligible, or (b) although not negligible, the effects tend to cancel each other and result in small error, or (c) the effects produce appreciable error, but the effects themselves can be minimized. In order to obtain at least a qualitative answer for the general case, mutual impedance effects have been and are being investigated for a few special cases.

The general procedure consists of measuring the impedances (both self and mutual) for a particular antenna system, calculating the antenna output voltages for a certain set of receiving conditions and then feeding this information into the RDF System Analyzer in order to find the bearing error caused by the mutual impedances. As an example of the results of this procedure the curves of Plate XXIV show the output voltage as a function of direction of arrival of the received wave, for one antenna of a circular array of 16 elements. The array diameter was approximately two wavelengths, corresponding to an antenna spacing of 0.383 wavelengths. Each antenna was a resonant length and was connected to a matched load impedance. The curves were obtained by considering the effects of the four nearest antennas only, since to have included all the elements would have necessitated evaluating a 16th order determinant. Comparing the solid curves with the dashed-line curves which correspond to the ideal case of no mutual impedance, it is apparent that the effects of mutual impedance on individual output voltages can be quite marked. However, there is symmetry about the reference axis (the curves are shown only for angles between $0^{\circ}$ and $180^{\circ}$) so that if this array were used for the Doppler system, for example, which utilizes only the fundamental component of the rotational frequency, the error resulting for one arriving wave would be theoretically zero. In practice, because of small inequalities in load impedances, there would be some error. Errors due to mutual impedance effects when more than one wave is arriving are being investigated with the RDF System Analyzer.

#### b. Distributed Small Aperture Systems

As with wide-aperture systems, theoretical analysis of distributed small-aperture systems, ordinarily neglects mutual impedance effects. In order to estimate the order of magnitude of the possible error which might be introduced, a simple calculation has been made of the maximum bearing error which could occur in the indication of a single system due to reradiation from a nearby system. The results are tabulated below.

| Spacing | Ratio of Reradiated to Direct Signal | Max. Bearing Error |
|---|---|---|
| 0.15 | 0.4 | $24^{\circ}$ |
| 0.5 | 0.2 | $12^{\circ}$ |
| 1.25 | 0.1 | $6^{\circ}$ |

These figures were obtained on the assumption that the reradiation from an individual system was equal to the reradiation from a resonant-length antenna connected to a matched load (it is probably considerably less than this). It is seen that this maximum error is quite large for small spacings. However it is entirely possible that in a large system having symmetrically disposed individual systems, errors will tend to cancel as in the wide-aperture case mentioned above. This point is being investigated further.

# VII. OVERALL COMPARISON OF SYSTEMS (INTERSYSTEM COMPARISON)

## A. Figure-of-Merit for Intersystem Comparison

The figure-of-merit calculations as used in Technical Report No. 8 of this project would require a large number of point by point calculations for curves such as those of Plate XXV. As an alternative procedure it is possible to find the average of the absolute error or the root mean square in continuous fashion, by integration with a planimeter. The average absolute error is somewhat the easier of these two techniques and can be expressed by the formula;

$$\int_0^{2\pi} \frac{|\xi(\varphi)|\, d\varphi}{2\pi},$$

where $\xi(\varphi)$ is a function (curve) of error vs angular separation for a given time phase difference.

In order to take into account time phase differences a curve of average absolute errors (each point being expressed by the equation of the previous paragraph) could be integrated with respect to time phase. This procedure would be tedious however and will not be used in this report.

It is felt that for most direction finders (due to the properties of the phase front) the maximum errors will tend to occur at or near $0^\circ$ and $180^\circ$ time phase. Hence, average absolute errors will be given for these two values of time phases.

The relative merits of a direction finder would be described reasonably accurately by the double average process discussed in the previous paragraph if for a typical rdf installation it were equally probable that any angular separation and any time phase would occur over a given period of operation and only two signals were present. This is definitely not the case and some sort of statistical weighting factors would be needed for a true figure of merit.

One of the biggest deviations from equally probable parameters would occur with the angular separation parameter. The probability of small angles of separation and fairly large relative magnitudes would outweigh other error conditions at most installations.

To show the accented tendencies under these conditions average absolute bearing error has been obtained considering only angles of separation between $0^\circ$ and $10^\circ$. A true figure of merit would probably lie somewhere between the $0^\circ$- to $10^\circ$ average and the $0^\circ$- to $360^\circ$ average, the exact location depending on the nature of the rdf equipment, the rdf site, and the nature (statistical) of the propogation conditions to be encountered.

## B. Tabulation of Average Absolute Bearing Error

Table 1 is a summary tabulation of the average absolute bearing error that has been determined for each of the several systems considered earlier. The table thus provides a means of intersystem comparison and is the primary basis for the conclusions which are drawn in the next section.

Table 1

| Type of System | | Time Phase Difference At System Center | Average Absolute Bearing Error | | | |
|---|---|---|---|---|---|---|
| | | | h = 0.5 | | h = 0.8 | |
| | | | \|ε\| Averaged Over | | \|ε\| Averaged Over | |
| | | | 0-10° | 0-360° | 0-10° | 0-360° |
| (A) Phase Front Normal | | Γ = 0° | 1.9° | 19.3° | 2.1° | 32.3° |
| | | Γ = 90° | 1.0° | 9.9° | 2.0° | 26.1° |
| | | Γ = 180° | 4.5° | 18.8° | 18.4° | 31.8° |
| (B) Adcock | | Γ = 0° | 1.9° | 19.3° | 2.1° | 32.3° |
| | | Γ = 90° | 1.0° | 5.6° | 2.0° | 13.04° |
| | | Γ = 180° | 4.5° | 18.8° | 18.4° | 31.8° |
| (C) Spaced Loop | | Γ = 0° | 1.5° | 9.8° | 2.0° | 17.9° |
| | | Γ = 90° | 0.9° | 2.2° | 1.5° | 7.1° |
| | | Γ = 180° | 4.2° | 9.2° | 14.4° | 15.6° |
| (D) Circular Distribution of 8 Adcock Systems | Diameter = 2λ | Γ = 0° | | | LVA 1.6° | LVA 9.9° |
| | | | | | VA 1.6° | VA 13.1° |
| | | Γ = 180° | | | LVA 4.8° | LVA 9.5° |
| | | | | | VA 0.4° | VA 11.1° |
| | Diameter = 10λ | Γ = 0° | | | LVA 0.8° | LVA 9.4° |
| | | | | | VA 2.8° | VA 11.2° |
| | | Γ = 180° | | | LVA 1.2° | LVA 10.6° |
| | | | | | VA 2.4° | VA 12.2° |
| (E) True Doppler System | Diameter = 2λ | Γ = 0° | 1.8° | 1.4° | 2.2° | 2.2° |
| | | Γ = 180° | 3.3° | 2.04° | 8.0° | 3.0° |
| | Diameter = 10λ | Γ = 0° | 0.6° | 0.2° | 0.8° | 0.3° |
| | | Γ = 180° | 0.8° | 0.2° | 0.9° | 0.3° |

## VIII. CONCLUSIONS

A study of the averaged absolute bearing error figures of Table 1 leads to the following conclusions:

1. Any small aperture system capable of giving a true indication of the phase from normal would yield the results shown in section (A) of this table. The bearing "error" refers to the deviation of the indicated bearing from the direction of arrival of the stronger wave. For waves that arrive from nearly the same direction ($|\xi|$ averaged over $0^\circ - 10^\circ$) the averaged absolute bearing error is small if the amplitude of the interfering wave is one-half that of the stronger wave ($h = 0.5$). When the interfering wave may have any direction relative to the stronger wave ($|\xi|$ averaged over $360^\circ$) the averaged absolute bearing error for $h = 0.5$ is about $18^\circ$. For large amplitudes of interfering waves arriving from nearly the same direction as the stronger wave ($h = 0.8$; $|\xi|$ averaged over $0^\circ - 10^\circ$ - a condition which assumed to apply in multipath sky-wave transmission) the bearing error is still small if the time phase angle between the signals does not exceed $90^\circ$. For time phase angles near $180^\circ$ the indicated bearing deviations increase sharply with an averaged absolute of about $19^\circ$ for the conditions mentioned.

2. Comparison of the results of section (B) with those of section (A) shows that as far as the averaged absolute bearing error is concerned a simple Adcock system gives results which do not differ significantly from a phase-front normal indicator. Of course, as was shown in the curves of Plate I, there are some particular sets of conditions where the two systems do give different indications.

3. Section (C) of table 1 shows that the spaced loop with its second-harmonic (4 lobe) pattern has an averaged absolute bearing error which is less (in some instances by as much as a factor of 2) than the simple Adcock. (This result, however, gives no consideration to the increased number of ambiguities).

4. Perhaps the most important conclusions are those which concern a distribution of narrow-aperture systems. Section (D) shows averaged bearing error figures on a distribution of 8 Adcock unit systems equispaced on circles of diameters of 2 wavelengths and 10 wavelengths respectively. These figures may be compared with the figures for a wide-aperture True Doppler system (the theoretical case of a single rotating element) shown in section (E). For the distribution of narrow aperture systems bearing error figures are shown for both vector average (VA) and limited vector average (LVA) summations. The following points should be noted:

   a. For small angles of separation, this is waves arriving from nearly the same direction, a medium diameter ($2\lambda$) distribution gives excellent results - even better than the True Doppler system of the same aperture. Because it is thought that this case ($h = 0.8$, $|\xi|$ averaged over $0^\circ - 10^\circ$) covers conditions met with in multipath sky-wave transmissions, this result is an important one.

b. When all possible angles ($|\varepsilon|$ averaged over $0^\circ - 360^\circ$) are considered the bearing improvement is much less than for the corresponding diameter True Doppler system. Thus, such a system is less effective in overcoming site-error reflections, although the averaging process does give some improvement as would be expected.

c. As the diameter of a circular distribution of 8 narrow aperture systems is increased, the bearing indication does not continue to improve as it does with the True Doppler (and presumably most other wide-aperture systems). It is probable, though not definitely established, that if the number of systems was increased with the diameter, the average absolute bearing error would decrease with increasing diameter. For 8 unit systems it can be shown that there are certain combinations of system diameter and angles $\Phi$ for which no bearing error improvement is obtained. This statement is probably ture for other numbers of units as well.

d. For small values of $\Phi$ the vector average summation gives somewhat better results than those given by the limited vector average summation. When all possible angles between arriving waves are considered equally probable, the limited vector average summation is slightly superior.

Operation of a distribution of narrow-aperture systems appears to be quite feasible and to have some advantages over a wide-aperture system. Under ordinary "guard" conditions the units could be operated individually, each covering its own share of the frequency spectrum. Whenever an accurate d-f on a particular transmission was desired, the units could be connected automatically according to one of the combining schemes suggested by Plates XXII and XXIII to give results comparable to those expected from a wide-aperture system. Because of these facts the bearing error results obtained on a distribution of narrow-aperture systems are believed to be of considerable significance.





END-Film-6-71

CONFIDENTIAL

ATI- 71240

TITLE: A Comparison of Radio Direction Finding Systems

REVISION (None)

ORIG. AGENCY NO. 9

AUTHOR(S) : Bailey, A.D.; Jordan, E.C.; Webb, H.D.
ORIG. AGENCY : University of Illinois, Dept. of Electrical Engineering, Urbana
PUBLISHED BY : (Same)

PUBLISHING AGENCY NO. (Same)

| DATE | U.S. CLASS. | COUNTRY | LANGUAGE | PAGES | ILLUSTRATIONS |
|---|---|---|---|---|---|
| Dec' 49 | Confd' l | U.S. | English | 51 | table, diagrs, graphs |

ABSTRACT:

Small and large aperture radio direction finding systems (existing and proposed), and limited space distributions of narrow aperture RDF systems are compared in terms of the bearing error which arises when a desired and an undesired signal of the same frequency but having arbitrary relative amplitude, arbitrary time phase difference and arbitrary relative directions of arrival are incident upon each of the several systems. The effect of increasing array diameter upon the bearing error of large aperture systems and distributions of small aperture systems is also considered. The comparative results are shown in the form of bearing error curves, photographs of simulated system performance and as a tabulation of average absolute bearing error for each system. A discussion of the practical problems involved in the realization of some of the proposed systems is included.

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DIVISION: Electronics (3)
SECTION: Navigational Aids (3)

SUBJECT HEADINGS: Direction finders, Radio
Direction finding - Fix evaluation
Direction finding techniques