UNCLASSIFIED

AD NUMBER

AD479781

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies only; Administrative/Operational Use; Nov 1963. Other requests shall be referred to Commanding General, U.S. Army Tank-Automotive Center, Attn: SMOTA-RTS, Warren, MI 48090.

AUTHORITY

USATAC ltr, 10 Apr 1974

THIS PAGE IS UNCLASSIFIED
ABSTRACT

Vehicle Concept Evaluation

Liston, Ronald A.

November 1963

8241 LL 94

5521.11.270

12. RELEASE STATEMENTS (For ASTIA use only)

Authors' Key Terms - Unclassified Only

1. Vehicles
2. Land locomotion
3. Cylindrical wheels
4. Hemispherical wheels
5. Cargo carriers
6. Soft-soil performance
7. Motion resistance
8. Cohesion of soil
9. Soil strength
10. Internal friction
11. Coefficient of friction
12. Soft soil

ASTIA DESCRIPTORS (For ASTIA use only)

A/2 scale model XM653 8x8, 5 ton Cargo Carrier
B/2 scale model 4x4 5 ton cargo carrier
C/2 scale model 4x4, 5 ton RHINO cargo carrier.

BEST AVAILABLE COPY
The purpose of the test is the evaluation of the soft-soil performance of a vehicle concepted based on an inclined hemispherical wheel. The performance of the concept was compared to a concept utilizing cylindrical wheels of similar size to the hemispherical wheel and to a conventional vehicle of equal payload capacity currently under development. Tests were conducted to establish the performance of 1/4 scale models of each of the vehicle concepts in the large soil bins located in the Land Ecosystem Laboratory. The models were tested in sand and in a sandy loam at three different moisture contents.

The models were given code designations and are identified as Concepts A, B, and C. The test results indicate that Concepts B and C have significantly better soft-soil performance than Concept A. The performance of Concept B is equal to that of Concept C. It is concluded that Concept C does not offer any improvement in soft-soil performance and should not be considered as a device to improve mobility on the basis of its soft soil characteristics.

The recommendations are that Concept C be given no further consideration for application to military vehicles.

Evaluation of soft-soil performance of a vehicle concept based on an inclined hemispherical wheel.
This document is the best quality available. The copy furnished to DTIC contained pages that may have the following quality problems:

- Pages smaller or larger than normal.
- Pages with background color or light colored printing.
- Pages with small type or poor printing; and or
- Pages with continuous tone material or color photographs.

Due to various output media available these conditions may or may not cause poor legibility in the microfiche or hardcopy output you receive.

☐ If this block is checked, the copy furnished to DTIC contained pages with color printing, that when reproduced in Black and White, may change detail of the original copy.
UNCLASSIFIED

COMPONENTS R & D LABORATORIES

LAND LOCOMOTION LABORATORY

Report No. 8241
LL No. 94

VEHICLE CONCEPT EVALUATION

By

Ronald A. Liston

November, 1963

DDC AVAILABILITY NOTICE

"U. S. MILITARY AGENCIES MAY OBTAIN COPIES OF THIS REPORT DIRECTLY FROM DDC. OTHER QUALIFIED USERS REQUEST THROUGH ARMY TANK-AUTOMOTIVE CENTER, ATTN: SMOTA-RTS."

Project No.: 5521.11.270
D/A Project No.: 1-D-5-21801-D-206

Authenticated: S. H. FULLER
Approved: JOHN W. WISS
Dep, Components R&D
Lt Colonel, Ordnance Corps
Laboratories
Chief, Components R&D
Laboratories

U.S. ARMY TANK-AUTOMOTIVE CENTER
WARREN, MICHIGAN

CODE SHEET INCLOSED

UNCLASSIFIED
"THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION"
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Abstract</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>iv</td>
</tr>
<tr>
<td>Symbols</td>
<td>v</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Object</td>
<td>3</td>
</tr>
<tr>
<td>Summary</td>
<td>4</td>
</tr>
<tr>
<td>Conclusions</td>
<td>8</td>
</tr>
<tr>
<td>Recommendations</td>
<td>11</td>
</tr>
<tr>
<td>Theoretical Analysis</td>
<td>12</td>
</tr>
<tr>
<td>Test Material</td>
<td>20</td>
</tr>
<tr>
<td>Test Equipment</td>
<td>23</td>
</tr>
<tr>
<td>Test Procedure</td>
<td>28</td>
</tr>
<tr>
<td>Results and Discussion</td>
<td>40</td>
</tr>
<tr>
<td>Accuracy of Results</td>
<td>48</td>
</tr>
<tr>
<td>References</td>
<td>50</td>
</tr>
<tr>
<td>Graphs, Figures and Tables</td>
<td>51</td>
</tr>
<tr>
<td>Appendix A</td>
<td>94</td>
</tr>
<tr>
<td>Figures 1a thru 5a</td>
<td>105</td>
</tr>
<tr>
<td>Appendix B</td>
<td>110</td>
</tr>
<tr>
<td>Appendix C</td>
<td>114</td>
</tr>
</tbody>
</table>
ABSTRACT

The report is concerned with the evaluation of the soft-soil performance of a vehicle concept based on an inclined hemispherical wheel. The performance of the concept was compared to a concept utilizing cylindrical wheels of similar size to the hemispherical wheel and to a conventional vehicle of equal payload capacity currently under development. Tests were conducted to establish the performance of 1/4 scale models of each of the vehicle concepts in the large soil bins located in the Land Locomotion Laboratory. The models were tested in sand and in a sandy loam at three different moisture contents.

The models were given code designations and are identified as Concepts A, B, and C. The test results indicate that Concepts B and C have significantly better soft-soil performance than Concept A. The performance of Concept B is equal to that of Concept C. It is concluded that Concept C does not offer any improvement in soft-soil performance and should not be considered as a device to improve mobility on the basis of its soft soil characteristics.

The recommendations are that Concept C be given no further consideration for application to military vehicles.
ACKNOWLEDGEMENTS

This test was conducted under the guidance of Mr. S. H. Fuller, Deputy, Components Research and Development Laboratories, ATAC. Dr. E. H. Jebe, Consultant in Statistics and Design of Experiments and Research Mathematician for the Institute of Science and Technology of The University of Michigan, reviewed the test plan and made many valuable suggestions to improve the test plan and testing procedure. Mr. P. Spanski, Land Locomotion Laboratory, was responsible for the design and construction of test models and equipment. Lt. R. T. Szwarc conducted the actual tests and designed some of the model vehicles. Mr. Z. Janosi prepared the analysis of the test wheel, and Mr. L. Martin prepared the statistical analysis of the data. In one way or another, every member of the Land Locomotion Laboratory assisted in the test program reported herein and their assistance is gratefully acknowledged.

RONALD A. LISTON
Chief, Land Locomotion Laboratory
Components R&D Laboratories
A. **Background:** This report is concerned with the testing of three 1/4 scale models of different 5 ton, Cargo Carrier concepts. The reason for conducting the tests was to obtain an experimental evaluation of the off-road performance of one of the concepts which was based on a hemispherical wheel tilted at 30° to the vertical. This concept was to be compared to a model having conventional wheels of a similar size as the hemispherical wheels and to a model of an 8x8, 5 ton vehicle currently under development for military use. The hemispherical wheel was not new to the Army since the inventor had offered it as a solution to off-road mobility problems on several occasions. In addition, a vehicle had been built for the inventor by a manufacturer, and a considerable amount of publicity had been given the machine. Because of the interest generated in the tilted hemispherical wheel, a contract was negotiated in 1957 between the Land Locomotion Laboratory and the Stevens Institute of Technology to conduct a study of the wheel (1).

B. **Tests at Stevens Institute of Technology:** The tests conducted by Stevens were concerned with an evaluation of the soft soil performance of a hemispherical wheel, rather than of a vehicle equipped with the wheels. In order to provide a basis for comparison, two conventional wheels were tested: one wheel having the same diameter and volume as the hemispherical wheel and the second having the same...
diameter and width as the hemispherical wheel. The tests consisted of: the determination of the relationship between load and static sinkage; the relationship between load and rolling resistance; and the relationship between load and drawbar-pull. In each case, the wheels were tested at tilt angles of 0°, 15°, and 30°. The wheels were tested in two artificial soils: a non-plastic synthetic clay (Cereclay) and a highly plastic clay-water mixture (Volclay).

The conclusions of the Stevens' test are quoted (1):

"On the basis of the foregoing (analysis) it must be concluded that tilted hemispherical wheels provide no advantage over conventional wheels from the viewpoint of wheel-soil interaction on soft ground. No improvement can be expected either in motion resistance or traction. This in no way intends to reflect on the possible advantages or disadvantages of a vehicle equipped with hemispheroidal wheels from other viewpoints such as vehicle stability, design considerations, etc."

The Stevens' tests were necessarily limited by the funds available so that performance in granular soils was not evaluated. The intent of the evaluation was to study the wheel itself so that obstacle and swimming performance was left to conjecture.

C. Land Locomotion Laboratory Tests: Discussions subsequent to the Stevens' investigation between the hemispherical wheel inventor, ATAC personnel and, at a later date, the Commanding General of MOCOM
resulted in an agreement that the Land Locomotion Laboratory conduct an evaluation of a 1/4 scale model of a proposed 5 ton, Cargo Carrier. The proposed vehicle was a 4x4, skid steered vehicle having 72 inch diameter tilted hemispherical wheels. It was agreed that a complete test plan would be prepared and approval of the test plan by the inventor would be obtained prior to initiation of testing. It was further agreed that the inventor and his technical advisor would be in attendance during the testing to permit changes to test procedures and the addition of other tests considered necessary by the inventor. The test plan attempted to include the evaluation of soft soil, obstacle, side-slope and water performance of the hemispherical wheel vehicle concept.

The test program was not completed since the inventor requested that the test be discontinued prior to completion of the soft-soil performance evaluation. The obstacle, side-slope and water performance phases of the test were not initiated.

OBJECT

The object of this test was to conduct an evaluation of the potential off-road performance of a proposed 5 ton, 4x4, Cargo Carrier. The proposed vehicle is unconventional in form because of the use of skid steering and tilted hemispherical wheels. The evaluation was to consider a range of off-road conditions to include sand, strong loam, weak loam,
obstacles such as vertical walls, ditches, side slopes and water. The evaluation was to be accomplished by use of 1/4 scale models of: the hemispherical wheel concept, a vehicle having cylindrical wheels but otherwise similar to the hemispherical wheel concept, and a military 5 ton, 8x8, Cargo Carrier currently under development. The latter two vehicle models were to provide criteria to judge the performance of the hemispherical wheel.

The cylindrical wheels used in this test were constructed to simulate the characteristics of conventional wheels. The designation "cylindrical wheel" is straightforward: the wheel consists of a section of a cylinder. The running surface of the cylindrical wheel was formed so that its surface was the center segment of a sphere, simulating the "crown" effect observed on a standard pneumatic tire. The dimensions of the cylindrical wheel were established by agreement between the inventor of the hemispherical wheel and ATAC personnel — to produce hemispherical and cylindrical wheels having equal diameters and volumes. (See Figure 9.)

The results of the test program were to be used as a basis to determine whether the proposed vehicle had adequate potential for further development.

SUMMARY

The test program originally scheduled was not completed. An adequate number of tests in sand and loam were completed to permit
conclusions to be drawn concerning the soft-soil performance of the three models. Any other conclusions that are offered are based on general observations of the performance of the three models, and very likely could have been made without benefit of the extensive soft-soil tests. All references to results, conclusions and recommendations will be made to Concept A, Concept B, or Concept C. Qualified readers are directed to the code sheet to establish the identity of the concepts.

The drawbar pull-slip test results indicate that there is no significant difference in the soft-soil performance of Concepts B and C. In extremely weak soil, Concept C performed considerably better than Concept B when wheel slip was in excess of 75%. At lesser wheel slip, the two concepts were essentially equal. The improved weak soil performance of Concept C is attributed primarily to a more favorable belly configuration and a more favorable weight transfer characteristic. The effect of weight transfer, resulting from the application of a drawbar load, can be observed by measurement of vehicle trim angle. If a drawbar is applied to a vehicle at a point above the line of action of the resultant of the rear wheel traction forces, a greater load is applied to the rear wheels than to the front wheels. The normal result is that the vehicle sinks further in the rear and a trim angle is assumed. The wheel form utilized on Concept C was shown experimentally by the Stevens test (1), to develop less sinkage for a given load than the wheel used on Concept B. The theoretical analysis by Janosi appear-
ing in Appendix A reached the same conclusion. This meant that for the same amount of weight transfer, a greater trim angle was assumed by Concept B than Concept C. Once a greater trim angle is assumed, the effect of the drawbar load is increased just as if the drawbar had been located at a higher point on the vehicle. This causes an increase in trim angle and so on. What normally happens in such a circumstance is that the drawbar pull reaches a maximum at a relatively low sinkage and the vehicle assumes a trim angle. The drawbar pull then decreases because the motion resistance acting against the rear wheel increases, resulting in a lower net tractive effort to balance the drawbar load. The characteristic described is important when vehicles are required to move towed loads. Concept A was inferior to Concepts B and C in each of the three moisture contents for the loam tests. There was no difference in the performance of Concepts B and C in sand; Concept A was slightly inferior in performance to the other two concepts. The performance of all three concepts in sand can be considered as no better than average for wheeled vehicles.

Concepts A and B were somewhat underpowered during the loam tests and on 18 and 19 July, it was not possible to achieve 100% slip conditions. For example, on 18 July only one point for Concept B was obtained for a slippage in excess of 65%. The gear reduction on all three concepts was increased during the later phases of the test to assure adequate torque to develop the 100% slip condition.
The results of the "free-run" tests* in weak loam did not indicate any difference in performance between Concepts B and C. Concept A performed so poorly in the initial weak-soil free-run tests that it was not included in the subsequent tests. The free-run test permitted the models to operate with no drawbar attached. Performance was measured by determining the maximum load that was required to produce immobilization in a given soil condition. Both Concepts B and C were immobilized by one load and mobile when the load was decreased by 10%, or fifty pounds in this particular test.

*No drawbar load applied.
CONCLUSIONS

On the basis of the broader range of soft-soil tests conducted by the laboratory, there is no justification to modify the conclusions of Stevens Institute concerning soft-soil performance. The variation in soil conditions from one point to another was sufficiently great so that a larger test sample in comparison to the sand tests was required to permit one to draw specific conclusions. A statistical analysis of the test results showed that there was no significant difference in the soft-soil performance of Concepts B and C in any of the soils tested. The analysis also indicated that Concepts B and C were superior in performance to Concept A.

If one merely examines the test results, the same conclusion is obvious except for the results of one test* in which Concept C is clearly superior to Concepts A and B. Examination of films recording the tests of that day indicate a considerably better trim attitude taken by Concept C compared to Concepts A and B. However, the results of the "free-run" tests in which soil conditions were quite similar indicated that the effect of the drawbar-load transfer was the most likely source of difference in performance between Concepts B and C. The superiority of Concept C in accepting a drawbar-load would only be significant when the concept was used to pull a towed load in very weak soil conditions.

It is concluded, therefore, that Concept C does not offer any advantage over Concept B. Both Concepts B and C are capable of considerably better performance than Concept A, but this conclusion does not *(See Figures 25 and 26 for 18 July 63)*
necessarily imply that Concepts B and C represent better solutions for a 5 ton, Cargo Carrier because off-road performance is obtained at the sacrifice of load carrying ability.

It is further concluded that Concept C does not represent a useful solution to fulfilling the military requirement for a 5 ton, Cargo Carrier. This conclusion is reached because of a combination of reasons:

a. The concept requires skid steering unless the vehicle is articulated. However, the proposed concept assumes skid steering which cannot be considered as an efficient method to steer a vehicle. If the concept were modified to accept articulated steering, the steering efficiency would be improved but the soft-soil performance would be no better than a conventional wheel of similar dimensions.

b. If the prototype vehicle is to be useful, it is mandatory that the wheels used on the concept be suspended. It cannot be said that it is impossible to suspend the wheels used on Concept C. It also cannot be said that the mechanism for suspending the wheels used on Concept C will be as simple as that used for Concept B. If a unique component requires increased mechanical complexity, it is essential that the increased complexity be reflected by a proportional improvement in performance. Concept C does not promise such a proportional increase in performance.

c. The form of Concept C does not adapt itself to a cargo carrying role. Unless bulk cargo is carried, it is apparent that the center of gravity of the vehicle will be raised by the available area
for cargo stowage. Once the center of gravity is raised, the stability characteristics of Concept C are reduced.

Concept C was carefully examined for its potential in the 500 pound load class to fulfill the remote area vehicle role. A rough layout was made to examine the possibilities of the concept for such a role. The results were disappointing since the stability was considerably compromised by a high center of gravity when loaded and by a relatively narrow tread width. Even in the light-weight category examined, a rudimentary suspension would be required since it is highly questionable that pneumatic running surfaces on the wheels utilized by Concept C would permit the wheel flexibility required to provide good off-road performance.

It could be argued that the potential of Concept C as a remote area vehicle is equal to that of other remote area vehicle ideas that have been tried. The model of Concept C was considered as a 1/2 scale model of a 500 pound carrier in order to make an analysis of a remote area vehicle. The analysis indicated that the concept was not practical.
RECOMMENDATIONS

It is recommended that no further effort be expended in the evaluation of Concept C. The concept offers no potential in the heavy category of vehicles on the basis of soft-soil performance since it does not provide mobility that cannot be achieved by means of conventional suspensions. When realistic center of gravity locations are assumed, the stability of the vehicle is not significantly better than conventional vehicles. Because of the latter point and because of the unfavorable cargo stowage area, the concept does not have potential as a medium or light weight vehicle.
THEORETICAL ANALYSIS:

A major portion of the theoretical analysis appears in Appendix A prepared by Mr. Z. Janosi, Chief of the Theoretical Land Locomotion Mechanics Section. Janosi's analysis is concerned with a description of the behavior of the tilted hemispherical wheel. The analysis of the performance of conventional wheels has been published by Bekker (2) and Janosi (3), among others, and is an accepted part of the literature of land locomotion mechanics.

When comparing the performance of the hemispherical wheel concept to that of a conventional wheel, it was necessary to develop a set of wheel-soil equations specifically for the hemispherical wheel. The equations that have been derived to describe conventional wheel performance assume a cylindrical wheel form. Thus each point across the face of the wheel is at a fixed sinkage and the only point to point variation in sinkage is along the contact length. When analyzing the tractive effort, sinkage, or motion resistance, it is relatively simple to integrate over the surface of the conventional wheel since the forces only vary in one direction, i.e., with depth. The problems of the conventional wheel can be reduced to two-dimensional at worst. However, when looking at a hemispherical wheel, the computations are much more complex because the point to point sinkage varies both along and across the contact surface. Thus, for the hemispherical wheel it is necessary that surface integrals rather than area integrals be evaluated. It is quite reasonable to evaluate surface...
integrals but the resulting expressions do not lend themselves to ready visual analysis. It is necessary to complete a set of calculations for both a hemispherical and conventional wheel and compare the results in graphical form.

The basis for comparing soft soil performance is normally taken as the drawbar pull-weight ratio plotted against wheel slip. In order to simplify the computations the performance of the hemispherical and conventional, or cylindrical, wheels was predicted on the basis of maximum drawbar-pull versus weight. However, in order to examine the characteristics of the hemispherical wheel in detail, the load-sinkage, motion resistance-weight, tractive effort-weight curves were prepared in addition to the drawbar-pull-weight ratio versus weight curve. The soil used in the analysis can be considered as a strong, purely frictional material having the following strength parameters:

\[
\begin{align*}
  c &= 0 \\
  \rho &= 38^\circ \\
  k_c &= 0 \\
  k_\rho &= 4.5 \\
  n &= .75 \\
  K &= 1
\end{align*}
\]

The load-sinkage relationships for the two wheels is presented in Figure 1. The sinkage of a conventional wheel is given by the equation (2):
and for the hemispherical wheel by Equation 29 in Appendix A:

\[ W_0 = \frac{kD}{2(n+1)} \left[ \pi z_o^{n+1} - \left( z_o - \frac{2}{n+1} \right) \left( \sum_{n+1}^{n+1} \frac{1}{2(n+1)} \right) \right] \]

The results of these computations shown in Figure 1 indicate that the hemispherical wheel does not sink as much as a conventional wheel for a given load. At loads less than 150 pounds in the soil selected there is no particular difference between the two wheel forms. If a weaker soil had been chosen for the analysis, the difference in the load sinkage curves would have occurred at a lower wheel load and if a stronger soil had been used, the difference would be evident at a higher load. However, the curves shown in Fig. 1 indicate that the hemispherical wheel has equal or less sinkage than a conventional wheel for any load.

The motion resistance is proportional to the sinkage so it is logical that the hemispherical wheel is shown to have lower motion resistance than a conventional wheel in Figure 2. The motion resistance for a conventional wheel is given by the Equation (2):

\[ R_c = \frac{(k_c + b k_p)(n+1)}{n+1} \]

which when combined with the sinkage equation produces:

\[ R_c = \frac{1}{2(n+1)^2} \left( \frac{3W}{V_D} \right)^{2(n+1)} \left( \frac{2n+2}{(3-n)^{2(n+1)}(n+1)(k_c + b k_p)^{2(n+1)}} \right) \]
The equation to predict motion resistance of the hemispherical wheel is given by:

\[ R = \frac{k}{n+1} \sqrt{D \left[ \frac{2-n}{3} (z_0)^\frac{n+3}{2} + (z_0) (z_1)^\frac{1}{2} \right] - \frac{n+1}{3} (z_0)^n (z_1)^{3/2}} \]

There does not seem to be any great significance to Figure 2 since the load-sinkage curves imply that the motion resistance of the hemispherical wheel should be equal to or less than that for a conventional wheel. However, Figure 2 indicates that the motion resistance of the hemispherical wheel is significantly less than that of a cylindrical wheel throughout the range of weights selected. If only Figure 1 were examined, it would be reasonable to conclude that the difference in motion resistance would not appear until a wheel load of 150 pounds was reached. This apparent disagreement between the two sets of curves results from the fact that the sinkage of the hemispherical is taken as the sinkage of the lowest point of the wheel in contact with the soil. The "average" sinkage of the wheel is approximately one-half the sinkage of the lowest point. The motion resistance is taken as proportional to the work involved in compacting the soil to the depth of the wheel sinkage. Since only a single point on the hemispherical sinks to the maximum value computed, the wheel compacts less soil and thus produces less rolling resistance than the cylindrical wheel.

In order to compute the tractive effort of both wheels, a graphical solution was devised by Lt. Col. A. D. Sela. This solution
appears as Graph No. 1 and is based on the determination of a "mean coefficient of friction". The coefficient of friction was defined as the ratio of the soil shear strength and normal pressure:

\[ \mu = \frac{s}{p} = \frac{c}{p + \tan \delta}(1 - e^{-K}) \]

The mean coefficient of friction for a given displacement, \( Jx \), is then given by:

\[ \bar{\mu} = \frac{Jx}{\mu} \]

Since \( K \) was taken as unity, the value of \( \bar{\mu} \) is:

\[ \bar{\mu} = \frac{c}{p + \tan \delta} [1 - \frac{1}{Jx} (1 - e^{-Jx})] \]

The value of \( Jx \) is established by the wheel contact length and slip:

\[ Jx = \frac{L}{d} \]

Contact length for conventional wheel = \( (L)_c = \sqrt{D (z - z^2)} \)

Contact length for hemispherical wheel = \( (L)_h = \frac{\pi}{4} \sqrt{D (z - z^2)} \)
In using Graph No. 1, a value of \( z_o \) is selected and the load established from Figure 1. The contact lengths, \( L_c \) and \( L_h \), are computed and the desired slip rate selected. Having the slip rate and contact length values it is possible to solve graphically for the mean value of \( \mu \). The lower portion of the graph is entered with the ordinate, \( L_c \), and the intersection of \( L_c \) and the curve for the selected slip rate determined. The abscissa of the intersection of \( L_c \) and the slip rate is the value of \( \frac{j_x}{K} \). Since \( K \) was taken as unity in this example, the abscissa is therefore \( j_x \). Entering the upper portion of the graph with this value of \( j_x \), the intersection is found between \( j_x \) and the curve representing the solution of the equation for \( \bar{\mu} \). \( \bar{\mu} \) for the cylindrical wheel is the ordinate of the intersection between \( j_x \) and the curve. The process is then repeated using the value of \( L_h \) to find \( \bar{\mu} \) for the hemispherical wheel.

The load required to produce the assumed sinkage had been obtained at the outset from Figure 1. The gross tractive effort is then taken as:

\[ H = \bar{\mu} W \]

The results of a graphical solution using the soil selected appear in Figure 3. The results indicate that the cylindrical wheel develops a slightly higher tractive effort than the hemispherical wheel in the soil selected and at a slip rate of 60%. This slip rate was selected since performance at a slip rate in excess of 60% is not
considered to have practical significance. This result is not surprising since the tractive effort was taken to be proportional to the contact length for a given load. Since the sinkage of the cylindrical wheel is greater for a given load, the contact length is greater and thus the tractive effort is greater.

The result of prime interest is, of course, the drawbar pull-load ratio as a function of load. This curve is shown in Figure 4 and indicates that the performance of the hemispherical wheel is slightly better than that of the cylindrical wheel in the soil selected. The difference in the ratios for wheel loads less than 800 pounds is less than 0.03 which produces a maximum difference in predicted performance of 15% ($\Delta P/P$). The range of wheel loads used in the test were 200 pounds or less. Figure 4 indicates that the predicted performance of the two wheels is essentially the same for both wheels. It is doubtful that an eighteen inch diameter wheel would be loaded in excess of 400 pounds. The difference in performance between the two wheels at that load is of the order of 11% which cannot be considered as a significant performance improvement.

The analysis of the hemispherical wheel and the cylindrical wheel assumed the wheels had equal diameters and equal volumes. The wheel dimensions were taken equal to those used on the models in order to eliminate any possible error due to size effects. The results of the analysis as indicated in Figures 1 through 4 show
that, on a theoretical basis, there is no reason to expect any significant difference in performance between the two wheels. The test results confirm the results of the theoretical analysis as will be shown later in this report.
TEST MATERIAL

The material for this test consisted of three 1/4 scale models of 5 ton truck concepts. Each model is described in detail below:

a. 1/4 Scale Model of a Current 5 ton Truck: This model, shown in Figure 5, was powered by means of four 1/2 horsepower electric motors. The original gear reduction was 140 to 1, but this was later increased to 225 to 1 in order to achieve 100 percent slip conditions in all soils tested. The model was 66 inches long and 24 inches wide. Pneumatic tires, 3-1/2 inches wide and 11-3/4 inches in diameter, were mounted on standard go-kart wheels. The tires had a standard truck tread and were operated at a pressure of 35 psi so that each tire behaved as a rigid wheel. The weight of the model without load was 275 pounds.

Steering was provided by means of conventional Ackermann steering of the two front sets of wheels. Since one of the other models did not have a provision for steering, the steering capability was not used.

b. 1/4 Scale Model of Hemispherical Wheel Concept: This model, shown in Figures 6 and 7, was powered by two 1 horsepower motors with a gear reduction of 300 to 1. The gear reduction was later increased to 360 to 1. The model was 38 inches long and 27 inches wide. The original wheels, as shown in Figure 6, were 18 inch diameter hemispheres tilted at 30° to the vertical. One half-inch depth grousers
were attached at 180° intervals and oriented at 30° to the wheel axle. The surface was painted aluminum with the exception of a 1/2 inch wide rubber running surface for operation on non-deforming soil. These wheels were later coated with rubber in order to meet the request of the wheel inventor, and the grousers were extended to include all areas of the wheel expected to be in contact with the soil. This second set of wheels is shown in Figure 7. The unloaded weight of the model was 225 pounds. Skid steering capability was incorporated in the model, but was not evaluated in these tests.

c. 1/4 Scale Model of Cylindrical Wheeled Vehicle: This model, shown in Figures 8 and 9, was powered by four 1/2 horsepower motors with an original gear reduction of 266 to 1. The gear reduction was later increased to 355 to 1 in order to assure reaching the 100 percent slip conditions in all soils to be used in the test. The model was 40 inches long and 24 inches wide. The original wheels, shown in Figure 8, were 8 inches wide and 15.6 inches in diameter. The wheels were constructed of wood and a tread was cut in the surface to approximate the non-directional, standard military mud and snow tread. The wheel size was established on the following basis: The diameter was taken as the "effective" diameter of the tilted hemispherical wheel from the relationship \( D = \frac{d}{\cos 30°} \); the width was taken so that the volume of the cylindrical wheel was equal to the volume of the hemispherical wheel.
The wheel described above was objected to by the inventor of the hemispherical wheel on the following grounds:

a. The surface of the conventional wheel was different from the surface of the hemispherical wheel.

b. The tread on the conventional wheel was not realistic and different from that on the hemispherical wheel.

c. The wheel shape was not correct since the running surface was at 90° to the wheel sides rather than the top surface being convex as on an actual tire.

d. The wheel form was incorrect in that the wheel diameter of the conventional wheel should be the same as the overall diameter of the hemispherical wheel. The width of the conventional wheel should have been taken as that which would produce an equal volume as the hemispherical wheel.

As a result of these objections raised by the inventor of the hemispherical wheel, a second conventional wheel was constructed that would meet his requirements. Both the hemispherical wheel and the conventional wheel were coated with rubber and had similar grouser configurations as shown in Figures 7 and 9. The wheel size was changed to an 18 inch diameter and 6 inch width. It should be emphasized that there was no agreement that the original wheel was not satisfactory. The inventor was obviously adamant in his demands. Even though the laboratory engineers contended that the changes would not significantly change the test results, the modifications were made. It was also pointed out that the increase in wheel diameter would likely result in
a slight increase in performance even though the diameter increase was accompanied by a width decrease. Figure 10 is a photograph of a commercially available tire having a similar form and tread as the original wooden wheel. Figures 11, 12 and 13 are drawbar-pull versus slip curves to show the equality in performance of the two wheels used on the model. The performance of the two wheels in sand can be considered as representing a significant difference. The poorer performance of the wooden wheel can be attributed to the difference in tread since performance in sand is inversely proportional to the aggressiveness of the tire grouser. However, even though the rubber coated grousered wheel performed somewhat better than the wooden wheel, the difference even in sand was not enough to modify test results or conclusions. The performance of the two wheels in loam can only be considered as identical.

The unloaded weight of the model was 325 pounds. No provision was made for steering.

TEST EQUIPMENT

The test equipment utilized for this test consisted of soil, soil bins, soil processing equipment, power supplies and controls for the models, wheel slip measurement equipment, data recording equipment and a dynamometer for the measurement of drawbar-pull. A description of these various items follows:
a. Soil: Two soils were used in the test program: a well-graded sand and a sandy loam. The mechanical analysis of the two soils is included as Figures 14 and 15. The sand was in the uncompacted state with a density of 0.06 pounds per cubic inch. No attempt was made to control the moisture content since experience has indicated that the moisture content does not vary significantly with changes in humidity. Slight variations in the soil processing procedure produce much greater differences in soil properties than does the natural variation in moisture content. The soil values of the sand during the test are given in Table 1.

It was desired to conduct tests at three levels of moisture content in the loam. It was anticipated from previous experience that precise control of the moisture content within normal limits would be extremely difficult. Therefore, ranges of moisture content were selected rather than specific values. This approach would require a greater number of tests for each moisture content range to obtain valid results. It was thought that the alternate approach of controlling moisture content precisely would, in fact, be more time consuming than the additional tests. The ranges of moisture content selected were 22 to 24 percent, 20 to 22 percent and 16 to 20 percent. The properties of the sandy loam are less sensitive to changes in moisture content at values less than 20 percent so the wider range was considered justified. The soil values of the loam are presented in Table 1.
A note should be made concerning difficulties experienced during the loam tests. The soil processing equipment and the method for adding moisture to the soil were developed for tests in which large wheels or full scale vehicles would be utilized. Although it is not possible to ignore variations in soil properties completely for a full scale vehicle test, it is possible to conduct valid tests without completely homogeneous soil conditions. A full scale vehicle normally operates in a large enough soil mass so that soil conditions are averaged. If a sample of soil values is obtained, the average of the samples can be used to predict or describe the averaged performance as produced by the measurements. When conducting scale model tests, one must either have homogeneous soil conditions or conduct a larger number of tests. The soil conditions were not homogeneous and varied sufficiently throughout the length of the soil bin so that one could not formulate an opinion concerning the performance of any model until a large number of tests were completed.

b. Soil Bins: Two soil bins were used in this test. One bin, shown in Figure 16, is 12 feet wide, 5 feet deep and 100 feet long. The bin is filled with 4 feet of Mason sand. A dynamometer carriage is mounted on the bin and rides on the side rails. The carriage is towed by a chain drive mounted on the side of the bin which also acts as a lock arrangement for the carriage. The dynamometer, power supplies and controls, and recording equipment are mounted on the carriage. The complete test setup is shown in Figure 16.
The second soil bin, shown in the left background of Figure 16, is 12 feet wide, 5 feet deep and 120 feet long. The bin contains 3 feet of loam, 6 inches of gravel and 6 inches of Mason sand. The sand and gravel which comprise the bottom two layers of material are required to ease the addition of water. Water is added to the soil by means of a network of pipes on the bottom of the bin which serve to add and remove water to and from the bin. The same dynamometer carriage was used on both the sand and the loam bin.

c. Soil Processing Equipment: The soil processing equipment used for the sand consisted of a simple rake arrangement and a mold board shown in Figure 17. The procedure for processing the sand consisted of a single pass with the rake to loosen the sand and eliminate any ruts. The rake was removed and a single pass made with the mold board to smooth the sand surface.

The soil processing equipment for the loam bin is shown in Figure 18. The device shown is a gyrotiller similar to agricultural equipment used to eliminate hard pan in cultivated soil. The gyrotiller is satisfactory for eliminating ruts, loosening compacted soil, and removing hard pans but is not effective as a device for the thorough mixing of soil.

d. Power Supplies and Controls: Power was supplied to all three models through the control devices shown in Figure 19. The control panels are shown in the right side of the photograph. The panel on the far right controlled Concepts A and B. The panel to its immediate left controlled Concept C. In both cases, control of power to
the models was achieved by means of rheostats. The power source was a 110 Volt A.C. line fed into the carriage by means of overhead bus bars which can be seen in the upper right hand portion of Figure 16.

e. Data Recording Equipment: All drawbar-pull and slip data were recorded on a standard 6 channel paper recorder. The recorder and amplifiers are shown in the left hand portion of Figure 19.

f. Dynamometer: The dynamometer is shown in Figure 20. A load cell, not shown in photograph, was connected between the load cable and the model under test. The dynamometer operated as follows: the loading cable was connected to the model, and as the model moved forward, the cable caused a rotation of the drum to which it was attached. The drum was connected directly to a gear box which stepped up the speed and was connected to a hydraulic pump by means of a pulley arrangement. The output of the pump was fed through a pressure regulating valve which acted as a load control. The output from the control valve was returned to an oil reservoir which also fed the pump. The control valve operated by changing the size of an orifice so that the valve was sensitive to the rate at which the cable unwound. It was not possible to place a predetermined drawbar-load since any particular valve setting would give a reading that was dependent on the rate at which the model was moving. The device had proven adequate during previous tests. However, it was demanded that each drawbar test be conducted in an identical way so that the dynamometer was a constant source of frustration. If each test had to follow an identical procedure, the technician controlling the dynamometer could not be sure that an adequate drawbar-pull
versus slip determination was obtained. If extensive model testing were anticipated, it would be wise to resort to a gravity dynamometer. This system is considerably more clumsy than that used in the test but does not require a knowledge of standard test procedures on the part of participants or observers.

g. Slip Measurement Equipment: Slip was measured by means of tachometer generators mounted on the models and a micro-switch mounted on the dynamometer. The tachometer generators were attached to the drive shafts of the models and measured the wheel revolutions permitting the determination of the theoretical distance travelled. The micro-switch provided a measure of the actual distance travelled by a model by counting the rotations of the dynamometer shaft.

TEST PROCEDURE

Since the test procedure consists of a fairly complicated set of steps, each step will be described separately. Upon completion of the description of each component of the test procedure, a general discussion of the overall test is presented.

a. Soil Preparation:

1. Sand Bin:

(a) The sand was processed before each test series by making one or two passes with the rake mounted on the dynamometer carriage. The sand was then leveled using the mold board mounted on
the dynamometer carriage.

(b) Compaction of the sand was not attempted due to the difficulty in obtaining uniform results from the compaction of such a large soil mass.

2. Loam Bin: The procedure for preparing the soil in the loam bin was concerned with two problems: the soil had to be processed to produce uniform, repeatable conditions, and moisture content had to be controlled.

(a) Moisture Content: The procedure, in general, consisted of the complete saturation of the soil mass by adding and draining water from the bottom of the bin. A standard garden hose was connected to the water inlet and water fed into the bin at a slow rate in an attempt to spread water evenly throughout the soil. Water was added until it was standing on the surface of the soil and then drained from the bin by means of the installed drainage system. Once drainage stopped, the soil was processed and moisture samples taken. Moisture samples were taken at the surface and at a 6 inch depth. Moisture content was established by weighing the sample and container, drying the sample in the laboratory oven until no further weight change was observed and recording the initial and final weights. The container weight was determined in order to establish the soil weight. The moisture content was taken as:

\[
\% \text{ Moisture} = \left( \frac{W_w - W_d}{W_d} \right) \times 100
\]
where: \( W_w = \) wet weight of sample
\( W_d = \) dry weight of sample

In each moisture content determination, nine surface samples and nine samples from a 6 inch depth were obtained to assure that an average value of the moisture content could be determined. The procedure to be followed in the control of moisture content was specified in the test plan but was not followed exactly due to a variety of reasons. The original procedure began with the complete flooding of the soil bin. If the moisture content was found to be in excess of 24%, the soil was to be processed and allowed to sit for eight hours, processed again and new moisture determinations made. Once the moisture content had reached a point between 22 and 24%, the soil bin was to be covered with a plastic sheet to prevent further evaporation. A series of tests were to be run at the 22 - 24% moisture content level and the soil then allowed to dry out to 18 - 20%. The evaporation rate was to be increased by repeated processing of the soil. Once the moisture content was reduced to approximately 20%, the soil was to be processed and drawbar tests run. The procedure was then to be repeated with a new moisture range of 16 to 20%. Due to changes in test procedures, model failures, and other unforeseen events, it was not possible to follow the schedule of variation of moisture content. An attempt was made to start testing with the soil at high moisture content and obtain lower moisture contents by means of evaporation and processing of the soil. As indicated by
Table 2, it was not possible to follow such a schedule. The moisture contents used did not fit the precise ranges that had been originally agreed upon because it was not possible to allow sufficient time for the soil-water system to reach equilibrium. That is, a given amount of water will affect the properties of soil differently if the soil properties are determined soon after the water is added, or if a considerable lapse of time is provided. Because this was the first "production line" test that had been conducted by the laboratory, it was often difficult to determine the best course of action to follow in varying soil properties. The usual test procedure allows adequate time for equilibrium conditions to be reached since the personnel conducting a test can reduce data while awaiting for the soil and water to be properly mixed. Since a rigid schedule was attempted, the soil properties rather than moisture content were taken as the guide.

(b) Soil Processing: In order that soil conditions could be as reproducible as possible, strict adherence to a set procedure for soil processing was required. The following method was used: The gyrotiller was mounted on the dynamometer carriage. The first pass was made on the north side of the bin with west to east direction. Only one such pass was made, and the tiller was lifted from the carriage at the east end of the bin. The carriage was returned to the west end of the bin and the tiller again mounted on the carriage. The second pass was made with the tiller located so that there was a slight overlap with the previous pass. A single pass was again made from west to east.
processing was repeated until the complete soil bin was processed. It was found that five passes constituted one mix. If the soil was processed for a run as opposed to processing for moisture control, the soil was covered with plastic if testing was not immediately scheduled. The subsequent mix began at the south side of the bin with an east to west direction. The direction of processing the soil was reversed for each mix so that the experimental units would be more homogenous.

(c) **Soil Value Measurement:** The soil values were measured by means of the carriage mounted Bevameter shown in Figure 20. Both sinkage and shear soil values were obtained by means of a random sampling procedure. Circular plates of 6 inches, 8 inches and 10 inches in diameter were used to determine the sinkage parameters and normal loads of 40, 60, 80 and 100 pounds were used to obtain the shear parameters.

(d) **Determination of Model Weight:** The test weight of each model was determined in such a way that the test results could be considered to be valid for the full scale prototypes of the vehicles. Since all of the models were constructed to be models of 5 ton vehicles, it was felt that a reasonable basis for comparison of performance would be the scaled-up performance of the prototype. In order to establish the weight to be used, we were faced with the problem that we had to scale up performance. A dimensional analysis reveals to us that normal scaling is not possible if soils other than pure sand or pure clay are used. The following proposition was therefore made: the performance of the full scale vehicle can be computed using soil-vehicle
relationships developed by the laboratory; the performance of the scale model can also be computed using the same relationships. If the computed performance of the model is equated to that computed performance of the prototype, it should be possible to establish the relationship between model size, soil conditioning and model weight. This approach was considered necessary since all of the parameters of the prototype are fixed and the soil properties are measured and fixed so that the only possible parameters that could be varied were model parameters. The geometry of the model is fixed so the only reasonable variable is the model weight. The performance is identified by the drawbar-pull ratio which is taken as:

\[
\frac{DP}{W} = \frac{H - R}{W}
\]

where:
- \(DP\) = Drawbar-pull
- \(W\) = Vehicle weight
- \(H\) = Gross Tractive Effort
- \(R\) = Motion Resistance

The gross tractive effort and the motion resistance are both functions of vehicle geometry and soil conditions. Knowing the prototype characteristics and the soil conditions, it is possible to compute the \(\frac{DP}{W}\) for the prototype. If the performance of the model is to be scaled up, the \(\frac{DP}{W}\) of the model must be the same as the prototype.
In order to establish the model weight, the following procedure was followed:

1. The soil values were measured and the \( \frac{DP}{W} \) for the prototype computed by the equation (2):

\[
\frac{DP}{W} = \frac{H - R}{W} = \frac{3}{2(3-n)} \sqrt{\frac{D}{z}} \cos^{-1} \left( 1 - \frac{2z}{D} \right) \left[ \frac{C}{kz^n} + \tan \beta \right]
\]

\[
= \frac{3}{(3-n)(n+1)} \sqrt{\frac{z}{D}}
\]

where:
- \( D \) = Wheel diameter
- \( b \) = Wheel width
- \( z \) = Wheel sinkage
- \( n \) = Exponent of sinkage
- \( k \) = Soil proportionality constant = \( \left( \frac{k_c}{b} + k_f \right) \)
- \( c \) = Cohesion
- \( \beta \) = Angle of internal friction of soil

2. It was required that:

\[
\left( \frac{DP}{W} \right)_{\text{model}} = \left( \frac{DP}{W} \right)_{\text{prototype}}
\]

3. In order to find the vehicle weight, a trial and error solution was used. The value of \( \frac{DP}{W} \) for the model was known and all the variables in the equation to determine the \( \frac{DP}{W} \) were known except the sinkage. It would have been possible to solve the \( \frac{DP}{W} \) equation for the sinkage but this would have required computer programming and solution of vast trial and error procedures. Therefore, a value
for the weight was assumed and sinkage computed from the equation:

\[
z = \left[ \frac{3w}{bk/\delta(3 \cdot n)} \right]^{2n+1}
\]

With the computed value of sinkage known, the \( \frac{DP}{W} \) equation was solved. Assuming that this solution did not produce the proper value of \( \frac{DP}{W} \) for the model so that it was not equal to the \( \frac{DP}{W} \) computed for the prototype, a new value for the model weight was assumed. The sinkage \( z \), was again computed and the \( \frac{DP}{W} \) was recomputed. This procedure was repeated until a close agreement was achieved between the model and prototype \( \frac{DP}{W} \).

A different model weight resulted for each soil condition in which the models operated since the values of \( c, \phi, k \) and \( n \) changed for each soil. It was not possible, for example, to compute the model weights for operation in sand and maintain these weights for other soil conditions. The result of this approach would be that only the performance of the models could be compared rather than the performance of the prototypes and the models.

As the tests progressed, it was found that the weight required to permit the scaling-up of performance became so large that it was feared that the models would fail due to overloading. A reverse
procedure to that described above was used. That is, a model weight was arbitrarily selected and the operating weight of the prototype computed. The test results would then be translated to prototype performance by means of a connection factor. Instead of plotting prototype drawbar-pull weight ratio versus slip, the $\frac{DP}{W}$ would be connected by a factor of $\frac{W_a}{W_t}$. $W_a$ was taken as the actual prototype payload as represented by the test results and $W_t$ was the rated payload of 10,000 pounds.

(e) Drawbar-Pull and Slip Measurement:

(1) Drawbar-Pull: The drawbar-pull was measured by means of a hydraulically operated dynamometer. The dynamometer consisted of a load cell, cable attached to a drum, a control valve, a hydraulic pump or motor, and an oil reservoir. The operation is as follows: A pintle was mounted on the rear of each vehicle located so that the load transfer at a trim angle of $5^\circ$ was approximately the same for each model. It was necessary to follow this procedure since the location of the center of gravity was different for each model making a single ratio of pintle height to wheel diameter impractical. A load cell was mounted directly to the pintle and the drawbar cable attached to the load cell. The load cell readings were recorded on a standard six-channel recorder mounted on the dynamometer carriage. The drawbar cable was wrapped around the drum and rotated the drum as the model moved forward. The drum axle was connected to a hydraulic pump that pumped fluid from the reservoir.
through a flow control valve and back to the reservoir. By restricting flow through the control valve, various loads could be applied to the cable.

The procedure for applying the loads was as follows: The models were started on a test run with no load applied by the dynamometer. The no-load condition was held for five seconds. At the end of five seconds, the first load increment was applied and held for five seconds. The load was increased at five-second intervals until approximately 100% slip was reached. The load increments were applied by turning the control valve to index marks installed on the valve to indicate the load. However, since the dynamometer was sensitive to load rate, the index marks actually only indicated the closing of the valve by a fixed amount rather than load increment.

It was considered that the dynamometer may have been the source of apparent anomalies in test results which were observed when the soil was very weak. The loading procedure was modified by test personnel; the technician operating the dynamometer observed the record of the drawbar readings and changed the settings of the control valve when he was satisfied that the drawbar load was constant for a long enough period to determine slip conditions. Careful observations of the behavior of the models indicated that the apparent malfunction in the dynamometer was in fact a result of load transfer because of the drawbar. That is, as the drawbar load was increased, the models assumed a large trim angle which increased the rolling resistance due to sinkage of the rear wheels. As the trim angle increased, the
drawbar-load required to maintain equilibrium with the net tractive effort developed by the model decreased in a direct proportion to the increased rolling resistance. Once a large trim angle was assumed, the soil was too weak to permit the model to climb out of its rut even though the drawbar-load was reduced. Once this was established, the original method of applying the drawbar-load was again used.

(2) Slip Measurement: Wheel rotation was measured by means of a tachometer generator mounted on the models with the output of the generators recorded on the six-channel recorder. The rotation of the wheel permitted the computation of the theoretical distance travelled. The actual distance travelled was measured by means of a micro-switch mounted so that the rotation of the cable drum was recorded. This record appeared as a series of pips on the recorder. Since the paper speed of the recorder was known, and the angle of rotation of the drum between each pip was known, it was possible to determine the actual distance travelled. The slip was then determined from the relationship:

\[ \text{slip} = \frac{S_t - S_a}{S_t} \]

where:
- \( S_t \) = Theoretical distance
- \( S_a \) = Actual distance
By and large, the test procedure established in the test plan was followed in the conduct of the soft soil tests. The soil was processed as the first step followed by measurement of the soil values and collection of moisture samples. The soil value data were reduced and model weights determined. Once the model weights were established, testing was started. Random sampling techniques were used to measure soil values, obtain moisture samples, and to conduct the drawbar-pull tests. Great care was taken to assure that no ordered effects crept into the test results due to test procedures.

However, to assure that test techniques remained consistent, a test team was established which permitted each task to always be accomplished by the same technician or engineer. For example, one engineer reduced soil value data so that any error due to data interpretation would affect each test in the same way. The same technician always applied the drawbar load, another always operated the instruments, etc.

The test schedule was essentially abandoned because it was found to be overly optimistic concerning failure of equipment and uniformity of test results. In testing full scale vehicles in the bins it had been found that it was quite simple to conduct tests in the loam at high moisture content. As has been indicated elsewhere in this report, great difficulty was encountered in the conduct of the loam tests. This required a considerable amount of reruns in order to obtain an adequate sample size to assure identification of any statistically significant differences in performance.
produced the effect of skid steering.

The average moisture content on 26 July was 24%. The soil in the west end of the bin was stronger than that at the east end and similarly, the soil on the south side of the bin was stronger than that on the north side. The bin was divided into three twenty-foot sections beginning approximately twenty feet from the west end in order to avoid the weak soil in the east end of the bin. It was agreed that the test would consist of a Go-No-Go determination in which Go was defined as negotiating 10 feet of the 20-foot course. A more appropriate measure would have been an upper limit of wheel slip as the criterion for successful operation. It was also agreed that each model would operate in each test section in order to eliminate the effect of variation in soil strength between the various test sections.

The results of the free-run tests are shown in Table 3. Only six runs were made on 26 July due to the failure of the drive shaft of Concept C. On the basis of the limited free-run tests of 26 July, one could conclude incorrectly that Concept C was considerably better than Concept A or B. Concept C negotiated the full twenty feet of its first run at a maximum slip of 31.5%. Concept A was not able to negotiate any of the test sections. Concept B negotiated more than one-half of one test section at a maximum slip of 83%. Although it is likely that Concept B could have negotiated the complete test section, a slip rate of 83% is so high that the vehicle can be considered as immobilized. It should be pointed out, however, that the tests of 26 July
In several cases, the models were not operated at the correct weight to produce prototype performance at rated payload. In those cases a corrected plot is given in the form of \( \frac{DP \cdot W_{ap}}{W} \) vs. Slip in which \( W_{ap} \) is the actual payload and \( W_{tp} \) is theoretical payload, i.e., 10,000 pounds. (See Figure 31)

a. 16 July 63: The tests on 16 July were conducted in the loam having a moisture content of approximately 22%. Concept A appeared somewhat underpowered as it could only develop 50% slip as shown in Figure 21. The averaged results shown in Figure 22 indicate that all three models performed equally well. There is no reason to expect that the performance of Concept A would have worsened relative to the other two concepts if adequate power had been available.

b. 17 July 63: The tests on 17 July were made in the loam with a moisture content of 22.7%. Again, no significant difference in performance between any of the three concepts is seen in either Figure 23 or Figure 24. The soil was slightly weaker so that 100% slip was achieved by Concept A. The slight improvement in Concept B over Concepts A and C at high slip rates is not significant.

c. 18 July 63: The tests on 18 July were conducted in the loam having a moisture content of 21.3%. Concept C performed considerably better than Concepts A or B, as shown in Figures 25 and 26. The results of this test indicated the favorable reaction of Concept C to drawbar load transfer to the rear wheels. Since the wheels on Concept C did not sink as much as those on Concept B for a given load, the
performance was considerably better in this particular soil condition. The lesser sinkage of the rear wheels provides for an improved performance in at least three ways: the vehicle assumes a lower trim angle so that it does not have to "climb as great a slope"; the rear wheels do not develop as much motion resistance; and the effect of a given drawbar load is not increased because of the trim attitude. It should be observed, however, that Concept C does not show a significant improvement over Concept B until a slip rate in excess of 50% is reached.

d. 19 July 63: The tests on this day were run in the loam with a moisture content of 19.7%. The soil was strong enough in bearing strength so that the effect of load transfer due to the drawbar was minimized. As seen in Figures 27 and 28, the performance of Concepts B and C were the same and both were significantly better than Concept A.

e. 22 July 63: The moisture content on 22 July was 17.9%. The results obtained on 22 July were essentially the same as on 19 July as shown in Figures 29 and 30. The comments for 19 July are also appropriate to those of 22 July. An inspection of the results for the two days would indicate that the values of the drawbar pull-ratio are somewhat higher on 22 July. This is due to the fact that the weights used on 19 July for all of the models were arbitrarily reduced for the tests on 22 July because it was evident on 19 July that the models were overloaded. The corrected results for the prototype performance appear in Figure 3.
f. 24 July: The tests on 24 July were run in Ottawa Sharp sand. The results shown in Figures 32 and 33 indicate no difference in the performance between Concepts B and C and that both concepts performed better than Concept A.

g. 25 July: The moisture content of the loam was 24.9% for the tests on 25 July. Although differences in performance are shown in Figures 34 and 35, they cannot be considered significant since analysis indicated that prototype performance would be zero in this soil condition. That is, the model weight selected was such that the scaled-up prototype weight was less than the curb weight of the prototypes. The analysis also indicated that the full scale vehicles could not have operated in the soil condition tested. The test of 25 July, therefore, was interesting but of no significance to this program.

h. 2 August: The moisture content of the loam was 20.7% on 2 August. The test results shown in Figures 36 and 37 indicate no difference in the performance of Concepts B and C. Both concepts are shown to be better than Concept A. However, the corrected curves shown in Figure 38 indicate minor differences in the performance among any of the three models. This is partly due to the fact that the arbitrary weights selected produced an overload on Concept A as compared to Concepts B and C. All three concepts were overloaded but Concept A had a relatively higher overload. The soil was sufficiently strong to permit the overload without a deterioration in performance.

f. Free-Run Tests: The free-run tests were conducted on two
days: 26 July and 5 August. The tests of 26 July were not completed because Concept C broke a drive shaft as it began its second run. However, the results obtained on 26 July will be discussed briefly before the complete test conducted on 5 August is described.

(1) 26 July: The free-run tests were conducted in the loam bin with a high moisture content. It had been found by experience that it was more difficult to obtain uniform soil conditions at very high moisture contents, because extensive mixing was required in order to achieve uniformity. The additional mixing of the soil was necessary because water tended to concentrate at several spots in the soil bin due to channels that had developed in the sand and gravel at the bottom of the bin. When water was added to the bottom of the bin through the pipe network, it would flow into the bin through the channels. This produced concentration of water at various points throughout the bin which could be eliminated either by allowing water to cover the soil for several days or by thoroughly mixing the soil. The soil processing equipment available at the time of the test was designed for eliminating hard pans and for local mixing of the soil. The equipment did not permit the movement of soil from one point to another so that when a particularly weak spot developed due to excessive moisture, it would remain weaker than the surrounding soil after mixing had been completed. The weak spots were identified and attempts were made to avoid them but this was not very effective since the models had no directional control. If one set of wheels encountered weaker soil, the model would turn in the direction of the weaker soil since the variation in tractive effort
produced the effect of skid steering.

The average moisture content on 26 July was 24%. The soil in the west end of the bin was stronger than that at the east end and similarly, the soil on the south side of the bin was stronger than that on the north side. The bin was divided into three twenty-foot sections beginning approximately twenty feet from the west end in order to avoid the weak soil in the east end of the bin. It was agreed that the test would consist of a Go-No-Go determination in which Go was defined as negotiating 10 feet of the 20 foot course. A more appropriate measure would have been an upper limit of wheel slip as the criterion for successful operation. It was also agreed that each model would operate in each test section in order to eliminate the effect of variation in soil strength between the various test sections.

The results of the free-run tests are shown in Table 3. Only six runs were made on 26 July due to the failure of the drive shaft of Concept C. On the basis of the limited free-run tests of 26 July, one could conclude incorrectly that Concept C was considerably better than Concept A or B. Concept C negotiated the full twenty feet of its first run at a maximum slip of 31.5%. Concept A was not able to negotiate any of the test sections. Concept B negotiated more than one-half of one test section at a maximum slip of 83%. Although it is likely that Concept B could have negotiated the complete test section, a slip rate of 83% is so high that the vehicle can be considered as immobilized. It should be pointed out, however, that the tests of 26 July
cannot be considered as valid since Concept C only operated in one test section and Concepts A and B only operated in two test sections.

In addition, the objective of the free-run test was to establish what maximum weight could be carried by one of the models and have it capable of negotiating all three test sections. The model that could carry the greatest payload would be considered as superior in this test to the other two models. In conducting the test it was anticipated that weights would be removed in fifty pound increments until one model could negotiate all test sections. It was not anticipated that the maximum model weight would be established for all three models.

(2) 5 August 63: The soil was at a moisture content of 23.1% on 5 August, and water had been added to the surface during soil processing. This resulted in better control of moisture content so that reasonably uniform soil conditions were available for the tests. It was determined that only Concepts B and C would be tested and that only two test lanes would be used. The section of the bin having the most uniform conditions was divided into two twenty-foot long test sections.

The test results are given in Table 3 and it can be seen that both concepts were immobilized when operating at a total weight of 500 pounds. Although both of the models were able to negotiate several inches more than one-half of the test section at least once, the maximum slip rate was in excess of 95%. This means that if the motors could have operated at heavy loads for a long enough period, the models could have negotiated the complete test sections. But one can hardly consider
negotiating weak soil at a slip rate of 97 to 99% as good performance. It was, and is concluded, therefore, that the models could not negotiate the test section at the 500 pound weight.

The weight was reduced to 450 pounds on both models and then both were able to negotiate the test section with no trouble. The average maximum slip rate for Concept B was 51% and for Concept C it was 57.5%. Performance at these slip rates can be considered "good" in weak soil conditions. No attempt was made to further define the maximum weight that could be carried by the models.

To summarize, the test results show that Concept B and C are essentially identical in soft soil performance as indicated by Table 4. Occasionally Concept B performed better than Concept C and vice-versa. In almost all cases, however, Concepts B and C performed considerably better than did Concept A. Concept C demonstrated that it was less sensitive to the effect of a drawbar load because of the more favorable load-sinkage relationship of the wheel used on the concept.
ACCURACY OF RESULTS

Soils usually exhibit considerably greater random variation in properties than most experimental materials. This random variation is associated with the size of the experimental units and the spacing or distance between points at which measurements are taken. In agricultural research it has been found that much of this random variation can be associated with the rows and columns of a field layout of the experimental units. Hence, it is helpful to utilize experimental designs which remove or isolate these row and column components. In this test program it was found that a Youden Square or an incomplete Latin Square provided a suitable plan for laying out the test strips in the large soil bins. Generally, three replicates of this plan, or six trials of each vehicle, were carried out to obtain estimates of model performance. Appendix B discusses the analysis of this experimental design and offers an interpretation of the results.

Table 4 presents a summary of the analysis of the test data. This table presents the coefficient of variation of the mean for each experiment (three replicates), the mean difference in performance for model comparisons, and a probability for the observed results (difference) or a more extreme result (greater difference) under the hypothesis that the true difference is zero. The latter, zero difference, is equivalent to stating that the models being compared are equal in performance.

*This Section prepared by Dr. E. H. Jebe

Reproduced From
Best Available Copy
The coefficient of variation is a general measure of the repeatability of the observed results, i.e., the standard deviation of the mean divided by the mean value itself.
REFERENCES


APPENDIX A.


APPENDIX B.

LOAD vs. SINKAGE.

FIG. 1.
MOTION RESISTANCE vs. LOAD.

FIG. 2.
TRACTIVE EffORT vs. LOAD

FIG. 5.
DRAWBAR PULL / LOAD vs. LOAD

FIG. 4.
Figure 8: 1/4 Scale Model, Conventional Wheel (Original Wheel)
TYLER STANDARD SIEVE NUMBERS

GRASS SIZE IN MILLIMETERS

PER CENT FINE BY WEIGHT

LOAM

MEDIUM GRAVEL FINE GRAVEL COARSE SAND MEDIUM SAND FINE SAND VERY FINE SAND SILT CLAY

U.S. BUREAU OF SOILS CLASSIFICATION

PROJECT__________________________ BORING NO ______ SAMPLE NO ______

DEPTH____ ELEVATION____ REMARKS______________________________

GRAIN SIZE DISTRIBUTION DIAGRAM

FIG.15.
Figure 18: Soil Processing Equipment. (Loam)
Figure 20: Soil Bin Bevameter

Figure 20-A: Load Dynamometer
17 July 1963 Sandy Loam

Concept A Concept B Concept C

Drawbar-Pull / Vehicle Weight

0 10 20 30 40 50 60 70 80 90 100

% Slip

Figure 23.
AVERAGE TEST DATA
17 July 1963 - SANDY LOAM

- - - - - CONCEPT A: GW = 760 lbs.
- - - - - CONCEPT B: GW = 580 lbs.
- - - - - CONCEPT C: GW = 580 lbs.

Figure 24.
CORRECTED TEST DATA
22 JULY 1953 - SANDY LOAM

- - - - CONCEPT A: GW = 1,000 lbs.
- - - - CONCEPT B: GW = 700 lbs.
- - - - CONCEPT C: GW = 700 lbs.

Figure 31.
24 July 1963 Sand

Concept A □□ Concept B □□ Concept C □□

Figure 32.
Figure 34.
AVERAGE TEST DATA
25 July 1963 - SANDY LOAM

- - - CONCEPT A1 GW = 400 lbs.
- - - CONCEPT B1 GW = 400 lbs.
- - - CONCEPT C1 GW = 400 lbs.

Figure 35.
AVG TEST DATA
2 August 1963 - SANDY LOAM

- CONCEPT A: GW = 1,100 lbs.
- CONCEPT B: GW = 700 lbs.
- CONCEPT C: GW = 700 lbs.

Figure 37.
CORRECTED TEST DATA
2 AUGUST 1963 - SANDY LOAM

CONCEPT A: G.V.W. = 1,100 lbs.
CONCEPT B: G.V.W. = 700 lbs.
CONCEPT C: G.V.W. = 700 lbs.

Figure 38.
<table>
<thead>
<tr>
<th>DATE</th>
<th>$k_c$</th>
<th>$k_\phi$</th>
<th>$n$</th>
<th>$c$</th>
<th>$\tan \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>28-29 May</td>
<td>0</td>
<td>4.5</td>
<td>0.77</td>
<td>0</td>
<td>0.813</td>
</tr>
<tr>
<td>15 July</td>
<td>7.5</td>
<td>3.0</td>
<td>0.8</td>
<td>0.4</td>
<td>0.425</td>
</tr>
<tr>
<td>16 July</td>
<td>3.3</td>
<td>1.5</td>
<td>0.74</td>
<td>0.1</td>
<td>0.51</td>
</tr>
<tr>
<td>17 July</td>
<td>7.0</td>
<td>0.94</td>
<td>0.67</td>
<td>0.5</td>
<td>0.44</td>
</tr>
<tr>
<td>18 July</td>
<td>4.0</td>
<td>4.5</td>
<td>0.54</td>
<td>0.5</td>
<td>0.54</td>
</tr>
<tr>
<td>19 July</td>
<td>9.9</td>
<td>4.3</td>
<td>0.45</td>
<td>0.45</td>
<td>0.615</td>
</tr>
<tr>
<td>22 July</td>
<td>32.4</td>
<td>0.3</td>
<td>0.8</td>
<td>0.43</td>
<td>0.59</td>
</tr>
<tr>
<td>24 July</td>
<td>0</td>
<td>4.5</td>
<td>0.77</td>
<td>0</td>
<td>0.813</td>
</tr>
<tr>
<td>25 July</td>
<td>5.5</td>
<td>0.57</td>
<td></td>
<td>0.25</td>
<td>0.215</td>
</tr>
<tr>
<td>26 July</td>
<td>0.75</td>
<td>3.65</td>
<td>0.50</td>
<td>0.6</td>
<td>0.35</td>
</tr>
<tr>
<td>31 July</td>
<td>9.0</td>
<td>2.1</td>
<td>1.08</td>
<td>0.38</td>
<td>0.73</td>
</tr>
<tr>
<td>2 Aug</td>
<td>7.8</td>
<td>1.55</td>
<td>1.39</td>
<td>0.5</td>
<td>0.61</td>
</tr>
<tr>
<td>5 Aug</td>
<td>2.02</td>
<td>2.1</td>
<td>0.51</td>
<td>0.43</td>
<td>0.195</td>
</tr>
<tr>
<td>6 Aug</td>
<td>7.87</td>
<td>1.27</td>
<td>1.04</td>
<td>0.68</td>
<td>0.37</td>
</tr>
<tr>
<td>9 Sept</td>
<td>0.15</td>
<td>3.8</td>
<td>1.28</td>
<td>0.82</td>
<td>0.55</td>
</tr>
<tr>
<td>10 Sept</td>
<td>5.25</td>
<td>2.75</td>
<td>1.31</td>
<td>0.52</td>
<td>0.67</td>
</tr>
<tr>
<td>19 Sept</td>
<td>6.0</td>
<td>2.8</td>
<td>0.83</td>
<td>0.45</td>
<td>0.47</td>
</tr>
<tr>
<td>20 Sept</td>
<td>8.25</td>
<td>0.65</td>
<td>0.96</td>
<td>0.68</td>
<td>0.35</td>
</tr>
<tr>
<td>DATE</td>
<td>MOISTURE CONTENT RANGE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28 - 29 May</td>
<td>Negligible (Dry Sand)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 July</td>
<td>18.5 - 21.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 July</td>
<td>21.1 - 22.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17 July</td>
<td>21.1 - 23.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 July</td>
<td>20.6 - 22.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 July</td>
<td>18.6 - 20.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 July</td>
<td>18.7 - 21.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24 July</td>
<td>Negligible (Dry Sand)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 July</td>
<td>24.2 - 25.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26 July</td>
<td>22.7 - 26.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31 July</td>
<td>18.9 - 22.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Aug</td>
<td>19.0 - 22.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Aug</td>
<td>22.1 - 24.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 Aug</td>
<td>21.6 - 24.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 Sept</td>
<td>15.2 - 16.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Sept</td>
<td>16.5 - 18.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 Sept</td>
<td>16.5 - 18.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 Sept</td>
<td>17.5 - 18.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Table 3.
### Summary of FRFF Runs

**Date:** 26 July 1963

<table>
<thead>
<tr>
<th>RUN NO.</th>
<th>VEHICLE CONCEPT</th>
<th>WEIGHT Lbs.</th>
<th>GO*</th>
<th>NO GO</th>
<th>DIST. TRAVELED</th>
<th>AVER. SLIP</th>
<th>MAX. SLIP</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>500</td>
<td>+</td>
<td>-</td>
<td>20' 7&quot;</td>
<td>22%</td>
<td>31.5%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>500</td>
<td>-</td>
<td>+</td>
<td>8' 11&quot;</td>
<td>96%</td>
<td>97%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>700</td>
<td>-</td>
<td>+</td>
<td>4' 4&quot;</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>500</td>
<td>+</td>
<td>-</td>
<td>12' 1&quot;</td>
<td>55%</td>
<td>83%</td>
<td>Stalled</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>700</td>
<td>-</td>
<td>+</td>
<td>4' 5&quot;</td>
<td>-</td>
<td>-</td>
<td>Stalled</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>500</td>
<td>-</td>
<td>+</td>
<td>11' 16&quot;</td>
<td>-</td>
<td>-</td>
<td>Shaft broke</td>
</tr>
</tbody>
</table>

**Date:** 5 August 1963

<table>
<thead>
<tr>
<th>RUN NO.</th>
<th>VEHICLE CONCEPT</th>
<th>WEIGHT Lbs.</th>
<th>GO*</th>
<th>NO GO</th>
<th>DIST. TRAVELED</th>
<th>AVER. SLIP</th>
<th>MAX. SLIP</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>500</td>
<td>-</td>
<td>+</td>
<td>1' 5&quot;</td>
<td>90%</td>
<td>97%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>500</td>
<td>-</td>
<td>+</td>
<td>8' 95&quot;</td>
<td>94.5%</td>
<td>98.2%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>500</td>
<td>-</td>
<td>+</td>
<td>7' 1&quot;</td>
<td>90%</td>
<td>98%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>500</td>
<td>+</td>
<td>-</td>
<td>11' 5&quot;</td>
<td>95%</td>
<td>99%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>500</td>
<td>-</td>
<td>+</td>
<td>9' 10.5&quot;</td>
<td>93%</td>
<td>98.3%</td>
<td>Pulled to the right</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>500</td>
<td>+</td>
<td>-</td>
<td>10' 6.5&quot;</td>
<td>90.2%</td>
<td>98%</td>
<td>Motor overheated</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>500</td>
<td>+</td>
<td>-</td>
<td>10' 1.5&quot;</td>
<td>72%</td>
<td>98%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>B</td>
<td>500</td>
<td>-</td>
<td>+</td>
<td>9' 7&quot;</td>
<td>84%</td>
<td>98%</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>B</td>
<td>450</td>
<td>+</td>
<td>-</td>
<td>19' 3&quot;</td>
<td>32.5%</td>
<td>62%</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>C</td>
<td>450</td>
<td>+</td>
<td>-</td>
<td>18' 6&quot;</td>
<td>43%</td>
<td>80%</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>C</td>
<td>450</td>
<td>+</td>
<td>-</td>
<td>18' 6&quot;</td>
<td>30%</td>
<td>35%</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>B</td>
<td>450</td>
<td>+</td>
<td>-</td>
<td>20' 1&quot;</td>
<td>28%</td>
<td>40.5%</td>
<td></td>
</tr>
</tbody>
</table>

*More than 10 ft. of travel*
<table>
<thead>
<tr>
<th>DATE</th>
<th>CONCEPT PERFORMANCE</th>
<th>ANALYSIS OF DIFFERENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Value</td>
</tr>
<tr>
<td>16 &amp; 17 July (60% Slp):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Sandy Loam: 22 and 22.7% moisture)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.022</td>
<td>71%</td>
</tr>
<tr>
<td>B</td>
<td>.050</td>
<td>32%</td>
</tr>
<tr>
<td>C</td>
<td>.073</td>
<td>22%</td>
</tr>
<tr>
<td>22 July (70% Slp):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Sandy Loam: 17% Moisture)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.182</td>
<td>29%</td>
</tr>
<tr>
<td>B</td>
<td>0.382</td>
<td>14%</td>
</tr>
<tr>
<td>C</td>
<td>0.362</td>
<td>15%</td>
</tr>
<tr>
<td>24 July (70% Slp):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Dry Sand)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.093</td>
<td>7.1%</td>
</tr>
<tr>
<td>B</td>
<td>0.190</td>
<td>3.5%</td>
</tr>
<tr>
<td>C</td>
<td>0.181</td>
<td>3.5%</td>
</tr>
<tr>
<td>25 July (70% Slp):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Sandy Loam: 24.9% Moisture)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.117</td>
<td>12%</td>
</tr>
<tr>
<td>B</td>
<td>0.096</td>
<td>15%</td>
</tr>
<tr>
<td>C</td>
<td>0.144</td>
<td>10%</td>
</tr>
<tr>
<td>2 August (60% Slp):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Sandy Loam: 20.7% Moisture)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.287</td>
<td>11.3%</td>
</tr>
<tr>
<td>B</td>
<td>0.432</td>
<td>7.8%</td>
</tr>
<tr>
<td>C</td>
<td>0.363</td>
<td>9.2%</td>
</tr>
</tbody>
</table>
APPENDIX A.

SINKAGE OF A TILTED HEMISPHERICAL WHEEL

The sinkage is calculated from the equation of equilibrium of the vertical forces. To develop this equation one has to sum the vertical components of the soil reaction force-elements. The normal pressure is expressed by Bekker's well known equation: (4)

\[ p = k_\alpha^n \]  \hspace{1cm} (1)

According to Figure 1-A:

\[ \gamma = z - z_0 \]  \hspace{1cm} (2)

The equation of a sphere is:

\[ F = x^2 + y^2 + z^2 - (\frac{D}{2})^2 = 0 \] \hspace{1cm} (3)

Where D is the diameter of the sphere.

The equation of equilibrium of the vertical forces is obtained by integrating \( p \cos \gamma \) over the pertinent surface region. The result has to be equal to the load, \( W \), which is the only active vertical force on the wheel. To evaluate the above integral we use the following expression, which can be found in any standard textbook on advanced calculus:

\[ W = \iint_S p \cos \gamma \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2} \, dx \, dy \] \hspace{1cm} (4)

*Prepared by Z. Janosi.*
Using Equation 3, Equation 4 becomes the following:

\[ W = \iint_S p \cos \phi \sqrt{x^2 + y^2 + z^2} \frac{1}{|z|} \, dx \, dy \]  \hspace{1cm} (5)

But according to Equation 3, the expression under the radical is \( \frac{D}{2}^2 \), so

\[ W = \iint_S p \cos \phi \frac{D}{2} \frac{1}{|z|} \, dx \, dy \]  \hspace{1cm} (6)

It will be convenient to transform Equation 6 to cylindrical coordinates. According to Figure 2:

\[ x = \frac{D}{2} \sin \gamma \cos \theta \]  \hspace{1cm} (7)
\[ y = \frac{D}{2} \sin \gamma \sin \theta \]  \hspace{1cm} (8)
\[ z = \frac{D}{2} \cos \gamma \]  \hspace{1cm} (9)

We need the following Jacobian:

\[
\begin{vmatrix}
\frac{\partial (x, y)}{\partial (\varphi, \theta)} & \frac{\partial (x, y)}{\partial (\psi, \phi)} \\
\end{vmatrix} = \begin{vmatrix}
\cos \varphi \cos \theta & -\sin \varphi \sin \theta \\
\cos \varphi \sin \theta & \sin \varphi \cos \theta \\
\end{vmatrix}
\]

which turns out to be:

\[ \left(\frac{D}{2}\right)^2 \sin \gamma \cos \gamma \]

Using Equations 7, 8, 9 and the Jacobian, Equation 6 becomes the following:

\[ W = \iint_S p \cos \phi \left(\frac{D}{2}\right)^2 \sin \gamma \, dy \, d\theta \]  \hspace{1cm} (10)
To express \( p \) by means of the new coordinates, we substitute Equation 2 into Equation 1 and express \( z \) by means of Equation 9. The result is Equation 11:

\[
p = k\left(\frac{D}{2}\right)^n (\cos \gamma - \cos \gamma_0)^n \tag{11}
\]

where \( \gamma_0 \) is shown in Figure 1-A.

Thus, Equation 10 becomes:

\[
W = \pi k\left(\frac{D}{2}\right)^n \int_0^{\gamma_0} (\cos \gamma - \cos \gamma_0)^n \cos \gamma \sin \gamma \, d\gamma \, d\theta \ldots (12)
\]

The integration with respect to \( \theta \) yields \( \pi \) so

\[
W = \pi k\left(\frac{D}{2}\right)^n \int_0^{\gamma_0} (\cos \gamma - \cos \gamma_0)^n \cos \gamma \sin \gamma \, d\gamma \ldots (13)
\]

Let \( u = \cos \gamma - \cos \gamma_0 \) and \( u_0 = \cos \gamma_0 \)

Then \( \cos \gamma = (u + u_0) \)

and \( -\sin \gamma \, d\gamma = du \)

So that Equation 13 becomes:

\[
W = \pi k\left(\frac{D}{2}\right)^n (u + u_0)^n (u + u_0) \, du
\]

The result of the integration is:

\[
W = \pi k\left(\frac{D}{2}\right)^n \frac{(1 - \cos \gamma_0)^{n+2}}{n+2} + \frac{(1 - \cos \gamma_0)^{n+1}}{n+1} \cos \gamma_0 \ldots (14)
\]

and since \( \frac{D}{2} (1 - \cos \gamma_0) = \gamma_0 \ldots \)
\[ W = w_k \frac{n^2}{n+2} + \frac{(\frac{\theta}{2}) - (\frac{\theta}{2})^2}{n+1} \cdots \cdots \cdots (15) \]

which can be rearranged so that:

\[ W = \frac{k\pi D}{2(n+1)} \int_0^{\theta_0} \left( 1 - \frac{\theta_0}{2} \cdot \frac{1}{n+2} \right) \cdots \cdots \cdots (16) \]

To express the sinkage we neglect the second term, which is small when compared to unity.

\[ \frac{\partial \theta_0}{\partial \theta} = \left[ \frac{2(n+1)}{\pi D k} \int_0^{\theta_0} \right] \cdots \cdots \cdots (17) \]

Equation 17 gives less sinkage than Equation 16, so it is "in favor" of the spherical wheel. When more accuracy is desired Equation 16 should be used.

The previous calculation dealt with a spherical wheel, however, thus, a certain portion of the supporting surface, considered so far was actually "missing". Therefore we will calculate the load which the "missing" part would support and subtract it from Equation 16.

To this end we use Figure 3A which is a bottom view of the "missing" surface. We make use of Equation 12. The limits of integration will be changed naturally.

\[ W_1 = k \left( \frac{D}{2} \right)^{n+2} \int_{\theta_1}^{\frac{\pi}{2}} \left( \cos \gamma - \cos \theta_0 \right)^n \cos \psi \sin \gamma d\psi d\theta \cdots \cdots (18) \]

The evaluation of Equation 18 is similar to that of Equation 12.
$$w = (\frac{n}{2} - \Theta_1) k(\frac{D}{2})^{n+2} \left[ \frac{\left(\cos \gamma_1 - \cos \gamma_0\right)}{n+2} + \frac{\left(\cos \gamma_1 - \cos \gamma_0\right)^{n+1}}{n+1} \cos \gamma_0 \right] \cdots \cdots (19)$$

The angle $\Theta_1$ is a function of the sinkage. From Figure 3, $A = z_0 \tan \gamma_1$

and the radius of the smaller circle, defined by $z_0$ is:

$$\frac{D}{2} \sin \gamma_0$$

So,

$$\sin \Theta_1 = \frac{z_0 \tan \gamma_1}{\frac{D}{2} \sin \gamma_0} = \frac{(\frac{D}{2} - z_0) \tan \gamma_1}{\frac{D}{2} \sin \gamma_0}$$

and since

$$\frac{D}{2} - z_0 = \frac{1}{\tan \gamma_0} \tan \gamma_1$$

(See Figure 3A)

$$\Theta_1 = \sin^{-1} \left[ \frac{1}{\tan \gamma_0} \tan \gamma_1 \right] \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS
So that:

\[ W_1 = \gamma_0 k \left( \frac{D}{2} \right) \left( \cos \gamma_1 - \cos \gamma_0 \right) + \frac{D}{2} \frac{\cos \gamma_0}{(n+1)} \]

denote

\[ \frac{D}{2} \left( \cos \gamma_1 - \cos \gamma_0 \right) = \frac{D}{2} \] (Figure 3-A) . . . . . (23)

Then:

\[ W_1 = \gamma_0 k \left( \frac{D}{2} \right)^{n+1} \left( \frac{\gamma_1}{n+2} + \frac{\gamma_0}{n+1} \right) \] . . . . . . . . (24)

The difference between \( \gamma_1 \) and \( \gamma_0 \) is constant, say \( M \) , so that

\[ \gamma_1 = \gamma_0 - M \] . . . . . . . . . (25)

Whence:

\[ \int = \gamma_0 k \left( \frac{D}{2} \right)^{n+1} \left( \frac{\gamma_1}{n+2} + \frac{\gamma_0}{n+1} \right) \] . . . . . . . . (26)

or

\[ W_1 = \gamma_0 k \left( \frac{D}{2} \right)^{n+1} \left[ - \frac{\gamma_0}{(n+2)(n+1)} + \frac{M(n+1)}{(n+2)(n+1)} + \frac{D}{2} \frac{(n+2)}{(n+1)(n+2)} \right] \]

or

\[ W_1 = \gamma_0 k \left( \frac{D}{2} \right)^{n+1} \left[ \frac{D}{2} \frac{(n+1)}{(n+2)} \right] \] . . . . . . . . (27)
or approximately:

\[ W_1 = \frac{2}{n} \frac{k(j_0 - j)^{n+1}}{n+1} \frac{D}{2} \]  

If we neglect the second term in the brackets in Equation 16 and subtract Equation 28 from Equation 16, we obtain the load:

\[ W_o = W - W_1 = \frac{kD}{2(n+1)} \left[ \pi j_0^{n+1} - \pi j_0 (j_0 - j)^{n+1} \right] \]  

The neglects employed in Equations 16 and 28 partially cancel the small errors introduced.

MOTION RESISTANCE ACTING AGAINST
A HEMISPHERICAL WHEEL

In the following it is assumed that the resistance of the wheel is the sum of the resistances acting against infinitely thin wheel-elements, whose diameter varies in accordance with the geometry of the sphere.

The resistance acting against the motion of a cylindrical wheel was shown by Bekker to be:

\[ R = \frac{k \pi j^{n+1}}{n+1} b \]  

Here \( b \) is the width of the wheel. (Figure 4A)

In the present problem \( b = dx \), let us first derive the resistance of the right hand part of the hemispherical wheel:
\[ R_1 = \frac{k}{n+1} \int_{J}^{J} j^{n+1} \, dx \quad \cdots \cdots \cdots \cdots \quad 31 \]

Let \( m = n + 1 \)

\[ R_1 = \frac{k}{m} \int_{j}^{j} j^{m} \, dx \quad \cdots \cdots \cdots \cdots \quad 32 \]

This type of integral has been solved by Bekker in his first book, "Theory of Land Locomotion".

According to Bekker, Equation 32 yields the following:

\[ R_1 = \frac{k}{m} \frac{3 - m}{3} \sqrt{D} \int_{j_0}^{j} j^{m + \frac{1}{2}} \cdots \cdots \cdots \cdots \quad 33 \]

or

\[ R_1 = \frac{k}{n+1} \frac{2 - n}{3} \sqrt{D} \int_{j_0}^{j} j^{n + \frac{3}{2}} \cdots \cdots \cdots \cdots \quad 34 \]

Next we tackle the left hand side of the hemispherical wheel.

The following derivation involves the same mathematical steps as those which were referred to, with respect to Equation 33:

\[ R_2 = \frac{k}{n+1} \int_{j_0}^{D} j^{n+1} \, dx \quad \cdots \cdots \cdots \cdots \quad 35 \]

and

\[ (j_0 - j_1) + \xi \cdots \cdots \cdots \cdots \quad 36 \]

where the meaning of the symbols is shown in Figure 5A.
From the geometry of the circle in Figure 5A.1.

\[ z = \sqrt{D(\beta_1 - \xi)} \]  

and

\[ dx = -\frac{1}{2} \frac{D}{D(\beta_1 - \xi)} \; d\xi = -\frac{1}{2} \sqrt{\frac{D}{\beta_1 - \xi}} \; d\xi \]  

so, Equation 35 becomes

\[ R_2 = -\frac{k}{m} \frac{1}{\sqrt{D}} \int_{\beta_1}^{\beta_0} \left[ \beta_0 - (\beta_1 - \xi) \right]^m \frac{1}{\sqrt{\beta_1 - \xi}} \; d\xi \]  

Let:

\[ t = \sqrt{\beta_1 - \xi} \quad \text{hence } 2t \; dt = -d\xi \]  

then

\[ R_2 = \frac{k}{m} \frac{1}{\sqrt{D}} \int_{\beta_1}^{\beta_0} (\beta_0 - t^2) \; dt \]  

Expand \((\beta_0 - t^2)^m\) into a binomial series and take the first two terms only. Then

\[ R_2 = \frac{k}{m+1} \frac{1}{\sqrt{D}} \left[ \beta_0^{m+1} \frac{1}{3} \beta_1^{3/2} - \frac{n+1}{3} \beta_0^{n+1} \beta_1^{3/2} \right] \]
and since the total resistance is \( R_1 + R_2 \); we add the right hand sides of Equations 34 and 43

\[
R = \frac{k}{n+1} \sqrt{D \left( \frac{2-n}{3} \delta_0 \frac{n+3}{2} + \delta_0 \frac{n+1}{3} 1 \right. \\
\left. - \frac{n+1}{3} \delta_0 \delta_1^{3/2} \right)} 
\]

NUMERICAL EXAMPLE

Let \( n = 0.75 \), \( k = 4.5 \) and use the dimensions of the actual model wheels, \( D = 18'' \); \( \gamma_1 = 30^\circ \); while \( b = 6'' \);
\( D = 18'' \) for cylindrical wheels.

Then the following table may be computed

<table>
<thead>
<tr>
<th>( \delta_0 ) (inch)</th>
<th>( W ) cylinder (lbs.)</th>
<th>( W ) Hemispherical (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85</td>
<td>72</td>
</tr>
<tr>
<td>2</td>
<td>203</td>
<td>232</td>
</tr>
<tr>
<td>3</td>
<td>340</td>
<td>431</td>
</tr>
<tr>
<td>4</td>
<td>485</td>
<td>655</td>
</tr>
<tr>
<td>5</td>
<td>644</td>
<td>925</td>
</tr>
<tr>
<td>6</td>
<td>807</td>
<td>1163</td>
</tr>
</tbody>
</table>

Reproduced From Best Available Copy
These numbers are plotted in Figure 1 of the report.

Table II-A... shows the motion resistance of the hemispherical wheels as a function of the sinkage.

<table>
<thead>
<tr>
<th>( z ) (in.)</th>
<th>( R_{\text{Hemisph.}} ) (lbs.)</th>
<th>( R_{\text{Cyl.}} ) (lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>15.4</td>
</tr>
<tr>
<td>2</td>
<td>43.4</td>
<td>51.0</td>
</tr>
<tr>
<td>3</td>
<td>111.0</td>
<td>104.0</td>
</tr>
<tr>
<td>4</td>
<td>206.0</td>
<td>174.0</td>
</tr>
<tr>
<td>5</td>
<td>327.0</td>
<td>255.0</td>
</tr>
<tr>
<td>6</td>
<td>489.0</td>
<td>354.0</td>
</tr>
</tbody>
</table>

One can pair the load and resistance data calculated at the same sinkage to obtain the resistance vs. load plot.

This is shown in Figure 2 of the Report.
FIGURE 2a.
FIGURE 4a.
FIGURE 5a.
DISCUSSION OF STATISTICAL ANALYSIS

Variations in soil consistency are inherent in the soil masses used in this test and a suitable experimental design is essential to eliminate this soil inconsistency, as well as other variabilities in experimental conditions.

An incomplete latin square, or Youden square, plan was selected to insure that the natural variability would be eliminated or "averaged out". This design would then permit vehicle performances to be evaluated with some degree of confidence. The experimental requirements for a Youden square plan in three replications involving three vehicle models are 3 soil mixes, arranged in 2 rows x 3 columns in each mix. This arrangement results in each vehicle model being tested a total of 6 times in each soil condition. The detailed statistical outline followed is described in Section 13.23, page 511, of "Experimental Designs" by Cochran and Cox. (5)

This "analysis of variance" for the Youden square produces an adjusted average for each vehicle model. These averages are compared with their estimated reliability or standard deviation. In addition, the coefficient of variation for each vehicle model provides a measure of the data "spread" for the vehicle under these experimental conditions.

*Prepared by L. Martin
A second numerical measurement resulting from this analysis is the calculated Student's 't' value. This value reflects the level of significance of the observed results in relation to the original hypothesis and tables are readily available to refer the Student's 't' value directly to its probability distribution for specified degrees of freedom. The original hypothesis was stated as "no performance differences among vehicle models". When the Student's 't' value is referred to its distribution (Prob $t \geq t_{obs}$), this value for the probability indicates the chance of securing the observed result, or a more extreme result, for the difference if in fact there were no real differences between the vehicles (hypothesis).

The following data taken from Table 4, 24 July will be used to illustrate the procedure for calculating the coefficient of variation and the Student's 't' value.

| Concept A: | 0.093 |
| Concept B: | 0.190 |
| Standard Deviation: | 0.00065 |
| Standard deviation of Difference: | 0.0093 |

\[
\text{Student's 't' value} = \frac{\text{Difference } (\bar{A} - \bar{B}) - 0}{\text{hypothesis of no difference}}
\]

\[
= \frac{(0.190 - 0.093) - 0}{0.0093} = \frac{0.097}{0.0093} = 10.4
\]

The "Analysis of Variance" also gives the degrees of freedom for the estimate of experimental error being used. In these experiments the degrees of freedom were 4 when based on 3 replications of the basic plan.
With these 2 values, 4 degrees of freedom and Student's 't' of 10.4 the table reads Prob (t ≥ t obs) = 0.01.

The interpretation of P = 0.01 in this case is that there is only a 0.01 probability of obtaining a result equal to or greater than 10.4 by "chance". This relatively low probability is an indication that the original hypothesis was in error and there are differences in performance between Concepts A and B.

It is customary to set a probability level of P = 0.05, equivalent to one chance in 20, of securing a significant result. While it is realized that the observed results may still be due to chance if a P ≤ 0.05 is obtained, most scientists and layman are willing to take such a risk (1 in 20) and declare that there are real differences between the methods or techniques being compared.

The coefficients of variation for the two concepts are calculated as follows:

\[
\begin{align*}
\text{coefficient of variation} & = \frac{\text{standard deviation of the difference}}{2 \times \text{concept average}} \\
\text{coefficient of variation (Concept A)} & = \frac{0.0093}{2 \times 0.093} = 7.1\% \\
\text{Coefficient of variation (Concept B)} & = \frac{0.0093}{2 \times 0.190} = 3.5\%
\end{align*}
\]

These coefficients of variation are very low which indicates uniform soil conditions. On 24 July, the day on which these data were taken, the test soil was dry sand which is the most uniform and manageable of our test soils.

The coefficient of variation is a measure of the uniformity of test conditions and may be used as a "yardstick" for determining the number of replications necessary in any one test condition.
In considering Table 4 as a whole it should be noted that 12 comparisons of differences are being made. Thus, even if there were no real differences for any of the three comparisons among vehicles it would not be surprising to obtain at least one apparently significant difference. More specifically in considering $C - B$ alone, there are four comparisons made. Only one of these four shows a $t$ value greater than two ($t>2$) and even this value has a probability greater than 1 in 20 (actually $P=0.08$). These results tend to give additional force to the general conclusion that under a variety of conditions Concept C offers no apparent advantage in performance as measured by the criterion $(DP/W)$. 
*APPENDIX C

SAMPLE WEIGHT CALCULATIONS

Equations Used

\[ z = \left[ \frac{3W}{bk \sqrt{D(3-n)}} \right] \frac{2}{2^{n+1}} \ldots \ldots 1. \]

\[ \frac{DP}{W} = \frac{3\sqrt{n}}{2(3-n)\sqrt{z}} \left[ \cos \left( \frac{1-2z}{D} \right) \left( \frac{c \tan \phi}{kn + 1} \right) \right] = \frac{3\sqrt{z}}{(3-n)(n+1)\sqrt{D}} \ldots 2. \]

Date, 16 July 1963

<table>
<thead>
<tr>
<th>Soil Values</th>
<th>Vehicle Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_0 = 4.0 )</td>
<td>( W = 2,940 \text{ lbs./wheel} )</td>
</tr>
<tr>
<td>( k_\phi = 4.5 )</td>
<td>( D = 43.6 \text{ in.} )</td>
</tr>
<tr>
<td>( n = 0.54 )</td>
<td>( b = 15 \text{ in.} )</td>
</tr>
<tr>
<td>( c = 0.5 )</td>
<td></td>
</tr>
<tr>
<td>( \tan \phi = 0.54 )</td>
<td></td>
</tr>
</tbody>
</table>

\[ k = \frac{k_0}{b} + \frac{k_\phi}{b} = \frac{4.0}{15} + 4.5 = 4.77 \]

Substitution in Equations 1 and 2 yields

\[ z = \frac{2}{2.08} \left( \frac{3 \times 2,940}{15 \times 4.77 \times 6.6 \times 2.46} \right) = 7 \text{ inches}. \]

\[ \frac{DP}{W} = \frac{3 \times 6.6}{2 \times 2.46 \times 2.65} \left[ \cos \left( \frac{1-14}{43.6} \right) \right] \left( \frac{0.5}{4.77 \times 2.06} + \frac{0.54}{1.54} \right) \]

\[ - \frac{3 \times 2.65}{2.46 \times 1.54 \times 6.6} \]

*Prepared by L. Martin
\[
\frac{DP}{W} \text{ Full size} = 1.52 \times 0.82 \times 0.387 = 0.164
\]

MODEL DATA

\[
D = 10.9 \quad k = \frac{4.9}{3.75} + 4.5 = 5.57
\]

\[
b = 3.75
\]

\[
W = \text{dependent variable}
\]

Sinkage of model = \[z_m = \left(\frac{3}{3.75 \times 5.57 \times 3.3 \times 2.46}\right)^{0.96}\]

\[
z_m = \left(\frac{W}{56.5}\right)^{0.96}
\]

\[
\frac{DP}{W} = \frac{3 \times 3.3}{2 \times 2.46 \times \sqrt{z}} \left[\cos^{-1}\left(\frac{1 - z}{10.9}\right) \left(\frac{0.5}{5.57 z^n} + \frac{0.35}{1.54}\right) - \frac{3 \sqrt{z}}{2.46 \times 1.54 \times 3.3}\right] = \frac{2}{\sqrt{z}} \left[\cos^{-1}\left(\frac{1 - \frac{z}{5.45}}{0.09}\right) + 0.350\right] - \frac{\sqrt{z}}{4.17}
\]

Trial and Error solution resulted in a wheel weight of 120 lbs. giving a \[
\left(\frac{DP}{W}\right)_\text{Model} = 0.164 = \left(\frac{DP}{W}\right) \text{ Full size}
\]

Therefore, the total model weight used was 120 lbs./wheel x 8 wheels = 960 lbs.
During the later tests the time consumed in calculating the correct model weights was too lengthy to complete the testing in the same day and the same soil conditions. Therefore, it was agreed to select an arbitrary model weight for test purposes and then calculate the corresponding full size gross vehicle weight (GVW). Then using a correction factor \( \frac{W}{W_t} \),

where \( W_a = \) calculated GVW - Curb weight

\[ W_t = \text{design payload (10,000 lbs.)} \]

the test data was corrected to

\[ \frac{DP}{W} \times \frac{W}{W_t} \]

to reflect a true picture of the expected vehicle performance.

The same equations used for the scale model weight were used for the "scaled" GVW. The curb weight was taken as a constant equal to the proposed concept weight - design payload.

A sample calculation follows:
DATE: 22 July 1963

Soil Values                      Model Data

\[ k_0 = 32.4 \]                        \[ D = 10.9 \text{ in.} \]
\[ k'_b = 0.3 \]                           \[ b = 3.75 \text{ in.} \]
\[ n = 0.6 \]                               \[ W = 125 \text{ lbs./wheel} \]
\[ e = 0.43 \]
\[ \tan \phi = 0.59 \]

\[ k = \frac{32.4}{3.75} + 0.3 = 8.95 \]

Model Sinkage = \[ \left( \frac{3 \times 125}{3.75 \times 0.95 \times 3.3 \times 2.2} \right)^{2.6} \]
\[ = 1.4 \text{ in.} \]

\[ \frac{D_P}{W} = \frac{3 \times 3.3}{2 \times 2.2 \times 1.18} \times \left[ \cos^{-1} \left( \frac{1 - 1.4}{5.45} \right) \right] \times \frac{0.43}{6.95 \times 1.3} + \frac{0.59}{1.8} \]
\[ = \frac{3 \times 1.18}{2.2 \times 1.8 \times 3.3} = 1.91 \times 0.73 \times 0.365 - 0.70 \]

\[ \frac{D_P}{W} \] Model = 0.240

Full Size                      Vehicle Data
\[ D = 43.6 \]                           \[ k = \frac{32.4}{15} + 0.3 = 2.46 \]
\[ b = 15 \]
\[ W = \text{dependent variable} \]
Trial and Error solution resulted in a wheel weight of 1,800 lbs., giving

\[ \frac{DP}{W} = 0.240 = \frac{DP}{W} \text{ Model} \]

Therefore, the "scaled" GVW is 1,600 lbs./wheel x 8 wheels = 14,400 lbs.

The curb weight for the designed concept is 23,500 lbs. - 10,000 lbs. = 13,500 lbs.

Thus the "scaled" payload \( (W_a) \) is 14,400 lbs. - 13,500 lbs. = 900 lbs.

The correction factor to be applied to the test data is

\[ \frac{W_a}{W_t} = \frac{900 \text{ lbs.}}{10,000 \text{ lbs.}} = 0.09 \]
A complete table of correction factors and dates:

<table>
<thead>
<tr>
<th>DATE</th>
<th>CONCEPT B</th>
<th>CONCEPT C</th>
<th>CONCEPT A</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 July</td>
<td>0.34</td>
<td>0.34</td>
<td>0.09</td>
</tr>
<tr>
<td>24 July</td>
<td>1.00</td>
<td>1.00</td>
<td>1.33</td>
</tr>
<tr>
<td>25 July</td>
<td>'scaled' GVW's &lt; design curb weight</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 August</td>
<td>1.2</td>
<td>1.2</td>
<td>2.65</td>
</tr>
</tbody>
</table>

As seen in the table on 25 July the 'scaled' GVW's for all three concepts was less than the design curb weight which was assumed to be the minimum possible vehicle weight. Therefore, no correction factor was calculable for 25 July.

The basic assumptions, and possible source of errors, necessary for the preceding calculations are enumerated:

1. Rigid wheel equations were used.
2. Bulldozing resistance was neglected
   \[ \Delta P = Ac + W \tan \phi - R_c \]
3. The wheel leaves a rut resulting in ground contact to the bottom of the wheel only.
4. Sinkage is static sinkage instead of dynamic.
5. The pressure between the wheel and the soil was averaged around the contact length.
6. The weight, in full size and models, was distributed equally on all wheels.

Reproduced From
Best Available Copy
Another source of error is that the wheel width \( b \) is incorporated in the soil strength parameter

\[
k = \frac{k_c}{b} + k_\phi
\]

The vehicle sinkage is inversely proportional to this parameter \( k \). In some soils when \( k_c \) was large and \( k_\phi \) small, the width \( b \) influenced \( k \) to the extent that the soil would be weak when seen by the full size vehicle and strong for the model. This resulted in non-realistic weights needed on the model to obtain enough sinkage to get equivalent \( \frac{DP}{W} \) as that of the full size vehicle.
This report is concerned with the evaluation of soft-sell performance of a vehicle concept based on an inclined hemispherical wheel. Performance of the concept was compared to a concept utilizing cylindrical wheels of similar size to the hemispherical wheel and to a conventional vehicle of equal pay load capacity currently under development. Tests were conducted to establish the performance of a scale model of each of the vehicle concepts in the large flat area located in the Land Locomotion Laboratory. The models were tested in sand and in a sandy loam at three different moisture contents.

The models were given code designations and are identified as Concepts A, B, and C. The test results indicate that Concepts B and C have significantly better soft-sell performance than Concept A. The performance of Concept B is equal to that of Concept C. It is concluded that Concept C does not offer any improvement in soft-sell performance and should not be considered as a device to improve mobility on the basis of its soft-sell characteristics.

The recommendations are that Concept C be given no further consideration for application to military vehicles.