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A COMPARISON OF DUCTED PROPELLER THEORY  
WITH BELL X-22A EXPERIMENTAL DATA 

by 

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ABSTRACT

For the forward flight regime, a limited comparison is made between theoretical predictions of duct pressure distributions and data obtained from one-third and full scale model tests of the Bell X-22A ducted propeller unit. The theoretical calculations are based upon studies of ducted propellers with finite blade number which have been undertaken at Therm Advanced Research, Inc. It is found that the theory is in reasonable agreement with experiment and generally tends to underestimate the measured pressures. Also, the characteristic shape of the predicted distribution agrees well with the measured distribution. More conclusive evaluation of the reliability and range of applicability of the theory is needed, not only for forward flight, but also for the static case and in transition.
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NOMENCLATURE

\( C_T \)  propeller thrust coefficient, \( T_p/\frac{\rho}{2} U^2 \pi R_p^2 \)

\( c \)  duct chord, ft

\( c_p \)  duct surface pressure coefficient, \( \frac{(p-p_0)}{\frac{\rho}{2} U^2} \), see also Eq. (1)

\( J \)  propeller tip advance ratio, \( \frac{U}{\Omega R_p} \)

\( p \)  static pressure on duct surface, lb/ft\(^2\)

\( p_a \)  free stream static pressure, lb/ft\(^2\)

\( R_p \)  propeller radius, ft

\( T_p \)  propeller thrust, lb

\( U \)  free stream velocity, ft/sec

\( u \)  total axial perturbation velocity on duct surface, ft/sec

\( x \)  axial distance from duct leading edge, ft

\( \Delta c_p \)  duct net loading, or inner surface pressure coefficient minus outer surface pressure coefficient

\( \rho \)  fluid density, lb sec\(^2\)/ft\(^4\)

\( \Omega \)  propeller angular velocity, rad/sec
A comprehensive theoretical study of ducted propellers with finite blade number was undertaken at Therm Advanced Research, Inc.\textsuperscript{1-9}, the major portion of the study being concerned with the determination of the steady loading on the duct in forward flight at zero angle of attack. In order to facilitate preliminary design applications, the theory as developed\textsuperscript{1,2,4} has been put into the form of a simple numerical procedure\textsuperscript{5,8} which permits the rapid calculation of effects of varying duct parameters such as camber, thickness and chord-to-diameter ratio, as well as propeller parameters such as thrust coefficient, blade number, advance ratio, tip clearance, and axial location in the duct.

An essential part of this study would have been the experimental evaluation of the reliability and range of applicability of the theory as developed. Unfortunately, a lack of suitable experimental data on ducted propellers precluded such an evaluation. However, with the current design and development of the X-22A Tri-Service VTOL Aircraft by Beli Aerosystems Co., both one-third scale and full scale test results of a powered ducted-propeller configuration became available. These tests were not, of course, designed specifically to provide a means for verification of our theory. However, by a judicious choice of parameters and a careful examination of the test data, it was possible to extract sufficient results to enable us to obtain at least a preliminary comparison with our theoretical predictions.

The purpose of this report, then, is to make such a comparison for the forward flight regime at zero angle of attack. The first Chapter outlines briefly the basic mathematical model, the assumptions made and the development of the theory. Chapter Two is devoted to the comparison of the one-third scale X-22A test data with theoretical predictions while Chapter Three gives corresponding results for the comparison with the full scale X-22A tests. Finally, the results are summarized and appropriate conclusions drawn as to the degree of validity of the theory.
CHAPTER ONE
BASIC THEORY

1.1 Formulation

The three-dimensional theory of ducted propellers in forward flight as developed at Therm Advanced Research, Inc. has been reported previously. However, it appears appropriate to review briefly the basic ideas underlying the theory before proceeding to the details of the limited comparison with experiment. The mathematical model for the ducted propeller is formulated as follows. A lightly-loaded ducted propeller in uniform motion at zero angle of attack in an inviscid, incompressible fluid is considered. Viewed in propeller-fixed coordinates, the duct is replaced by suitable distributions of sources and vortices which are used to represent the effects of thickness and camber. These singularities are placed on an appropriate mean reference cylinder. The propeller is represented by the classical model of a bound, radial vortex line of varying circulation for each blade accompanied by associated trailing helical vortex sheets. The path of these trailing vortices is determined solely by the incoming free stream and the angular rotational velocity of the propeller.

If the duct geometry is specified and the form of the propeller circulation distribution is assumed, the formulation of the governing equations follows the procedures of thin lifting-surface theory. That is, we require the flow to be tangent everywhere to the duct surface and satisfy the Kutta condition at the trailing edge. Within the limitations of the theory, the problem of determining the steady duct loading is equivalent to the problem of a ring wing with the same thickness distribution but having an "effective" camber composed of the original duct camber plus the camber required to cancel the radial velocities induced by the propeller trailing vortices as well as by the sources used to provide duct thickness.

The duct source strength is calculated directly from the known rate of change of the duct thickness and the corresponding induced camber may be expressed as a simple matrix operation on known thickness distribution coefficients. With regard to the propeller induced camber, this is determined once the propeller circulation distribution is specified. The duct vortex strength is then obtained by correcting the value associated with the effective camber for the effect of duct chord-to-diameter ratio. This operation has also been expressed in simple matrix form.

In order to present the analytic solution in a form suitable for detailed aerodynamic evaluation, certain calculations requiring the use of a digital

* The propeller circulation was assumed to be reasonably approximated by the Betz optimum with a Goodman tip correction for these calculations.
computer were carried out for representative ducted propeller configurations, an engineering design procedure and accompanying charts were prepared and the results were combined in report form\textsuperscript{5,8}. Thus, a simple procedure is available whereby the steady duct net loading, surface pressure distributions, thrust and sectional radial force, pitching moment and center of pressure can be calculated by hand. Configurations whose parameters fall somewhere in-between or outside the tabulated parameter values can be evaluated by interpolation and/or extrapolation.

1.2 Application to X-22A Calculations

The calculation of the duct net loading and surface pressure distributions was carried out for the Bell X-22A one-third scale and full scale models at several flight conditions, the hand-calculation procedures of Ref. 5 being used for this purpose. In general, the X-22A parameters did not correspond exactly with those for which computational tables had been prepared. As a result, interpolation was required in several of the tables given in that report. The duct thickness and geometric camber distributions were approximated by seven and thirteen term power series respectively.

Two changes were made in the calculation procedure of Ref. 5, one being in the final calculation of the surface pressure coefficient, and the other in the inclusion of hub effects.

If \( u \) is the total axial perturbation velocity on the assumed reference cylinder which represents the duct and \( U \) is the free stream velocity, the full Bernoulli equation for the surface pressure coefficient \( c_p \) is given by

\[
c_p = \frac{-2u}{U} - \left(\frac{u}{U}\right)^2
\]  

(1)

For the calculations contained in this report, Eq. (1) was used for \( c_p \) with \( u \) composed of the contributions from the singularities representing the duct, propeller and hub. This method differs from that employed in Ref. 5. There, either the well-known linear form of Bernoulli's equation or the form of Eq. (1) with the term of order \( u^2 \) containing only the contribution from the duct source distribution was used.

The second change was to permit the inclusion of the effect of the propeller hub. For this, the hub was replaced simply by an appropriate distribution of line sources along the hub axis. The effective induced camber and the contribution to the axial perturbation velocity on the shroud were then determined straightforwardly in accordance with the methods as given in, say, Ref. 12.
2.1 Model Data and Tests

The powered one-third scale model of the Bell X-22A ducted propeller was tested in the 8' by 10' subsonic wind tunnel at the David Taylor Model Basin. The tests were carried out in the period from the Fall of 1963 to the Spring of 1964 and were designed to obtain propulsion data in order to verify the estimated performance. Data was recorded for the various combinations of angle of attack, forward flight speed and power input that were anticipated for the complete flight regime of the X-22A.

The model geometry was scaled down exactly, from its full scale counterpart, by a factor of three except for the propeller hub which was somewhat larger than one-third scale in order to house the driving motor. Dimensions of the principal parameters for both the one-third scale and full scale units are listed below. A three bladed propeller was used; further information regarding the model and test runs may be found in Ref. 13.

<table>
<thead>
<tr>
<th>Principal Parameters</th>
<th>1/3-Scale Model</th>
<th>Full Scale Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duct chord</td>
<td>16.3&quot;</td>
<td>49.0&quot;</td>
</tr>
<tr>
<td>Maximum duct thickness</td>
<td>2.8</td>
<td>8.4</td>
</tr>
<tr>
<td>Radius of reference cylinder</td>
<td>15.5</td>
<td>46.6</td>
</tr>
<tr>
<td>Propeller radius</td>
<td>14.0</td>
<td>42.0</td>
</tr>
<tr>
<td>Propeller tip clearance</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>Propeller axial location</td>
<td>4.7</td>
<td>14.0</td>
</tr>
<tr>
<td>Hub length</td>
<td>26.8</td>
<td>43.9</td>
</tr>
<tr>
<td>Maximum hub radius</td>
<td>9.4</td>
<td>17.5</td>
</tr>
</tbody>
</table>

Static pressure distributions were measured at five azimuthal stations on the duct with sixteen taps on the inner surface and fourteen on the outer surface. In view of the close proximity of several azimuthal stations to the elevons and supporting struts, there was a noticeable variation in many readings and, accordingly, considerable care had to be exercised in selecting what could be considered as reliable data. Consequently, the data points labelled "experimental" in the comparison curves represent the azimuthal average of the readings retained at any axial location on the shroud.
2.2 Results

In Fig. 1 we see a comparison of the theoretical predictions and experimental results for the duct inner and outer surface pressure coefficients. For this case, the propeller tip advance ratio, \( J = \frac{U}{\Omega R_p} \), was 0.302 and the propeller thrust coefficient, \( C_T = \frac{T_p}{\rho U^2 \pi R_p^2} \), was 0.129, values which correspond to customary cruise flight. Despite the many differences between the mathematical model used in the theory and the actual model used in the tests, the theoretical predictions and experimental measurements are in reasonable agreement, the theory generally underestimating the magnitude of the pressure coefficient. However, note that the characteristic shapes of the two curves are very similar.

Variation of the net duct loading, \( \Delta c_p \), namely, the inner surface pressure coefficient minus the outer surface pressure coefficient, is shown in Fig. 2. Here again, the theoretical curve follows the experimental data thus indicating that the duct vortex distribution (which is simply proportional to the net loading) is predicted fairly well.

The calculated pressure coefficients on the duct and hub units without propeller are shown in Fig. 3 along with the corresponding one-third scale data. To illustrate the effect of the hub, the contribution of duct alone is also shown. It is seen that aft of approximately the one-quarter chord, or equivalently, the propeller plane, the hub has little effect on pressure coefficient. Comparison of Figs. 1 and 3 reveals that, for this particular propeller thrust coefficient, the propeller contribution is at most about 10% of the total value of the pressure coefficient.

A comparison of theory and experiment for a larger propeller thrust coefficient is shown in Fig. 4 for \( J = 0.242 \) and \( C_T = 0.486 \). Reasonable agreement between the two results is obtained, particularly with respect to the characteristic shapes of the curves, although the theory still tends to underestimate the measured values. In this case, the propeller contribution has now risen to about 20-30% of the total value of the pressure coefficient.
FIGURE 1

DISTRIBUTION OF DUCT SURFACE PRESSURES
COMPARISON OF THEORY AND ONE-THIRD SCALE EXPERIMENTAL DATA

$J = 0.302$, $C_T = 0.129$
FIGURE 3

DISTRIBUTION OF DUCT SURFACE PRESSURES
COMPARISON OF THEORY AND ONE-THIRD SCALE EXPERIMENTAL DATA
No Propeller
Figure 4
DISTRIBUTION OF DUCT SURFACE PRESSURES
COMPARISON OF THEORY AND ONE-THIRD SCALE EXPERIMENTAL DATA
\( J = 0.242 \), \( c_T = 0.486 \)
CHAPTER THREE
COMPARISON WITH FULL SCALE EXPERIMENTS

3.1 Model Data and Tests

The tests of the powered full scale model of the X-22A ducted propeller unit, conducted at the NASA Ames Research Center in the 40' by 80' wind tunnel in August 1964 and March 1965, are reported in Ref. 14. The overall objective of these tests was essentially the same as those undertaken for the one-third scale model. In addition, a model which had a modified inlet camber and duct diffuser ratio was tested to determine if any benefits in performance could be obtained as a result of this type of configuration change. The essential model dimensions are listed in Chapter Two.

For these tests, the static pressure distribution was measured at three azimuthal stations on the duct with eleven pressure taps on the inner surface and eight on the outer surface. Again, a selected average of the pressure readings was used to determine the experimental points shown in the comparisons.

Two full scale test runs were compared with the corresponding theoretical predictions. As opposed to the one-third scale model test cases, the elevons were not mounted in the duct unit for these runs.

3.2 Results

The results for $J = 0.200$ and $C_T = 0.276$ are shown in Fig. 5 and the results for $J = 0.249$ and $C_T = 0.496$, in Fig. 6. The values of $C_T$ were calculated on the assumption that the propeller thrust is simply the difference between the total thrust and the shroud thrust, both of which were measured.

Generally, the agreement between theory and experiment is quite good for the outer surface but not as favorable for the inner surface where the differences at several stations are as great as 40%. However, the characteristic shape of the theoretical curve again matches well with the test results.

Further calculations at higher propeller thrust coefficients show that the discrepancy between theory and experiment increases with increasing thrust coefficient. This is to be expected since the linearized theory begins to break down at large thrust coefficients.

3.3 Modifications to Theory

Two modifications to the theory were investigated in order to determine if better agreement with test data could be obtained. First, the duct boundary layer displacement thickness was estimated, based on the known inviscid velocity
FIGURE 5
DISTRIBUTION OF DUCT SURFACE PRESSURES
COMPARISON OF THEORY AND FULL SCALE EXPERIMENTAL DATA
\( J = 0.200 \), \( c_T = 0.276 \)
field, and then added to the duct thickness to yield a new "effective" duct thickness. The calculation of $c_p$ was then carried out as before but based on this new thickness distribution. In view of the extremely small viscous layer, the results were virtually unchanged, thus indicating that inclusion of the boundary-layer thickness in the previous calculations is not warranted.

Secondly, we investigated the effect of duct reference cylinder location with respect to the propeller tip. That is, for a duct of large thickness — as was used in the X-22A — it appears more appropriate to distribute the duct vortices on reference cylinders more nearly approximating the true inner and outer duct surfaces. This was in fact carried out for run #39-29 where we found that a relocation of the reference cylinder produced better agreement between theory and experiment for the inner surface pressure distribution. This is particularly so ahead of the propeller plane where the theoretically obtained values were increased by approximately 20%. Aft of the propeller, the effect of relocating the reference cylinder diminishes rapidly. The entire outer surface pressure distribution is relatively unaffected by appropriate relocation of the mean reference cylinder.
CONCLUSIONS

As a first step in evaluating the applicability of the three-dimensional theory of ducted propellers which had been developed at Therm Advanced Research, a comparison was made between theoretical predictions of the duct surface pressure distributions and appropriate data obtained from one-third and full scale model tests of the Bell X-22A aircraft for the forward flight regime. Despite the very limited quantity of suitable data for comparison purposes, it was still possible to draw the following conclusions:

The theoretical predictions are in reasonable agreement with experiment and generally tend to underestimate the measured pressures.

The characteristic shape of the predicted distribution agrees well with the measured distribution.

It is important to determine accurately the contribution to the pressure coefficient of the duct separately as this comprised from 60 to 90 percent of the total value.

Improved agreement between theory and experiment was obtained when — to make allowances for the relatively thick duct — mathematical singularities were distributed on individual reference cylinders representing the inner and outer surfaces, rather than on a single mean reference cylinder.

In order to consider all regions of practical interest, more conclusive evaluation of the reliability and range of applicability of the theory is needed, not only for forward flight, but also for the static case and in transition.
REFERENCES


For the forward flight regime, a limited comparison is made between theoretical predictions of duct pressure distributions and data obtained from one-third and full scale model tests of the Bell X-22A ducted propeller unit. The theoretical calculations are based upon studies of ducted propellers with finite blade number which have been undertaken at Therm Advanced Research, Inc. It is found that the theory is in reasonable agreement with experiment and generally tends to underestimate the measured pressures. Also, the characteristic shape of the predicted distribution agrees well with the measured distribution. More conclusive evaluation of the reliability and range of applicability of the theory is needed, not only for forward flight, but also for the static case and in transition.
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