Application of the Chisholm-Laird two phase flow correlation to flow of varying quality.
APPLICATION OF THE CHISHOLM-LAIRD
TWO PHASE FLOW CORRELATION TO
FLOW OF VARYING QUALITY

DONALD E. ECKELS
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Donald E. Eckels
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by

Donald E. Eckels

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

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ABSTRACT

For the purpose of studying the effect of varying quality upon the pressure drop accompanying isothermal, two phase, two component flow, a horizontal, one-inch flow channel of circular cross section, into which air could be injected at various points, was designed and constructed.

Preliminary tests were made using water and air injected at a single position. The resultant data confirmed the findings of Chisholm and Laird for turbulent flow in a smooth tube.

Tests were then conducted in which the quality, the percent mass of gas phase of the total mass flowing, was varied by injecting known amounts of air at seven injection points along the 42-foot test section. The Chisholm-Laird correlation provided pressure drop predictions to within 25% of the measured values.
ACKNOWLEDGMENT

The author wishes to express his appreciation to Professors C. D. G. King and P. F. Pucci for their help and guidance during this investigation.

The technical excellence of the machine work by Mr. Kenneth Mothersell of the Mechanical Engineering Department machine shop, and his many practical suggestions contributed significantly to the successful completion of this project.
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1. Introduction.

Accurate prediction of the pressure drop experienced by two phases of a substance (or substances) flowing concurrently through a pipe or channel is required in many engineering situations. Some common uses of two phase flow are: pipeline gathering systems in the gas and oil industry; heat exchangers; horizontal tube evaporators; steam generation lines; reactor coolant channels; pipeline contactors in chemical plants; and condensate lines in refrigeration plants.

The term, 'two phase flow', used herein, will mean either two component, two phase flow, or single component, two phase flow, according to the context.

As fluids pass through horizontal, closed channels or pipelines, they experience a loss of pressure through energy dissipation due to turbulence and friction between the walls and the fluid. For single phase flow the pressure drop may be predicted with adequate precision using the Darcy-Weisbach equation and a friction factor obtained either from the Moody Diagram or, if the pipe is smooth and the flow turbulent, calculated from the Blasius formula. Unfortunately, a simple summation of the pressure drops due to each of the phases flowing separately does not predict the actual loss of pressure.

The increased complexity of computations encountered is due to one or more of the following reasons.

The hydraulic diameter of each of the phases is variable and generally undefinable as flow progresses. The cross section available for flow of one phase is reduced as the second phase is introduced. The unsteady, intensely turbulent flow, which is characteristic of high mass flow rates in
two phase flow, is an efficient energy absorber. Much energy is irrecoverably expended creating ripple, wave, plug, and slug flow. No longer is the flow boundary a relatively smooth pipe wall, having become instead a rough, mobile, irregular interface between the two phases. In view of such problems, considerable effort has been required to obtain good correlations.

The history of two phase flow investigation extends back to 1939 when Boelter and Kepner /18/ first attempted to obtain a workable correlation. In 1942 Dittus and Hildebrand /19/ presented a method which used both mathematical equations and graphical solutions for the determination of pressure drop in oil vapor-oil mixtures flowing through furnace coils. Then in 1944 came the first generally accepted correlation for the prediction of two phase flow pressure drop, that of Martinelli /3/. During succeeding years, Martinelli and his associates attacked the problem of pressure drop during the forced circulation boiling of water /5/, and produced a revised correlation in which the liquid fraction (the fraction of the cross-sectional area of the pipe occupied by liquid) was integrated into the prediction /4/. In 1954 Alves /20/ reported experimental results within 15% of the predicted pressure drop, and classified the seven types of flow that were observed. White and Huntington /7/ in 1955 proposed a new correlation which brought the predicted values to within 12% of the measured values, for the very restricted type of flow known as "ripple flow". In 1955 Chenoweth and Martin /6/ presented a correlation that came within 20% for turbulent flow. Chisholm and Laird /1/ modified Martinelli's flow-rate ratio parameter, introduced the effect of roughness, and obtained a correlation to within 15% without recourse to a graphical solution. A two phase friction factor developed by Bertuzzi, Tek and Poettmann in 1957 /21/.

*/ denotes reference in Bibliography.
provided the basis for a correlation which produced a standard deviation of 20.8%. In more recent years, Hatch and Jacobs /22/ investigated the prediction of pressure drop in the two phase flow of hydrogen, and Kordyban /23/ presented a flow model that provided a logical explanation for the applicability of the Chenoweth-Martin correlation /6/.

A preliminary library search produced no reports which dealt with the pressure drop associated with flow of varying quality. Consequently a horizontal, one-inch, flow channel of circular cross section, into which air could be injected at various points, was designed and constructed.

Preliminary tests were made using water and air injected at a single position. The resultant data confirmed the findings of Chisholm and Laird for turbulent flow in a smooth pipe.

Tests were then conducted in which the quality, the percent mass of gas phase of the total mass flowing, was varied by injecting known amounts of air at seven injection points along the 42-foot test section. The Chisholm-Laird correlation provided pressure drop predictions within 25% of the measured values.
2. Description of Equipment.

a. **Flow Channel Apparatus.** The 42-foot test section, Figs. 1 and 2, were made of extruded, acrylic plastic tubing of a nominal one-inch inside diameter. Seven sections of the tubing, six feet in length, were joined in station connector blocks (Fig. 3) using O-ring seals. This construction provided very satisfactory sealing with a minimum reduction in the smoothness of the tube. The actual inside diameter varied from 0.995 to 1.025 inches both in the diametral plane and along the tube.

An effective diameter of 1.013 inches was used. By assuming the tube to be smooth, using equations (1) through (8) (listed in Section 3-b) with the values obtained from the single phase water runs, and treating the diameter, D, as the unknown variable, the resultant equation was solved for the effective diameter. This diameter was then used, as required, in subsequent calculations.

Figure 4 displays the results obtained from the single phase water runs using this diameter. It is evident from Fig. 5 that the single phase air runs suffered no adverse effects from the use of the diameter thus calculated. Included in the effective diameter were the effects of the eight, station connector blocks. Slight discontinuities in the smoothness of the tube were introduced by the two tubing ends butting against an insert of the same tubing glued inside the block, and by the annular series of holes for each of the pressure taps. Distances between stations are shown in Appendix I.

b. **Auxiliary Equipment.** An overall view of the test equipment is provided in Fig. 1. A more detailed view of the auxiliary equipment shown in Fig. 6. In this photograph the following pieces of equipment are evident:
Number  Item
1. Water Drain Stop Cock
2. Rotameter Selector Switch Valve
3. Rotameter Selector Switch Block
4. Injection Air Rotameter
5. Water Rotameter
6. Differential Manometer Inlet Pulsation Damper
7. Station Pressure Manometer Pulsation Damper
8. Manometer Line Bubble Trap
9. Water Main Inlet to Bubble Trap (with air bleeder valve directly above)
10. Precision Potentiometer
11. Electric Timer
12. Timer Control Box
20. Cathetometer
21. Manometer Bank for Measuring Rotameter Pressure
22. Differential Mercury Manometer
23. Air Pressure Regulators in Series
24. Differential Water Manometer

Where these pieces of equipment are indicated on the general apparatus schematic, Fig. 7, the numbers correspond.

The main features of the auxiliary equipment, the selector switches, may be seen in Figs. 8 and 9. In these figures most details of their construction are apparent. Two details not evident are:

1. Whenever gluing of the plastic was required, a mixture of thin acrylic shaving in ethelyene dichloride was inviscid enough to penetrate the thinnest crack or abutment of the pieces being joined. This
method provided joints of more than adequate strength.

2. The aluminum alloy (24ST) slide valve, item 2, Fig. 9 had slots milled along half of its length, 3/16 inch steel tubing laid therein, and was returned to its cylindrical shape by filling with epoxy cement. This eliminated the difficult task of successfully drilling small diameter holes for a long distance in a relatively soft material. The holes which are evident in the valve, lead directly to the tubing connections at the right hand end of the valve. These in turn are connected to the inlet and outlet of a rotameter.

The rotameter selector switch, Fig. 9, allowed insertion of the rotameter into each station's air supply line successively, only momentarily disrupting the continuity of any of the supply lines, and the "steady" state of the flow.

The station pressure selector switch, Fig. 8, was used to transfer the manometer pressure leads from one station to the next, and could also be used for easily obtaining differential pressures between successive pairs of stations.

The bubble trap, item 8, Fig. 10, kept air bubbles from entering the manometer pressure lines. Water was admitted to the top of the bubble trap through the valve, item 9. Entrapped air was then bled off the top of the trap as it accumulated. Station pressures were measured in all cases by mercury or water manometers vented to the atmosphere. All differential pressures between stations were measured with a mercury or water differential manometer. In the case of differential pressures, all liquid levels were measured to within 0.05 millimeters using a Gaertner cathetometer.

Temperatures were measured using copper-constantan thermocouples referred to an ice bath. The wiring diagram for this circuit is indicated in
Fig. 11. Temperature measurements made at the inlet (thermocouple #1) and at the outlet (thermocouple #2) indicated that the flow was isothermal to within plus or minus 0.1°F.

The high pressure air source fluctuated between 80 and 100 psig. This fluctuation was reduced to plus or minus 0.1 inches of water by the use of two pressure regulators connected in series.

The water main pressure was found to be sufficiently constant, varying in mass flow rate by only 0.007 lbm/sec, when the tests were conducted after hours of normal water usage.

The weigh tank scale was checked with dead weights.

Rotameter readings were converted to cubic feet per minute using the curves supplied by the manufacturer.

The test section was horizontally aligned, using a K&E Tilting Level, to within plus or minus 0.03 inches. A chalk line, snapped between the extremities of the test section support, provided a straight reference line for longitudinal alignment. See Fig. 2.

Figs. 12 and 13, with the wiring schematic, Fig. 14, show the method employed to measure the time required for a set mass of water to flow. Movements of the tank switch contact arm caused this mass of water to vary from 148 to 154 lbm, and consequently the "set" mass frequently required redetermination.
3. Procedure.

   a. General. Except as modified in subsequent sections, each test run was conducted in a similar manner. Temperatures were stabilized by running both the water and air for an hour before the test. Atmospheric temperature and pressure were recorded. The quick closing valve on the weigh tank was closed and the weighing proceeded automatically. Water and air injection temperatures were recorded. The overall pressure drop, station pressures, and air rotameter pressure were then noted.

   b. Single Phase Qualifying Tests. In order to establish the validity of measurements made, tests were conducted to determine the friction pressure drop for single phase water flow and single phase air flow. The results of these tests are displayed in Figs. 4 and 5. In all cases in this investigation, data reduction was accomplished using a digital computer.

   The determination of friction factors, Reynolds numbers, and pressure drops for the case of a fluid flowing alone was made using the following equations:

(1) \[ \omega_L = \rho A \rho V_L, \]

where:

- \( \omega_L \) = Mass flow rate, lbm/sec,
- \( \rho L \) = Density of liquid, lbm/ft³,
- \( A \rho \) = Area of tube cross section, ft²,
- \( V_L \) = Average velocity of flow, ft/sec, the velocity of the liquid which would occur if only the single phase were flowing;

rearrangement gives,

(2) \[ V_L = \frac{\omega_L}{\rho L A \rho}. \]
Using this velocity, the Reynolds number was calculated by:

\[ N_R = \frac{V L D \rho}{\mu_L} \]

where:
- \( N_R \) = Reynolds number, dimensionless,
- \( D \) = Effective diameter of tube, ft,
- \( \mu_L \) = Absolute viscosity of liquid, lbm/ft·sec.

Since the test section was a smooth tube, the Blasius formula provides the applicable friction factor:

\[ f = \frac{0.316}{N_R^{2/5}} \]

One form of Bernoulli's equation states:

\[ g \frac{Z_1}{\rho_0} + \frac{p}{\rho} + \frac{V^2}{2 \rho_0} = g \frac{Z_2}{\rho_0} + \frac{p}{\rho} + \frac{V_2^2}{2 \rho_0} + h_f. \]

In the case of horizontal flow with constant area and fluid density, \( \rho \), \( Z_1 = Z_2 \), and equation (5) reduces to the form:

\[ \frac{p_1 - p_2}{\rho} = \frac{\Delta p}{\rho} = h_f \]

where:
- \( \Delta p \) = Pressure drop due to liquid flowing alone in the tube,
- \( h_f \) = Friction head, ft-lbf/lbm.

The Darcy-Weisbach equation states:

\[ h_f = \frac{L}{D} \frac{V^2}{2 \rho_0} \]

Equating (6) and (7):

\[ \frac{\Delta p}{\rho} = f \frac{L}{D} \frac{V^2}{2 \rho_0} \]

or,

\[ \frac{\Delta p}{L} = \Delta P = \frac{\rho f V^2}{2 D \rho_0} \]
where: \( \Delta P \) = Pressure drop per unit length, psf/ft.

c. Two Phase Flow.

i. Background of the Chisholm and Laird Correlation.

Use is made in this paper of a two phase flow pressure drop correlation developed for smooth and rough tubes by D. Chisholm and A. D. K. Laird /1/ in 1956. The reasons for choosing this correlation are twofold; the data could be correlated to within 20%, and the portion of the correlation used, was completely in mathematical terms, which facilitated data reduction by a digital computer.

When correlations which were available to Chisholm and Laird were used, the percent error in prediction of pressure drop increased as a given set of flow conditions deviated farther from the conditions of flow in a smooth tube. These errors were commonly of the magnitude of 30%.

To produce a better correlation Chisholm and Laird collected data from concurrent, horizontal, isothermal flow of water and air through smooth and rough tubes of approximately one-inch bore. The ranges of experimental data which they correlated, and the types of tubes used, are summarized, with those of the present investigator, in Tables 1 and 2 on the following pages.
TABLE 1
RANGES OF DATA

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>CHISHOLM &amp; LAIRD</th>
<th>AUTHOR</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass velocity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>39 to 600</td>
<td>51 to 236</td>
<td>1bm/sec-ft²</td>
</tr>
<tr>
<td>air</td>
<td>0.1 to 20</td>
<td>0.589 to 2.10</td>
<td>1bm/sec-ft²</td>
</tr>
<tr>
<td>Reynolds number</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>4500 to 80,000</td>
<td>5790 to 26,000</td>
<td></td>
</tr>
<tr>
<td>air</td>
<td>1000 to 140,000</td>
<td>4090 to 14,580</td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>59 to 74</td>
<td>52.3 to 61.8</td>
<td>degrees F.</td>
</tr>
<tr>
<td>air</td>
<td>59 to 74</td>
<td>64.8 to 68.3</td>
<td>degrees F.</td>
</tr>
<tr>
<td>Viscosity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>77x10⁻⁵ to 63x10⁻⁵</td>
<td>85x10⁻⁵ to 73x10⁻⁵</td>
<td>1bm/ft-sec</td>
</tr>
<tr>
<td>air</td>
<td>1.2x10⁻⁵ to 1.23x10⁻⁵</td>
<td>1.2x10⁻⁵ to 1.21x10⁻⁵</td>
<td>1bm/ft-sec</td>
</tr>
</tbody>
</table>

Tube Surfaces

\( \epsilon/D \), pipe roughness ratio

Acrylic plastic* 0.000
Smooth brass** 0.000
Commercial galvanized tube** 0.0025
Brass tube, concrete internal surface** 0.013
Brass tube, internal thread** 0.028 (measured) 0.037 (apparent)
Brass tube, non-uniformly distributed sand** 0.045
Brass tube, uniformly distributed sand** 0.068

General Remarks

Flow in both phases was considered turbulent throughout the ranges considered. Mean arithmetic values of the water temperature at the inlet and the outlet were used when determining water properties. Air properties were determined with no appreciable error by assuming that the air was dry and the temperature quickly became the same as the mean water temperature upon mixing.

* Author's tube
** Chisholm & Laird's tube
### TABLE 2
SPECIFIC RANGES OF MASS FLOW RATE 
FOR EACH TYPE OF FLOW 
STUDIED IN THIS INVESTIGATION

<table>
<thead>
<tr>
<th>TYPE OF FLOW</th>
<th>WATER, 1bm/sec</th>
<th>AIR, 1bm/sec</th>
</tr>
</thead>
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<tr>
<td>Single phase water</td>
<td>0.508 to 1.32</td>
<td>-</td>
</tr>
<tr>
<td>Single phase air</td>
<td>-</td>
<td>0.00330 to 0.0118</td>
</tr>
<tr>
<td>Two phase single injection</td>
<td>0.286 to 1.27</td>
<td>0.00348 to 0.00950</td>
</tr>
<tr>
<td>Two phase multiple injection</td>
<td>0.694 to 1.30</td>
<td>0.000426 to 0.00142 (per injection point)</td>
</tr>
</tbody>
</table>
Through introduction of the effects of roughness into the Martinelli correlation Chisholm and Laird arrived at the following correlation of their data. They define a flow-rate-ratio parameter, $\bar{X}$, by equation

$$\bar{X} \equiv \left( \frac{G_L}{G_G} \right)^{0.875} \left( \frac{\mu_L}{\mu_g} \right)^{0.125} \left( \frac{\rho_L}{\rho_g} \right)^{0.5}$$

where:
- $G_G$ = Liquid-mass velocity based on tube cross section, lbm/sec-ft$^2$,
- $G_L$ = Gas-mass velocity based on tube cross section, lbm/sec-ft$^2$,
- $\mu_L$ = Absolute viscosity of liquid, lbm/sec-ft,
- $\mu_g$ = Absolute viscosity of gas, lbm/sec-ft,
- $\rho_L$ = Density of liquid, lbm/ft$^3$,
- $\rho_g$ = Density of gas, lbm/ft$^3$.

The pressure-drop parameter, $\frac{\Delta P_{fp}}{\Delta P_L}$, is then correlated by equation

$$\frac{\Delta P_{fp}}{\Delta P_L} = 1 + \frac{C}{\bar{X}^m}$$

where:
- $\Delta P_{fp}$ = Total (friction) pressure drop over an increment of length
- $\Delta P_L$ = Friction pressure drop for liquid flowing alone in tube, psf/ft,
- $C = \frac{\lambda}{\lambda_s} \sqrt{N_{RPL}}$
- $m = \frac{\lambda}{\lambda_s}$
- $N_{RPL}$ = Reynolds number where the liquid flows alone,
- $\lambda$ = Pipe friction factor for rough tube,
- $\lambda_s$ = Pipe friction factor for smooth tube.

Equation (10) may now be expressed as

$$\frac{\Delta P_{fp}}{\Delta P_L} = 1 + C \left( \frac{G_L}{G_G} \right)^{0.875m} \left( \frac{\mu_L}{\mu_g} \right)^{0.125m} \left( \frac{\rho_L}{\rho_g} \right)^{0.5m}$$
The liquid fraction, $R_L$, is correlated with the pressure-drop parameter, $\frac{\Delta P_{fp}}{\Delta P}$, as follows:

\[
(12) \quad \frac{\Delta P_{fp}}{\Delta P} = \frac{0.8}{R_L^{1.75}} \quad \text{(for the smooth tube)},
\]

where: $R_L$ = Liquid fraction, the area occupied by the liquid divided by the cross-sectional area of the tube.

A correlation of the liquid fraction, $R_L$, with the flow-rate-ratio parameter, $\bar{X}$, for the smooth tube is:

\[
(13) \quad \frac{0.8}{R_L^{1.75}} = 1 + \frac{2\bar{X}}{X} - \frac{1}{X^2} \quad \text{(for the smooth tube)}.
\]

ii. Single Injection Tests. After the single phase tests were completed, a series of runs was made in which water and air were injected upstream of the test section. The correlated pressure drop was obtained in the following manner:

1. $\bar{X}$ was determined using equation (9).
2. $R_L$ was obtained from equation (13).
3. $\frac{\Delta P_{fp}}{\Delta P}$ could then be calculated from equation (12).

The results of these single injection tests, together with the results of the multiple injection tests (described in the next section), are presented graphically in Figs. 15, 16, and 17, where the coordinates used are those found to be the most useful by Chisholm and Laird.

iii. Multiple Injection Tests. Having thus shown that the apparatus is capable of producing data comparable to that presented by
Chisholm and Laird, test runs were conducted in which air was injected in known amounts at seven points along the test section.

The purpose of this multiple injection of air was to provide portions of the test section with two phase flow whose liquid fraction was able to be determined. In addition, it was desired that the liquid fraction vary from injection point to injection point. The design of the test section allowed injection of air at each station connector block. Since the amount of air injected at each station was measured, the liquid fraction was calculable, and constant for each portion of the test section between injection points. The correlation process, set forth previously in section 3-c-ii, was then used and the pressure drop found for each constant-liquid-fraction section. The pressure drops were then summed over the whole test section, and the overall pressure drop obtained. The results of these multiple injection tests are displayed in Figs. 15, 16, and 17.
4. Results and Conclusions.

A horizontal channel suitable for multiple air injection into the primary flow of water was designed, constructed, and tested.

The results of single phase tests, Figs. 4 and 5, indicated that the test section behaved as a smooth tube. Consequently, the friction factor computed by the Blasius formula was the only one employed during subsequent reduction of the two phase runs.

These Figures show that the pressure drop accompanying two phase flow of varying quality may be predicted to the same degree of accuracy as constant quality two phase flow, by a suitable selection of the "constant quality" sections. A logical next step would be the prediction of pressure drop in a two phase, varying quality, isothermal situation, such as a saturated steam generating tube, by applying the correlation to sections of the tube small enough to be of "constant quality", from one end to the other. This could be done on a digital computer and the results compared with experimental data. A further extension of this line of investigation might well start with mounting the variable quality test section vertically.


ADDITIONAL REFERENCES


Kuong, J. F., "Figure heat transfer for two-phase flow," (nomograph), Hydrocarbon Processing & Petroleum Refiner, Vol. 41, October 1962, pp. 147-148.


Thomsen, E. G., "Pressure drop accompanying two-component flow in a closed conduit with various liquids and air," University of California, Berkeley, California, MS Thesis 1941.

APPENDIX I

LOCATION OF PRESSURE TAPS ALONG THE TEST SECTION

<table>
<thead>
<tr>
<th>Station Number</th>
<th>Distance to next station, in inches</th>
<th>Distance to station indicated from station one, in inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71.7</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>71.72</td>
<td>71.7</td>
</tr>
<tr>
<td>3</td>
<td>71.75</td>
<td>143.42</td>
</tr>
<tr>
<td>4</td>
<td>71.8</td>
<td>215.17</td>
</tr>
<tr>
<td>5</td>
<td>71.75</td>
<td>286.97</td>
</tr>
<tr>
<td>6</td>
<td>71.8</td>
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<td>7</td>
<td>71.7</td>
<td>430.52</td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>502.22</td>
</tr>
</tbody>
</table>

Distance of station 1 from inlet --37.22 inches
Distance of station 8 from outlet --35.53 inches
APPENDIX II

DRAWINGS, PHOTOGRAPHS, AND GRAPHS
Fig. 2 Overall View of Test Section
Fig. 4 Friction Factor vs. Reynolds Number
Single Phase Water Flow.

\[ f = \frac{316}{N_r^{0.25}} \]

(Smooth Tube)
Fig. 5 Friction Factor vs. Reynolds Number
Single Phase Air Flow.
Fig. 8. Pressure Station Selector Switch Detail

29
Fig. 13. Timer Control Box

34
Fig. 15 Experimental Pressure Drop Ratio vs. Reciprocal Liquid Fraction in Turbulent Flow of Air and Water.
\[ \frac{\Delta P_{TP}}{\Delta P_{L}} - 1 = \frac{2}{\bar{X}} + \frac{1}{\bar{X}^2} \]

Fig. 16 Experimental Pressure Drop Ratio Minus One vs. Chisholm-Laird Flow Parameter in Turbulent Flow of Air and Water.
Fig. 17 Experimental Pressure Drop Ratio vs. Predicted, Pressure Drop Ratio in Turbulent Flow of Air and Water.
Application of the Chisholm-Laid two ph