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AIR FORCE MATERIALS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
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(6) MEASUREMENT OF DIELECTRIC PARAMETERS OF HIGH-LOSS MATERIALS IN DISTRIBUTED CIRCUITS

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FOREWORD

This report was prepared by the Massachusetts Institute of Technology, Laboratory for Insulation Research, Cambridge, Massachusetts, under USAF Contract AF 33(615)-2199. This contract was initiated under Project No. 7371, "Exploratory Development in Electrical, Electronic, and Magnetic Materials," Task No. 737101, "Dielectric Materials." The work was administered under the direction of the AF Materials Laboratory, Research and Technology Division, with W. G. D. Frederick acting as project engineer.

This technical report has been reviewed and approved.

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Materials Physics Division
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Illustrations

MEASUREMENT OF DIELECTRIC PARAMETERS OF HIGH-LOSS MATERIALS IN DISTRIBUTED CIRCUITS

by

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Cambridge, Massachusetts

Abstract: Measurement techniques for determination of the complex dielectric constant and complex permeability are given a short review with emphasis on the problems encountered in practical high-loss materials. Typical calculations and descriptions of special sample holders are included.

Introduction

For purposes of discussing measurement techniques, we can divide electromagnetic energy-absorbing materials for perpendicular impedance into several categories.

1. In uniform materials the macroscopic parameters ϵ^* and μ^* are invariant along the axis of wave propagation. They may be (a) isotropic or (b) anisotropic, depending on whether or not these parameters are the same in other directions also. When reflected as well as traveling waves are considered, the question of reciprocity arises, and additional classifications are: (c) for reciprocal materials, (d) for nonreciprocal.

2. Uniform-layered materials are composites of layers, each layer being uniform, and the thickness of the junction is very small compared to wavelength in adjacent layers.

3. Nonuniform materials have ϵ^* and/or μ^*, varying gradually along the axis of propagation. The nonuniformity may arise from external shaping (tapers) or gradual change in chemical composition or loading. Combinations of uniform and nonuniform layers constitute a nonuniform material.

Obviously all layered materials may be symmetric or unsymmetric in reference to their center plane.

Composite materials of mixed ingredients are uniform if particle sizes are small compared to wavelength and if the materials are homogeneous. Clumping of particles causes electrical inhomogeneities; the measured values of ϵ^* and μ^* will then be dependent on sample thickness and how the measurement techniques are affected by scattering.

Measurement Techniques

The general methods for determining both ϵ^* and μ^* are (a) the input impedance measurements with sample in two positions using resonant cavities or standing-wave detectors; (b) perturbation measurements with a small sample in two positions of resonant cavity; (c) combination of input impedance and transmission measurement.

In (a) the use of resonant cavities is limited by the need for at least moderately high loaded Q's (50 or more) to obtain accurately measurable amplitude resonance characteristics. When approximate sample properties are not known before measurement, this is a severe restriction. The standing-wave method limitations arise at much higher values of loss (lower values of impedance) when the sample in both positions has the same impedance, that of an infinite-length sample. Then only the ratio ϵ^* / μ^* will be measured with poor accuracy, because the separation of node and sample face will be very small. Figure 1, for nonmagnetic materials, illustrates the difficulties. The x_o values are generally small fractions of a wavelength, and inverse standing-wave ratios are high except for materials having very high κ' values and/or very high $\tan\delta$ values. Also the fractional errors in dielectric constant and loss are always nearly twice the error in experimental quantities because of the square-law dependence. For example, for κ' and $\tan\delta = 10$, $E_{min}/E_{max} = .075$ and $x_o/\lambda = .012$ or approximately $\frac{1}{2}(\Delta x/\lambda)$. If the node position is located to within 1/20 of Δx, a reasonable error, and E_{min}/E_{max} is accurate to 5%, the resultant error in κ'' is 14%.

In the two-position measurement system, good results depend mainly on having a thin enough sample to avoid "infinite-length" conditions and

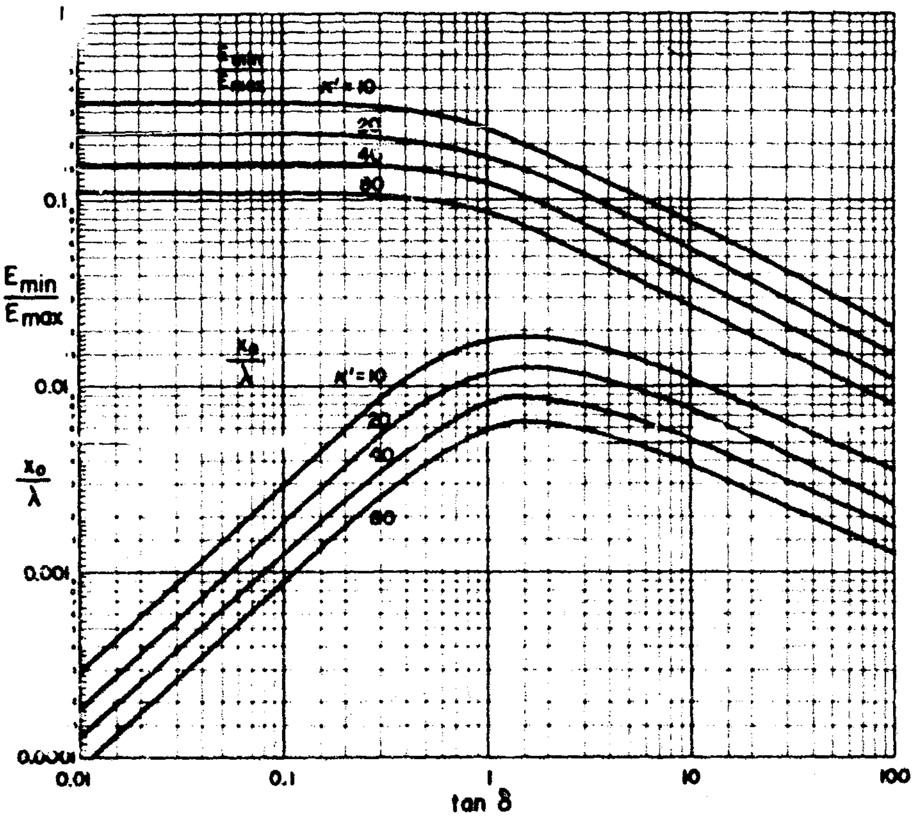


Fig. 1. Inverse standing-wave ratio and x_0/λ for "infinite-length" nonmagnetic materials (TEM mode).

making good contact with the metallic conductor(s). If the approximate magnitude of the product $\epsilon^*/\epsilon_0 \cdot \mu^*/\mu_0$ is known, the physical thickness of sample for an electrical thickness of 0.3 radian can be readily determined (Fig. 2). This thickness assures good measurements in the sense that the experimentally measured quantities vary almost linearly with sample parameters, and nearly separate measurements of electric and magnetic parameters are achieved. Also higher order modes can hardly exist. Limitations of the method arise when sample thickness must be so small to achieve electrical thinness that the sample no longer is representative of bulk material; e.g., evaporated films often have different electrical parameters.

High-constant and/or high-loss samples require good contact with the conductor(s) in the measurement system. For example, in a 1-inch

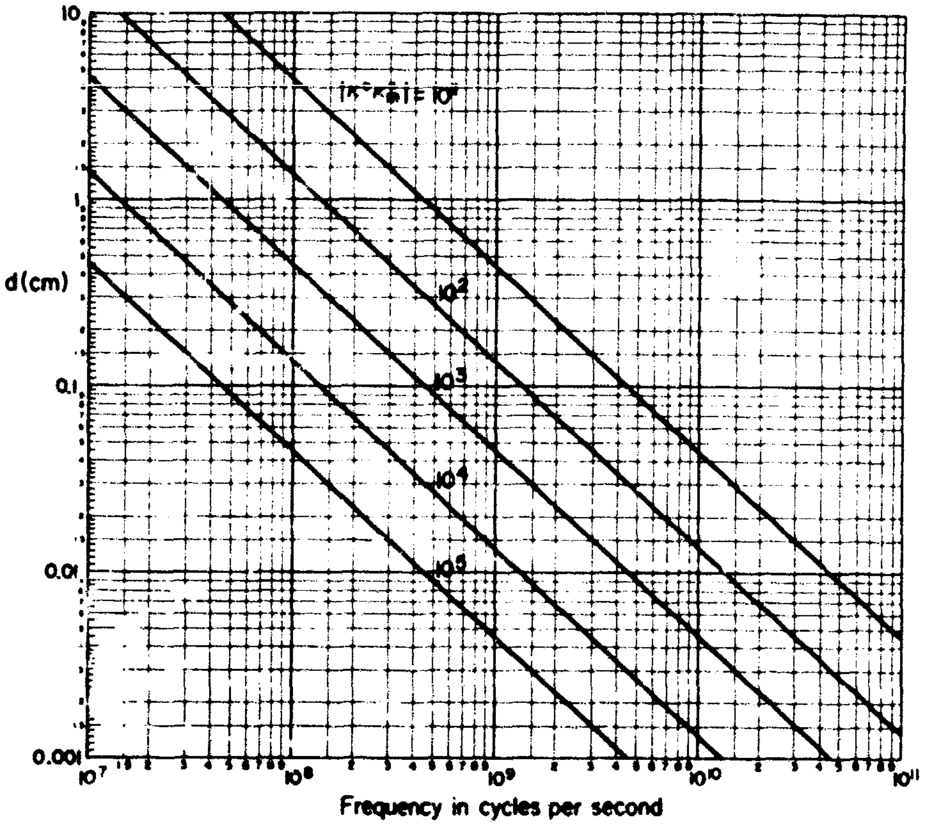


Fig. 2. Sample thickness d (in cm) for electrical thickness of 0. 3 radians for various magnitudes of $(\epsilon^*/\epsilon_o)(\mu^*/\mu_o)$ versus frequency.

60-ohm coaxial line a clearance of 0. 0005 inch on the center conductor causes a 13% error in measuring a material with $\kappa' = 100$. Correction equations and charts are given in the Appendixes II and IIIC. Usually the clearance cannot be defined with sufficient accuracy for good measurements and intimately bonded electrodes should be added. To avoid leakage and insure contact the sample holder of Fig. 3 is used. A peripheral coat of silver paint is allowed to overflow onto the faces which are clamped between silver conductors, one of which is thin-walled and collapses to accommodate variations in sample thickness. The samples must be strong enough to withstand the compressive strain; repeated use dulls the tubing edge which can be reformed by lathe turning. The silver on

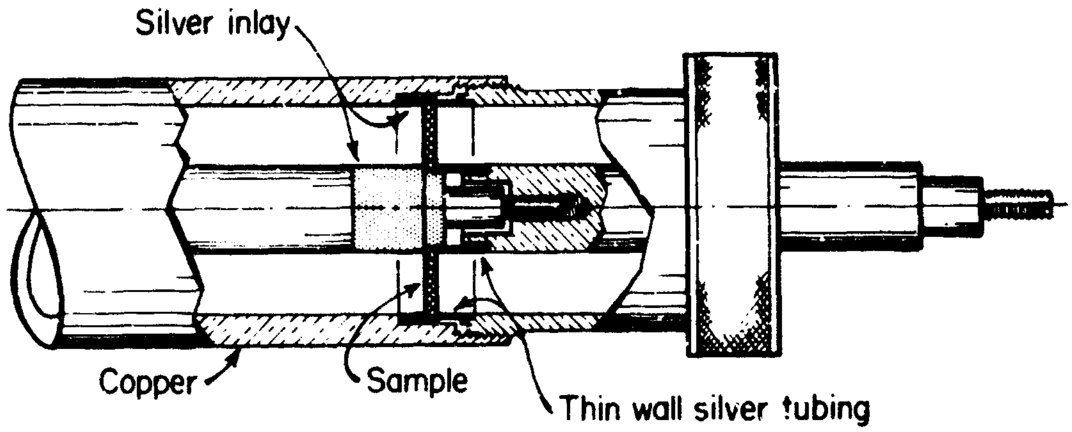


Fig. 3. Method of clamping silvered sample in coaxial line.

the faces introduces additional fringing field and limits the accuracy of measurements to about 5%. A better arrangement uses coated samples soldered to thin-walled tubing as shown in Fig. 4.

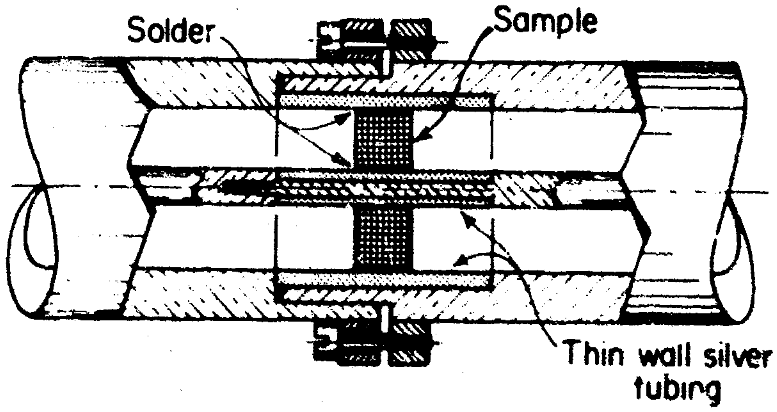


Fig. 4. Soldered sample in a thin-walled (0.005 to 0.010") tubing.

In measuring the magnetic properties of materials, ring-shaped samples are sometimes used in a coaxial line or cavity.[1] This procedure leads to a different and more subtle source of error. If the electrical thickness along either the axial or radial direction exceeds 0.3 radian,

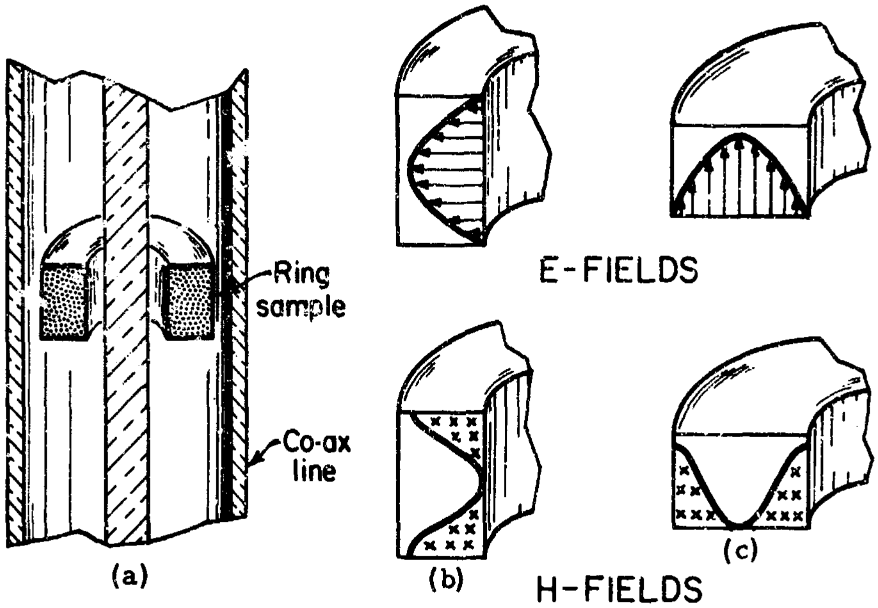


Fig. 5. Magnetic ring sample in coaxial line: (a) physical arrangement, (b) electric and magnetic fields in radially thin sample, (c) electric and magnetic fields in longitudinally thin sample.

other modes exist which are readily coupled by the magnetic field as shown in Figs. 5a and 5b. Good contact at coaxial conductors by a full-sized sample reduces chance for this type of error. Highly conducting samples can give false diamagnetic readings caused by eddy current shielding of the sample interior.

In hollow waveguide the same ideas apply, but in addition one can possibly deform the center of the broad faces of rectangular guide to make contact with a sample coated on the longer edges only (Fig. 6).

The two-position, thin-sample technique gives best separation of electric and magnetic parameters. When thick samples only can be used, the input-impedance measurement gives the ratio κ'_m/κ' and the difference in loss angles. Since this is essentially a measurement of the ratio of magnetic to electric field strengths, namely, ratio of two different kinds of units, this determination will be done with less precision than achieved in measurement of the propagation function. The latter can be measured

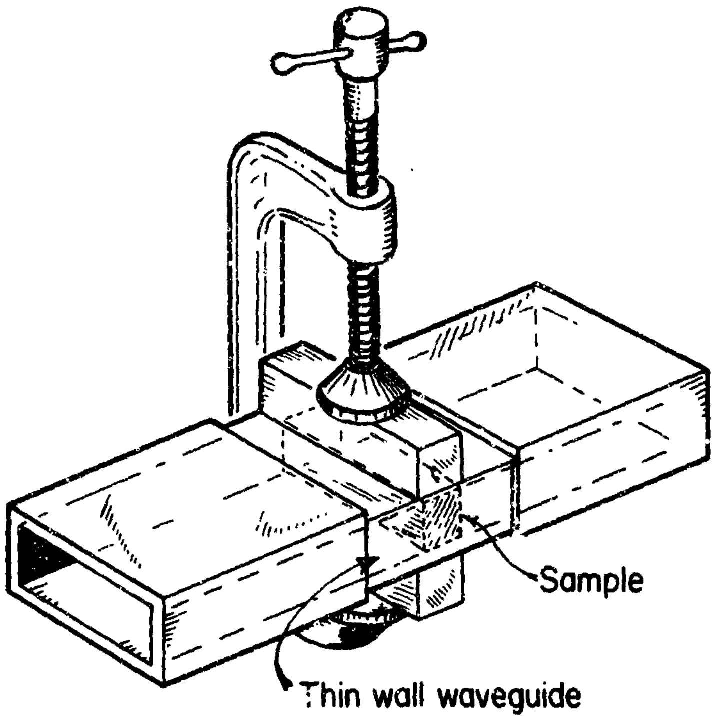


Fig. 6.

Sample clamped in a thin-walled (< 0.10") waveguide.

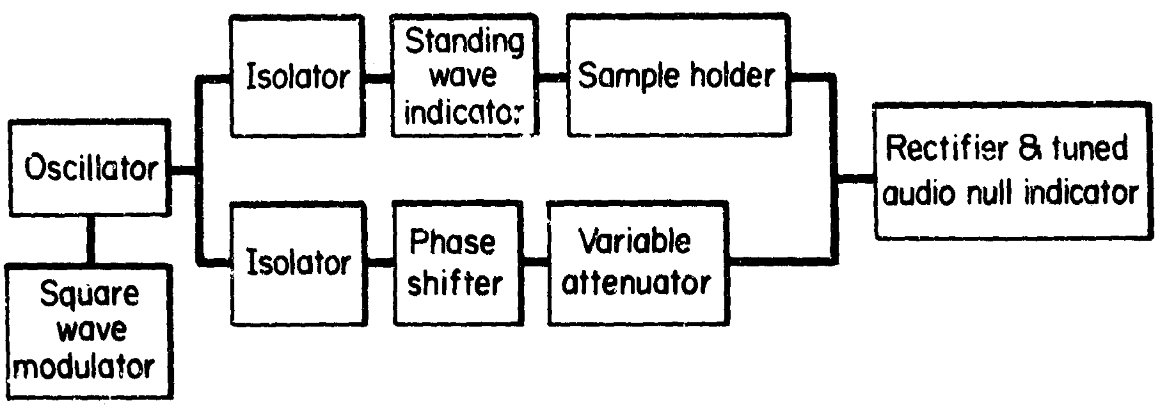


Fig. 7. Equipment for measurement of input impedance and transmission.

with either electric or magnetic indicators. Figure 7 is a schematic diagram of equipment for combination measurement of input impedance and transmission. A suitable phase shifter for coaxial line is shown in Fig. 8. The calculation procedure follows the two-position method

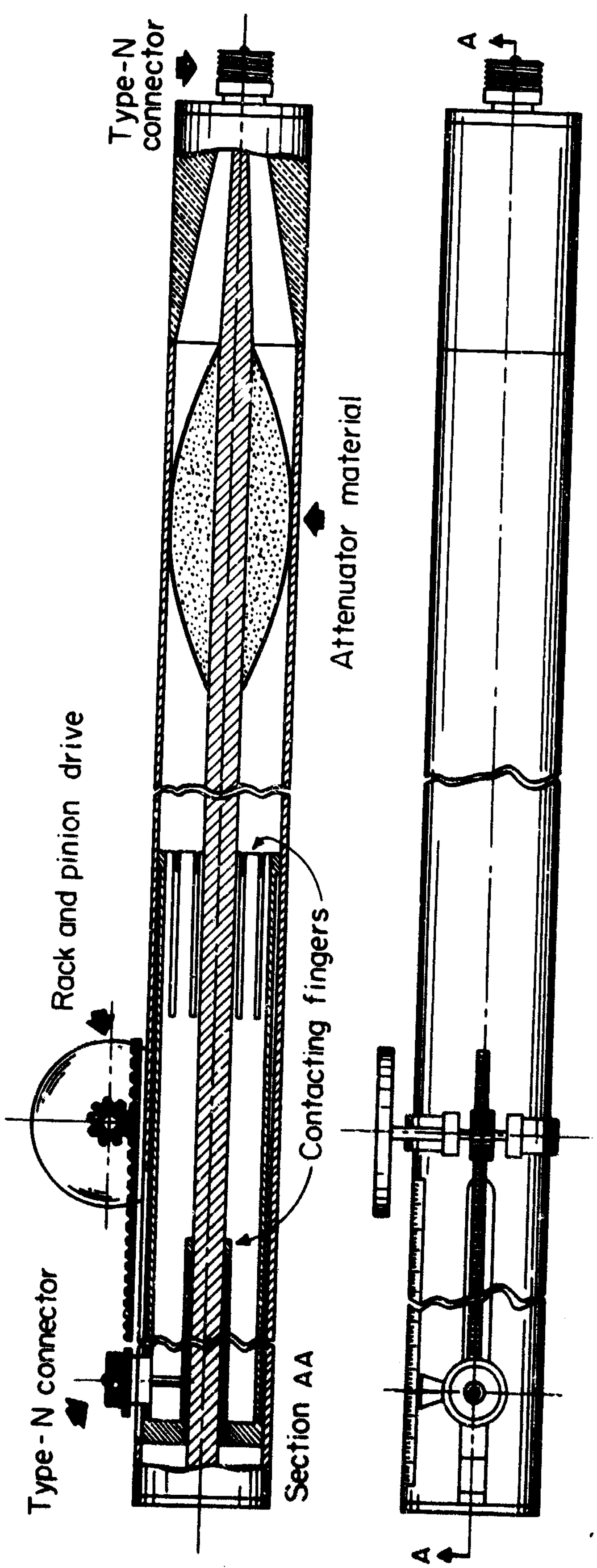


Fig. 8. Coaxial phase shifter for fixed frequency.

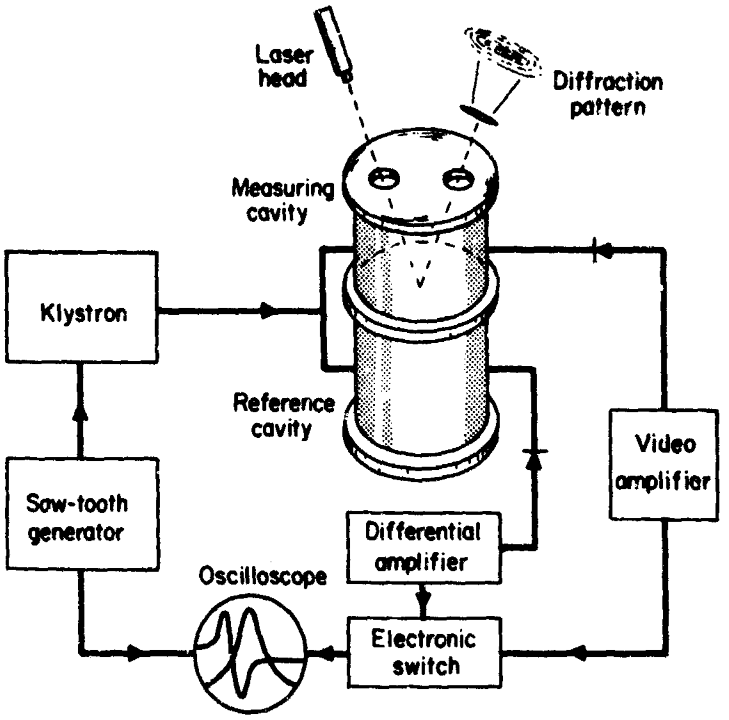


Fig. 9.

Suggested cavity measurements for determining μ^* of conductors. Laser head would be mounted on cavity. Sample surface is bottom of measuring cavity.

except that Z_{02}/Z_{01} and γ_2 (e. g., 12 and 15, Appendix ID) are given directly by the input impedance measurement and the change in amplitude and phase per unit length of sample increase.

Metallic materials, even as thin foils, have attenuations greater than can be readily measured in transmission. The sample can serve as a conducting wall in a waveguide, a resonant cavity, or surface-guided wave system. A proposed measurement system is shown in Fig. 9. A gas laser interferometer is suggested to measure the position of the cavity wall, so the small changes in resonant frequency due to the μ' term can be detected. Measurements of the complex permeability of iron were made some years ago by Dr. Rado at NRL.[2)] He superimposed a saturating magnetic field to reduce μ^* to μ_o to establish a nonmagnetic reference condition in a coaxial cavity with the sample serving as center conductor.

The small-sample technique is similar to common ferromagnetic reso-

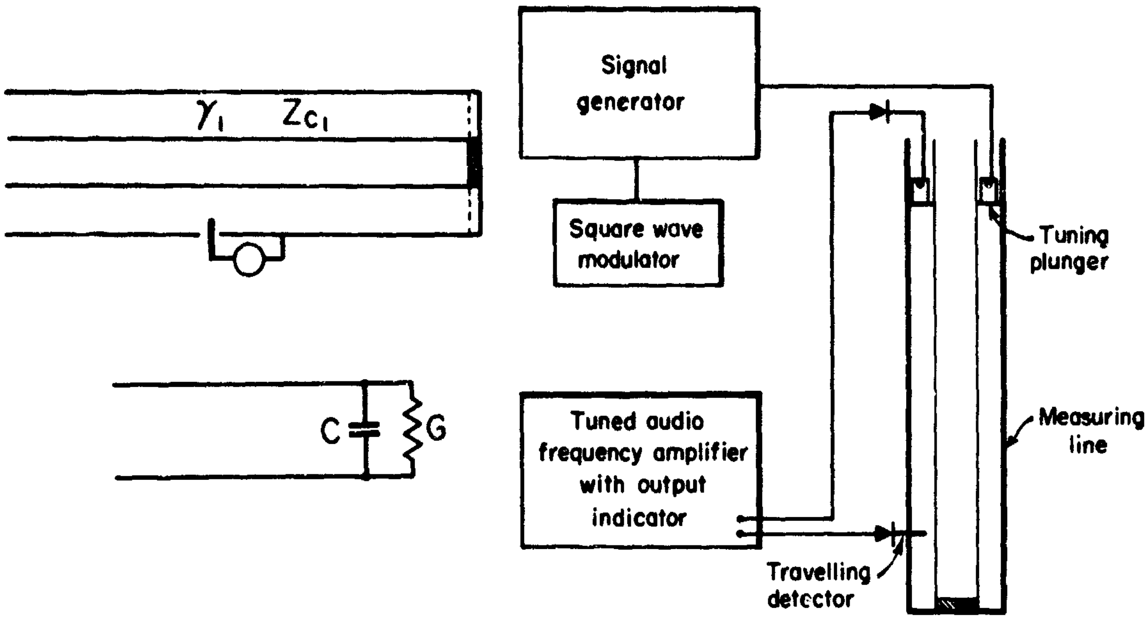


Fig. 10. Lumped-capacitor sample at end of coaxial line: (a) sample in line, (b) equivalent circuit, (c) equipment arrangement.

nance experiments[3] with external d-c field. A spherical sample placed in two positions in a cavity, or in two different cavities, allows both ϵ^* and μ^* to be determined. Since the dipole field corrections[4] used are derived from magneto- and electrostatics, the diameter of the sphere should be 0.1 radian or less. At high frequencies very small spheres (0.005-inch) are required, and surface finish becomes important. To cover a wide range of frequencies, a series of spheres are required. For nonmagnetic materials, rodlike samples can be used as lumped capacitors at the end of a coaxial line (Fig. 10).

Measurements on uniform-layered materials with parallel electric fields to determine ϵ^* and μ^* have progressively less meaning when the electrical thickness along layer rises above 0.3 radian.

With uniform-layered materials, the two-position method for magnetic dielectrics or the single-position measurement of nonmagnetic dielectrics have progressively less meaning as frequencies rise, so that the total electrical thickness exceeds 0.3 radian. In an extreme case the measurements are meaningless as an example will show. Figure 11 shows a symmetrical-layered sample having nonmagnetic parameters

	$\kappa' = 3$ $\delta = 0$	$\kappa' = 100$ $\tan\delta = 0.1$	$\kappa' = 3$ $\delta = 0$
Thickness (in radians)	0.512	0.300	0.512
Relative physical thickness	10	1	10

Fig. 11. A nonmagnetic three-layer dielectric.

for simplicity. Measurements against the short circuit place the high-constant portion in a region of high field strength and and give an effective κ' value of 9.22 and an equivalent length of 1.88 radians. In the open-circuit measurement, the high-κ center receives lower field strength, the effective constant is 7.78 and the length is 1.73 radians. The combined two-position calculation gives $\kappa' = 7.4$ and $\kappa_m = 0.44$. The average κ value in a uniform field is $2d_1\kappa_1' + d_2\kappa_2' = 7.68$. If many of these three-layer laminates are stacked, the phase change and attenuation per unit length will vary as each laminate is added, but if enough layers are used, an average value can be defined (if only one mode exists). Because of internal reflections between successive high-κ layers, the κ and $\tan\delta$ values obtained will not be the same as the uniform-field value. With a thicker high-κ section, the difference would be appreciable. The same reasoning regarding the necessity for overall thinness applies to nonuniform layers.

Thus absorbers are best defined by input impedance (or reflection factor) as a function of angle of incidence, not by attempts to measure or define ϵ^* and μ^*.

APPENDIX I

Calculation Procedures for Standing-Wave Methods

A. General Notation:

x_o = distance from face of sample (Fig. AI. 1) to first minimum.

ΔN = node shift toward short with introduction of sample (N_a-N_s).

Δx = width of minimum with measured at twice minimum power points (2/1 current ratio with square-law detector) corrected for line loss between minimum and sample face.

N_s = node reading, sample in.

N_q = node reading without sample, $\lambda/4$ spacer in.

u = $(\lambda_1/\lambda_c)^2$ = 0 for TEM modes.

w = 1 + u.

λ_1 = wavelength in air-filled section of line.

λ_c = cutoff wavelength = 3.412586 times radius of guide for TE_{11} mode, = twice broad dimension in rectangular waveguide TE_{10} mode.

E_{min}/E_{max} = inverse standing-wave ratio = $\frac{\pi \Delta x}{\lambda_1}$ - C_1.[5)]

B. General Relations for Nonmagnetic Sample $\lambda/4$ From Short

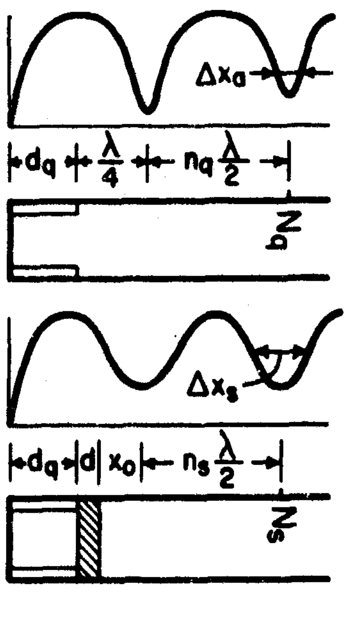


Fig. AI. -1.

$$x_o = N_s - N_q + n_q\left(\frac{\lambda_1}{2}\right) - n_s\left(\frac{\lambda_1}{2}\right) + \frac{\lambda}{4} - d,$$

$$\Delta x = \Delta x_s - \left[x_o + n_s\left(\frac{\lambda_1}{2}\right)\right]\frac{\Delta x_a}{n_a\left(\frac{\lambda_1}{2}\right)} .$$

$$\frac{Z_B}{Z_{01}} = \frac{\frac{E_{min}}{E_{max}} - j\tan\frac{2\pi x_o}{\lambda_1}}{1 - j\frac{E_{min}}{E_{max}}\tan\frac{2\pi x_o}{\lambda_1}} , \qquad \text{(AI. 1)}$$

$$\frac{\coth \gamma_2 d}{\gamma_2 d} = \frac{1}{\gamma_1 d}\frac{Z_B}{Z_{01}} . \qquad \text{(AI. 2)}$$

$\gamma_2 d$ is determined from charts or tables of cothx/x in literature or Appendix IIIA if sample is less than $\lambda_s/4$ in thickness:

$$\kappa^* = \frac{u - \left(\frac{\lambda_1}{2\pi d} \gamma_2 d\right)^2}{1 + u} \qquad \text{(AI.3)}$$

C. Thin Sample Calculations

Restriction 1:

$\gamma_2 d < 0.1^r$ for 0.5% error,

$< 0.15^r$ for 1% error,

$< 0.3^r$ for 5% error.

$$\kappa' - j\kappa'' = \frac{1}{w}\left[\frac{\lambda_1}{j2\pi d} \cdot \frac{Z_{01}}{Z_B} + u\right] \qquad \text{(AI. 4)}$$

Restriction 2:

If $(E_{min}/E_{max})^2 << 1$ and $\tan \frac{2\pi x_o}{\lambda} > 1$, then $\frac{Z_{01}}{Z_B} = \frac{\pi \Delta x}{\lambda_1} + j \cot \frac{2\pi x_o}{\lambda_1}$

and

$$\kappa' = \frac{1}{w}\left[\frac{\lambda_1}{2\pi d} \tan \frac{2\pi(\Delta N + d)}{\lambda_1} + u\right], \qquad \text{(AI. 5)}$$

$$\kappa'' = \frac{\Delta x}{2dw} . \qquad \text{(AI. 6)}$$

Restriction 3:

$\tan \frac{2\pi(\Delta N + d)}{\lambda_1} < 0.1$ for 3% error,

< 0.15 for 1% error,

< 0.3 for 3% error.

$$\kappa' = 1 + \frac{\Delta N}{wd} . \qquad \text{(AI. 7)}$$

If after calculating κ^* with Restrictions 2 or 3 satisfied, Restriction 1 is not satisfied, the value of κ^* already calculated can be corrected if if $\gamma_2 d < 0.5^r$ by noting that

$$\frac{Z_{01}}{Z_B} = \frac{\gamma_2}{\gamma_1} \tanh \gamma_2 d \cong \frac{\gamma_2}{\gamma_1}\left[\gamma_2 d - \frac{1}{3}(\gamma_2 d)^3\right]. \qquad \text{(A. I. 8)}$$

Then

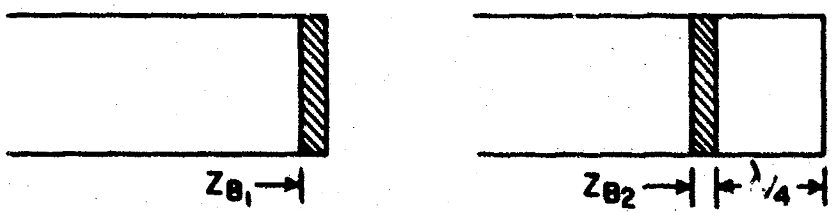
$$\kappa^*_{\text{corrected}} = \kappa^*_{\underset{(s.c.)}{\text{short calc.}}} + \Delta\kappa' - j\Delta\kappa'' , \qquad \text{(A. I. 9)}$$

where

$$\Delta\kappa' = \frac{1}{3w}\left(\frac{2\pi d}{\lambda_1}\right)^2 \left[(\kappa'_{s.c.} w - u)^2 - (\kappa''_{s.c.} w)^2\right] \qquad \text{(A. I. 10)}$$

$$\Delta\kappa'' = \frac{2}{3w}\left(\frac{2\pi d}{\lambda_1}\right)^2 \kappa''_{s.c}(\kappa'_{s.c.} w - u) \qquad \text{(A. I. 11)}$$

D. <u>General Relations for Magnetic Samples, TE or TEM Waves</u>



$$Z_I = \frac{Z_{B_1}}{Z_{01}} \qquad Z_{II} = \frac{Z_{B_2}}{Z_{01}}$$

$$\frac{Z_{02}}{Z_{01}} = (Z_I Z_{II})^{1/2} = B\underline{/\phi} \qquad \text{(A. I. 12)}$$

$$\tanh \gamma_2 d = \left(\frac{Z_I}{Z_{II}}\right)^{1/2} = r \underline{/\phi} \qquad \text{(A. I. 13)}$$

From charts of tanh (a + jb) = $r \underline{/\theta}$

$$\gamma_2 d = a + jb \qquad \text{(A. I. 14)}$$

$$\frac{\gamma_2}{\gamma_o} = \frac{\gamma_2 d}{j\frac{2\pi d}{\lambda}} = A \underline{/-\psi} \qquad \text{(A. I. 15)}$$

$$\tan \delta_m = -\tan (\phi - \psi) \qquad \text{(A. I. 16)}$$

$$\frac{\mu'}{\mu_o} = \frac{AB}{(1 + \tan^2 \delta_m)^{1/2}} \qquad \text{(A. I. 17)}$$

$$\frac{\epsilon'}{\epsilon_o} = \frac{A}{Bw} \cos (\phi + \psi) + \frac{u}{ABw} \cos (\phi - \psi) \qquad \text{(A. I. 18)}$$

$$\frac{\epsilon''}{\epsilon_o} = \frac{A}{Bw} \sin (\phi + \psi) + \frac{u}{ABw} \sin (\phi - \psi) \qquad \text{(A. I. 19)}$$

For TEM mode only

$$\frac{\epsilon'}{\epsilon_o} = \frac{A}{B} \cos (\phi + \psi) \qquad \text{(A. I. 18a)}$$

$$\tan \delta_d = \tan (\phi + \psi) \qquad \text{(A. I. 19a)}$$

Two sample calculations follow in Appendix III.

E. <u>Thin Magnetic Sample Calculations</u>

With Restrictions 1 and 3 satisfied for sample in both positions:

$$\frac{\mu'}{\mu_o} = 1 + \frac{\Delta N_1}{d} \qquad \text{(A. I. 20)}$$

$$\frac{\mu''}{\mu_o} = \frac{\Delta x_1}{2d} \tag{A. I. 21}$$

$$\frac{\epsilon'}{\epsilon_o} = \frac{\Delta N_2 + d}{dw} + \frac{u}{w}\,\frac{\frac{\mu'}{\mu_o}}{\left(\frac{\mu'}{\mu_o}\right)^2 + \left(\frac{\mu''}{\mu_o}\right)^2} \tag{A. I. 22}$$

$$\frac{\epsilon''}{\epsilon_o} = \frac{\Delta x_2}{2dw} + \frac{u}{w}\,\frac{\frac{\mu''}{\mu_o}}{\left(\frac{\mu'}{\mu_o}\right)^2 + \left(\frac{\mu''}{\mu_o}\right)^2} \tag{A. I. 23}$$

For TEM modes:

$$\frac{\epsilon'}{\epsilon_o} = 1 + \frac{\Delta N_2}{d} \tag{A. I. 22a}$$

$$\frac{\epsilon''}{\epsilon_o} = \frac{\Delta x_2}{2d} \tag{A. I. 23a}$$

If Restriction 3 is satisfied only for sample at short,

$$\frac{\epsilon'}{\epsilon_o} - j\frac{\epsilon''}{\epsilon_o} = \left[\frac{\lambda_1}{j2\pi d}\,\frac{1}{Z_{II}} + u\,\frac{\mu_o}{\mu' - j\mu''}\right]\frac{1}{w} \quad . \tag{A. I. 24}$$

which reduces if Restriction 2 is satisfied to

$$\frac{\epsilon'}{\epsilon_o} = \frac{1}{w}\left[\frac{\lambda_1}{2\pi d}\cdot \tan\frac{2\pi\Delta N_2 + d}{\lambda_1} + u\frac{\frac{\mu'}{\mu_o}}{\left(\frac{\mu'}{\mu_o}\right)^2 + \left(\frac{\mu''}{\mu_o}\right)^2}\right] \tag{A. I. 25}$$

$$\frac{\epsilon''}{\epsilon_o} = \frac{1}{w}\left[\frac{\lambda_1}{2\pi d}\cdot\frac{\pi\Delta x_2}{\lambda_1} + u\,\frac{\frac{\mu''}{\mu_o}}{\left(\frac{\mu'}{\mu_o}\right)^2 + \left(\frac{\mu''}{\mu_o}\right)^2}\right] \tag{A. I. 26}$$

If after calculating ϵ^*/ϵ_o and μ^*/μ_o from Eq. 20 through 26 Restriction 1 is not satisfied, the values already calculated can be corrected if $\gamma_2 d < 0.5^r$ by making use of the series expansion as in Eq. A.I.18. Then

$$\frac{\epsilon^*}{\epsilon_o} = \kappa = \kappa_{\text{short calc.}} + \Delta\kappa' - j\Delta\kappa'' \qquad \text{(A.I.9)}$$

$$\frac{\mu^*}{\mu_o} = \kappa_m^* = \kappa_{m\ \text{short calc.}}^* + \Delta\kappa_m' - j\Delta\kappa_m'' \qquad \text{(A.I.27)}$$

where, for TEM mode,

$$\Delta\kappa' = \frac{1}{3}\left(\frac{2\pi d}{\lambda_1}\right)^2\left[\kappa_m'\left((\kappa')^2 - (\kappa'')^2\right) - 2\kappa_m''\kappa'\kappa''\right] \qquad \text{(A.I.28)}$$

$$\Delta\kappa'' = \frac{1}{3}\left(\frac{2\pi d}{\lambda_1}\right)^2\left[\kappa_m''\left((\kappa')^2 - (\kappa'')^2\right) + 2\kappa_m\kappa'\kappa''\right] \qquad \text{(A.I.29)}$$

$$\Delta\kappa_m' = \frac{1}{3}\left(\frac{2\pi d}{\lambda_1}\right)^2\left[\kappa'\left((\kappa_m')^2 - (\kappa_m'')^2\right) - 2\kappa''\kappa_m'\kappa_m''\right] \qquad \text{(A.I.30)}$$

$$\Delta\kappa_m'' = \frac{1}{3}\left(\frac{2\pi d}{\lambda_1}\right)^2\left[\kappa''\left((\kappa_m')^2 - (\kappa_m'')^2\right) + 2\kappa'\kappa_m'\kappa_m''\right] \qquad \text{(A.I.31)}$$

F. Input Impedance of Infinite Length Sample (TE or TEM waves)

For nonmagnetic dielectrics

$$\kappa' - j\kappa'' = \frac{1}{w}\left[u + \frac{1}{\left(\frac{Z_B}{Z_{01}}\right)^2}\right]. \qquad \text{(A.I.32)}$$

For conductors $\kappa'' \gg \kappa'$; E_{min}/E_{max} and $\tan \frac{2\pi x_c}{\lambda}$ are small and equal in magnitude. Then

$$\kappa'' = \frac{1}{2w\left(\frac{E_{min}}{E_{max}}\right)^2} \qquad \text{(A.I.33)}$$

$$\sigma = \frac{1}{4\pi w f \epsilon_o} \left(\frac{E_{max}}{E_{min}} \right)^2 = \frac{\lambda_o}{4\pi w Z_o} \left(\frac{E_{max}}{E_{min}} \right)^2 . \qquad \text{(A. I. 34)}$$

σ will be in reciprocal ohm-cm when ϵ_o is given in fd/cm and λ_o in cm.

APPENDIX II

Corrections for Sample Fit in Coaxial Line

Letting D_1 = diam. of center conductor,
D_2 = diam. of hole in sample ,
D_3 = outside diam. of sample ,
D_4 = inside diam. of outer conductor ,

and defining

$L_1 = \log D_2/D_1 + \log D_4/D_3$,
$L_2 = \log D_3/D_2$,
$L_3 = \log D_4/D_1$,

gives:

$$\kappa'_{cor.} = \kappa'_{meas.} \frac{1 - \frac{L_1}{L_3} \kappa'_{meas.} (1 + \tan^2 \delta_{meas.})}{\frac{L_3}{L_2} - 2\frac{L_1}{L_2} \kappa'_{meas.} + \frac{L_1^2}{L_1 L_3} (\kappa'_{meas.})^2 (1 + \tan^2 \delta_{meas.})} , \qquad \text{(A. II. 1)}$$

$$\tan \delta_{cor.} = \tan \delta_{meas.} \frac{1}{1 - \frac{L_1}{L_3} \kappa'_{meas.} (1 + \tan^2 \delta_{meas.})} . \qquad \text{(A. II. 2)}$$

When $\tan^2 \delta_e \ll 1$, the above equations simplify to

$$\kappa'_{cor.} = \kappa'_{meas.} \frac{L_2}{L_3 - \kappa'_{meas.} L_1} , \qquad \text{(A. II. 3)}$$

$$\tan \delta_{cor.} = \tan \delta_{meas.} \left[1 + \kappa'_{meas.} \frac{L_1}{L_2} \right] = \tan \delta_{meas.} \frac{L_3}{L_2} \frac{\kappa'_{cor.}}{\kappa'_{meas.}} . \qquad \text{(A. II. 4)}$$

In addition, when clearances are small, they may both be referred to the same diameter and added

$$\kappa'_{cor.} - \kappa'_{meas.} = \frac{\left[\left(\kappa'_{meas.}\right)^2 - \kappa'_{meas.}\right] \frac{\text{clearance}}{\text{diameter}}}{2.30 \log_{10} \frac{D_4}{D_1}} \qquad \text{(A. II. 6)}$$

$$\tan \delta_{cor.} = \tan \delta_{meas.} \left[\frac{1 + \kappa'_{meas.} \frac{\text{clearance}}{\text{diameter}}}{2.30 \log_{10} \frac{D_4}{D_1}} \right] \qquad \text{(A. II. 7)}$$

APPENDIX III

Charts and Sample Calculations

A. Charts of the complex function cothx/x. These have been refined and extended since previously given[6] and show values of the function

$$\frac{\coth T \angle \tau}{T \angle \tau} = C \angle -\zeta .$$

Chart XVI. T, 0 to 0.8^r; τ, 38° to 90° (abscissa is 1/C)

XVII. T, 0.8 to 1.5^r; τ, 50° to 90°

CHART XVI

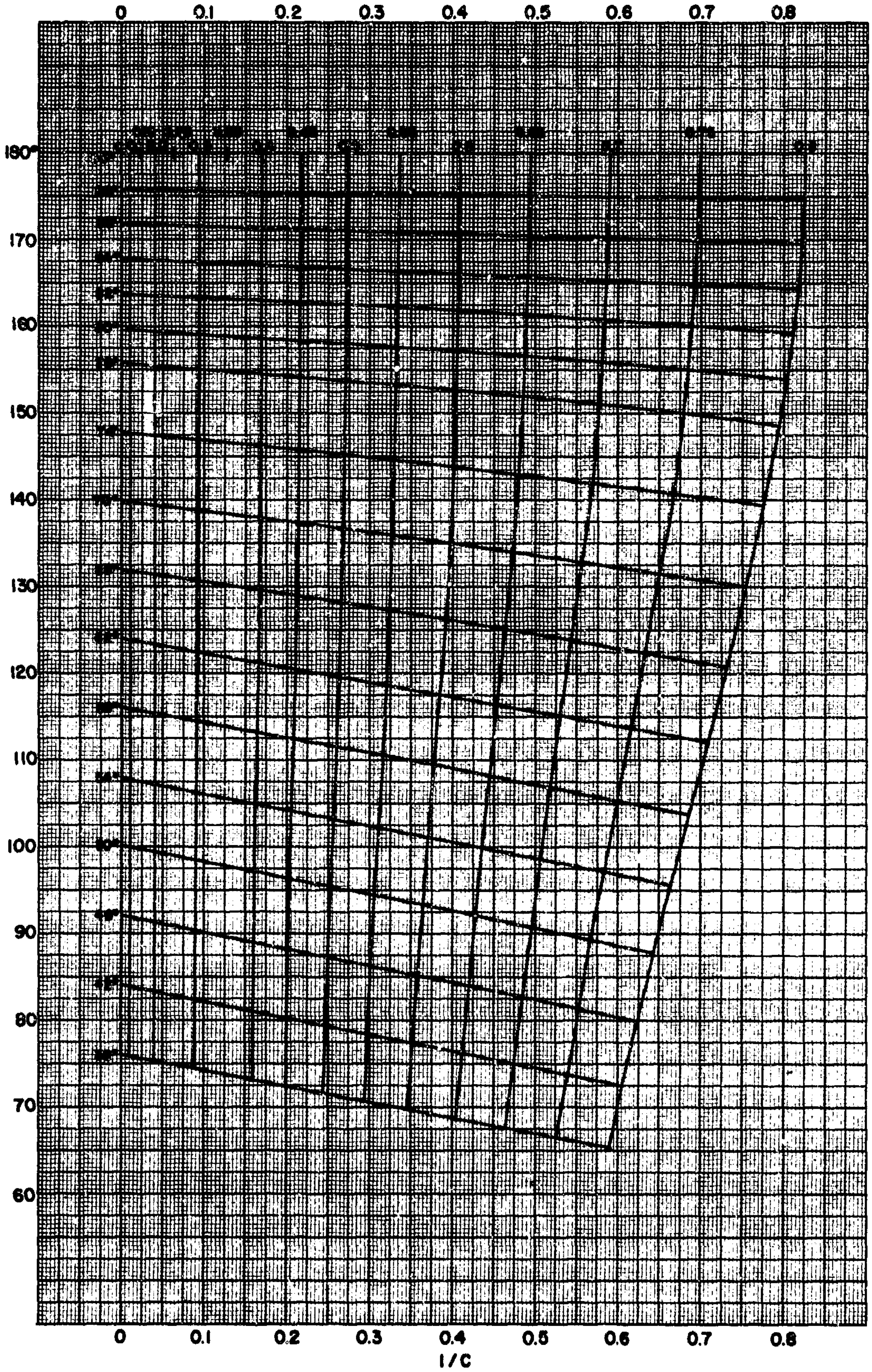


Chart XVI. T, 0 to 0.8, τ, 38° to 90°

CHART XVII

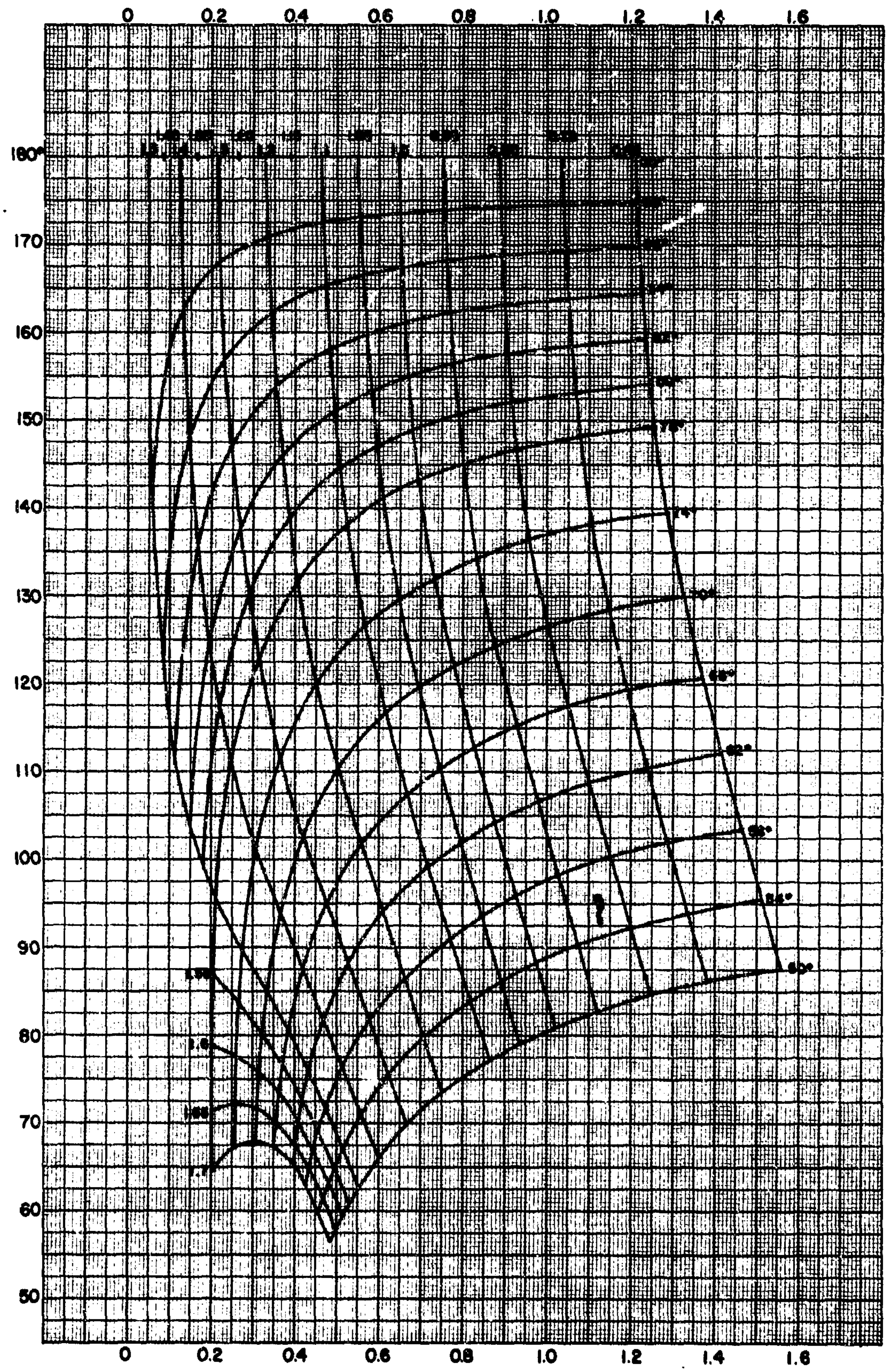


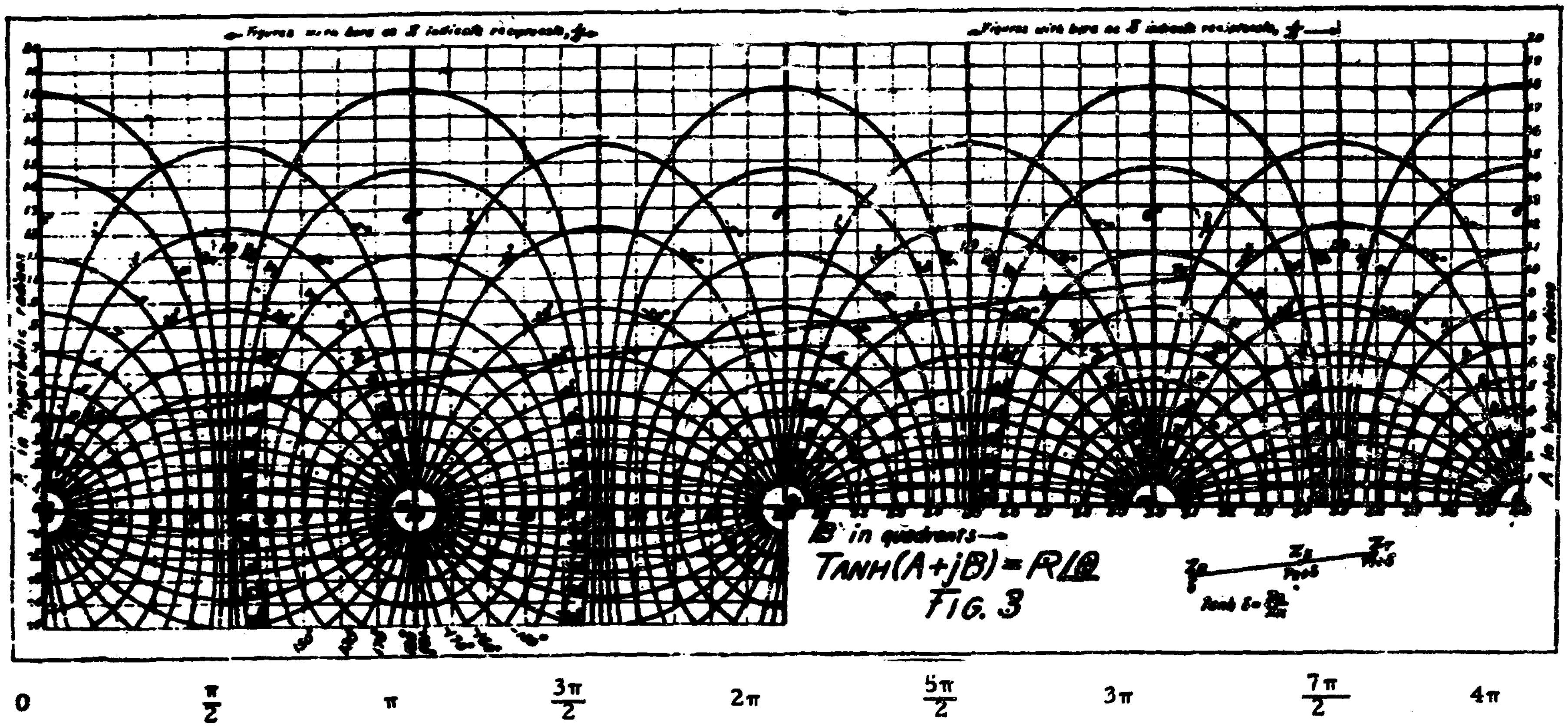
Chart XVII. T, 0.8 to 1.5, τ, 50° to 90°

B. Charts of the complex function tanhx. These are shown in the form $\tanh (a + jb) = r\underline{/\theta}$. Chart I is a survey chart[7] with b in quadrants, showing the symmetry properties. Chart II shows the first half-quadrant of Chart I in enlarged form with b in radians. Chart III shows an enlarged section of Chart II near the origin. Charts IV, b and c, show parts of the first quadrant for low-loss samples only $(a \leq 0.1)$.

Acknowledgments

We are grateful to Barbara B. East and John J. Mara for preparation of all charts shown in this section.

CHART I



B in radians

Note: regarding the following charts: with $r \underline{/\theta}$ use b as read; $\frac{1}{r}\underline{/\theta}$ use 1.5708 - b; $\frac{1}{r}\underline{/-\theta}$ use 1.5708 + b; $r\underline{/\theta}$ use 3.1416 - b. Multiples of π may be added to any of these values.

CHART II

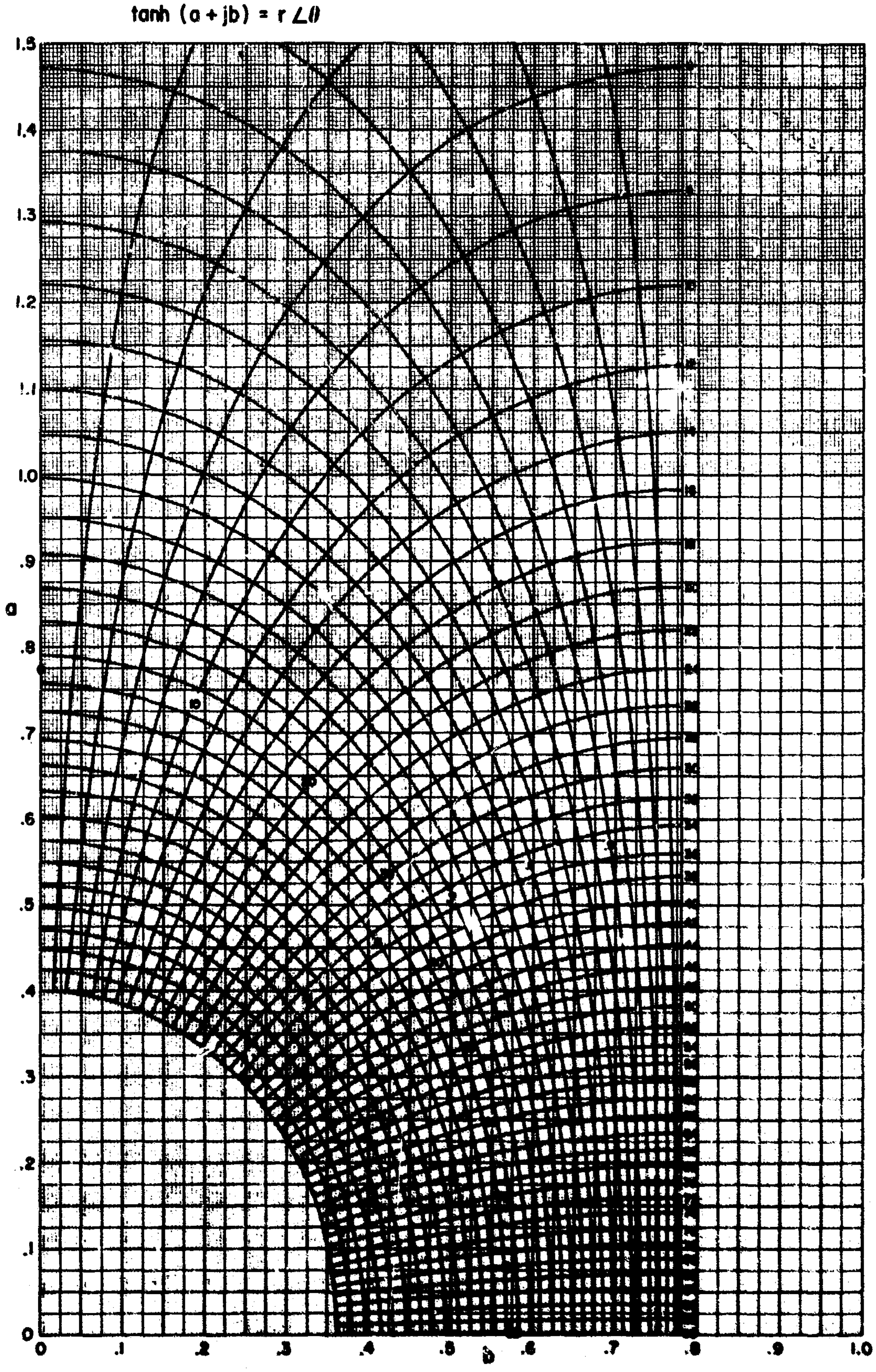


CHART III

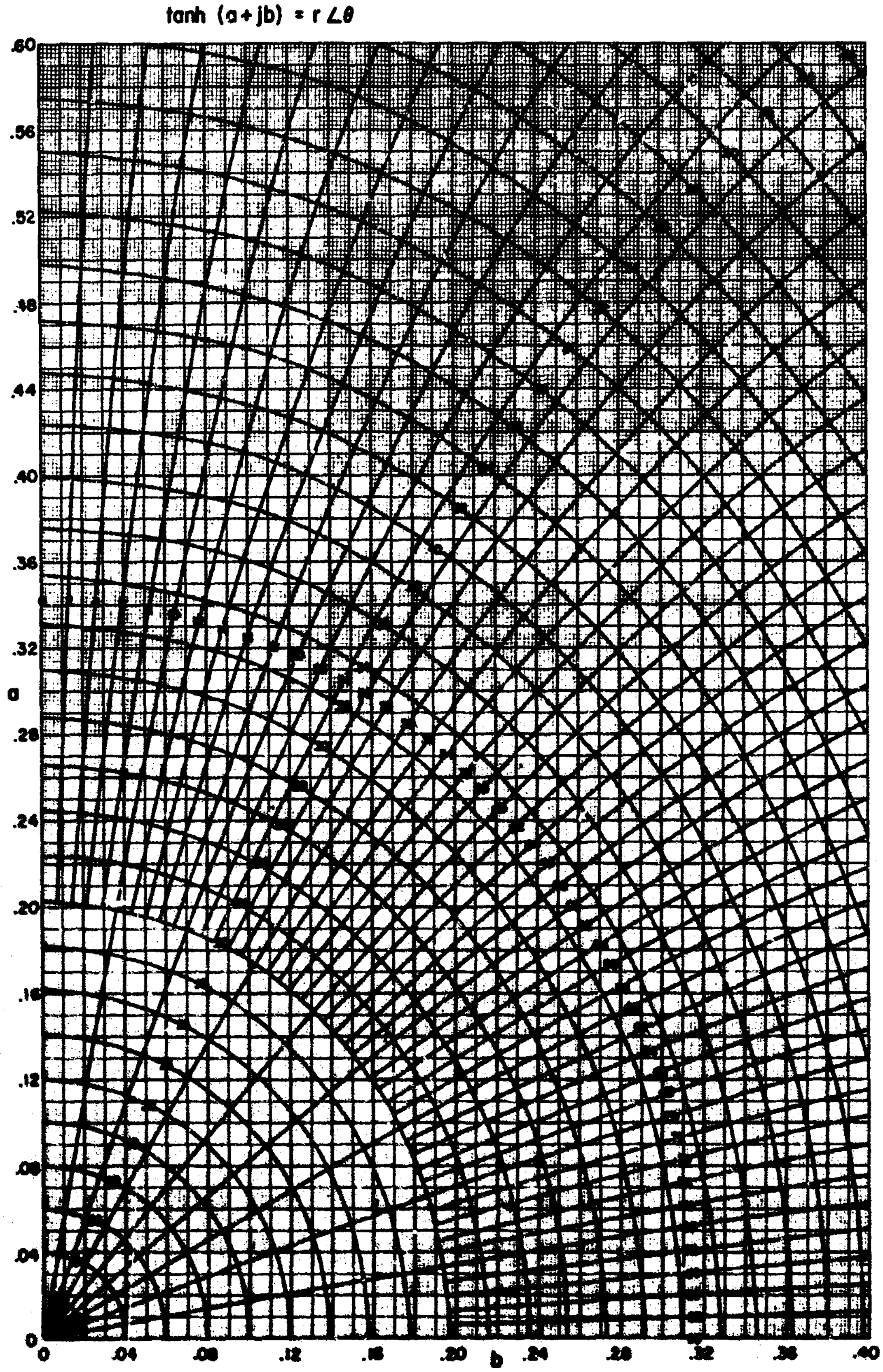

tanh (a+jb) = r ∠θ
.60
.56
.52
.48
.44
.40
.36
.32
a
.28
.24
.20
.16
.12
.08
.04
0
0
.04
.08
.12
.16
.20
b
.24
.28
.32
.36
.40

CHART IVb

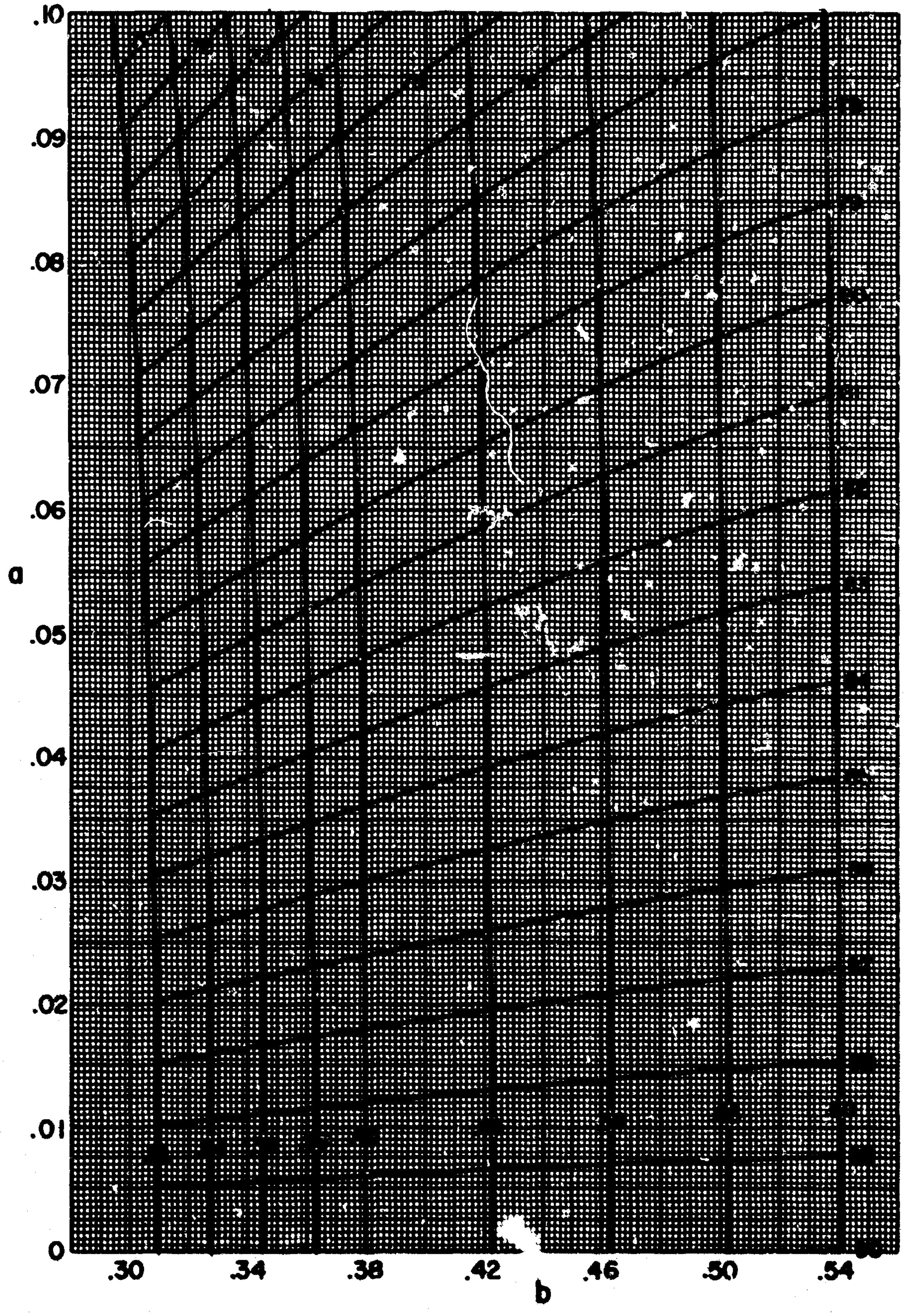
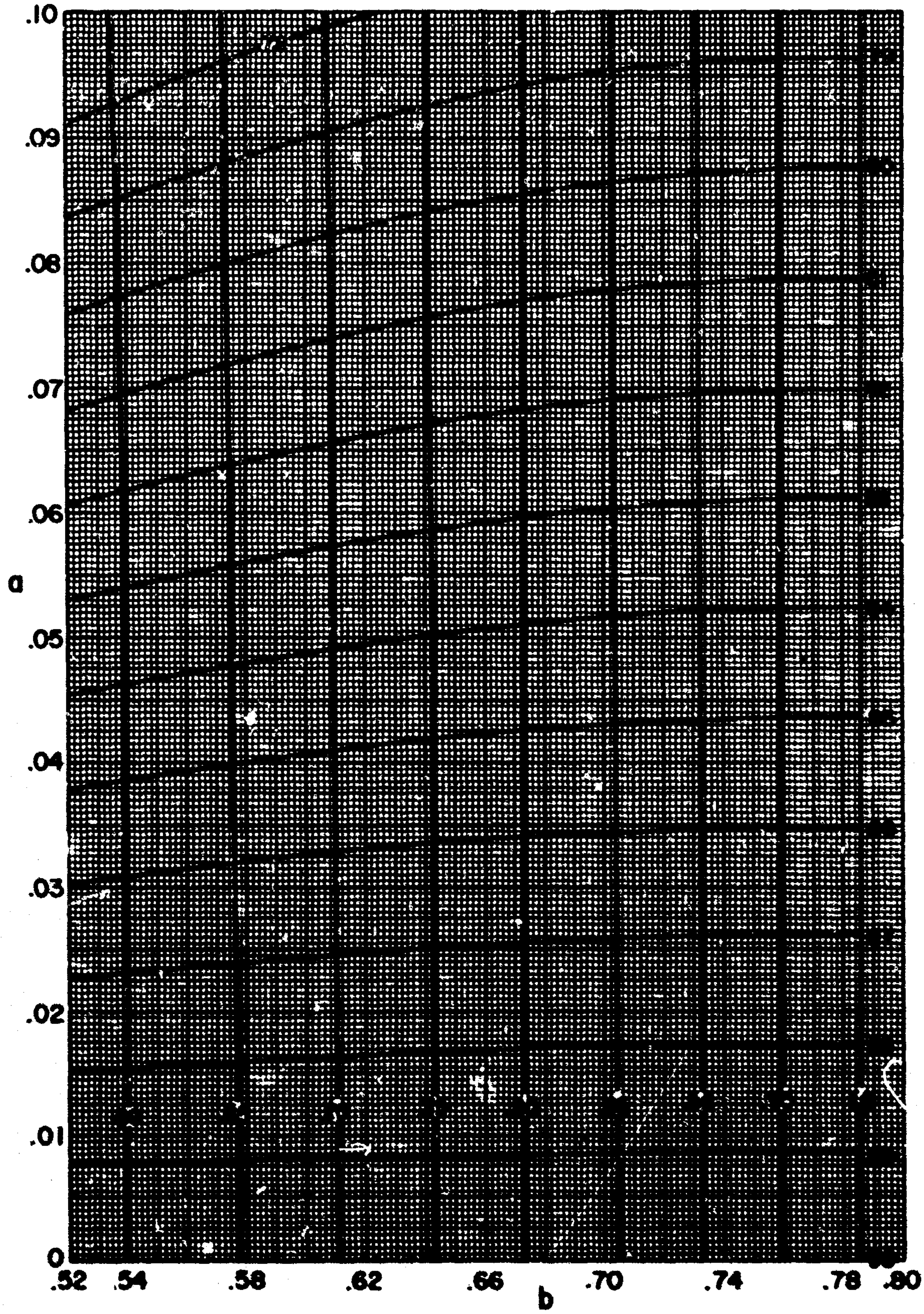
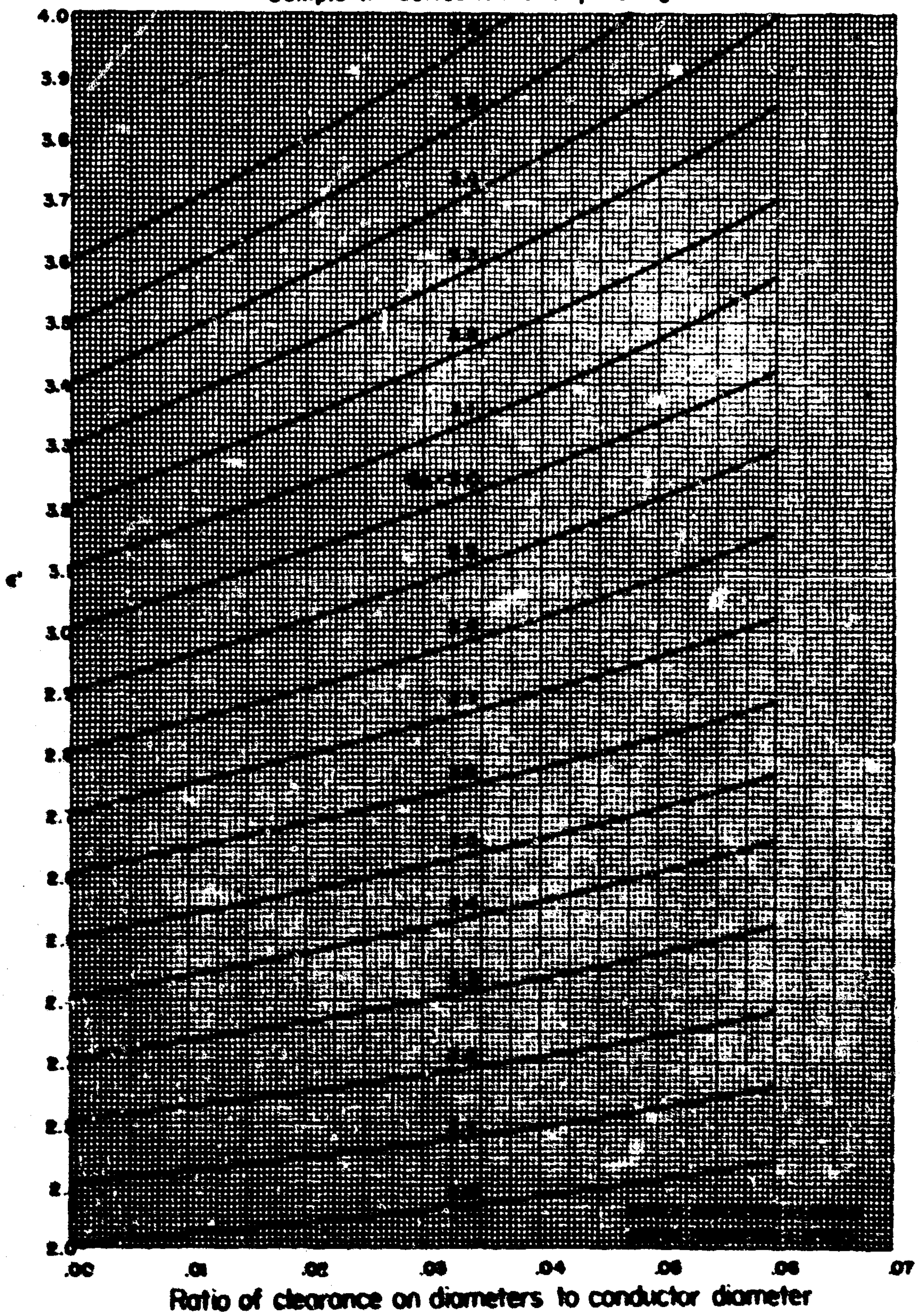


CHART IVc

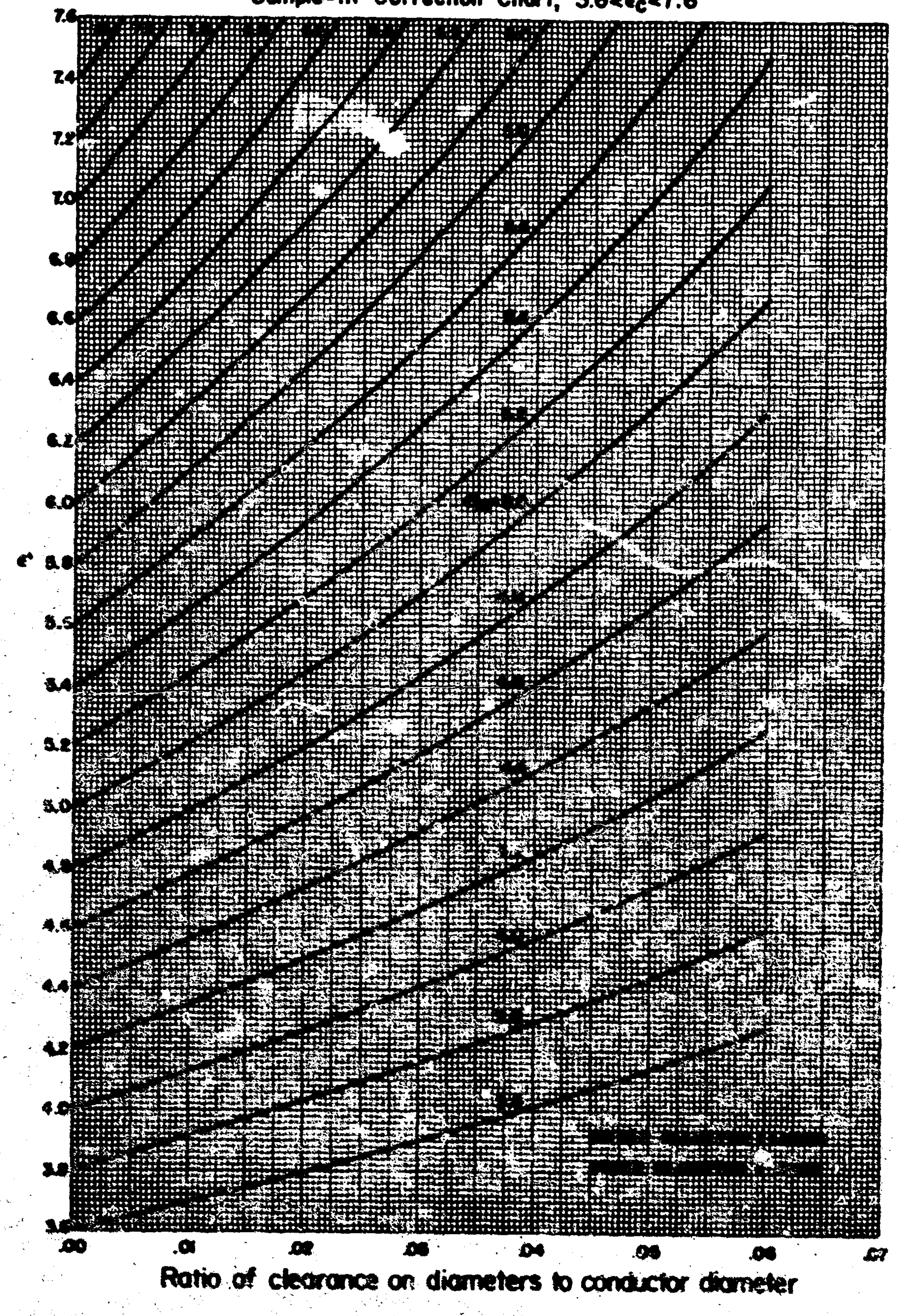


C. Charts for Sample Fit Corrections in Coaxial Line with Diameter Ratio 8/3

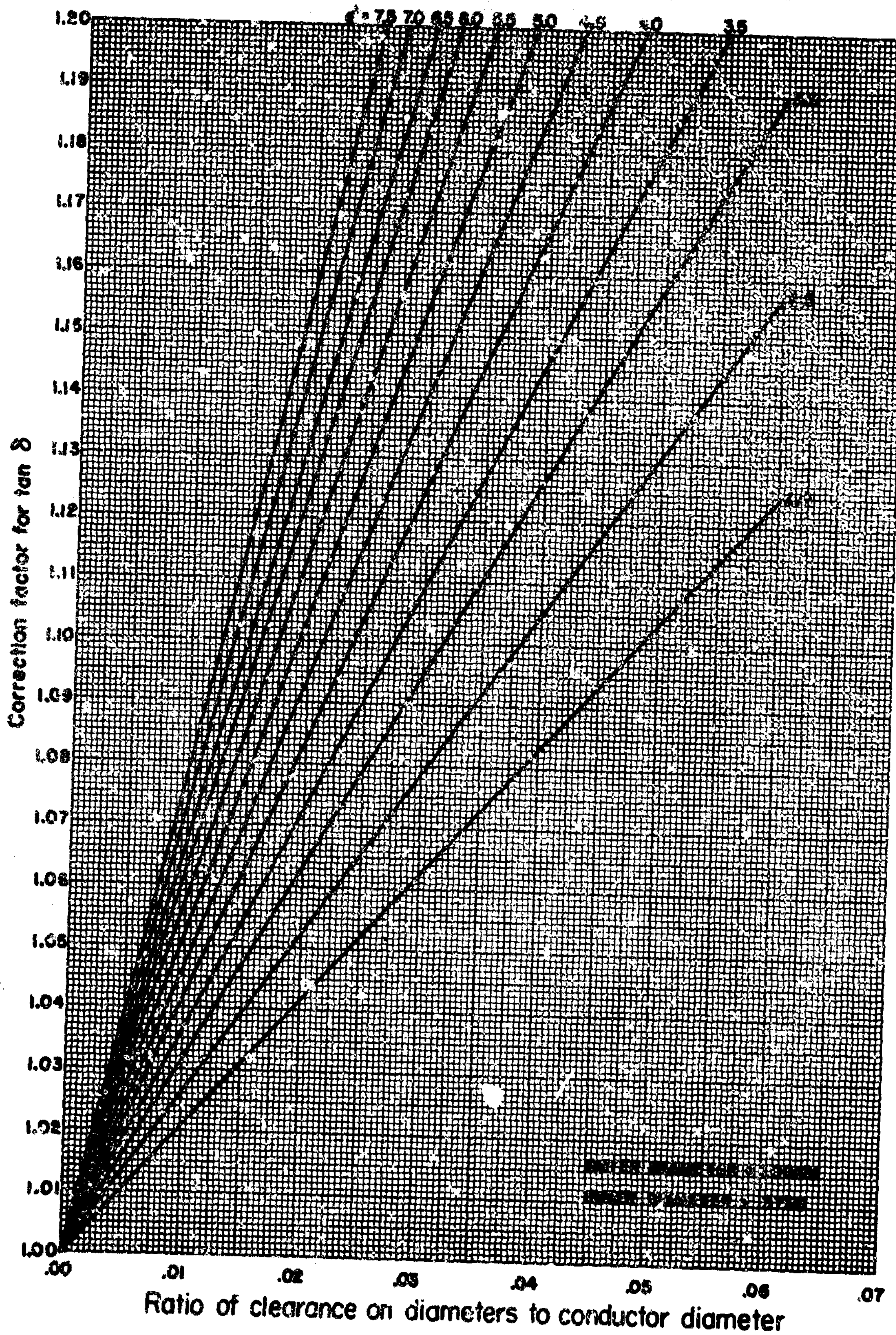
Sample-fit Correction Chart, $2.0 < \epsilon_c < 4.0$



Sample-fit Correction Chart, $3.6 < \epsilon_c < 7.6$



Sample-fit Correction Chart, $\tan \delta_c$ for $\epsilon'_c = 2.0$ to 7.5



D. 1. Sample Calculation for an Epoxy-Loaded with Al in Coaxial Line

Sample #	λ = 30.000 coaxial line	$\varepsilon'/\varepsilon_0$ = 7.90
Run #	Date =	μ'/μ_0 = 0.825
Sample: Aluminum-loaded epoxy	Temp =	$\tan\delta_d$ = 0.0154
	Thickness = 1.8665 cm	$\tan\delta_m$ = 0.0764

I Sample on terminal plate

(1) $x_0 + \text{mic.} = \begin{bmatrix} N_s = 14.8040 \\ H_s = \\ \frac{n_a\lambda}{2} = 30.0000 \\ S_a = \end{bmatrix} - \begin{bmatrix} N_a = 15.0948 \\ H_a = \\ \frac{n_s\lambda}{2} = \\ S_s = \\ d = 1.8665 \\ \hline 16.9613 \end{bmatrix}$

$= 30 - 2.1573$

(2) $\Delta x = \begin{array}{r} .4775 \\ 5 \\ \hline .4725 \end{array}$

(3) $\frac{E_{min}}{E_{max}} = \frac{\pi\Delta x}{\lambda} - C_1^{*)} = 0.0495$

(4) $\tan\frac{360 x_0}{\lambda} = -\tan 25.82^\circ = -.484$

(5) $\left[(3)^2 + (4)^2 \right]^{\frac{1}{2}} = \begin{array}{r} 234 \\ .00243 \\ \hline .2364^{\frac{1}{2}} \end{array}$

(6) $\tan^{-1}\frac{(4)}{(3)} = -90^\circ + 5^\circ 5'$

(7) $\tan^{-1} (3) \times (4) = -.02392^r = -1^\circ 22'$

(8) $\left[1 + (3)^2 \times (4)^2 \right]^{\frac{1}{2}} = 1.0012$

(9) $Z_I = \frac{(5)}{(8)} e^{j\left[-(6) + (7)\right]}$

$= .487 \; 90^\circ - 7^\circ 12'$

II Sample $\lambda/4$ from terminal plate

(1) $x_0 + \text{mic.} = \begin{bmatrix} N_s = 10.2820 \\ H_s = \\ \lambda/4 = 7.5000 \\ \frac{n_a\lambda}{2} = \\ S_a = \\ \hline 17.7820 \end{bmatrix} - \begin{bmatrix} N_q = 14.8535 \\ H_a = \\ \frac{n_s\lambda}{2} = \\ S_s = \\ \lambda/4 = \\ d = 1.8665 \\ \hline 16.7200 \end{bmatrix}$

$= 1.0620$

(2) $\Delta x = \begin{array}{r} .1324 \\ 3 \\ \hline .1321 \end{array}$

(3) $\frac{E_{min}}{E_{max}} = \frac{\pi\Delta x}{\lambda} - C_1^{*)} = .01382$

(4) $\tan\frac{360 x_0}{\lambda} = \tan 12.77^\circ = .2268$

(5) $\left[(3)^2 + (4)^2 \right]^{\frac{1}{2}} = \begin{array}{r} 0514 \\ 00019 \\ \hline .05159^{\frac{1}{2}} \end{array}$

(6) $\tan^{-1}\frac{(4)}{(3)} = 90^\circ - .0609^r = 90^\circ - 3^\circ 29.4'$

(7) $\tan^{-1} (3) \times (4) = .00314^r = 0^\circ\, 10.8'$

(8) $\left[1 + (3)^2 \times (4)^2 \right]^{\frac{1}{2}} = 1$

(9) $Z_{II} = \frac{(5)}{(8)} e^{j\left[-(6) + (7)\right]}$

$= .227 \; \angle -90^\circ + 3^\circ 40'$

*) Page 65, T.R. 182.

(10) $u = \left(\frac{\lambda_1}{\lambda_c}\right)^2 =$ (11) $v = 1 + u =$

(12) $\frac{Z_2}{Z_o} = \left[Z_I \cdot Z_{II}\right]^{1/2} = \frac{Z_p}{Z_o} \angle\varphi = B \angle\varphi = \sqrt{0.1104} = 0.333 \angle{-1^\circ 46'}$

(13) $\tanh \gamma_2 d_2 = \left[\frac{Z_I}{Z_{II}}\right]^{1/2} = 0.2148 = 1.465 \angle{90^\circ} - 5^\circ 26' = \frac{1}{.683}$

from Chart IVc

(14) $\gamma_2 d_2 = 0.04425 + j\,\frac{\overset{1.5708}{0.598}}{0.9728} = 0.973 \angle{90^\circ} - 0.0455^r$

(15) $\frac{\gamma_2 d_2}{j\,\frac{2\pi d_2}{\lambda_1}} = A \angle{-\psi} = 2.49 \angle{-0.0455^r}$

(16) $(\varphi + \psi) = 0.01465^r$ (17) $\sin(\varphi + \psi) =$ (18) $\cos(\varphi + \psi) = 0.9999$

(19) $(\varphi - \psi) = 0.07635^r$ (20) $\sin(\varphi - \psi) =$ (21) $\cos(\varphi - \psi) =$

(22) $\tan \delta_m = -\tan(\varphi - \psi) = 0.0764$

(23) $\frac{\mu'}{\mu_o} = \frac{AB}{(1 + \tan^2\delta_m)^{1/2}} = 0.825$

(24) $\frac{\epsilon'}{\epsilon_o} = \frac{A}{B}\ \frac{(18)}{w} + \frac{u}{wAB}\ (21) =$

(25) $\tan \delta_d = \frac{\frac{A}{B}\ \frac{(17)}{w} + \frac{u}{wAB}\ (20)}{\frac{A}{B}\ \frac{(18)}{w} + \frac{u}{wAB}\ (21)} =$

For coaxial line omit steps 17, 20, 21, 24, 25 above

(24) $\frac{\epsilon'}{\epsilon_o} = \frac{A}{B}$ (18) $= \frac{\overset{7.47}{0.43}}{7.90}$ ←correction for sample fit

(25) $\tan \delta_d = \tan(\varphi + \psi) = 0.0146$

-0.0003 ← wall loss

$0.0143 \times 1.076 = 0.0154$

↑sample fit

D. 2. Sample Calculation for a Ferrite in Hollow Wave Guide

Sample # 2310	λ = 5.748	$\varepsilon'/\varepsilon_o$ = 18.7
Run # 1	Date = 5/23/50	μ'/μ_o = 0.51
Sample Ferramic C	Temp = 25°C	$\tan\delta_d$ = 0.58
	Thickness =	$\tan\delta_m$ = 1.26

I Sample on terminal plate

(1) x_o = +mic. $\left[N_s = ;\ H_s = ;\ \frac{n_a\lambda}{2} = ;\ S_a = \right] - \left[N_a = ;\ H_a = ;\ \frac{n_s\lambda}{2} = ;\ S_s = ;\ d = \right]$

=

(2) Δx = .2016 − $\frac{2}{3}$ .0028 = .1997

(3) $\frac{E_{min}}{E_{max}} = \frac{\pi\Delta x}{\lambda} - C_1{}^{*)}$ = .109

(4) $\tan \frac{360 x_o}{\lambda}$ = −.00366

(5) $\left[(3)^2 + (4)^2 \right]^{\frac{1}{2}}$ = .109

(6) $\tan^{-1} \frac{(4)}{(3)}$ = $\tan^{-1}$ −.3355 = 161°27.2'

(7) $\tan^{-1}$ (3) x (4) = 1

(8) $\left[1 + (3)^2 \times (4)^2 \right]^{\frac{1}{2}}$ = 1

(9) $Z_I = \frac{(5)}{(8)} e^{j\left[-(6) + (7)\right]}$

= .109 18°32.5'

II Sample $\lambda/4$ from terminal plate

(1) x_o = $\lambda/4$ + mic. $\left[N_s = ;\ H_s = ;\ \frac{n_a\lambda}{2} = ;\ S_a = \right] - \left[N_q = ;\ H_a = ;\ \frac{n_s\lambda}{2} = ;\ S_s = ;\ \lambda/4 = ;\ d = \right]$

=

(2) Δx = .1531 − $\frac{2}{3}$ .0028 = .1512

(3) $\frac{E_{min}}{E_{max}} = \frac{\pi\Delta x}{\lambda} - C_1{}^{*)}$ = .0826

(4) $\tan \frac{360 x_o}{\lambda}$ = tan 4.03° = .7045

(5) $\left[(3)^2 + (4)^2 \right]^{\frac{1}{2}}$ = .1082

(6) $\tan^{-1} \frac{(4)}{(3)}$ = $\tan^{-1}$ .00582 = 20'

(7) $\tan^{-1}$ (3) x (4) = 1

(8) $\left[1 + (3)^2 \times (4)^2 \right]^{\frac{1}{2}}$ = 1

(9) $Z_{II} = \frac{(5)}{(8)} e^{j\left[-(6) + (7)\right]}$

= .1082 −40°6'

*) Page 65, T.R. 182.

(10) $u = \left(\frac{\lambda_1}{\lambda_c}\right)^2 = 2.33$ (11) $w = 1 + u = 3.33$

(12) $\frac{Z_2}{Z_o} = \left[Z_I \cdot Z_{II}\right]^{1/2} = \frac{Z_p}{Z_o} \underline{/\varphi} = B\underline{/\varphi} = \sqrt{.1086} = \frac{1}{2} -21^o33' = \underline{/-10^o47'}$

(13) $\tan \gamma_2 d_2 = \left[\frac{Z_I}{Z_{II}}\right]^{1/2} = 1.004 \underline{/\frac{1}{2}(58^o39')} = 29^o20'$

from Chart II

(14) $\gamma_2 d_2 = .675 + .785^r = 1.033 \underline{/\tan^{-1} 1.163} = 49^o19'$

(15) $\frac{\gamma_2 d_2}{j \frac{2\pi d_2}{\lambda_1}} = A\underline{/-\psi} = \left(\frac{1.033 \times 5.74}{6.28 \times .1250} = 7.55\right) \underline{/-40^o41'}$

(16) $(\varphi + \psi) = 29^o54'$ (17) $\sin(\varphi + \psi) = .498$ (18) $\cos(\varphi + \psi) = .867$

(19) $(\varphi - \psi) = -51^o28'$ (20) $\sin(\varphi - \psi) = -.623$ (21) $\cos(\varphi - \psi) = .782$

(22) $\tan \delta_m = \tan(\varphi - \psi) = 1.256$

(23) $\frac{\mu'}{\mu_o} = \frac{AB}{(1 + \tan^2\delta_m)^{1/2}} = \frac{.1086 \times 7.55}{(1 + 1.58)^{1/2}} = 0.510$

(24) $\frac{\epsilon'}{\epsilon_o} = \frac{A}{B} \frac{(18)}{w} + \frac{u}{wAB} (21) = 69.5 \frac{.867}{3.33} + \frac{2.33 \times .782}{3.33 \times .818} = 18.06 + .668 = 18.73$

(25) $\tan \delta_d = \frac{\frac{A}{B}\frac{(17)}{w} + \frac{u}{wAB}(20)}{\frac{A}{B}\frac{(18)}{w} + \frac{u}{wAB}(21)} = \frac{10.4 - .532}{18.73} = .527$

For coaxial line omit steps 17, 20, 21, 24, 25 above.

(24) $\frac{\epsilon'}{\epsilon_o} = \frac{A}{B}\, 18 =$

(25) $\tan \delta_d = \tan(\varphi + \psi) =$

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7. H. E. Hartig, Physics 1, 386 (1931).

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13. ABSTRACT

Measurement techniques for determination of the complex dielectric constant and complex permeability are given ~~in a short review~~ with emphasis on the problems encountered in practical high-loss materials. Typical calculations and descriptions of special sample holders are included.

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