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TECHNICAL MEMORANDUM NO. 3

PHYSICAL PHENOMENA AFFECTING THE DYNAMIC BEHAVIOR OF FINNS

BY

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PREPARED UNDER CONTRACT N6-Onr-25129
(Hr-041-113)
FOR
OFFICE OF NAVAL RESEARCH

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STANFORD, CALIFORNIA

MAY 9, 1961
MEMORANDUM NO. 3

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I. INTRODUCTION

A. Purpose

This memorandum can be considered an extension of Technical Memorandum No. 2. In that memo we discussed stabilization by fins, postulating an "idealized" case and a "non-idealized" case. The non-idealized case was defined in order to take into account certain effects neglected in the ideal case. Specifically, we noted that in practice the equations describing the lift and moment of the fins: (1) had certain non-constant parameters (e.g., \( \frac{\partial L}{\partial \alpha} \) varies as \( V^2 \)); (2) had certain non-linearities (e.g., \( L_{\text{max}} \) = finite number); and (3) had certain higher-order terms previously neglected (e.g., the so-called "frequency effects").

In this memorandum we will consider in greater detail the physical phenomena or "causes" out of which the above-mentioned "effects" arise.

B. Certain Physical Phenomena

One would arrive at the "idealized" case of Memo #2 by using the classic theory of airfoils in steady motion in an unbounded medium; by assuming the ship's velocity to be always the same; by assuming that the capacity of the system
is never exceeded; and by making a one-degree-of-freedom assumption for the ship (which last will not be commented on in this memo). We will discuss four complications or additional phenomena, which affect the actual dynamic behavior of fins:

1. The Velocity Effect
2. The Free-Surface Effect
3. Cavitation Effects
4. Unsteady Motion Effects
II. THE VELOCITY EFFECT

This phenomenon is very simple physically. It only has importance because of the closed-loop type control being used. We have the following situation. Within the working range of the fins,

\[ L = C_L \left( \rho \frac{v^2}{2} \right) \text{Area} = k \text{c} \cdot V^2 \quad (1) \]

Thus, \[ \frac{dL}{dc} = k \cdot v^2 \quad (2) \]

This means that the gain of the fins varies as \( V^2 \). Now a high-performance closed-loop system is always quite sensitive to gain. Thus the variation above must be compensated for, when the ship changes speed. If the range of operating speeds is large, this adjustment becomes more and more difficult.

The variation of \( \frac{dL}{dc} \) may not be quite square law with speed, due to Reynolds number effects or cavitation. Hence it is desirable to know the exact law of variation of \( \frac{dL}{dc} \) with velocity.
III. THE FREE-SURFACE EFFECT

The stabilizing fin of a ship actually moves in a fluid medium which has a free-surface boundary in the vicinity of that fin, as below:

![Diagram of stabilizing fin and free-surface boundary](image)

It has been shown both theoretically and experimentally that to a first approximation, the free-surface simply reflects and inverts the load lines of the foil to form an equivalent biplane as above.¹ A comparison of Glaucott's biplane coefficient (versus the depth/chord ratio) and the experimentally determined lift of several wings is shown in Figure 1, taken from Cannon.

Clearly the ratio depth/chord should be greater than three if free-surface effects are to be negligible. Furthermore, if this ratio becomes less than 1.5 at any time (e.g., as the ship rolls in waves) the foil may begin to "ventilate", that is, to suck down surface air intermittently, causing the lift to be erratic.

IV. CAVITATION EFFECTS

A. General

If hydrofoils are operated under conditions such that heavy cavitation occurs, the center of pressure will be seriously shifted and hence the moment on the fins will be seriously affected. Assuming, on the other hand, that the foils are not seriously overloaded, the principal effect of cavitation is to limit maximum lift, and it is this side of the question which we will discuss here.

By definition, we say that the cavitation limit has been reached, when the minimum local pressure at any point on the foil is just reduced to the vapor pressure (i.e., essentially to zero). Thus cavitation depends on the pressure loading of the foil and specifically on the peak of negative pressure loading. One feels intuitively that the chance for cavitation must increase with greater total loading, and with the non-uniformity of pressure distribution. These intuitions can be made much more precise.

B. The Cavitation Number and the Cavitation Index

A cavitation situation is most commonly characterized by the non-dimensional scaling number,

\[ Q = \frac{P_0 - P_x}{q} = \frac{P_{cr}}{q} \]  

This is shown clearly in J. F. Allen, "The Stabilization of Ships by Activated Fins," Trans. Inst. Naval Architects, Vol. 87, 1946, Figure 16, page 134.
Where,
- \( p_c \) = "free-stream" pressure of undisturbed fluid
- \( p_v \) = vapor pressure of fluid
- \( q \) = kinetic head = \( \sqrt{2} \) \( \alpha V^2 \)

Note that,
\[
    p_c = p_a + \gamma h = (\text{atmos. press.}) + \gamma (\text{depth})
\]
\[
    \gamma = \text{specific weight of fluid}
\]

Now to every cavitation situation and scaling of that situation there corresponds a \( Q = Q_{cr} \) at which the cavitation limit is reached. Unfortunately, \( Q_{cr} \) is quite dependent on the geometry of the situation, and in particular for foils, \( Q_{cr} \) is quite dependent on the angle of attack. For our purposes we would like to find another non-dimensional number, characterizing cavitation, which is less dependent on the angle of attack. Let us make the following definition,

Cavitation index \( C_L \) \( = \frac{C_L}{C_{cr}} = \frac{q}{F_{cr}} \)

Again we may define \( C_L = C_{cr} \) at the cavitation limit. It is not hard to convince oneself that \( C_L \) is less dependent on angle of attack or \( C_L \) (which is to say almost the same thing) than is \( Q_{cr} \). In fact, insofar as cavitation depends on the total loading, \( C_{cr} \) is independent of \( C_L \), for \( C_L \) equals nothing more or less than the ratio of
actual loading to allowable peak negative pressure drop, that is,

\[ \mathbf{C} = \frac{\text{Lift/Area}}{P_{cr}} \]  \hspace{1cm} (6)

Insofar as cavitation depends on the distribution of pressure, \( \mathbf{C}_{cr} \) is a function of \( C_L \). Hence \( \mathbf{C}_{cr} \) essentially characterizes the favorableness of the pressure distribution for a given foil at a given angle of attack. This makes it quite a useful number as we shall see. In any event, given \( \mathbf{C}_{cr} \) we immediately have the critical loading, from equation (6) above,

\[ (\text{Lift/Area})_{cr} = P_{cr} \times \mathbf{C}_{cr} \]  \hspace{1cm} (7)

The following manufactured examples will give a pretty good idea of the values \( \mathbf{C}_{cr} \) may be expected to have in practice.

1) \textbf{Kin loaded only with positive pressure}

\[ \begin{array}{c}
\text{pressure} \\
\end{array} \]

But the positive pressure can increase without limit without causing cavitation, hence \( \mathbf{C}_{cr} = \frac{\mathbf{C}}{P_{cr}} = \mathbf{C} \).
2) **Fin loaded equally and uniformly top and bottom**

![Pressure Diagram]

Then at the cavitation limit, \( \Delta p = -P_{cr} \) and \( C_{cr} = 2.0 \).

However, it is well known that the under surface of a foil contributes little lift, so perhaps a more realistic case is,

3) **Fin loaded uniformly on top only**

![Pressure Diagram]

Then at cavitation limit, \( \Delta p = -P_{cr} \), and \( C_{cr} = 1.0 \).

That this is realistic is indicated by experimentally determined values of \( C_{cr} \), which approach but almost never exceed 1.0, even under the most optimized conditions. If the pressure distribution is not uniform, \( C_{cr} \) must come down. Specifically, the pressure distribution becomes less uniform at the very high and the very low angles of attack. Thus if we wish to operate a foil to its separation limited angle of attack we must accept a \( C_{cr} \) of approximately 0.3, while for very low angles of attack, \( C_{cr} \) tends to zero (naturally).

C. **Anti-Cavitation Design and the Mach Number Analogy**

The most obvious approach to anti-cavitation design is
to find symmetrical foil sections which lead to relatively uniform pressure distributions, hence to higher $C_{cr}$, and thence to higher allowable loading. We are greatly aided in this search by an interesting coincidence. It happens that the prediction of the cavitation limit for hydrofoils is very closely analogous to the prediction of the compressibility burble point (critical Mach number) for airfoils. Because of the monotonic, one-one correspondence between velocity and pressure, anti-burble sections must have the same characteristics as anti-cavitation sections, i.e., tend to produce uniform pressure distribution. Hence, one might expect to find good hydrofoil sections among the already developed anti-burble sections of the NACA. As a matter of fact, NACA has already studied the use of certain of these sections for hydrofoil purposes.\(^3\)

As a most specific aid, all the information on critical Mach number predictions in NACA's useful compendium "Summary of Airfoil Data" (NACA Report 942) may be converted over to critical cavitation index or to critical cavitation speed predictions, by one-to-one relationships, as follows.

D. Converting Critical Mach Number to Cavitation Index

We first define the Mach number for the foil as:

$$M = \frac{\text{speed of foil}}{\text{speed of sound in undist. fluid}}$$  \(8\)

\(^3\) For example, John Stack, NACA Report 763, discusses the NACA 16-series sections designed to delay the compressibility burble. J. M. Benson and R. S. Land, NACA Wartime Report L-758, test one of these sections as a hydrofoil.
Define the **compressibility limit** as the point at which the highest local velocity over the foil just equals the local speed of sound. Then for every compressibility situation there exists a $M = M_c$ at which the compressibility limit is reached.

$$M_c = \frac{\text{speed of foil}}{\text{speed of sound}}$$ (9)

Let $V$ denote the highest local velocity over the foil by the quantity $(V + \Delta V)$. By definition, at the limit this just equals the local speed of sound. If we chose to neglect the effect of compressibility on the local speed of sound, we would obtain the result,

$$M_c = \frac{V}{V + \Delta V} = \frac{1}{1 + \frac{\Delta V}{V}}$$ (10)

Further, if we neglect compressibility, we find that $(\Delta V/V)$ is a function of $\frac{\Delta P}{q}$, in fact,

$$\frac{\Delta V}{V} = -1 + \sqrt{1 + \frac{\Delta P}{q}}$$ (11)

Now, it has been shown by Karman and others that even if the effect of compressibility on the local speed of sound and the local pressure is taken into account, $M_c$ still may be expressed as a function of $\frac{\Delta P}{q}$, where $\Delta P$ is the maximum change of pressure (in the suction sense) on the foil, predicted by low-speed or incompressible theory.

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Thus, \[ M_{cr} = M_{cr} \left( \frac{\Delta p}{q} \right) \] (12)

Or, \[ \frac{\Delta p}{q} = f(M_{cr}) \] (13)

Equation (13) is plotted in "Summary of Airfoil Data", or rather, one may find there a plot of \( S \) versus \( M_{cr} \), where,

\[ S = 1 - \frac{\Delta p}{q} \] (14)

Equation (13) is true for any airfoil. Now a given airfoil will have a given pressure distribution at a given angle of attack, hence one may plot \( M_{cr} \) as a function of \( C_L \) (the low-speed lift coefficient). There are many such plots in "Summary of Airfoil Data". From these plots and equation (13) we may have \( \frac{\Delta p}{q} \) as a function of \( C_L \), for numerous different foils. We may further obtain the critical cavitation index and the critical cavitating speed for these foils by the following means.

Remember that by definition at the cavitation limit,

\[ P_{cr} = -\Delta p = -q f(M_{cr}) \] (15)

From equation (15) it follows (again by definition) that,

\[ \Delta \theta_{cr} \equiv \frac{qC_L}{P_{cr}} = -\frac{C_L}{f(M_{cr})} \] (16)
and,
\[ v_{cr}^2 = q \times \frac{2}{\rho} = \frac{P_{cr}}{f(M_{cr})} (\frac{2}{\rho}) \]  

(17)

Note that the curves of \( M_{cr} \) versus \( C_L \) in "Summary of Airfoil Data" have been computed from theoretical (low-speed) pressure distributions, but that these theoretically calculated pressure distributions are generally quite accurate.

The above treatment is of necessity somewhat sketchy. However, the essential point is simple and should not be obscured by the semblance of difficulty which the reference to compressibility effects may tend to generate. The essential point is this. There are in "Summary of Airfoil Data" numerous curves showing \( M_{cr} \) as a function of low-speed lift coefficient, \( C_L \) for various foil sections. These may be converted into curves of peak normalized suction pressure (\( \Delta p/q \)) versus \( C_L \) by means of equation (13), which appears in modified form in the same NACA report. These last curves in turn may be converted into curves showing \( \mu_{cr} \) and \( V_{cr} \) versus \( C_L \) by means of equations (16) and (17).

E. Theoretical and Experimental Results

Figure 2 shows some theoretical curves converted from the "Summary of Airfoil Data" (NACA Report 824). These curves show critical cavitation speed versus lift coefficient for symmetrical 6-series foils of various thickness ratios. Note the difference in behavior between the thicker foils and the thin foils. Fortunately, the characteristics
of the thicker foils are more suited to our purposes. The 6-series foils, by the way, have the most uniform pressure distribution of the more or less standard NACA foils, but the newly originated 16-series has even superior characteristics.

Certain experimental results from Cannon and NACA\(^5\) are shown in Figure 3. These results indicate that even for cambered foils of special type \(c_r\) will not much exceed 0.3 at the higher angles of attack (or higher lift coeff.\). The actual limiting load for \(c_r = 0.3\) may be found by the following calculation. For a depth of about fifteen feet,

\[
p_{cr} = p_o - p_v = p_d + e^h - p_v = 2p_o + e^h = 3000/#/ft^2
\]

Hence for \(c_r = 0.3\) the critical loading is,

\[
\frac{(Lift/AREA)}{c_r} = 0.3 \times 3000 = 1000/#/ft^2 = 1/2 \text{ ton/ft}^2
\]

This is the origin for our figure of 1/2 ton/ft\(^2\) cavitation loading.

It is true that further lift can be obtained even after cavitation begins, but if this process is carried very far the center of pressure will be seriously shifted, as mentioned before, and the law of diminishing returns will begin to exercise itself.

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\(^5\) Cannon, op. cit., NACA Wartime Reports L-766 and L-758.
F. Other Ways to Improve Loading

After the best symmetrical section has been found, it is clear from the various references that further improvement at a given angle of attack may be had by the use of camber. Unfortunately, a variable angle of attack requires a correspondingly variable camber. It is as yet a moot point whether or not a variable flap could simulate variable camber satisfactorily from the cavitation point of view.

By analogy to compressibility treatments, sweepback and boundary layer control might also be useful, but here again the question is as yet moot.
All the previous comments have applied most exactly to so-called steady motion, in which the fin moves at a constant velocity and a constant angle of attack. If the fin is undergoing unsteady motion (e.g., has an oscillating angle of attack) certain higher order terms appear in the lift and moment equations. There is by now an extensive literature on this subject. The best discussion of what these terms mean physically is perhaps Karman and Sears.

The theory of the 2-dimensional (incompressible) case may be found in many sources, beginning with Glauert and ending with Theodorsen. All these authors arrive at strictly comparable results, but presented in more or less convenient form. Theodorsen's results are about the most usable.

The theory of the 2-dimensional case has been quite thoroughly verified by the experimental work of Reid and others.


The theory of the 3-dimensional case (incompressible) is treated at length (with tables of the appropriate functions) in Biot and Boehnlein.\textsuperscript{10}

We have not as yet been able to find a definitive experimental verification of the 3-dimensional results, but they do not differ markedly from the 2-dimensional results and hence are verified indirectly by Reid's work.

In either case, the form of the lift and moment equations depends on the choice of axis of rotation, and the principal argument of the equations is the so-called reduced frequency.

\[
\text{reduced frequency} = k = \frac{\omega b}{V} \quad (18)
\]

where,

\( \omega \) = frequency of oscillation in radians/second
\( b = \frac{1}{2}\)-chord in feet
\( V = \) velocity in feet/second

B. Equations for Axis at the Quarter Chord

If we arbitrarily take our axis of rotation at the \( \frac{1}{4}\)-chord, the equations for section lift and moment reduce to the following transfer function form:

2-dimensional case

\[
\frac{L}{A} = 2\pi \left( \frac{b V^2}{2} \right) 2b \left[ C + (2C + 1) \left( \frac{4k}{2} \right) + \left( \frac{4k}{2} \right)^2 \right] \quad (19)
\]

\[ M = 2\pi \left( \frac{c'^2}{2} \right) b^2 \left[ 0 + 2\left( \frac{1}{2} \right) + \frac{3}{2} \left( \frac{1}{2} \right)^2 \right] \]  

(20)

**3-dimensional case**

\[ M = 2\pi \left( \frac{c'^2}{2} \right) b^2 \left\{ P_{AR} + (2P_{AR} + 1) \left( \frac{1}{2} \right) \right\} \]  

(21)

\[ M = 2\pi \left( \frac{c'^2}{2} \right) b^2 \left\{ P_{AR} - Q_{AR} + [2(P_{AR} - Q_{AR}) + 2] \left( \frac{1}{2} \right) \right\} \]  

(22)

Where \( C, P_{AR}, \) and \( Q_{AR} \) are complex functions of \( k \). But \( P_{AR} \) and \( Q_{AR} \) are also functions of aspect ratio such that,

\[ P_{\infty} = Q_{\infty} = C \]  

(23)

Hence the 2-dimensional and 3-dimensional equations above are compatible in the limit \( (AR \to \infty) \).

Tables of \( C = P + jQ \) are given in Theodorsen; and tables of \( P_{AR} = P_{AR} + jQ_{AR} \) and \( Q_{AR} = P_{AR} + jQ_{AR} \) are given in Biot and Boehnlein, for various aspect ratios.

**C. Vector Locus of Relative Lift**

If, in either case, we define the quantity inside the brackets as \( L/L_0 = \text{relative lift} \) or \( M/M_0 = \text{relative moment} \), then we have a comparison between the lift (or moment) of the given foil at a given frequency and the lift (or moment) of a 2-dimensional foil at zero frequency. Figure 4 shows \( L/L_0 \) plotted as a vector locus (in terms of the argument \( k \)) for \( AR = \infty \) and \( AR = 5 \). Notice that the effect of finite aspect ratio is to remove the lagging kink in the
vector locus. In fact, the locus for \( AR = 5 \) can be closely approximated by a quadratic in \( k \) as shown in Figure 5. For \( AR = 5 \) it is clear that the lift, if anything, tends to lead the angle of attack. How much, depends of course on \( k \).

Typical values of \( k \) might be as follows. Let \( c = 4 \) ft, then \( b = c/2 = 2 \) ft. Let \( V = 30 \) ft/second. Then,

\[
k = \frac{gb}{V} = \frac{2}{15} = \frac{27}{15} = \frac{f}{2} \times 4
\]

Now at \( f = 0.5 \) cycle/second \( \equiv \) 5 x (ship's natural freq.)

\[
k \equiv 0.2
\]

Referring this value to Figure 4 it appears that the frequency effects in the lift equation, while noticeable, will not produce drastic changes.

D. **Frequency Effects in Moment Equation**

The frequency effects in the moment equation will be much more important, because the moment would otherwise be zero or nearly zero. To make the moment equation complete we should increase the coefficient of its inertia term to take account of the fin's own (metal) inertia; the term shown in equations (20) and (22) is only the so-called "induced" inertia. Calculation indicates that (for numbers as above) the self-inertia will probably be less than one/half the induced inertia.

Assume that the moment equation is corrected for self-inertia. Assume further that the axis of rotation for the
finite aspect ratio foil is adjusted to be zero at \( k = 0 \).

Then at \( k = 0.1 \) (\( f = 0.5 \) using \( b = 2 \) ft and \( V = 30 \) ft/sec) it appears that the moment due to angular velocity is about ten times the moment due to static angle or to angular acceleration. This indicates that the angular acceleration loading will not be very important (for design chords and speeds similar to those mentioned). The static angle loading also will be generally smaller than the angular velocity loading, perhaps gaining importance at the higher speeds where the static term is accentuated (by the change of \( k \)) and where cavitation may shift the center of pressure.
VI. CONCLUSIONS

In this memorandum, we have discussed a number of physical phenomena which affect the dynamic behavior of fins. At the moment it appears that a knowledge of these phenomena, including a knowledge of classic aerodynamic theory for steady motion, structural requirements, will constitute a reasonable working basis upon which to begin the design of fins.

We have considered four arbitrary categories of effects:
(1) The velocity effect; (2) The free-surface effect;
(3) Cavitation effects; and (4) Unsteady motion effects.
Of these, the last two are the most critical, but each has important and obvious implications concerning the design of fins and/or positioning motors. In subsequent memoranda we will discuss some of the relations between the body of knowledge mentioned above and practical fin and positioning motor design.
VIII. BIBLIOGRAPHY

Cavitation and the Free Surface Effect:


Unsteady Motion Effects:


PHYSICAL PHENOMENA AFFECTING THE DYNAMIC BEHAVIOR OF FINS, by Joseph H. Chadwick, Jr. 9 May 51, 22p., illus. (Tech Memo No. 3) (Contract N6onr-25129)

SUBJECT HEADINGS
DIV: Fluid Mechanics (9)
Fins
SEC: Dynamics (1).
Hydrofoils - Hydrodynamics (4)
(Copies obtainable from AS(TIA-DSC)
(NR-041-113)