

UNCLASSIFIED

AD NUMBER
AD476352
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative and Operational Use; Nov 1965. Other requests shall be referred to the Office of Naval Research, 800 North Quincy St, Arlington, VA 22217.
AUTHORITY
ONR, per ONR Notice dtd 27 Jul 1971

THIS PAGE IS UNCLASSIFIED

15 NOVEMBER 1965



476352

DIVISION OF ENGINEERING RESEARCH AND DEVELOPMENT
DEPARTMENT OF ELECTRICAL ENGINEERING

Report 396 (10)/2

Electromagnetic Fields In The Ocean Near A Shoreline

J. E. SPENCE, E. SULLIVAN, J. BEVILLE

Project: Extremely-Low and Sub-Audio Frequency Electromagnetic
Signals Generated by Natural and Man-Made Effects.

Office of Naval Research
Contract Nonr 396 (10)
NR 371-490

UNIVERSITY OF RHODE ISLAND
KINGSTON, RHODE ISLAND

University of Rhode Island
Department of Electrical Engineering
Kingston, Rhode Island

ELECTROMAGNETIC FIELDS IN THE OCEAN NEAR A SHORELINE

by

J.E. Spence,^{*} E. Sullivan,^{**} and J. Beville^{***}

Technical Report 396(10)/2
Project: Extremely-Low and Sub-Audio Frequency
Electromagnetic Signals Generated by Natural
and Man-Made Effects.

Research sponsored by the Office of Naval Research

[Electronics Branch, Code 427]

under contract
Nonr 396(10)
NR 371-490

Report date: 15 November, 1965

Reproduction in whole or in part is permitted for any purpose of the United States Government

* Associate Professor of Electrical Engineering

** NASA Trainee

*** N.S.F. Undergraduate Research Participant

ELECTROMAGNETIC FIELDS IN THE OCEAN NEAR A SHORELINE

ABSTRACT

The electromagnetic fields in the ocean resulting from ground wave excitation over land by natural electromagnetic noise such as Micropulsations or ELF Atmospherics are investigated analytically. A simplified two dimensional model is used. The purpose of the study is to determine the relative importance of electromagnetic energy propagation via air into the ocean versus propagation through the soil and ocean bottom into the sea. Vertical profiles of the horizontal electric field are plotted for several values of frequency, ocean depth, and the distance from the shoreline. The relative importance of the soil path is seen to be greater for shorter distances from the shore and for deeper oceans.

Introduction.

The purpose of this report is to present the results of an analytical study on the relative importance of electromagnetic energy propagation via air into the ocean versus propagation through the soil and ocean floor into the sea employing the highly simplified, two dimensional, model shown in figure 1. More complicated analytical models which may represent a closer approximation to the real three dimensional situation will be discussed in future reports.

In the present model all fields are considered to be independent of z (i.e. $\frac{\partial}{\partial z} = 0$) and the magnetic field \bar{H} has a z component only. The complete electromagnetic field can thus be expressed in terms of H_z as

$$\begin{aligned} \text{a) } H_z &= h \\ \text{b) } E_x &= \frac{+1}{\sigma - i\omega\epsilon} \frac{\partial h}{\partial y} \\ \text{c) } E_y &= \frac{-1}{\sigma - i\omega\epsilon} \frac{\partial h}{\partial x} \end{aligned} \quad (1)$$

where σ is the conductivity and ϵ is the permittivity. The permeability, μ_0 , of all media is taken to be that of free space. A time factor of $\exp(-i\omega t)$ is assumed throughout. The electromagnetic noise from natural sources over land is assumed to produce a nearly vertically polarized plane wave propagating in the $+x$ direction. The field established in the ocean when this wave is incident on the shoreline is to be determined.

The incident field is given by (1)

$$h_{inc} = e^{ik_{oy}y + ik_x x} \quad \text{for } y \geq 0 \quad (2)$$

$$h_{inc} = e^{-ik_{2y}y + ik_x x} \quad \text{for } y \leq 0 \quad (3)$$

where

$$k_{oy} = \frac{\omega}{c} \sqrt{\frac{-i\omega\epsilon_0}{\sigma_2}}$$

$$k_{2y} = \sqrt{i\omega\mu\sigma_2}$$

$$k_x = \sqrt{1 + \frac{i\omega\epsilon_0}{\sigma_2}}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

(4)

In order to obtain the field in the ocean we will treat the problem in the following manner.

We consider the field to be a superposition of two fields, one being the field due to the energy entering the ocean through the air-sea boundary and the other being the field due to the energy entering through the horizontal land-sea boundary. The energy entering through the vertical land-sea boundary is neglected in this treatment.

The magnetic field on the surfaces of the ocean (both top and bottom) will first be obtained under the assumption that the ocean is a perfect conductor.

To find the field at some point in the ocean the assumption is made that wave enters the ocean normal to the surface.

Contribution through Air-Sea Boundary.

In this section we shall determine an approximate expression for the magnetic field in the ocean due to the energy entering the air-sea boundary only. We begin by determining the magnetic field at the top surface of the ocean.

Consider equations (2) and (4) which describe the incident wave in the air above the ground. For the range of frequencies of interest here the quantity $[\omega\epsilon_0/\sigma_2] \ll 1$. We note then that the magnetic field in air is not very sensitive to a change in conductivity and therefore will propagate past the shore line relatively unperturbed.

Neglecting the attenuation (i.e. considering $\sigma = \infty$) we have the following approximate expression for the magnetic field at the top surface of the ocean

$$h_{ts} \approx e^{ik_0 x} \quad (5)$$

where $k_0 = \frac{\omega}{c}$

The assumption of infinite conductivity used in order to obtain h_{ts} is now dropped. The conductivity, however, is still considered high enough so that the energy will enter the ocean normal to the surface. The field at some point in the ocean will thus be approximately of the form

$$h_t = A \left[e^{-ik_1(y+d)} + \rho_b e^{+ik_1(y+d)} \right] \quad (6)$$

where

$$k_1 = \omega \sqrt{\mu_o \left(\epsilon_1 - \frac{\sigma_1}{i\omega} \right)} \approx \sqrt{i\omega\mu_o\sigma_1} \quad (7)$$

The subscript t indicates that this is the magnetic field in the ocean due to the energy entering the top surface only. The quantity ρ_b is the reflection coefficient of the magnetic field at the bottom of the ocean and is given by

$$\rho_b = \frac{\sqrt{\frac{\mu_o}{\epsilon_1 - \frac{\sigma_1}{i\omega}}} - \sqrt{\frac{\mu_o}{\epsilon_2 - \frac{\sigma_2}{i\omega}}}}{\sqrt{\frac{\mu_o}{\epsilon_1 - \frac{\sigma_1}{i\omega}}} + \sqrt{\frac{\mu_o}{\epsilon_2 - \frac{\sigma_2}{i\omega}}}} \approx \frac{\sqrt{\sigma_2} - \sqrt{\sigma_1}}{\sqrt{\sigma_2} + \sqrt{\sigma_1}} \quad (8)$$

For the values of the parameters involved here ρ_b is very nearly equal to -1. In (6) the coefficient A is chosen so that the magnetic field is continuous across the air-sea

boundary. Thus

$$A = \frac{h_{ts}}{e^{-ik_1 d} + \rho_b e^{+ik_1 d}} \quad (9)$$

Contribution Through Land-Sea Boundary.

In this section we find the magnetic field in the ocean due to the energy entering the Land-Sea boundary only.

To first obtain an approximate expression for the magnetic field at the bottom surface of the ocean, due to the energy entering this interface, Green's Theorem will be used.

Consider the region bounded by the closed dotted line path l' in figure 2. It can be shown² that the magnetic field at some point x, y in this region is given by

$$h(r) = \oint \frac{-1}{i\omega\epsilon_2} \left[G(r|r') \frac{\partial}{\partial n'} h(r') - h(r') \frac{\partial}{\partial n'} G(r|r') \right] dl' \quad (10)$$

where $G(r|r')$, the Green's function, is a solution to the inhomogeneous wave equation

$$(\nabla^2 + k_2^2) G(r|r') = i\omega\epsilon_2 \delta(|\bar{r} - \bar{r}'|) \quad (11)$$

and

$$k_2 = \omega \sqrt{\mu_0 \left(\epsilon_2 - \frac{\sigma_2}{i\omega} \right)} \approx \sqrt{i\omega \mu_0 \sigma_2} \quad (12)$$

This Green's function represents the magnetic field at r due to a line source at r' described by the Dirac delta function $\delta(|\bar{r} - \bar{r}'|)$. The operator $\frac{\partial}{\partial n'}$ denotes the derivative normal to the boundary l' .

The Green's function will be chosen so that its normal derivative will vanish along $y = -d$ for all x . In addition if we assume that the ocean is a perfect conductor, $\frac{\partial h}{\partial n'}$ vanishes for $y = -d, x > 0$. In this case the integral along $y = -d, 0 \leq x \leq \infty$ will be zero. By introducing small losses the integral over the infinite

arc will also be zero.

Thus (10) reduces to an integral along the y axis from $y = -\infty$ to $y = -d$.

The solution to (11) for $G(r|r')$ along $y = -d$ subject to the condition that $\frac{\partial}{\partial y} G(r|r) = 0$ at $y = -d$ is well known and is given by⁽³⁾

$$G(r|r') = \frac{\omega \epsilon_2}{2} H_0^{(i)} (k_2 \sqrt{(x-x')^2 + (y'+d)^2}) \quad (13)$$

In order to evaluate (10) we now need to know the magnetic field h along the path of integration. This, of course, is not known exactly. As is often the case in problems of this type we shall approximate h over this path by assuming it is equal to the incident magnetic field given in (3).

Substitution of (13) and (3) into (10) yields the following expression for the magnetic field along the bottom surface of the ocean

$$h_{bs} \cong \int_{-\infty}^{-d} \frac{-e^{-ik_2 y y'}}{2} \left\{ k_x H_0^{(1)} (k_2 \sqrt{x^2 + (y' + d)^2}) + \frac{ik_2 x}{\sqrt{x^2 + (y' + d)^2}} H_1^{(1)} (k_2 \sqrt{x^2 + (y' + d)^2}) \right\} dy' \quad (14)$$

This integral cannot be readily evaluated for all values of x . However, for sufficiently large x , say for $x > 6 \delta_2$ where $\delta_2 = \sqrt{\frac{2}{\omega \mu_0 \sigma_2}}$ is the skin depth in the earth, we show in the appendix that (14) is approximately equal to

$$h_{bs} \approx \frac{-ie^{ik_2 y d}}{2k_2 y} \left\{ k_x H_0^{(1)} (k_2 x) + ik_2 H_1^{(1)} (k_2 x) \right\} \quad (15)$$

We now drop our assumption of infinite conductivity and write that the magnetic field in the sea is approximately of the form

$$h_b = B \left[e^{ik_1 y} + \rho_t e^{-ik_1 y} \right] \tag{16}$$

The subscript b indicates that this is the magnetic field in the ocean due to the energy entering through the bottom of the ocean only.

The reflection coefficient at the top of the ocean is

$$\rho_t = \frac{\frac{\mu_o}{\sqrt{\epsilon_1 - \frac{\sigma_1}{i\omega}}} - \frac{\mu_o}{\epsilon_o}}{\frac{\mu_o}{\sqrt{\epsilon_1 - \frac{\sigma_1}{i\omega}}} + \frac{\mu_o}{\epsilon_o}} \tag{17}$$

This reflection coefficient is also nearly equal to -1 for our purposes.

The coefficient B is given by

$$B = \frac{h_{bs}}{e^{-ik_1 d} + \rho_t e^{+ik_1 d}} \tag{18}$$

which makes the magnetic field continuous at the bottom of the sea.

Total Magnetic Field in the Ocean

The total magnetic field in the ocean is given by the sum of (6) and (16).

Substituting for A and B from (9) and (18) we obtain

$$h_{total} = \frac{e^{-ik_1(y+d)} + \rho_b e^{+ik_1(y+d)}}{e^{-ik_1 d} + \rho_b e^{+ik_1 d}} h_{ts} + \frac{e^{ik_1 y} + \rho_t e^{-ik_1 y}}{e^{-ik_1 d} + \rho_t e^{+ik_1 d}} h_{bs} \tag{19}$$

If we set ρ_t and ρ_b equal to -1 we have finally

$$h_{total} \cong \frac{\sin k_1(y + d)}{\sin k_1 d} h_{ts} - \frac{\sin k_1 y}{\sin k_1 d} h_{bs}$$

where h_{ts} and h_{bs} are given by (5) and (15) respectively.

Electric Field.

Now that we have an expression for h , the electric field is easily obtained from (1). In particular the horizontal component of the electric field is

$$E_x = \frac{k_1}{\sigma_1} \left| \frac{\cos k_1 (y + d)}{\sin k_1 d} h_{ts} - \frac{\cos k_1 y}{\sin k_1 d} h_{bs} \right| \quad (21)$$

where again we are making use of the fact that $\omega \epsilon_1 / \sigma_1 \ll 1$.

Numerical Results.

Numerical values for the x component of the electric field were determined by digital evaluation of equation (21). The results are plotted in Figs. 3 to 12 as a function of depth and distance from shore for frequencies of 10, 100, and 1000 c/s. Depths range from 50 meters to 800 meters and distances from shore range from 3 km to 40 km. The depth is plotted as the abscissa and the various values of x (distance from shore) determine a family of curves.

The contribution to the field due to the energy passing through the air-sea boundary only is shown as a dotted line. This enables one to see more explicitly the effect that the energy passing through the horizontal land-sea boundary has on the total field.

In Figs. 13 and 14 δ_1 and δ_2 , the skin depth in the sea and land respectively are plotted as a function of frequency. This allows interpretation of the curves in terms of skin depth instead of absolute distance.

Discussion.

Before discussing the results it must be admitted that both the model and the incident field which we have chosen, and the manner in which they approximate the actual situation leave much to be desired. Nevertheless, the approximations used were felt to be sufficient to obtain a first order solution to our problem, namely to establish the relative importance of the contributions from the top and bottom of the ocean.

Upon examination of the curves, it can be seen that the contribution to the net field which enters through the ocean-land interface is quite prominent in the region near the shoreline. As the distance from the shore increases, this contribution becomes less and less noticeable. This is due to the fact that k_x , the wave number which governs the propagation in the x direction for the contribution through the air-ocean interface, has a small imaginary part, whereas the attenuation of the contribution through the ocean-land interface is quite high due to the conductivity of the earth.

The effect of the depth of the ocean on the electric field is also quite noticeable. For the region near the shoreline the curves show the earth propagated contribution more distinctly as the depth of the ocean increases. This is understandable since for a deep ocean the energy entering the top surface will be greatly attenuated before reaching the bottom. The contribution through the bottom will, therefore, be more evident.

Since an approximation was used to simplify the arguments of the Hankel function (see appendix) our expressions are not valid for $x < 6 \delta_2$. The model itself is, however, not very reasonable in this region.

Appendix A

 h_{bs} for large X

Consider equation (14). Because of the decaying exponential character of the integrand (k_{2y} is complex) the contribution to the integral is appreciable only for that portion of the integration path within a few skin depths of the bottom of the ocean. The skin depth in the earth is given by

$$\delta_2 = \sqrt{\frac{2}{\omega \mu_0 \sigma_2}} \quad (A1)$$

If we assume the integrand to be negligible for $|y'|$ greater than say $2\delta_2$, and restrict ourselves to values of x greater than $6\delta_2$ then the argument of the Hankel function can be greatly simplified over that portion of the integrand that is most important, i.e.

$$\sqrt{x^2 + (y' + d)^2} \approx x \quad (A2)$$

Since the values of d considered are much less than δ_2 its presence in the argument does not affect the approximation to any great extent.

With this approximation the Hankel functions are no longer functions of y' and may be moved outside of the integral sign leaving a simple exponential term to integrate.

Upon integrating we obtain

$$h_{bs} \approx \frac{-ie^{ik_2 y d}}{2k_{2y}} k_x H_0^{(1)}(k_2 x) + ik_2 H_1^{(1)}(k_2 x) \quad (A3)$$

References.

1. E.C. Jordan, "Electromagnetic Waves and Radiating Systems," pp. 205-7
(Prentice-Hall Inc., Englewood Cliffs, N.J. 1950)
2. P.M. Morse and H. Feshback, "Methods of Theoretical Physics," Vol. I, pp. 804-6
(McGraw-Hill Book Co. Inc., 1953.)
3. J.R. Wait, "Electromagnetic Waves in Stratified Media," p. 32
(The MacMillan Co., New York, 1962.)

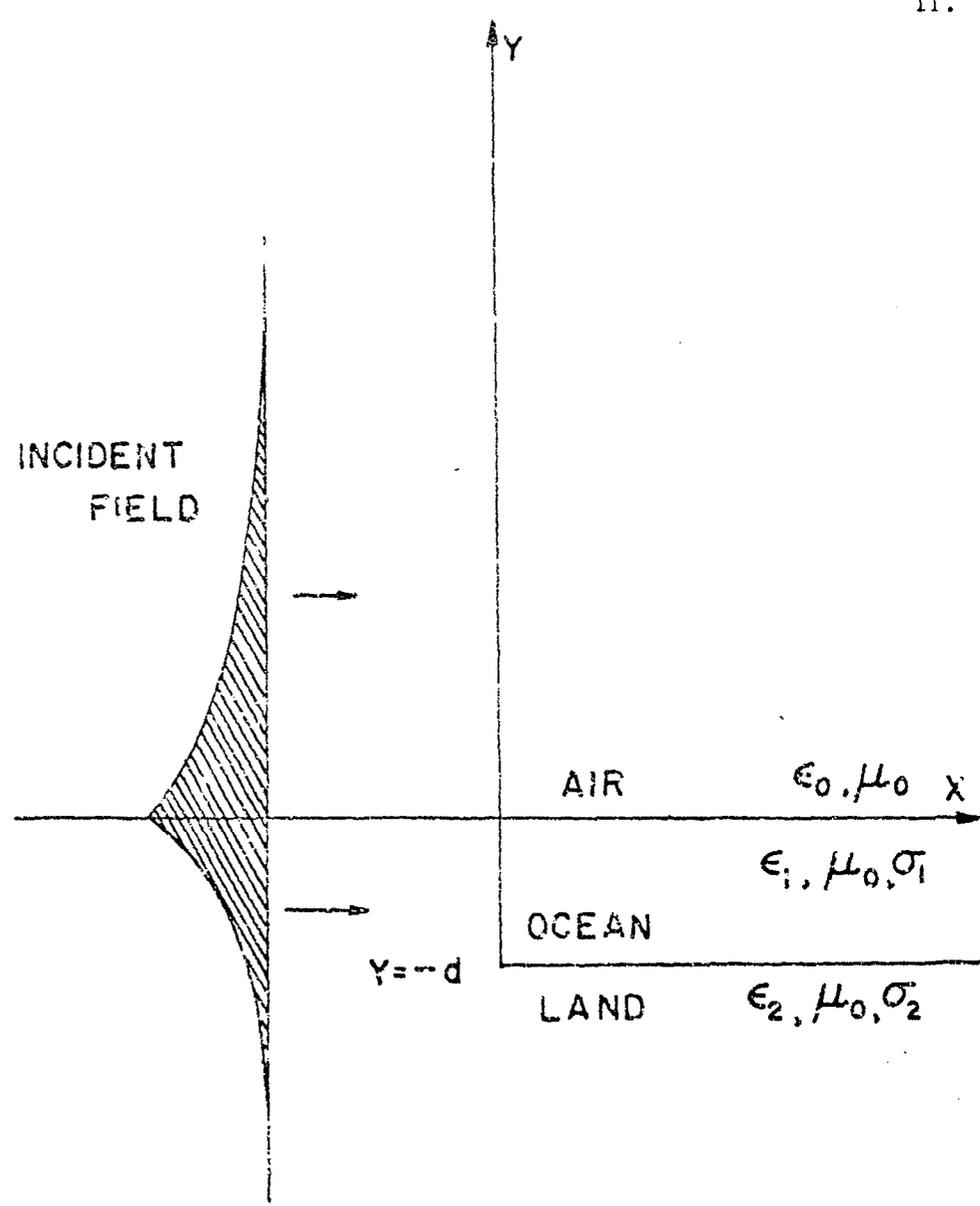


FIGURE - I

SIMPLIFIED TWO DIMENSIONAL MODEL

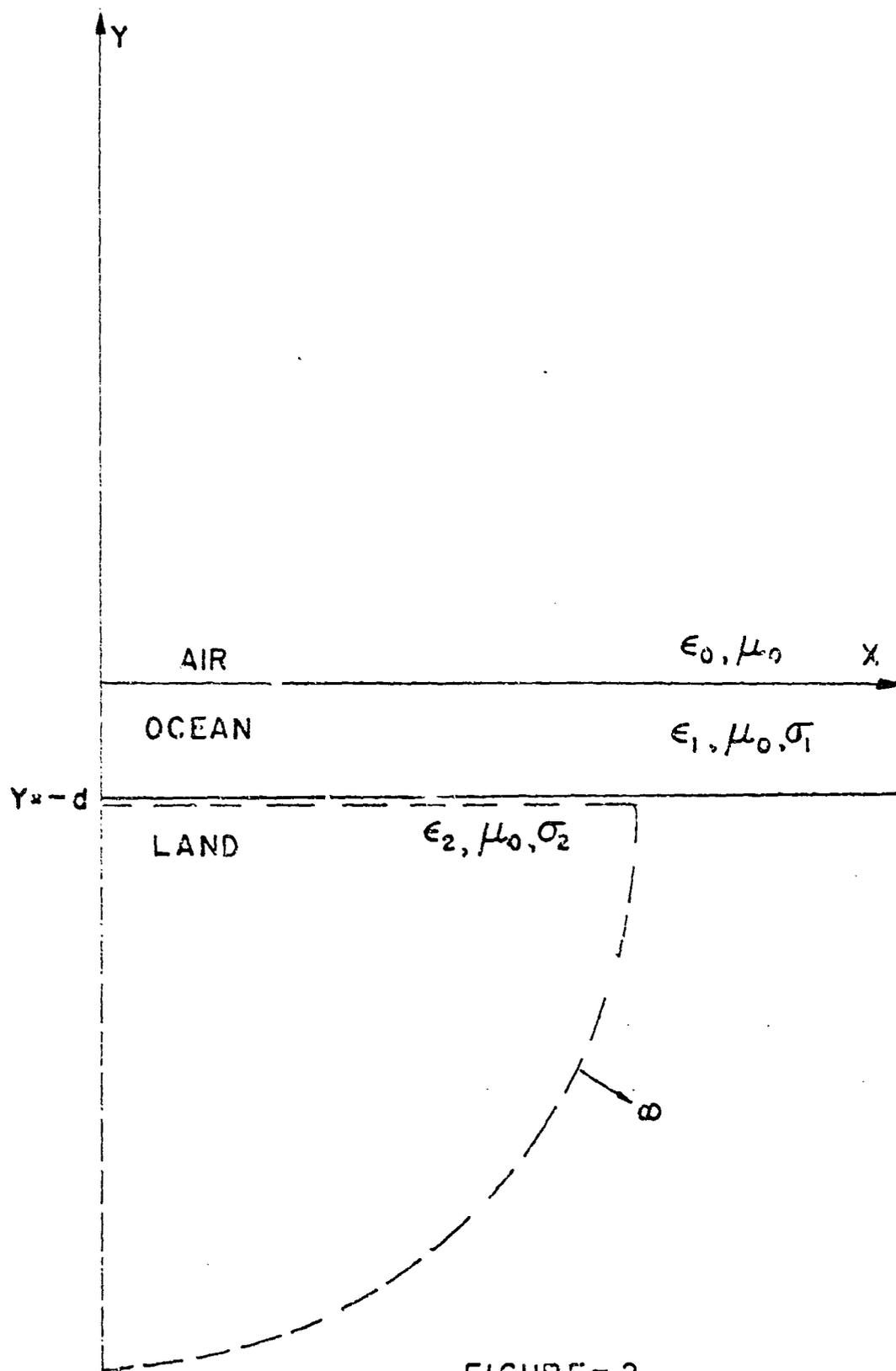


FIGURE-2

REGION TO WHICH GREEN'S THEOREM
IS APPLIED

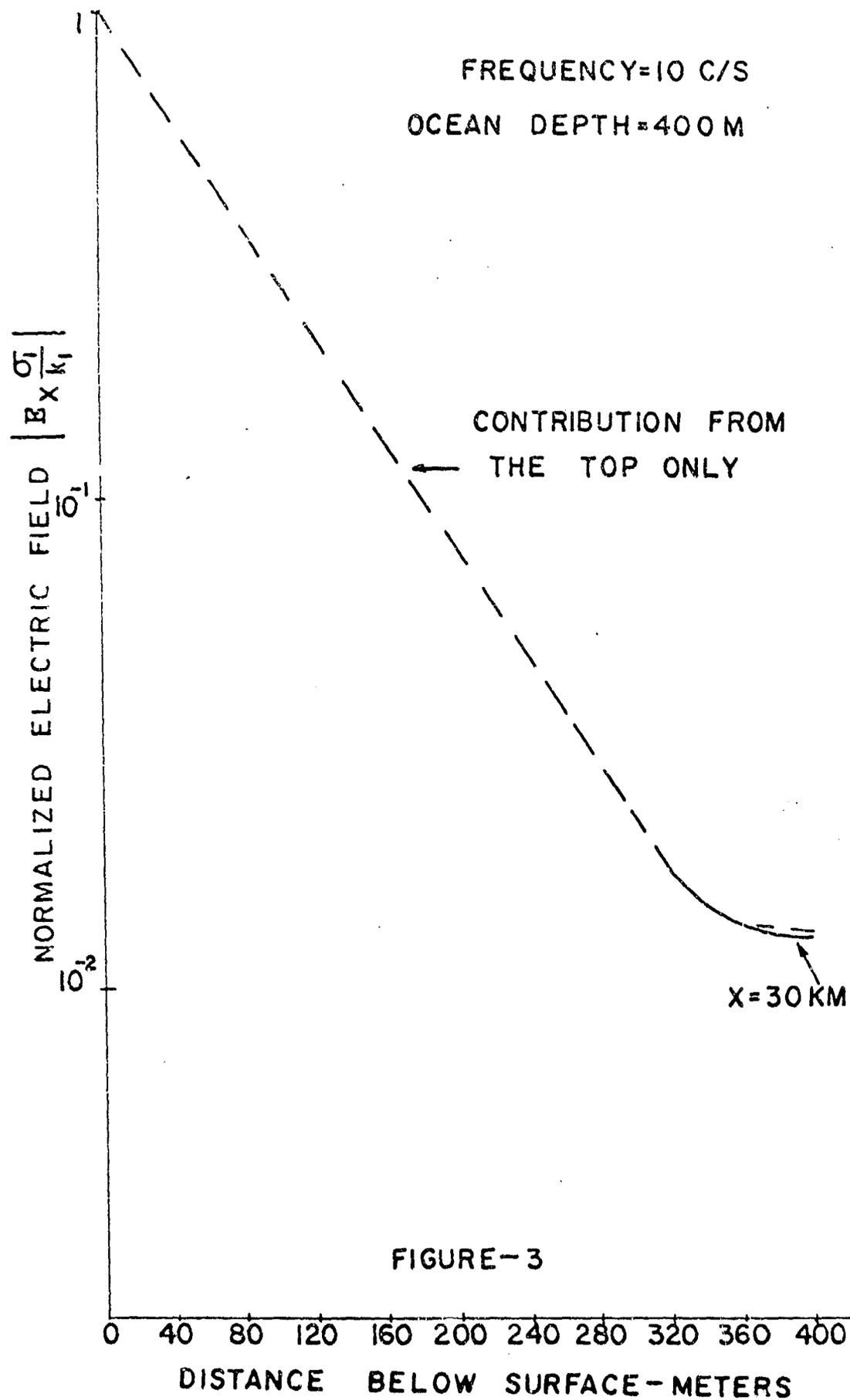
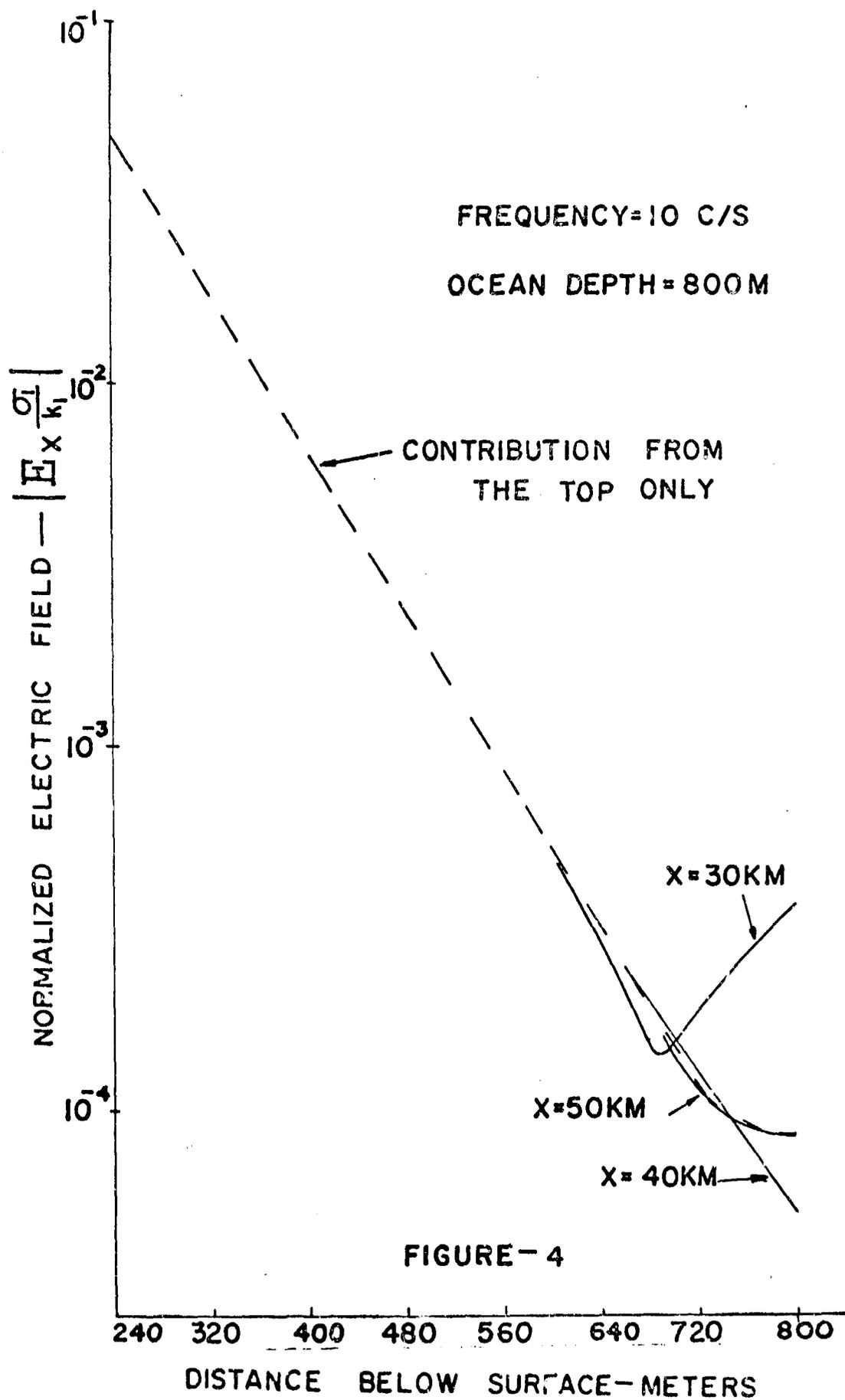
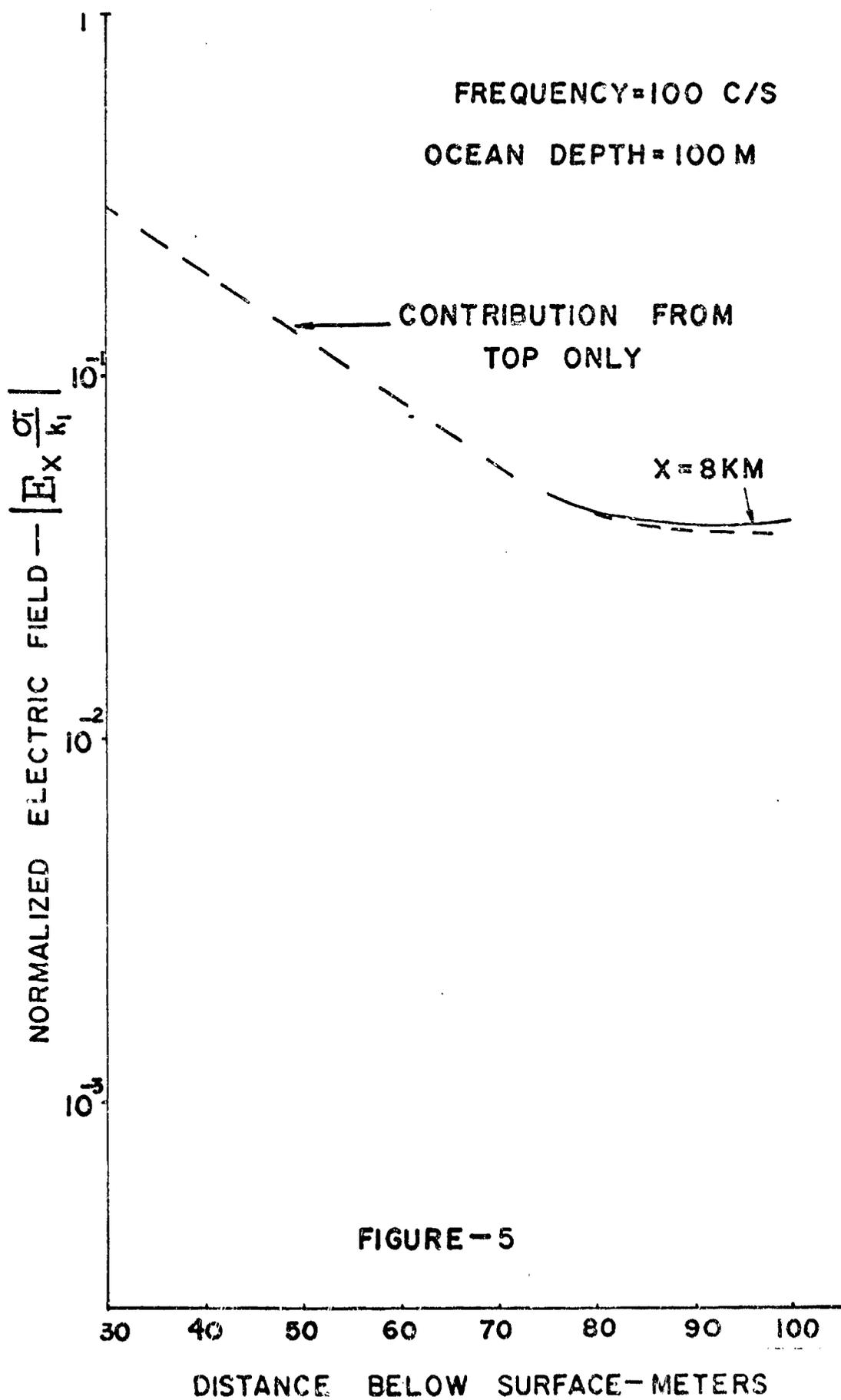
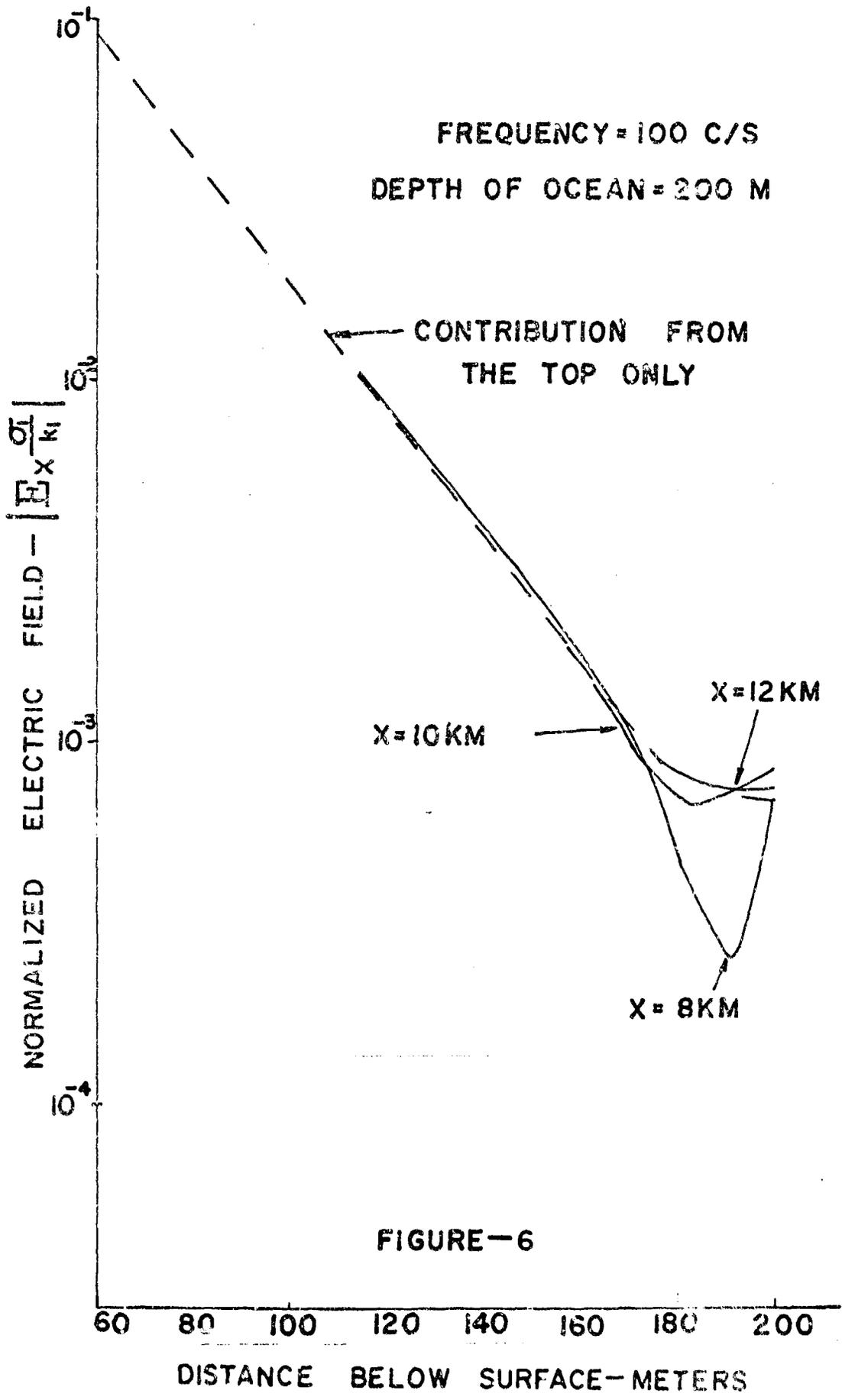
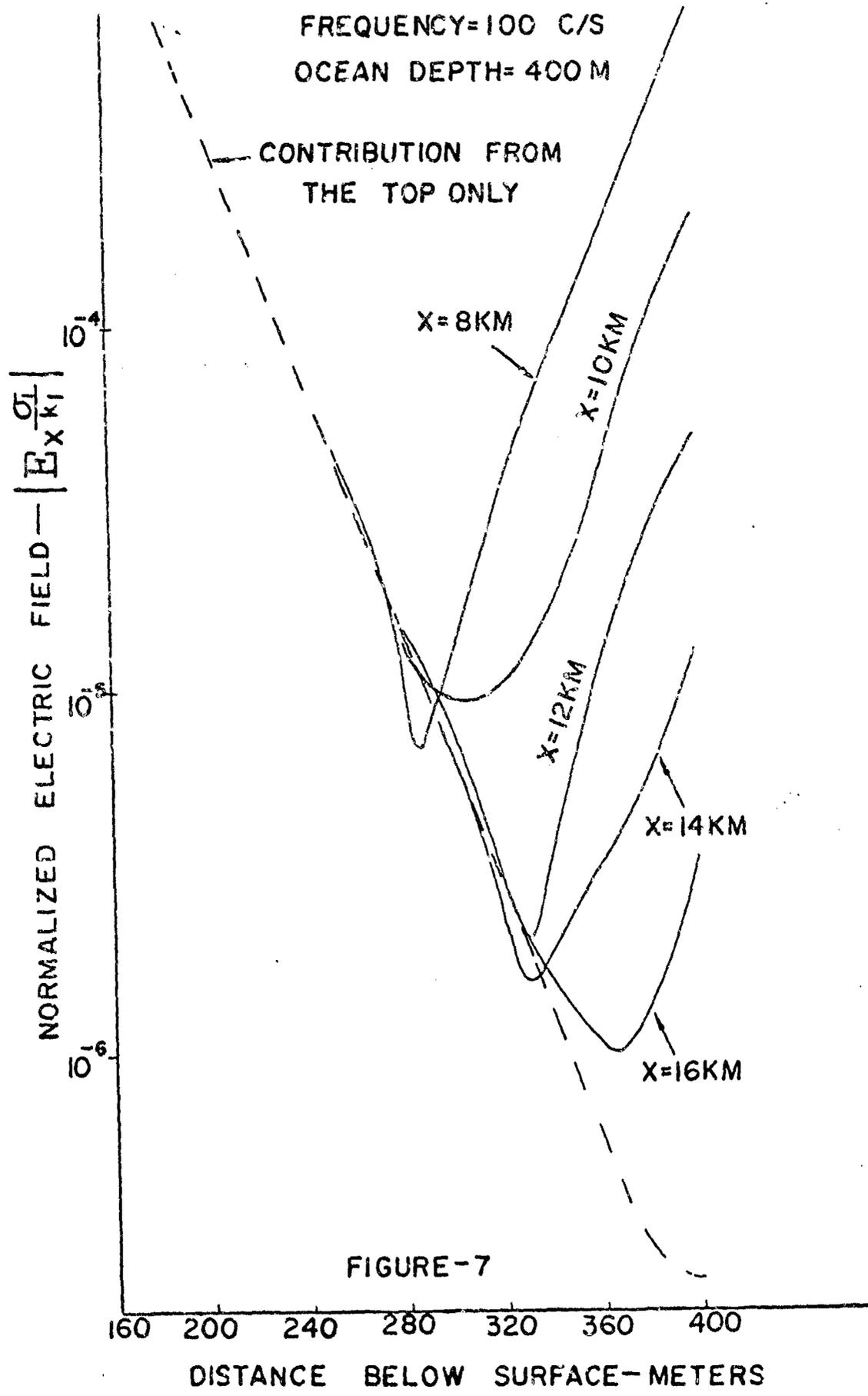


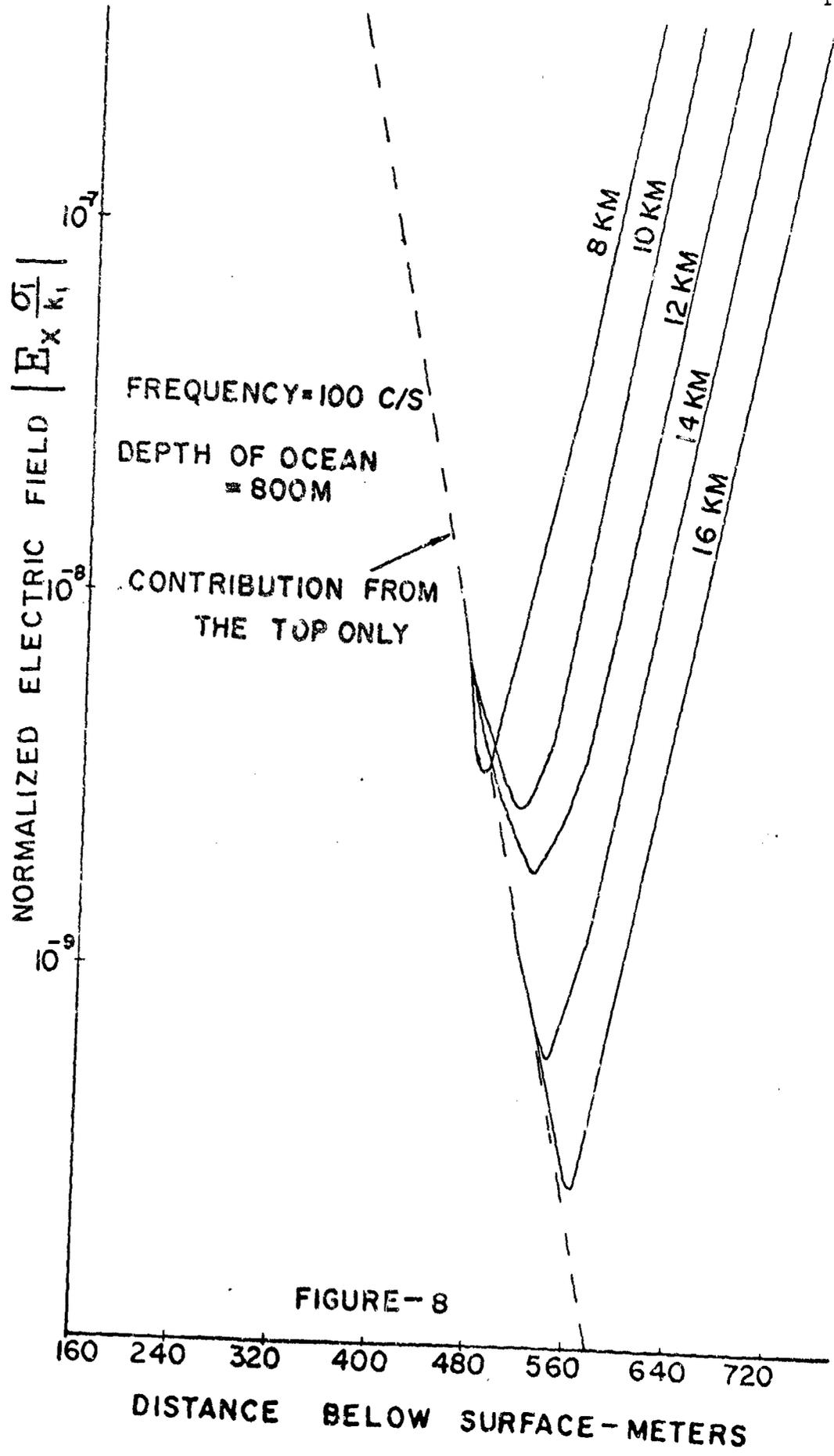
FIGURE-3

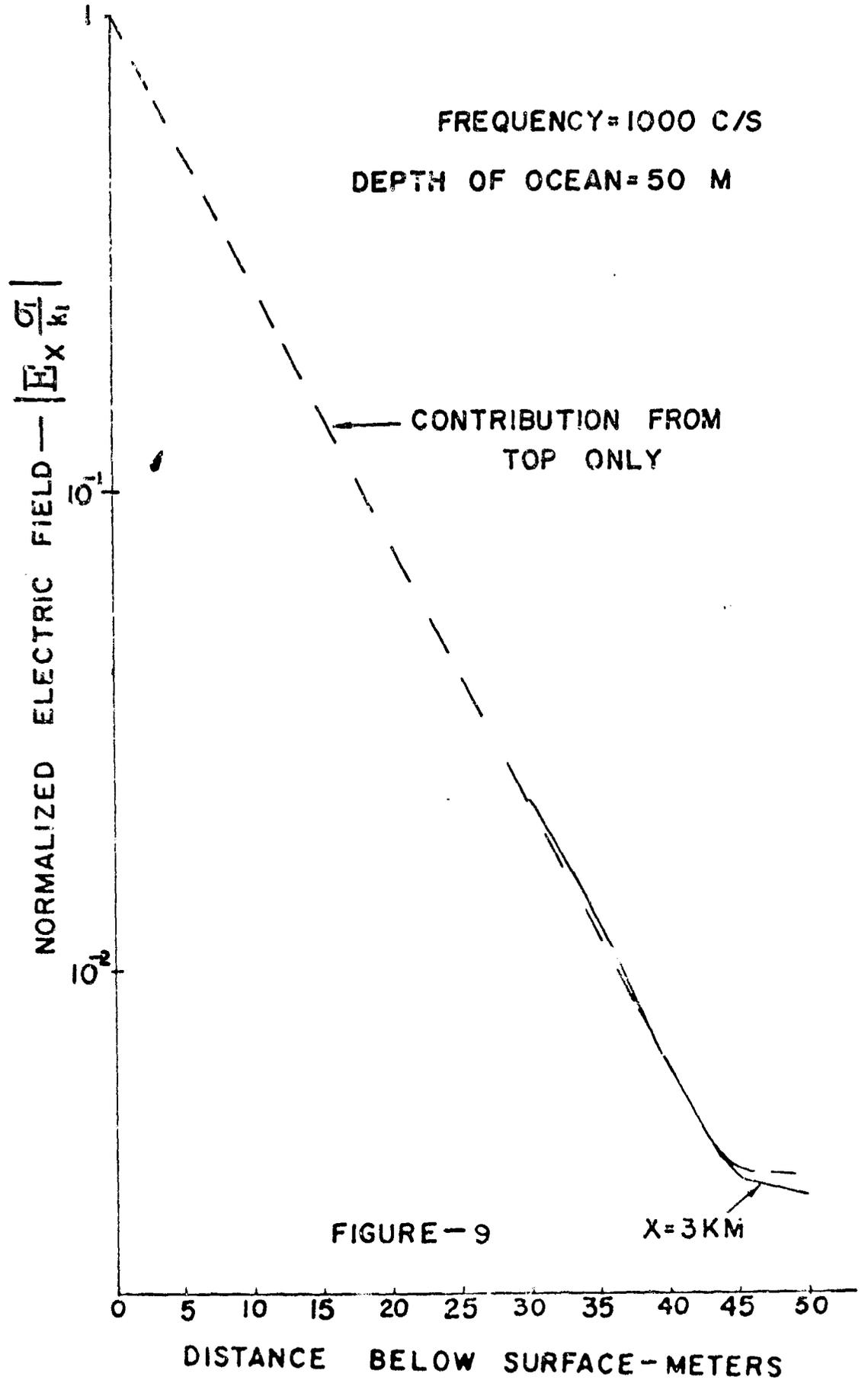


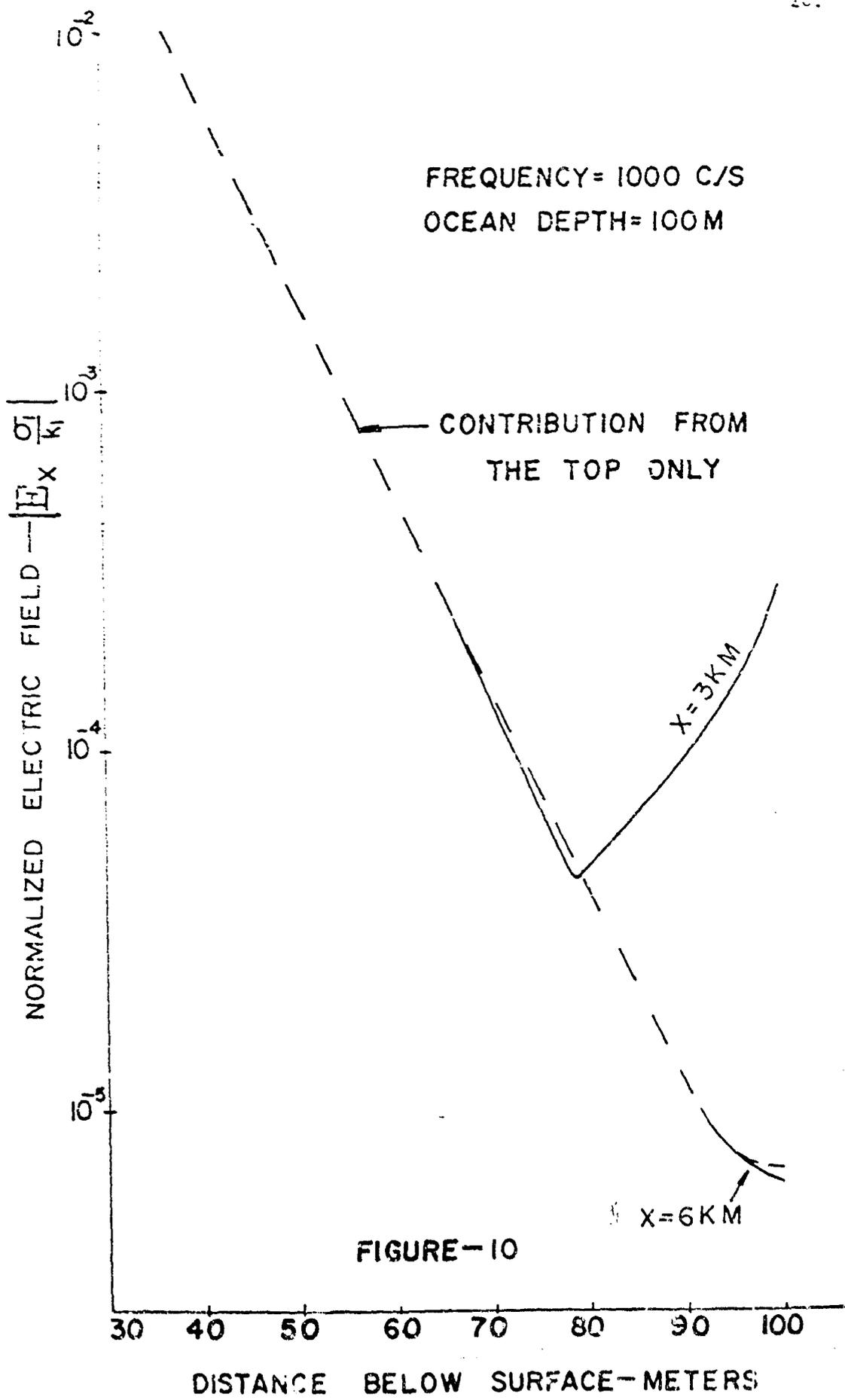












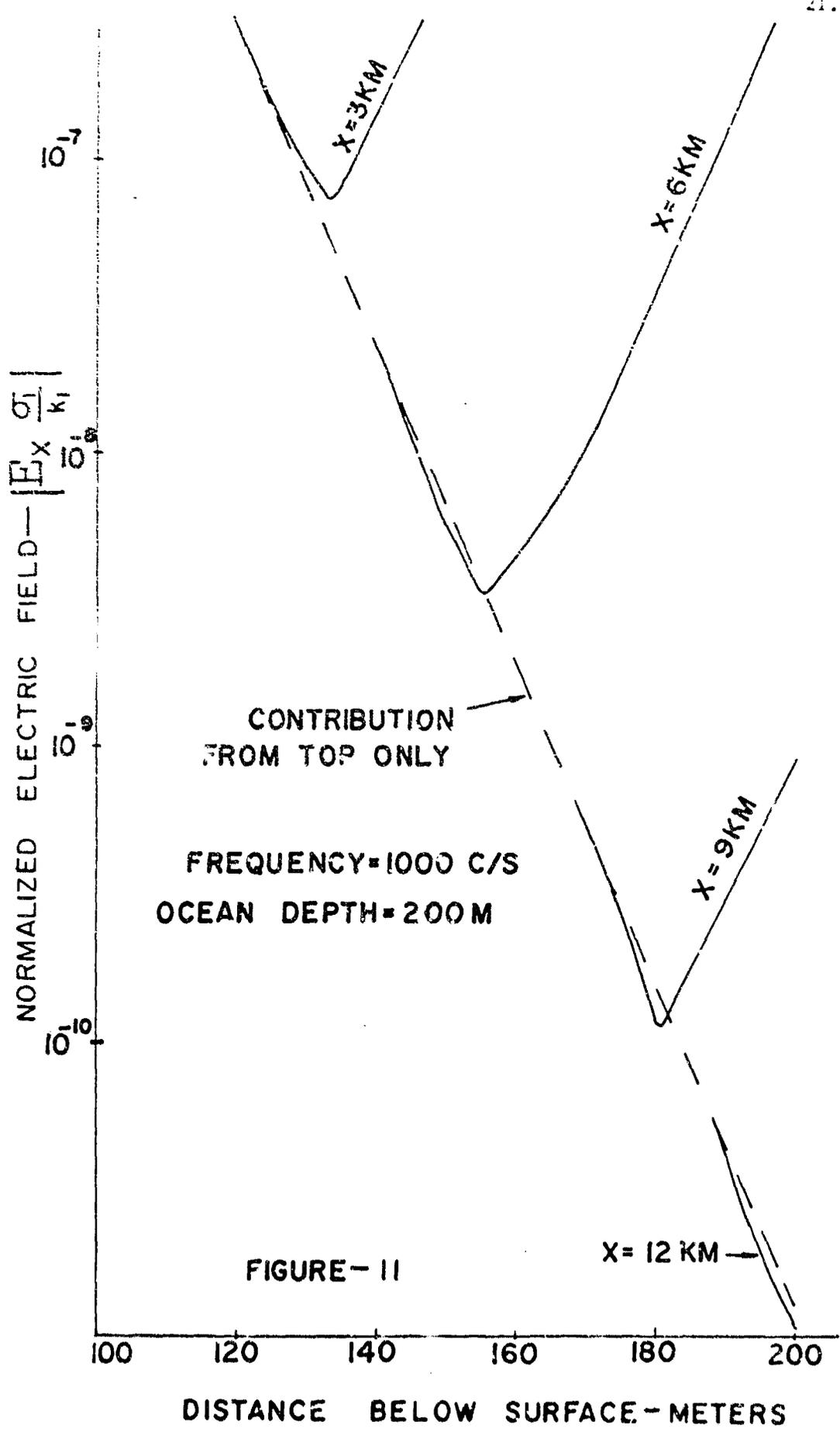


FIGURE - 11

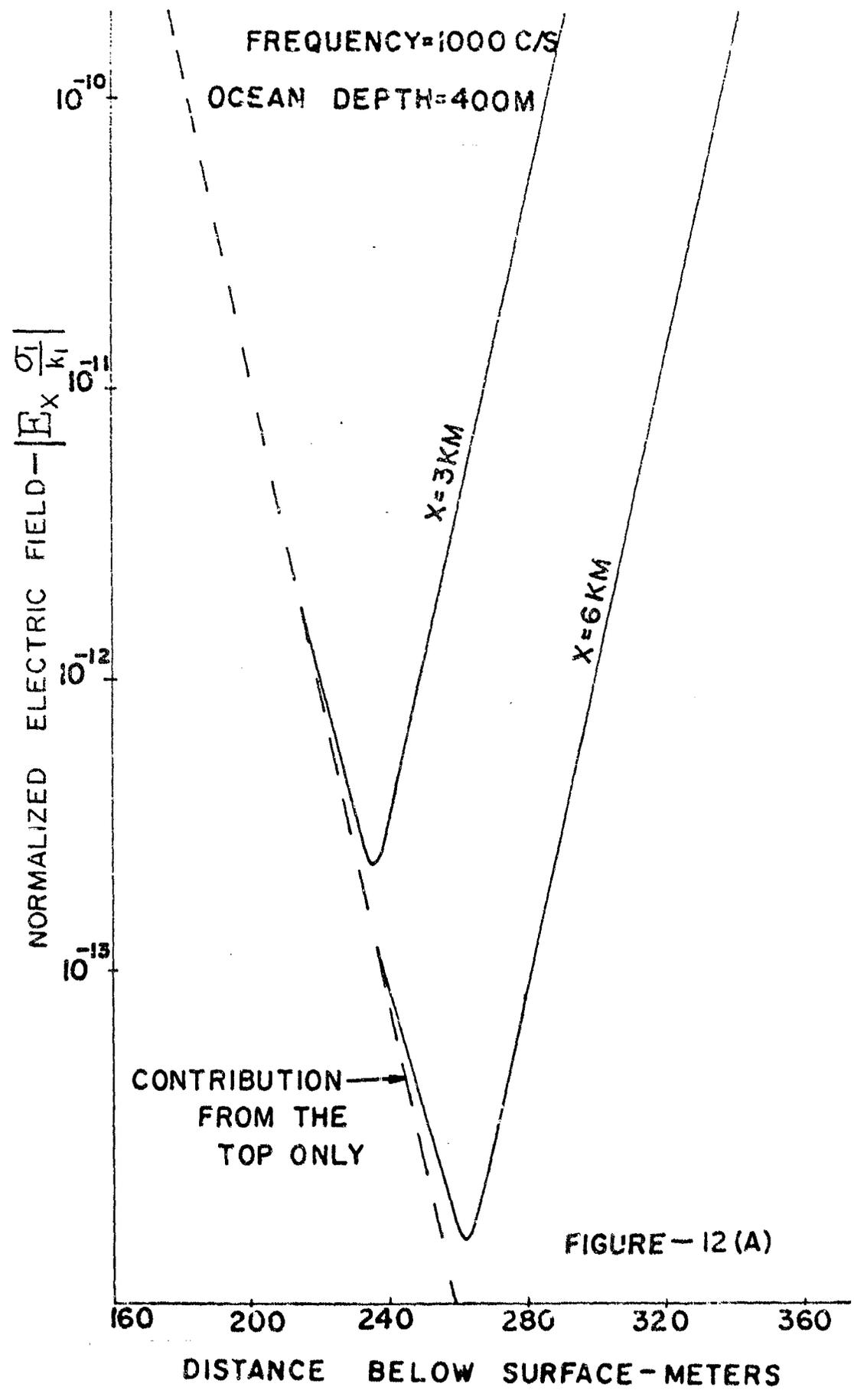
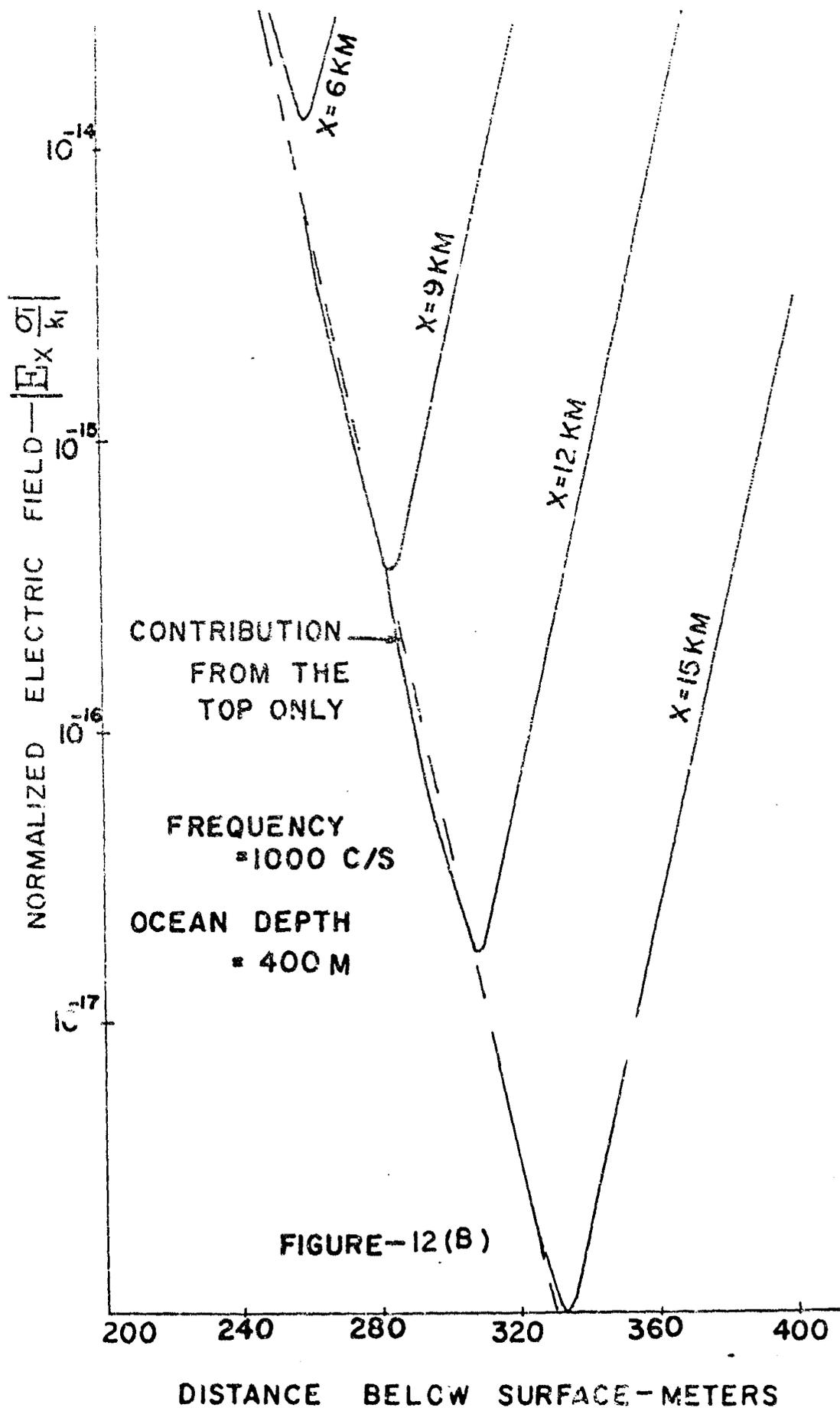
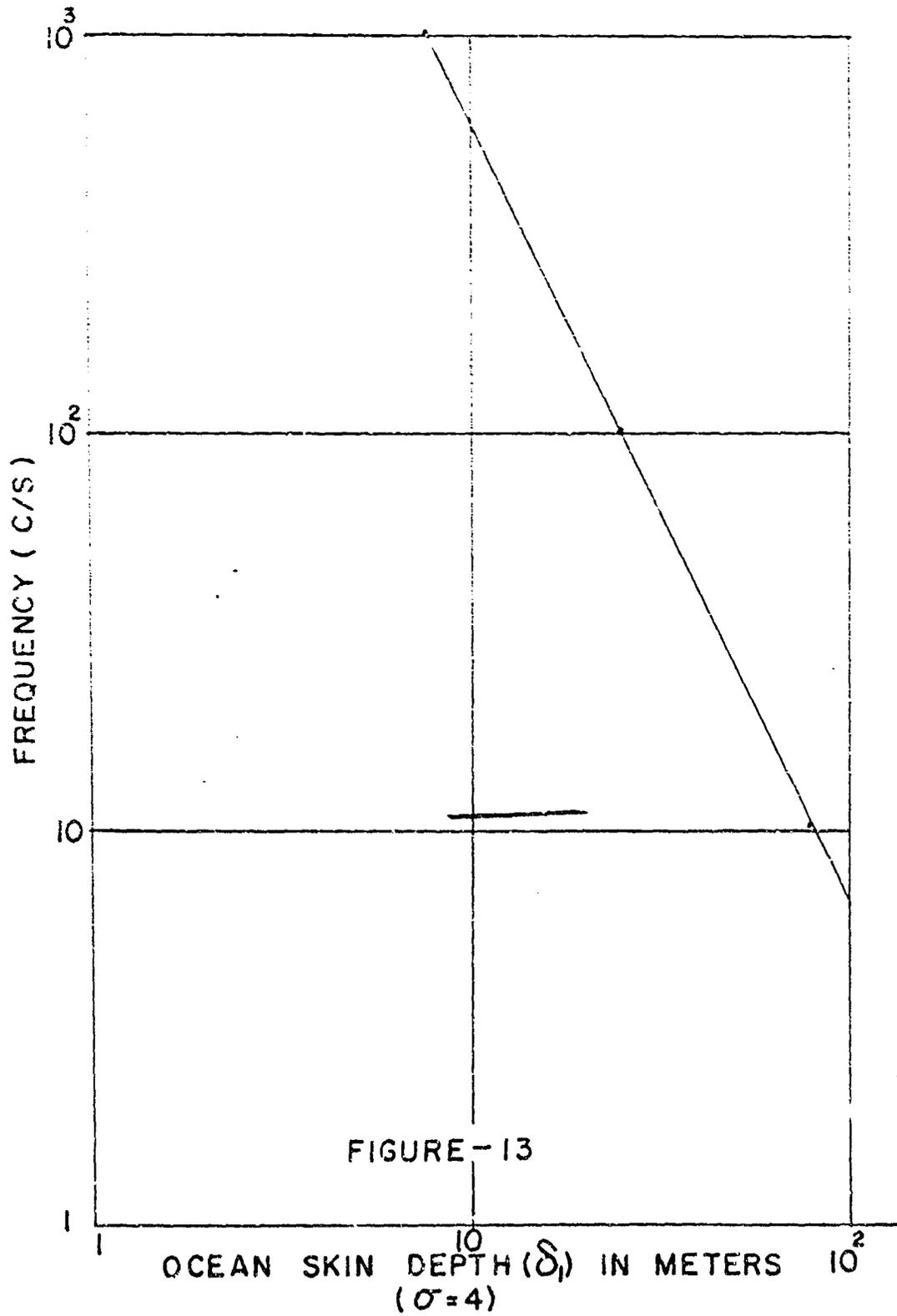
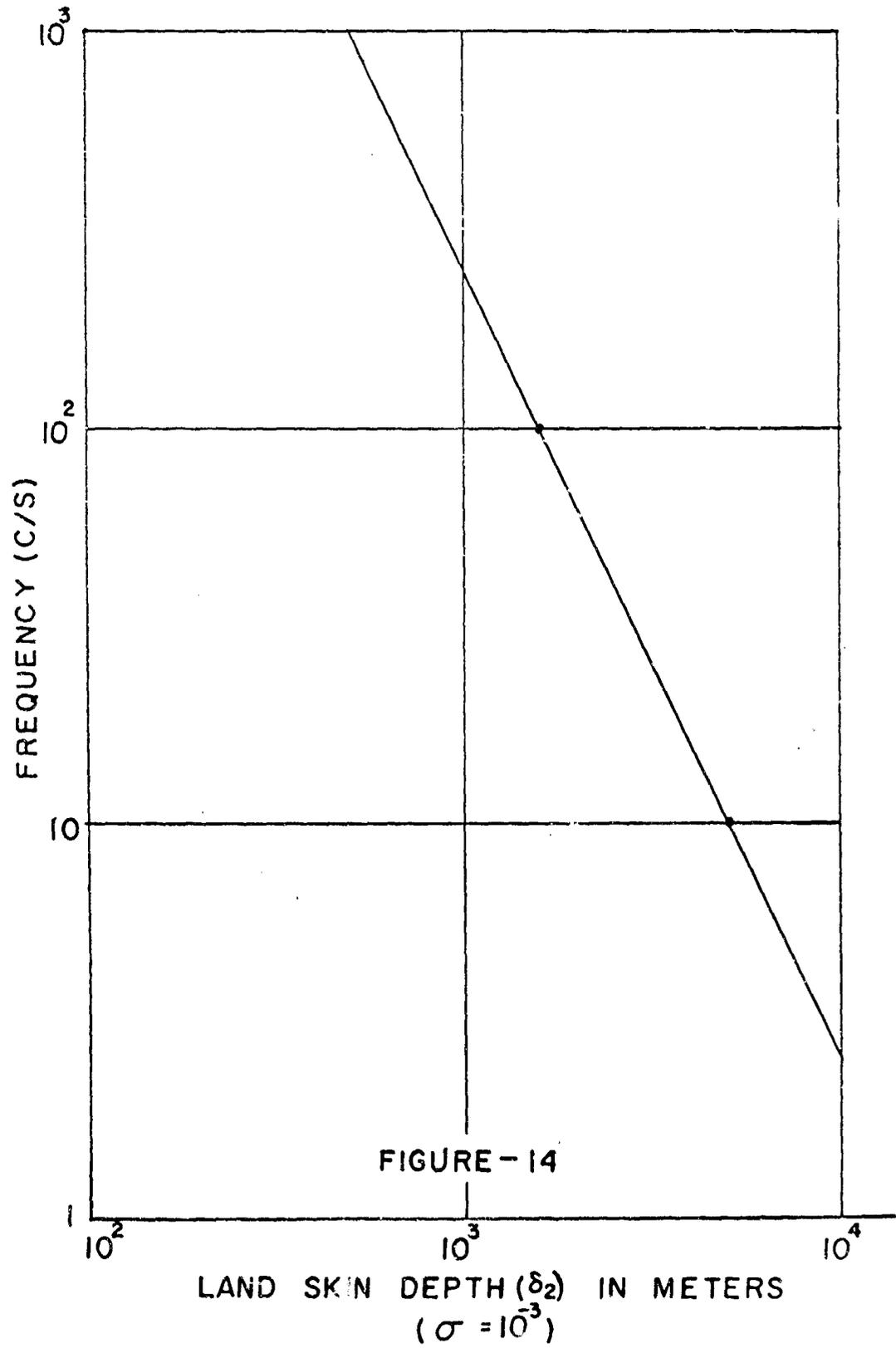


FIGURE - 12 (A)







Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Rhode Island Department of Electrical Engineering		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP	
3. REPORT TITLE ELECTROMAGNETIC FIELDS IN THE OCEAN NEAR A SHORELINE			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Report			
5. AUTHOR(S) (Last name, first name, initial) Spence, John E; Sullivan, Edmund J., and Beville, James J.			
6. REPORT DATE 15 November 1965		7a. TOTAL NO. OF PAGES 27	7b. NO. OF REFS 3
8a. CONTRACT OR GRANT NO. Nonr-396(10)		8a. ORIGINATOR'S REPORT NUMBER(S) 396 (10)/2	
b. PROJECT NO. NR 371-490		8b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
c.			
d.			
10. AVAILABILITY/LIMITATION NOTICES Qualified requesters may obtain copies of this report from DDC			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Office of Naval Research	
13. ABSTRACT The electromagnetic fields in the ocean resulting from ground wave excitation over land by natural electromagnetic noise such as Micropulsations or ELF Atmospherics are investigated analytically. A simplified two dimensional model is used. The purpose of the study is to determine the relative importance of electromagnetic energy propagation via air into the ocean versus propagation through the soil and ocean bottom into the sea. Vertical profiles of the horizontal electric field are plotted for several values of frequency, ocean depth, and the distance from the shoreline. The relative importance of the soil path is seen to be greater for shorter distances from the shore and for deeper oceans.			

Unclassified

Security Classification

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Electromagnetic fields in ocean; Electromagnetic wave propagation in ocean; Noise, electromagnetic in ocean; Electromagnetic field distribution with depth in ocean; Electromagnetic wave propagation through soil; Electromagnetic fields in ocean near a coastline.						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.
- 2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.
- 2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.
3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.
4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.
5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.
6. **REPORT DATE:** Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.
- 7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.
- 7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.
- 8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.
- 8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.
- 9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.
- 9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).
10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.