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COMPUTER EXPERIMENTS ON
ION-MOLECULE REACTIONS

by
Fred A. Wolf

October 6, 1965
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A computer program has been developed which follows Light's phase-space theory of three-body ion-molecule reactions. The program predicts cross sections for 10 possible reaction branches as functions of the relative kinetic energy of the colliding particles in the range from thermal energies up to 20 eV. It also yields partial cross sections for the populations of the vibrational levels of the diatomic molecule in the product channel. Included in the results are reactions for the following ion-molecule pairs: \([\text{O}^+ + \text{N}_2], [\text{N}^+ + \text{O}_2], [\text{O} + \text{N}_2^+], [\text{He}^+ + \text{N}_2],\) and \([(\text{O}^{1b})^+ + \text{O}_2]\). The effects of varying the level of excitation, both vibrationally and electronically, of the reactants are presented. Increasing the excitation level always results in greater cross sections for reactions which are otherwise endothermic and slightly smaller cross sections for reactions which are otherwise exothermic.

All endothermic cross sections reach maximum values as functions of the barycentric kinetic energy of the reactants. All exothermic cross sections decrease with increasing barycentric kinetic energy at rates faster than predicted by the Gioumousis and Stevenson theory. This is due to the greater competition between the products for the available phase space which occurs at elevated energies. Good agreement is found between the theory and experiment in the high energy region above 5 eV and in some cases throughout the whole energy range. Disagreement is most pronounced for low energies. It is felt that this indicates the presence of a small activation energy or, in the case where the products differ from the reactants by a charge transfer \(\text{A}^+ + \text{BC} \rightarrow \text{A} + \text{BC}^+\), the neglect of including resonance forces in the calculation.
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1. INTRODUCTION

Experimental data giving cross sections as functions of the reactant ion energy, appearing in the literature on ion-molecule reactions, have up to now been compared only with the theory of Gioumousis and Stevenson (G and S). This theory is based on the Langevin model which yields orbits of a charged particle moving in the induced dipole potential which exists between the charged and neutral particles. When the impact parameter is less than a certain critical value, an inward spiraling orbit takes place. Hence if the impact parameter is less than critical, a reaction can be expected to occur, whereas impact parameters greater than critical cannot give rise to reaction. This reaction cross section is given by

\[ \sigma(E_{\text{tr}}) = \pi \left( 2e^2 \alpha / E_{\text{tr}} \right)^2, \]

where \( e \) is the charge of an electron, \( \alpha \) is the polarizability of the neutral particle, and \( E_{\text{tr}} \) is the usual kinetic energy of the reduced mass particle in the center of mass coordinate system.

While this theory has been successfully compared to many of the experimental data, it has not been able to predict such features as the appearances of maxima in the cross section versus energy curves or the rapid fall off of the reaction cross sections with higher energy, nor has it been able to deduce what fraction of the products are in excited vibrational and electronic states. In fact, there is no way that the G and S theory can explain which of the products are most likely to be found after reaction or the fact that in some cases no reaction occurs at all.

This is not surprising when one realizes that this theory gives only an upper bound to the reaction cross section. Hence one should not expect to find any "structure" in the cross section-energy curve predicted by it. Because calculations of cross sections for ion-molecule reactions are practically intractable quantum mechanically and because many experimental data indicate something other than that predicted by Gioumousis and Stevenson, we feel that the phase-space theory of Light deserves special attention. To study this theory we have investigated the following ion-molecule reactions:

\[ \text{O}^+ + \text{N}_2 \rightarrow \text{products}, \]
\[ \text{N}^+ + \text{O}_2 \rightarrow \text{products}, \]
$$\text{(O}^{18}\text{)}^+ + \text{O}_2 \rightarrow \text{products},$$

$$\text{O} + \text{N}_2^+ \rightarrow \text{products},$$

$$\text{He}^+ + \text{N}_2 \rightarrow \text{products}.$$  \hspace{1cm} (2)

These reactions are of some importance in the atmosphere. Before giving the results we shall first review the Light theory.
2. DESCRIPTION OF THE LIGHT THEORY

Light's phase-space theory is based upon the postulate that: "The probability of formation of any given product in a 'strong coupling' collision is proportional to the ratio of the phase space available to that product divided by the total phase space available with conservation of energy and total angular momentum." The strong coupling is necessary to ensure that the complex formed by the reactants "loses all memory" of its initial state, and consequently that its decomposition be governed by phase space available.

The classical three-particle phase-space element is the usual

\[ d\Gamma = \prod_{i=1}^{3} r_i^3 p_i^3, \]

where \( r_i \) and \( p_i \) are the position and momentum coordinates of particle \( i \).

By separating out the center of mass motion, writing \( J_T = J_{ORB} + J_{ROT} \), and taking \( J_T \) to lie along the \( z \)-axis, we find after a Jacobian transformation, the reduced phase-space element

\[ d\Gamma = (J_T^2 + J_{ROT}^2 - 2J_T J_{ZROT})^{-\frac{3}{2}} J_T^3 dJ_T dJ_{ZROT} d\alpha_{ORB} \]

\[ dE_{T} d\beta_{ROT} d\alpha_{ROT} d\beta_{ROT} d\alpha_{VIB} dE_{VIB} dt_{VIB}, \]

where the symbol \( J \) refers to angular momentum, \( E \) refers to energy, \( t \) to time, \( \alpha \) to the angle conjugate to \( J \), and \( \beta \) to the angle conjugate to the \( Z \) component of \( J \). The subscript \( T \) means total, \( ORB \) means orbital and refers to the orbital motion of the single particle about the molecule, \( ROT \) means rotational and refers to the rotation of the diatomic molecule about its mass center, and \( VIB \) means vibrational and refers to vibration of the diatomic molecule. By noticing that the integrals over \( \alpha_{ORB}, \beta_{ROT}, \beta_{T}, \) and \( t_{VIB} \) yield constants independent of channel and vibrational energy, we can write for the relevant phase space available with conservation of total energy \( E_T \), vibrational energy \( E_{VIB} \), and total angular momentum \( J_T \),

\[ \Gamma = \iint \left[ 1 + \left( \frac{J_{ROT}}{J_T} \right)^2 - 2J_{ZROT}/J_T \right]^{-\frac{3}{2}} dJ_{ROT} dJ_{ZROT}. \]
It should also be noted that the integral given by Eq. (9) of Reference 7 is incorrect, as suggested by Pechukas and Light in a later paper. The correct expression is given by our Eq. (5).

The limits of integration of Eq. (5) are determined by the following physical constraints:

a. Since energy must be conserved we have

\[ E_T = E_{tr}^o + E_v^o + E_{ROT}^o = E_{tr}^i + E_v^i + E_{ROT}^i - Q_{oi} \]  

where \( E_{tr} \) is the translational kinetic energy \( \mu v^2/2 \), \( \mu \) is the reduced mass of the diatomic molecule in channel \( k \), and \( E_{ROT}^k \) is its rotational energy. This is given by

\[ E_{ROT} = J_{ROT}^2/2I , \]  

where \( I \) is the moment of inertia of the diatomic molecule. The superscripts \( o \) and \( i \) refer to the input and \( i \)th output channel respectively, and \( Q_{oi} \) is the exothermicity for the reaction. Thus an endothermic channel will have a negative \( Q \). Because \( E_{tr} \) must be positive, Eq. (6) yields for each channel \( i \) an upper bound on the rotational angular momentum

\[ J_{ROT}^2/2I \leq E_T - E_v^i + Q_{oi} = \epsilon . \]  

b. Since the molecule in the output channel must be stable, we must also limit the molecule from dissociating rotationally. Therefore

\[ J_{ROT}^2/2I < D_v^i , \]  

where \( D_v^i \) is the dissociation energy of the diatomic molecule from the vibrational level \( v \).

c. The final requirement is that the products must separate. When the products are an ion and a neutral particle, the effective one-dimensional central-field potential is given by

\[ V(r) = -e^2 a/2r^4 + E_{bf}^2/r^2 \]  

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where \( \alpha \) is the polarizability of the neutral particle, \( E_f \) is the final translational kinetic energy, and \( b_f \) is the impact parameter for the products. This interaction potential can be expected to be physically valid only for long ranges. Clearly the short-range repulsion is not included; however, it would only be important for large values of the mass reduced orbital angular momentum, \( J_{ORB}^2/2\mu = E_f b_f^2 \). For low relative kinetic energies, the centrifugal barrier occurs in the region of the attractive induced dipole potential. For intermediate values of \( E_f \), quantum mechanical resonance forces must be taken into account.\(^1\)\(^2\) Present calculations are underway which include resonance potentials. The results will be reported soon for several pertinent ion-molecule reactions.

The charge-quadrupole interaction which leads to a potential varying as \( r^{-3} \) can be neglected on the basis that it is usually much smaller than the induced dipole-charge interaction. This has been investigated by Arthurs and Dalgarno.\(^1\)\(^3\) Applying their calculation to \( \text{H}_2 + \text{H}_2^+ \) leads to the cross section

\[
\sigma_{AD} = 1.08 \left( E_{\text{tr}} \right)^{-2/3} \left( \hat{\lambda}^2 \right), \tag{11}
\]

where \( E_{\text{tr}} \) is the relative kinetic energy of the reactants in \( \text{hp} \).

The G and S cross section for this reaction is

\[
\sigma_{GS} = 15.08 \left( E_{\text{tr}} \right)^{-\frac{5}{6}} \left( \hat{\lambda}^2 \right); \tag{12}
\]

hence, it is clear that the charge-quadrupole interaction is only important at extremely low energies, well below room temperature.

The restriction that the products must separate can be written very simply as

\[
E_f \geq V(r^*) , \tag{13}
\]

where

\[
r^* = \left( \frac{e^2 \alpha/E_f b_f^2}{\mu} \right)^{\frac{1}{\hat{\lambda}^2}}, \tag{14}
\]
and

\[ V(r^*) = \left( \frac{E_1 b_1}{2e^2 \alpha} \right)^2 \]  

are the position and value of the potential, respectively, for the centrifugal barrier. This can be converted into a condition which limits the range of values taken by \( \text{J}^\text{ZROT} \).  

Using conditions a and c, we find for the total phase space available to products \( i \) in vibrational state \( v \), the result

\[ \Gamma(J_1, J_2, J_T; \epsilon) = I_1 + I_2 S(-J_1) + I_3 \text{sgn}(J_T - J_2) \]

\[ + I_4 S(J_1) \text{sgn}(J_T - J_1) + I_5 S(J_2 - J_T) S(J_T - J_1), \]  

(16)

where in the above

\[ S(A - B) = \begin{cases} 1, & A > B \\ 0, & A < B \end{cases}, \quad \text{sgn}(A - B) = \begin{cases} +1, & A > B \\ -1, & A < B \end{cases}, \]

(17)

and

\[ I_1 = (8\mu^2 e^2 \alpha)^{1/4} \int_{|J_1|}^{J_2} \left( \epsilon - J_{\text{ROT}}^2 / 2l \right)^{1/4} dJ_{\text{ROT}}, \]

\[ I_2 = J_T |J_1| + \frac{1}{2} J_1^2, \]

\[ I_3 = -J_T J_2 + \frac{1}{2} J_2^2, \]

\[ I_4 = J_T |J_1| - \frac{1}{2} J_1^2, \]

\[ I_5 = -J_T^2. \]

(18)
The values \( J_1 \) and \( J_2 \) correspond to the roots \( X_1 \) and \( X_2 \) given in Reference 7. Briefly, they are bounds on the values that \( J_{\text{ROT}} \) can assume when the inequality \( E_f \geq V(x^*) \) is applied. They are given by

\[
J_i = (2l_i)^{1/2} X_i, \quad i = 1, 2,
\]

and always satisfy the inequality \( J_2 \geq |J_1| \). The total phase space available to all products is given by summing \( \Gamma_i \) over both vibrational and product (electronic) states.

\[
\Gamma(E_f, J_T) = \sum_i \sum_v \Gamma_i(J_i, J_2^*, J_{\text{T}}, \epsilon).
\]

By not restricting \( \Gamma \) through use of condition b, we are including the phase space available to rotational dissociation.

Since the product channel contains a molecule, the product phase space available must be restricted by condition b. Writing \( J_3 = \langle 2lD_v \rangle \) for brevity, we find for the product phase space

\[
\Gamma_i(J_1, J_2, J_T, \epsilon) = \Gamma_i(J_1, J_2, J_T, \epsilon) S(J_3 - J_2)
\]

\[
+ \Gamma_i(J_1, J_3, J_T, \epsilon) S(J_2 - J_3) S(J_3 - |J_1|)
\]

\[
+ [(2J_T J_3 - J_T^2) S(J_3 - J_T) + J_3^2 S(J_T - J_3)]
\]

\[
\times S(|J_1| - J_3)
\]

\[
= \Gamma_i(J_1, J_2, J_T, \epsilon) S(J_3 - J_2)
\]

\[
+ \Gamma_i(J_1, J_3, J_T, \epsilon) S(J_2 - J_3) S(J_3 - |J_1|)
\]

\[
+ [\langle 2lD_v \rangle J_3 J_T - J_3^2] S(J_3 - J_T) + J_3^2 S(J_T - J_3)
\]

\[
\times S(|J_1| - J_3)
\]

\[
\Gamma_i(J_1, J_2, J_T, \epsilon) = \Gamma_i(J_1, J_2, J_T, \epsilon) S(J_3 - J_2)
\]

\[
+ \Gamma_i(J_1, J_3, J_T, \epsilon) S(J_2 - J_3) S(J_3 - |J_1|)
\]

\[
+ [\langle 2lD_v \rangle J_3 J_T - J_3^2] S(J_3 - J_T) + J_3^2 S(J_T - J_3)
\]

\[
\times S(|J_1| - J_3)
\].

The probability for the production of molecule \( i \) in the \( v \)th vibrational state is then given by

\[
P(E_f, J_{\text{T}}, E_v^i) = \Gamma_i(J_1, J_2, J_T, \epsilon) S(J_3 - J_2)
\]

\[
+ \Gamma_i(J_1, J_3, J_T, \epsilon) S(J_2 - J_3) S(J_3 - |J_1|)
\]

\[
+ [\langle 2lD_v \rangle J_3 J_T - J_3^2] S(J_3 - J_T) + J_3^2 S(J_T - J_3)
\]

\[
\times S(|J_1| - J_3)
\].

The cross section for this production is then

\[
\sigma(E_{\text{tr}}, i, v) = 2 \pi \int_{0}^{b_{\text{max}}} P(E_f, J_{\text{T}}, E_v^i) bdb
\].
where \( b \) is the impact parameter in the reactant channel and

\[
\beta_{\text{max}} = \left( \frac{2 e^2 \alpha}{E_{\text{tr}}^0} \right)^{1/4}.
\]

By using Eq. (23), we are assuming that the initial energy and angular momentum of rotation of the reactant molecule is negligible in comparison with the total energy and total angular momentum respectively. This allows us to take \( J_T^2 = 2 \mu E_{\text{tr}}^0 b^2 \). We point out that if \( P = 1 \), the cross section reduces to that predicted by the G and S theory; therefore, the phase-space theory enables us to predict what fraction of the total reaction cross section results in the production of a particular product channel \( i \) and vibrational state \( v \).

The cross section for dissociation is found by subtracting the sum of the formation cross sections from the maximum, or

\[
\sigma_{\text{Diss}} = \pi b_{\text{max}}^2 - \sum_{i,v} \sigma(E_{\text{tr}}^0, i, v). \tag{24}
\]

Our calculations have been carried out on a fast computer, using an expanded version of Light's original program to include 10 output channels. This includes a sub-program which yields the roots \( X_1 \) and \( X_2 \) of Eq. (19) in Reference 7 and calculates the integral \( I_1 \) by a simple numerical quadrature.

Our calculations make use of many data already available in the literature on atomic and molecular constants. Molecular potential energy curves, interatomic separations, dissociation energies, and vibrational energy levels, when given, were taken from the work of Gilmore. Atomic polarizabilities were taken from the work of Parkinson, and molecular polarizabilities were obtained from the experimental results of Rothe and Bernstein. In all cases, vibrational levels were obtained using the formula

\[
E_v^k = (v + 1/2) \hbar \nu_k - (v + 1/2)^2 \left( \frac{\hbar \nu_k}{2} \right)^2 / 4 D_e^k, \tag{25}
\]

where \( D_e^k \) is the dissociation energy from the bottom of the potential well of the product molecule \( k \) and fitting the frequencies \( \nu_k \) to the potential energy curves given by Gilmore.
3. DISCUSSION OF THE RESULTS

The results of the calculations are shown in Figures 1 through 19. We have plotted cross sections in units of $10^{-16}$ cm$^2 = 1 \text{ a.u.}$ versus the incoming relative kinetic energy of the reactants, $\mu v^2/2$, in eV units. For most of the energy range shown, a simple conversion from barycentric to laboratory ion energies is found by writing

$$E_{\text{lab}} = \frac{m}{\mu} E,$$

where $m$ is the incoming ion's mass and $\mu$ is the reduced mass of the reactants. For example, in Figure 1 the incoming reduced mass is $\mu = 16.28/44$ amu, while the ion's mass is 16. Hence $E_{\text{lab}} = 1.57 E$.

The straight negative sloped line labeled G and S in all of the figures represents the Gioumousis and Stevenson cross section for each set of reactants. Thus it appears as shown when plotted on the log-log graph. Furthermore, this cross section is the maximum obtainable when the induced dipole potential is used. By summing over all production cross sections, one obtains the G and S result.

Each curve, unless otherwise indicated, represents a composite in that it is a sum over all possible vibrational levels of the diatomic molecule in the output channel depicted. Thus Curve A1 in Figure 2 represents the total cross section for NO$^+$($X^1\Sigma^+$) production. Figure 5 shows the vibrational distribution of the product A1.

Curves A2 and A3 of Figure 2 represent, respectively, the production of the excited neutral states N($^2D$) and N($^3P$) together with NO$^+$($X^1\Sigma^+$). Since these reactions are endothermic, their cross sections are zero at low energies. Curve A of Figure 1 is the sum of A1, A2, and A3, and thus represents the total N + NO$^+$ production.

Curve B of Figure 1 represents the cross section for production of the same products as the reactants. For low energies, this cross section is non-reactive in that the products are also in the same vibrational state as the reactants. For higher energies, however, higher vibrational states will appear. This effect can be seen in Figure 5, which shows the vibrational distribution of the cross section for the production of NO$^+$($X^1\Sigma^+$ $v=0, 1, \ldots, 11$). In all the figures, at higher energies all vibrational levels are excited and the cross sections are seen to approach each other.
asymptotically. Low-energy cross sections for particular vibrational levels only appear for those vibrational levels which are exothermic.

Comparing Figures 7 and 10, we see that for low energies, agreement is best for the reaction

\[ \text{N}^+ (^3\text{P}) + \text{O}_2 (X^3\Sigma^{-}_g, v=0) \rightarrow \text{O} (^3\text{P}) + \text{NO}^+ (X^1\Sigma^+) \],

while for energies above 5 eV, the experimental results agree with the total \( \text{NO}^+ \) production (the sum of Curves A1 through A6, represented as A on Fig. 7). We point out that the above reaction is more than 6-eV exothermic, and hence one may not expect to find any important effects, due to activation energies. The other reactions, A2 through A6 in Figure 10, are not as exothermic (A5 and A6 are endothermic); therefore, if activation is present, these reactions may not be observed at low energies. This could explain the good agreement of experimental results with A1 at low energies. At higher energies, the experimental results best fit the total \( \text{NO}^+ \) production shown as A in Figure 7. This agreement is reasonable in light of our previous discussion of phase-space competition at higher energies.

Discrepancies loom large, as a usual rule, whenever a resonant charge-transfer process is observed to take place; i.e., our results yield cross sections for rearrangement resulting in charge transfer which is usually smaller than observed experimentally. By taking into account resonance forces, we hope to be able to predict cross sections for low-energy charge transfer which give better agreement with experiment.\textsuperscript{19} This will appear in a forthcoming paper. In the event that the reactant channel "sees" a resonance force, we could expect to find total cross sections which would exceed the G and S maximum.\textsuperscript{12} In the event of actual rearrangement, the results seem promising. Agreement is expected to be best when the output channels are highly exothermic, i.e., when the effects of any activation energy are expected to be small. Thus, see the results for the reaction \( \text{N}^+ + \text{O}_2 \rightarrow \text{O} + \text{NO}^+ \). Here (Fig. 7) good agreement is found between theory and experiment throughout the energy range shown.

The most significant feature of the theory is the fact that our results yield cross sections for exothermic reactions which fall off faster with increasing barycentric kinetic energy than that predicted by Gioumousis and Stevenson when the energy exceeds 5 eV or so. This is in good agreement with experimental measurements\textsuperscript{2,3,20,21} (witness Figs. 1, 7, 10, and 16). Our results indicate that this is due to phase-space competition. As the incoming energy increases, more channels open for the reaction path to follow. Thus each channel shares in the output. Finally, when the incoming energy is high enough, the exothermicities become small portions
of the total available energy. Hence all products become, roughly, equally likely. The small value for each cross section is attributed to the fact that more phase space is available to dissociated products at high energies than to stable vibrating molecules, since statistically a continuous energy distribution wins out over a discrete one.

The disagreement at low energies is significant and indicates either that an activation energy and/or "selection rule" restriction is in operation. Bates and Lynn point out that even though a reaction may be favorable energetically, it may be ruled out if an electronic transition is required to "make it go". Thus they would predict low cross sections for asymmetric resonant systems. In the reaction \( O^+ + N_2 \rightarrow NO^+ + N \) (Fig. 1), this is, however, not the case. The dissociated products and reactants match, i.e., \( O^+ + N_2 \rightarrow O^+ + N + N \rightarrow NO^+ + N \), hence no electronic transition is required. Thus we are at a loss to explain the experimental data on this basis. Recently, Schmeltekopf and his associates at the National Bureau of Standards, Boulder, Colorado, observed an increase of about a factor of 20 in the reaction rate for \( O^+ + N_2 \rightarrow NO^+ + N \) when the \( N_2 \) was initially in a higher vibrational state. Our results are shown in Figures 4 and 6. More vibrational levels are excited in the \( NO^+ \) channel as a result of increasing the reactant vibrational energy. The measured results in this case do seem to agree with that predicted here, thus confirming the speculation that the theory best fits reactions which are strongly exothermic.

The results predicted by the reaction (Figs. 16-18)

\[
He^+ + N_2 \rightarrow He + (N_2^+)^* \rightarrow He + N + N^+
\]  

(27)

deserve special mention. Here there is enough ionization energy available so that the dissociative reaction appears as an exothermic channel with the cross section increasing with decreasing energy. This is in contrast to any of the other cases shown, where dissociation, being an endothermic channel, does not take place until the translational kinetic energy exceeds the energy defect. Our results for dissociative charge transfer in this case fall much below the experimental results because the process is one in which resonance forces are probably quite large. Thus the accidental resonance reaction

\[
He^+ (^2S) + N_2 (X^1Σ_g^+ v=0) \rightarrow He (^1S) + N_2^+ (C^2Σ_u^+ v=4) + O \text{ eV} 
\]

(28)

could predominate, as may be deduced from the theory of Rapp and Francis. The phase-space theory as presented will always predict cross sections smaller than the G and S theory, and such a reaction as
(28) constitutes statistically a small part of the total Gioumousis and Stevenson cross section. It is, however, of interest to see the vibrational distribution of the output cross section for this reaction, and this is shown in Figure 18. By including resonance forces, we might expect the cross section for production of the v=4 level of \( \text{N}_2^+ \text{C}_2^E \text{E}^+ \) to be much larger than shown here. The potential energy curve crossing of the \( \frac{4\pi}{3} \) state with the \( \text{C}_2^2 \text{C}_u^+ \) state would then lead to dissociation of the \( \text{N}_2^+ \) molecule.

We have also calculated cross sections for the reactions \( \text{N}_2^+ + \text{O} \) and \( (\text{O}^{18})^+ + \text{O}_2 \). The former are shown in Figures 13 through 15 and the latter in Figure 19. The latter reaction is of interest for future experimentation, since it predicts a rather simple isotope effect. If we consider reactions of the type \( A^+ + B_2 \), we have as possible products

(a) \( A + B_2^+ \)
(b) \( AB + E^+ \) (2 ways to form this)
(c) \( AB^+ + B \) (2 ways to form this)
(d) \( A^+ + B_2 \)

Consider products (a) and (b). If A is an isotope of B, the reaction for (b) will occur statistically twice as many times as that for (a). Hence rearrangement charge transfer (b) for the reaction \( (\text{O}^{18})^+ + \text{O}_2 \rightarrow \text{O}^+ + (\text{O}^{16}\text{O}^{18})^+ \) will occupy twice the phase space available to simple charge transfer (a). Thus this reaction should prove a good test for the phase-space theory.

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Fig. 1--Production of NO$^+$ from the reaction O$^+$ + N$_2$. The experimental NO$^+$ production results of Giese$^{21}$ and of Stebbings et al.$^2$ are shown by curves XX and YY respectively.
Fig. 2--Electronic distribution of the products born of the reaction $O^+ + N_2$. The products involved in dissociation (D) are probably $2N + O^+$. The theory does not always predict which particle is ionized.
Fig. 3--Electronic distribution of the products from the reaction $\left( O^+ \right)^* (2D) + NO^+$. The increase in the number of products follows from the greater energy available when one of the reactants is in an excited state. More channels are exothermic and therefore competitive for the available phase space.
Fig. 4--Electronic distribution of the products from the reaction of \( \text{O}^+ + \text{N}_2 (v=4) \). The increased vibrational excitation of the reactant molecule produces a larger cross section (de-excitation) for formation of \( \text{O}^+ + \text{N}_2 \) (same products as reactants) because the de-excitation is exothermic.
Fig. 5--Vibrational distribution of the products \( \text{NO}^+ (v=0, 1, \ldots, 11) \) + \( N \) from channels \( v=0, 1, 2, 3 \) are exothermic. Channels \( v \geq 4 \) are endothermic.
Fig. 6--Vibrational distribution of the products $\text{NO}^+ (v=0, 1, \ldots, 11)$ + N from the reaction $\text{O}^+ + N_2 (v=4) \rightarrow$ 
$\text{N} (^4S)+\text{NO}^+ (X^1\Sigma^+ v=0,1,\ldots,11)$ 
Channels $v \leq 7$ are exothermic, $v > 7$ are endothermic.
Fig. 7--Products NO$^+$, O$^+$, O$_2^+$ from the reaction N$^+$ + O$_2$. The experimental results shown are those of Stebbings et al. Extrapolation is made to the low-energy experimental data of Fehsenfeld et al.
Fig. 8--Product distribution NO⁺, N⁺, O₂⁺ and O⁺ from the reaction (N⁺)⁺ + O₂. The de-excitation cross section is given by B.
Fig. 9--Product distribution NO⁺, N⁺, O₂⁺, and O⁺ from the reaction N⁺ + O₂ (v=4)
Fig. 10--Electronic distribution of the products O + NO$^+$ from the reaction N$^+$ + O$_2$. The experimental results are those of Stebbings et al.,$^2$ at higher energies. These have been extrapolated to the room temperature data of Fehsenfeld et al.$^{20}$
Fig. 11--Electronic distribution of the products O + NO\textsuperscript{+} from the reaction (N\textsuperscript{+})\textsuperscript{*} + O\textsubscript{2}
Fig. 12--Electronic distribution of the products O + NO⁺ from the reaction N⁺ + O₂ (v=4)
Fig. 13 -- Product distribution $\text{NO}^+$, $\text{O}^+$, $\text{N}_2^+$, and $\text{N}^+$ from the reaction $\text{O} + \text{N}_2^+$
Fig. 14--Product distribution NO⁺, O⁺, N₂⁺, and N⁺ from the reaction O⁺ + N₂⁺
Fig. 15--Product distribution NO\(^{+}\), O\(^{+}\), N\(_{2}\)^{+}, and N\(^{+}\) from the reaction O + N\(_{2}\)^{+} (v=4)
Fig. 16--Product distribution $N_2^+$, $N^+$, and $He^+$ from the reaction $He^+ + N_2$. The experimental data for $N_2^+$ production are taken from Stebbings et al. 3
Fig. 17 -- Electronic distribution of the products $\text{He} + \text{N}_2^+$ from the reaction $\text{He}^+ + \text{N}_2$
Fig. 18—Vibrational distribution of the products $N_2^+(v=0, 1, \ldots, 7)$ + He from the reaction $He^+(2S) + N_2(X^2\Sigma_g^+ v=0) \rightarrow He^+(1S) + N_2^+(C^2\Sigma_u^+ v=0, 1, \ldots, 7)$.
Fig. 19—Distribution of the products $O_2^+$, $(O^{16}O^{18})^+$, $(O^{16})^+$, and $(O^{18})^+$ from the reaction $(O^{18})^+ + O_2$. The ratios of A1 to A2, A3 to A4, B1 to B2, and B3 to B4 are roughly 2 to 1. This follows from the phase-space theory that $O^{16}O^{18}$ occupies twice the phase space of $O^{16}O^{18}$. That is, reactions of the type $A + B_2 \rightarrow AB + B$ have cross sections $2\sigma$. 

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REFERENCES AND FOOTNOTES


6. Assuming that the induced dipole force is the only long range force.


8. This theory not only predicts which products are present after reaction, but also the vibrational and electronic states in which they are to be found. The effects of changing the vibrational or electronic state of the reactants can also be determined. This is desirable from the experimental viewpoint since techniques are now in use which enable the experimenter to distinguish among the possible excitation states of the reactants (see Ref. 9).


10. Integrating over these 12 remaining coordinates would yield the total phase available for a particular three-body combination consisting of a single particle and a diatomic molecule.

REFERENCES AND FOOTNOTES (Cont'd.)


14. See p. 3224 of Ref. 7 for a complete description of this.


21. Giese, C. F., private communication. These results are preliminary; final results will be published soon.


24. Dr. L. Friedman of the Brookhaven Laboratory, Long Island, New York, has recently pointed out the possibility that the reaction \( \text{He}^+ + \text{N}_2 \to \text{He} + \text{N}^+ + \text{N} \) could proceed by the intermediary step \( (\text{HeN}^+) \) + N. This is under investigation and the results will be published soon.
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