NEW LIMITATION CHANGE

TO
Approved for public release, distribution unlimited

FROM
Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; May 1965. Other requests shall be referred to the Director, Air Force Aero Propulsion Laboratory, Attn: RTD, Wright-Patterson AFB, OH 45433.

AUTHORITY

AFAPL ltr, 21 Jul 1971
AFAPL-TR-65-45
Part IV

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY
Part IV: Ball Bearing Design Data

P. Lewis
S. B. Malanoski
Mechanical Technology Incorporated

TECHNICAL REPORT AFAPL-TR-65-45, PART IV

May 1965

Air Force Aero Propulsion Laboratory
Research and Technology Division
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio

Best Available Copy
NOTICES

When U.S. Government drawings, specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, or in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patent invention that may in any way be related thereto.

Qualified users may obtain copies of this report from the Defense Documentation Center.

Defense Documentation Center release to the Clearinghouse for Federal Scientific and Technical Information (formerly OTS) is not authorized. Foreign announcement and dissemination by the Defense Documentation Center is not authorized. Release to foreign nationals is not authorized.

DDC release to OTS is not authorized in order to prevent foreign announcement and distribution of this report. The distribution of this report is limited because it contains technology identifiable with items on the strategic embargo lists excluded from export or re-exports under U. S. Export Control Act of 1949 (63 Stat. 7), as amended (50 U.S.C. App. 2020, 2031), as implemented by AFR 400-10.

Copies of this report should not be returned to the Research and Technology Division unless return is required by security considerations, contractual obligations or notice on a specific document.

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY.
Part IV: Ball Bearing Design Data.

by P. Lewis and S.B. Malanoski

Mechanical Technology, Incorporated, Latham, N.Y.

Technical Report, MTI-65-TR-26

Contract AF-19(600)-1272,
F4162-64-C-0002
Task 814511

Air Force Aero Propulsion Laboratory
Research and Technology Division
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio
FOREWORD

This report was prepared by Mechanical Technology Incorporated, 968 Albany-Shaker Road, Latham, New York 12110, under USAF Contract No. AF 33(615)-1895. The contract was initiated under Project No. 3145, "Dynamic Energy Conversion Technology", Task No. 314511, "Nuclear Mechanical Power Units". The work was administered under the direction of the Air Force Aero Propulsion Laboratory, Research and Technology Division, with Mr. John L. Norris (APFL) acting as project engineer.

This report covers work conducted from 1 April 1964 to 1 April 1965.

This report was submitted by the authors for review on 18 March 1965. It is Part IV of final documentation issued in multiple parts. This report also is identified by the contractor's designation MTI-65-TR-35.

This technical report has been reviewed and is approved.

ARTHUR V. CHURCHILL
ARThUR V. CHURCHILL, Chief
Fuels and Lubricants Branch
Technical Support Division
Air Force Aero Propulsion Laboratory
This Part IV of the Final Report presents design data for the stiffness characteristics of ball bearings for use in analyzing the dynamical performance of a rotor. The dynamic characteristics of fluid film bearings are given in Part III which also gives the methods for performing the analysis of the rotor-bearing system.

Design data are presented for the extra-light and light group of deep-grooved and angular contact bearings undergoing either a pure radial load, pure axial load, or combined radial load with axial preload. The data are given in graphical form and cover both radial stiffness and load-carrying capacity. A nominal damping value for ball bearings, obtained from experimentation, is suggested.

Some of the general guide rules for the selection of ball bearings are given. These are concerned with fatigue life, limiting speeds, design, and lubrication. Safe load levels are indicated.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ILLUSTRATIONS</td>
<td>v</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. DESIGN REQUIREMENTS</td>
<td></td>
</tr>
<tr>
<td>Types of Bearings</td>
<td>2</td>
</tr>
<tr>
<td>Load Level</td>
<td>2</td>
</tr>
<tr>
<td>Ball Bearing Damping</td>
<td>2</td>
</tr>
<tr>
<td>Race Curvatures</td>
<td>2</td>
</tr>
<tr>
<td>Ball Bearings - Life</td>
<td>4</td>
</tr>
<tr>
<td>Bearing Centrifugal Loading</td>
<td>5</td>
</tr>
<tr>
<td>Effects of Cyclic Loading on Bearing Life</td>
<td>6</td>
</tr>
<tr>
<td>Lubricant Life</td>
<td>12</td>
</tr>
<tr>
<td>III. DESIGN DATA</td>
<td></td>
</tr>
<tr>
<td>Description and Discussion of Charts</td>
<td>14</td>
</tr>
<tr>
<td>Table I Deep Groove Bearings</td>
<td>17</td>
</tr>
<tr>
<td>Table II Angular Contact Bearings</td>
<td>18</td>
</tr>
<tr>
<td>Table III Axial Loaded Deep-Grooved Bearings</td>
<td>19</td>
</tr>
<tr>
<td>Design Charts</td>
<td></td>
</tr>
<tr>
<td>Pure Radial Load</td>
<td>20</td>
</tr>
<tr>
<td>Pure Thrust Load</td>
<td>25</td>
</tr>
<tr>
<td>Radial Load with Axial Preload</td>
<td>34</td>
</tr>
<tr>
<td>IV. SAMPLE PROBLEMS TO ILLUSTRATE USE OF DESIGN CHARTS</td>
<td></td>
</tr>
<tr>
<td>1. Pure Radial Loaded Bearing</td>
<td>59</td>
</tr>
<tr>
<td>2. Unidirectional Thrust Loaded Bearing</td>
<td>59</td>
</tr>
<tr>
<td>3. Double-Acting Thrust Loaded Bearing</td>
<td>59</td>
</tr>
<tr>
<td>4. Radial Loaded, Axial Preloaded Angular Contact Bearing</td>
<td>60</td>
</tr>
<tr>
<td>V. REFERENCES</td>
<td>62</td>
</tr>
<tr>
<td>VI. APPENDIX</td>
<td></td>
</tr>
<tr>
<td>A. Analysis</td>
<td>63</td>
</tr>
<tr>
<td>B. Computer Program, PN0182</td>
<td>68</td>
</tr>
<tr>
<td>C. Nomenclature for Analysis</td>
<td>76</td>
</tr>
<tr>
<td>iv</td>
<td></td>
</tr>
<tr>
<td>FIGURE</td>
<td>DESCRIPTION</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>A</td>
<td>Ball Bearing Schematic</td>
</tr>
<tr>
<td>B</td>
<td>Equivalent Load Ratio as a Function of Cyclic Load Ratio</td>
</tr>
<tr>
<td>C</td>
<td>Bearing Life Ratio as a Function of Equivalent Load Ratio</td>
</tr>
<tr>
<td>D</td>
<td>Bearing Life versus Temperature</td>
</tr>
<tr>
<td>1</td>
<td>Radial Stiffness for Deep Groove Ball Bearing - Pure Radial Load</td>
</tr>
<tr>
<td>2</td>
<td>Radial Stiffness for Deep Groove Ball Bearings - Pure Radial Load</td>
</tr>
<tr>
<td>3</td>
<td>Radial Stiffness for Deep Groove Ball Bearings - Pure Radial Load</td>
</tr>
<tr>
<td>4</td>
<td>Radial Stiffness for Deep Groove Ball Bearings - Pure Radial Load</td>
</tr>
<tr>
<td>5</td>
<td>Axial Stiffness versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>6</td>
<td>Axial Deflection versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>7</td>
<td>Axial Stiffness versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>8</td>
<td>Axial Deflection versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>9</td>
<td>Axial Stiffness versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>10</td>
<td>Axial Deflection versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>11</td>
<td>Axial Stiffness versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>12</td>
<td>Axial Deflection versus Axial Load - No Radial Load</td>
</tr>
<tr>
<td>13</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light $\beta_o = 25^\circ$</td>
</tr>
<tr>
<td>14</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light $\beta_o = 25^\circ$</td>
</tr>
<tr>
<td>15</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light $\beta_o = 25^\circ$</td>
</tr>
<tr>
<td>16</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light $\beta_o = 25^\circ$</td>
</tr>
<tr>
<td>17</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate $\beta_o = 25^\circ$</td>
</tr>
<tr>
<td>18</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate $\beta_o = 25^\circ$</td>
</tr>
<tr>
<td>19</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate $\beta_o = 25^\circ$</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate (\beta_0 = 25^\circ)</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>20</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre-Load - Preferred Heavy (\beta_0 = 25^\circ)</td>
</tr>
<tr>
<td>21</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 25^\circ)</td>
</tr>
<tr>
<td>22</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 25^\circ)</td>
</tr>
<tr>
<td>23</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 25^\circ)</td>
</tr>
<tr>
<td>24</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 25^\circ)</td>
</tr>
<tr>
<td>25</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>26</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>27</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>28</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Selected Light (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>29</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>30</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>31</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>32</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Moderate (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>33</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>34</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>35</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 15^\circ)</td>
</tr>
<tr>
<td>36</td>
<td>Radial Stiffness for Angular Contact Bearing, Pre Load - Preferred Heavy (\beta_0 = 15^\circ)</td>
</tr>
</tbody>
</table>
I
INTRODUCTION

The purpose of this report is to present design data for typical deep-groove and angular contact bearings of the commonly used light and extra-light series. The data may be used for various rotor-ball bearing system designs, in conjunction with our (MTI) critical speed and unbalance response programs.

The information is presented in graphical form. It consists of load carrying capacity, radial and axial stiffness, and load levels.

A complete description is given for all the variables used. A section entitled "Design Requirements" is written to describe various parameters and to present design considerations, guidelines and limitations.

A number of examples on the application of the curves to specific cases are included.

The analyses used are written in the Appendix, along with a computer program listing of the calculational procedure.

If any particular case is not covered by the included curves, more data may be generated by computer program PWO-182, IBM 1620-60E.
II

DESIGN REQUIREMENTS

A ball bearing schematic is shown as Fig. A to illustrate standard nomenclature.

**Types of Bearings**

The single row, deep groove ball bearing will sustain radial loads and in addition a substantial thrust load in either direction. When using this type of bearing, careful alignment between the shaft and housing is essential.

The angular contact ball bearing is designed to support a thrust load in one direction or a thrust load (preload) combined with a radial load. These bearings can be mounted singly or, when the side surfaces are flush ground, in multiple, either face-to-face or back-to-back for all combinations of thrust and radial loading. The basic difference between the two is the larger clearance and greater shoulder height of the angular contact bearing. Generally, this will permit operation with higher thrust loads and at higher speeds than the deep groove bearing.

**Load Level**

The load levels shown (C/P = 5 and C/P = 10) correspond to normally encountered Hertz Stress levels of 230,000 and 186,000 psi respectively.

**Ball Bearing Damping**

The only available damping information was from non-rotating tests on a grease packed ball bearing (A-2) system. The measured value was in the order of 15-20 pounds sec/in. This should be used only as a "ballpark" since it should be much higher in the rotating condition, and for larger bearing sizes.

**Race Curvatures**

Since the question of stiffness and rotor dynamics will be a major factor at high speeds, some design guidelines for this aspect are in order. Normally, more open curvatures, one piece machined retainers, and generous internal clearances are preferred. The normal curvatures are 51.6 percent for inner race and 53 percent for outer. Open curvatures, for high speed range, between 54 percent and 57 percent for both inner and outer are used. The 57 percent curvature is widely used and is included in the design discussion.
Fig. A  Ball Bearing Schematic
Ball Bearings - Life

The selection of ball bearings for various applications consider such factors as load, speed, temperature, environment, design, and lubrication. However, the initial sizing and selection is usually based upon the fatigue rating of the bearing.

Based upon a statistical distribution proposed by Weibull and the analytical and experimental work of Lundberg and Palmgren the life of the bearing for a given probability of survival has been found to vary inversely as the cube of the applied radial load. For other than radial loading, an equivalent radial load is defined. A Specific Dynamic Capacity (C) is defined as that radial load which will result in a life of one million inner race revolutions with a 90 percent probability of survival. The AFEMA (Anti Friction Bearing Manufacturers Association) has standardized on the following formula for (C):

\[ C = f_c (i \cos \beta_o)^{0.7} Z^{-2/3} d^{1.8} \]  \hspace{1cm} (1)

where  
- \( i \) = the number of rows of balls in any one bearing  
- \( Z \) = the number of balls per row  
- \( \beta_o \) = the angle of contact  
- \( d \) = the ball diameter, inch  
- \( f_c \) = a factor depending on oscillation and material

For normal bearing proportions \( f_c \approx 1500 \). The life (90 percent probability of survival) at any other radial load or equivalent radial load (P) is related to the Specific Dynamic Capacity (C) as follows:

\[ L = (C/P)^2 \]  \hspace{1cm} (2)

It is normally assumed that speed affects life in a linear fashion; that is, life varies inversely with speed.* For a given operating speed of \( N \) rpm, the number of revolutions which correspond to \( H \) hours of life is

\[ L = 60 NH \]  \text{revolutions}

* Based upon experimental data, equation (3) is too conservative at high speeds.
and
\[ \left( \frac{C}{P} \right)^3 \times 10^6 = 60 \text{ NH} \]

\[ H = \frac{(C/F)^3 \times 10^6}{60N}, \text{ hour} \]  

Catalog ratings are generated in this fashion, usually for some given number of hours of life with a 90 percent probability of survival. Five hundred hours is a common catalog rating. Note that a rating at 33-1/3 rpm for 500 hours is the Specific Dynamic Capacity.

Since this is available in the catalogs of most of the bearing companies, no further explanation of this aspect is included in this report.

**Bearing Centrifugal Loading**

In some instances, it is desired to estimate the life of a ball bearing at extremely high speed with little or no externally applied loading. In this case, the fatigue life is determined by the centrifugal loading of the balls on the outer ring. (Ref. 4)

The outer ring capacity is given by

\[ C_o = A \left( \frac{2f_0}{2f_0 - 1} \right)^{0.41} (1 + \gamma)^{1.39} 1.8 d^{-1/3} \left( \frac{1 - \gamma}{1/3} \right)^{1/3} \]

where

- \( A \) = material constant, usually 7140
- \( f_0 \) = outer race curvature factor (ratio curvature radius to ball diameter)
- \( \gamma \) = ball diameter to pitch diameter ratio, \( d/E \)
- \( d \) = ball diameter - inch
- \( Z \) = number of balls
- \( E \) = pitch diameter - inch

The life of the outer ring is given as

\[ \left( \frac{C_o}{P_{c.f.}} \right)^3 = 90\% \text{ life in } 10^6 \text{ revs.} \]
where

\[ P_{c.f.} = \text{centrifugal ball loading} \]
\[ P_{c.f.} = 5.257 \times 10^{-7} d^3 \frac{K}{\lambda^2 (1-\eta)^2} \]

**Effects of Cyclic Loading on Bearing Life**

As was previously shown, the fatigue life of a rolling element bearing is defined in terms of a 90 percent probability of survival. A specific dynamic capacity \( C \) is defined as that radial load which will result in a life of \( 10^6 \) inner race revolutions with 90 percent survival probability. The 90 percent life at any other load \( P \) is related to the specific dynamic capacity as follows:

\[ L = (C/P)^3 \times 10^6 \text{ Rev.} \quad (2) \]

When the load varies in a series of known steps, some equivalent or mean load is defined as follows:

\[ P_m = \left( \frac{P_1^3 N_1 + P_2^3 N_2 + \ldots + P_n^3 N_n}{N_1 + N_2 + \ldots + N_n} \right)^{1/3} \quad (6) \]

where \( P_1, P_2, P_n \) are loads applied for \( N_1, N_2, N_n \) cycles.

For the case of vibratory loading, an integral form of Equation (6) can be used:

\[ P_m = \left( \frac{1}{N} \int dN \right)^{1/3} \quad (7) \]

In the general case, the loading will consist of some steady load \( P_o \) and a sinusoidal load \( P_1 \sin \omega t \).

The bearing loading \( P \) is given as

\[ P = P_o + P_1 \sin \omega t \quad (8) \]

Using this in Equation (7) yields the following:

\[ P_m = \left[ \frac{1}{N} \int \left( P_o + P_1 \sin \omega t \right)^3 d(\omega t) \right]^{1/3} \quad (9) \]
Expansion of Equation (9) gives

\[
P_m = \left[ \frac{1}{x} \int_0^x \left( P_0^3 + 3P_0^2P_1 \sin \omega t + 3P_0P_1^2 \sin^2 \omega t \right.ight.
\left. + P_1^3 \sin^3 \omega t \right) d \omega t \right]^{1/3}
\]

\[
P_m = \left[ \frac{1}{x} \left( P_0^3 \omega t - 3P_0^2P_1 \cos \omega t + 3P_0P_1^2 \frac{\omega t}{2} \right.ight.
\left. - 3P_0P_1^2 \frac{\sin 2\omega t}{4} - P_1^3 \cos \omega t + P_1^3 \frac{\cos^3 \omega t}{3} \right) \right]^{1/3}
\]

\[
P_m = \left[ \frac{1}{x} \left( P_0^3 \omega t + \frac{3}{2} P_0^2P_1 x \right) \right]^{1/3}
\]

\[
P_m = \left[ P_0^3 \left( 1 + \frac{3}{2} \frac{P_1}{P_0} \right) \right]^{1/3}
\]

\[
\frac{P_m}{P_0} = \left[ 1 + \frac{3}{2} \left( \frac{P_1}{P_0} \right)^2 \right]^{1/3}
\]

The results of Equation (12) are plotted in Figure 5 as a function of the cyclic load ratio.

The life may be found from Equation (2) using the equivalent load \( P_m \).

Often the steady state load \( P_o \) is known and the effect of various cyclic loads is desired. The life due to the steady state load \( P_o \) is

\[
L_o = \left( \frac{C}{P_o} \right)^3
\]
Fig. B  Equivalent Load Ratio as a Function of Cyclic Load Ratio
While the life due to the equivalent load \( \text{Pe} \) is

\[
L = (C/\text{Pe})^3
\]  \hspace{1cm} (14)

The ratio of the lives is

\[
\frac{L}{L_0} = \left(\frac{C/\text{Pe}}{C/\text{Po}}\right)^3 = \left(\frac{1}{\text{Pe}/\text{Po}}\right)^3
\]  \hspace{1cm} (15)

The life ratio as a function of the equivalent load ratio is shown in Figure C.

In summary, the results apply to the following:

1. \( P_1 \leq P_o \)
2. Radial Loading
3. \( P_o \) is unidirectional
4. \( P_1 \) is the single amplitude of the cyclic disturbance.

The above can be used with manufacturers’ catalog data by noting that these are set up for some given life (usually 500 hours) at various speeds. The corresponding load is tabulated.

The case where a cyclic load only is applied is sometimes encountered. The bearing load is then

\[
P = P_1 \sin \omega t
\]  \hspace{1cm} (16)

Equation (7) now becomes

\[
P_m = \left[ \frac{1}{\pi} \int_0^\pi (P_1 \sin \omega t)^3 \sin(\omega t) \; d(\omega t) \right]^{1/3}
\]

\[
= \left[ \frac{1}{\pi} P_1^3 \left( \cos^3 \omega t + \frac{1}{3} \cos^5 \omega t \right) \right]^{1/3}
\]

\[
= 0.752 P_1
\]
Fig. C  Bearing Life Ratio As A Function of Equivalent Load Ratio
The ratio of equivalent loading to the single amplitude of the cyclic loading is

\[ \frac{P_e}{P_1} = 0.752 \]  

(18)

Bearing life is determined from \( P_e \).

As was previously noted, Equation (12) and Figure-B were derived for radial loading. However, the relations can be adapted for use with thrust loads, if the thrust load is represented by

\[ T = T_0 + T_1 \sin \omega t \]  

(19)

where \( T_0 \) is the steady thrust load and \( T_1 \) is the single amplitude of the cyclic thrust loading. This gives a similar relation to Equation (12) as follows:

\[ \frac{T_e}{T_0} = \left[ 1 + \frac{3}{2} \left( \frac{T_1}{T_0} \right)^2 \right]^{1/3} \]  

(20)

Figure B can be used to obtain either a mean radial or thrust load. However, in the case of thrust loading, bearing life must be calculated using an equivalent radial load with the specific dynamic capacity. For calculation purposes (preliminary engineering calculations) the equivalent radial load is given by:

\[ \text{Equiv. Rad. Load} = 0.37 P + 2T \]  

(21)

where \( P \) is the radial load and \( T \) is the thrust load. Either \( T \) or \( P \), or both are replaced by \( P_e \) and \( T_e \) where cyclic loading is involved. More accurate relationships for the various bearing types are found in the manufacturers' catalogs or the AFEMA standards.
Lubricant Life

In many instances, fatigue life is not the major consideration since the loading is light. The lubricant is usually the limiting item insofar as life is concerned. The first consideration is to be sure that lubricant and the lubrication system are adequate for the speed range.

Unfortunately, there are no exact guidelines that can be set. However, some generalizations are possible with respect to normal applications.

<table>
<thead>
<tr>
<th>System</th>
<th>Speed Limit $D \times N$ (bore in mm x speed in RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grease</td>
<td>250,000 (ribbon retainer)</td>
</tr>
<tr>
<td>Oil Level</td>
<td>300,000 (ribbon retainer)</td>
</tr>
<tr>
<td>Mist</td>
<td>700,000 (machined retainer)</td>
</tr>
<tr>
<td>Jet Oil</td>
<td>$&gt;10^6$ (machined retainer)</td>
</tr>
</tbody>
</table>

Above 300,000 dN, the usual ribbon retainer would be replaced by a machined retainer of metal or phenolic. For normal temperatures, the phenolic retainer is commonly used. With special greases, retainer design, and light loading, grease lubrication has been used to speeds of 750,000 dN.

With respect to grease lubrication, there is evidence that life is reduced in some logarithmic fashion with increasing dN value. This is similar to the effect of temperature. Figure D shows a typical behavior of life with respect to temperature. A reasonable rule of thumb is that life is cut in half for each 10°C rise over 100°C.
Fig. D  Bearing Life Versus Temperature
III

DESIGN DATA

Description and Discussion of Charts

There are basically three separate sets of design charts included in this report, namely:

a. Pure Radial Loaded Bearings (Deep Groove) Contact Angle $\beta_o = 0^\circ$
b. Pure Thrust Loaded Bearings (Deep Groove) Contact Angle $\beta_o = 10^\circ$
c. Angular Contact Bearings with Axial Preload and Applied Radial Load $\beta_o = 25^\circ$, $15^\circ$

Table I describes the dimensions and symbols used for the deep-grooved ball bearings. Table II contains information pertaining to the angular contact bearings.

The first set of four charts contains graphs of radial stiffness versus radial load. Load levels are indicated on the curves. The effects of bearing size and race curvatures are illustrated by these four charts. In general, a bearing with tighter raceway curvatures is a stiffer bearing. For example, a bearing with curvatures of $f_1 = .516$, $f_o = .530$ is stiffer than the same bearing operating with curvatures of $f_1 = f_o = .570$, for the same radial load. Radial stiffness is higher for a bearing with a larger bore diameter and/or a greater number of balls. Note, for pure radial load, the linear relationship between $\log S_R$ and $\log P$.

The second set of eight charts contains graphs of axial stiffness and axial deflection versus axial thrust applied load. Load levels are tabulated in Table III for these particular bearings undergoing a pure thrust load since cross plots of C/P will add confusion when reading the curves. Deflection curves are included to aid in analyzing a double acting thrust bearing set. The deflection curve for a double acting thrust bearing set is constructed from the deflection curve of a single bearing by adding increments of deflection to one bearing and subtracting from the other. The corresponding load differences equal the externally applied load. A similar observation, as given
above for radially loaded bearings, can be made for the thrust loaded bearing, i.e., a bearing operating with curvatures of \( f_1 = .516, f_0 = .530 \) is stiffer than the same bearing operating with curvatures of \( f_1 = f_0 = .570 \), for the same axial load. For all practical purposes, however, an average curve may be drawn for axial stiffness versus axial load for all bearing sizes. In particular, the bearing with the smaller bore and less balls is less stiff at light loads and more stiff at heavy loads as compared to the larger bore bearing. There is an approximate linear relationship between \( \log S_A \) and \( \log T \).

The third set of (24) charts contain graphs of radial stiffness versus radial load. Load levels are indicated on the curves. The effects of bearing size, race curvatures, initial contact angle, and axial preload are illustrated by these 24 charts. For the same radial load and axial preload, a bearing operating with curvatures of \( f_1 = .516, f_0 = .530 \) is stiffer than the same bearing operating with curvatures of \( f_1 = f_0 = .570 \). The radial stiffness level is higher for a bearing with a larger bore diameter and/or a greater number of balls, and the smaller initial contact angle. \( (\beta_o = 15^\circ) \). In general, the radial stiffness-radial load curve for an angular contact bearing is composed of three different behaving regions. One region shows the stiffness to be constant with varying radial load. (This is the light radial load region.) The middle, or moderate radial load region shows a minimum value for radial stiffness. The heavily radial loaded region shows a linear relationship between \( \log S_R \) and \( \log P \). The third region is similar in behavior to that of the characteristics of a pure radially loaded deep grooved bearing. The basic cause for this curve having three separate regions is due to the axial preload. In region one, the axial preload has a great effect in holding the radial stiffness constant. In region two, where the applied radial load becomes equal in magnitude to the axial preload, the radial stiffness tends to decrease with increasing applied radial load to a minimum value. In the third region, the axial preload has little or no effect, and the angular contact bearing reflects the behavior of a pure radially loaded bearing, i.e., a linear \( \log S_R \) versus \( \log P \) relationship.

Thus another point one is led to observe is the role of axial preload magnitude on the three regions of a typical stiffness versus load curve. Three different preloads are represented in these charts and are tabulated in Table II. These
preloads are given the names selected light, moderate, and preferred heavy. The effect of increased preload is to increase the region one load range and decrease region three load range. Thus, the ultimate is a constant radial stiffness with varying radial load obtained with an infinite preload. The increased preload also has the effect of increasing the level of stiffness in regions one and two. However, it should be noted particularly that the level of stiffness in region three, for the same radial load, is the same for all preload values. This, as mentioned above, is because the axial preload effect is relieved entirely above a certain (radial load/axial preload) ratio. (Approximately $P/T = 3$ for $\beta_o = 25^\circ$ and $R/T = 4$ for $\beta_o = 15^\circ$.)

In general, the light and extra light deep grooved ball bearings examined here will have a radial stiffness ranging from $10^5$ to $2 \times 10^6$ for radial loads of from 10 to 2,000 lbs. The angular contact bearings will have radial stiffness values from $2 \times 10^5$ to $2 \times 10^6$ for radial loads of from 10 to 2,000 lbs. The deep grooved ball bearings will have an axial stiffness per bearing of from $2 \times 10^6$ to $4 \times 10^6$ for thrust loads of from 10 to $10^4$ lbs. As in the case of the preloaded radial bearing, preloading will increase these values of axial stiffness.
<table>
<thead>
<tr>
<th>Bearing Symbol</th>
<th>Bore (Inch)</th>
<th>Bore (mm)</th>
<th>O.D. (Inch)</th>
<th>Ball Diameter, Inch</th>
<th>Number of Balls</th>
<th>$f_1$</th>
<th>$f_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>.5906</td>
<td>15</td>
<td>1.2598</td>
<td>.1875</td>
<td>9</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>A2</td>
<td>.5906</td>
<td>15</td>
<td>1.2598</td>
<td>.1875</td>
<td>9</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>B1</td>
<td>.9843</td>
<td>25</td>
<td>1.8504</td>
<td>.250</td>
<td>10</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>B2</td>
<td>.9843</td>
<td>25</td>
<td>1.8504</td>
<td>.250</td>
<td>10</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>C1</td>
<td>1.378</td>
<td>35</td>
<td>2.4409</td>
<td>.3125</td>
<td>11</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>C2</td>
<td>1.378</td>
<td>35</td>
<td>2.4409</td>
<td>.3125</td>
<td>11</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>D1</td>
<td>2.1654</td>
<td>55</td>
<td>3.5433</td>
<td>.40625</td>
<td>13</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>D2</td>
<td>2.1654</td>
<td>55</td>
<td>3.5433</td>
<td>.40625</td>
<td>13</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>E1</td>
<td>2.9528</td>
<td>75</td>
<td>4.5276</td>
<td>.46875</td>
<td>15</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>E2</td>
<td>2.9528</td>
<td>75</td>
<td>4.5276</td>
<td>.46875</td>
<td>15</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>A1</td>
<td>.5906</td>
<td>15</td>
<td>1.378</td>
<td>.2345</td>
<td>8</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>A2</td>
<td>.5906</td>
<td>15</td>
<td>1.378</td>
<td>.2345</td>
<td>8</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>BB1</td>
<td>.9843</td>
<td>25</td>
<td>2.0472</td>
<td>.3125</td>
<td>9</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>BB2</td>
<td>.9843</td>
<td>25</td>
<td>2.0472</td>
<td>.3125</td>
<td>9</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>CC1</td>
<td>1.378</td>
<td>35</td>
<td>2.8346</td>
<td>.4375</td>
<td>9</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>CC2</td>
<td>1.378</td>
<td>35</td>
<td>2.8346</td>
<td>.4375</td>
<td>9</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>DD1</td>
<td>2.1654</td>
<td>55</td>
<td>3.937</td>
<td>.5625</td>
<td>10</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>DD2</td>
<td>2.1654</td>
<td>55</td>
<td>3.937</td>
<td>.5625</td>
<td>10</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>EE1</td>
<td>2.9528</td>
<td>75</td>
<td>5.1181</td>
<td>.6875</td>
<td>11</td>
<td>.516</td>
<td>.530</td>
</tr>
<tr>
<td>EE2</td>
<td>2.9528</td>
<td>75</td>
<td>5.1181</td>
<td>.6875</td>
<td>11</td>
<td>.570</td>
<td>.570</td>
</tr>
<tr>
<td>Basic Static Load (Lb)</td>
<td>Bearing Number</td>
<td>Bore (Inch)</td>
<td>Bore (mm)</td>
<td>O.D. (Inch)</td>
<td>d (in)</td>
<td>Number Of Balls</td>
<td>$f_1$</td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------------</td>
<td>-------------</td>
<td>-----------</td>
<td>-------------</td>
<td>--------</td>
<td>----------------</td>
<td>-------</td>
</tr>
<tr>
<td>630</td>
<td>PA 1</td>
<td>.5906</td>
<td>15</td>
<td>1.2598</td>
<td>.1875</td>
<td>11</td>
<td>.516</td>
</tr>
<tr>
<td></td>
<td>PA 2</td>
<td>.9843</td>
<td>25</td>
<td>1.8504</td>
<td>.2500</td>
<td>13</td>
<td>.516</td>
</tr>
<tr>
<td>1400</td>
<td>PB 1</td>
<td>1.3780</td>
<td>35</td>
<td>2.4409</td>
<td>.3125</td>
<td>15</td>
<td>.516</td>
</tr>
<tr>
<td></td>
<td>PB 2</td>
<td>2.1654</td>
<td>55</td>
<td>3.5433</td>
<td>.40625</td>
<td>18</td>
<td>.516</td>
</tr>
<tr>
<td>2600</td>
<td>PC 1</td>
<td>2.9528</td>
<td>75</td>
<td>4.5276</td>
<td>.46925</td>
<td>21</td>
<td>.516</td>
</tr>
<tr>
<td></td>
<td>PC 2</td>
<td>3.750</td>
<td>58</td>
<td>4.3745</td>
<td>.6076</td>
<td>23</td>
<td>.520</td>
</tr>
<tr>
<td>5100</td>
<td>PD 1</td>
<td>4.5276</td>
<td>70</td>
<td>5.1181</td>
<td>.6875</td>
<td>26</td>
<td>.520</td>
</tr>
<tr>
<td></td>
<td>PD 2</td>
<td>5.625</td>
<td>90</td>
<td>6.2500</td>
<td>.7788</td>
<td>28</td>
<td>.520</td>
</tr>
<tr>
<td>760</td>
<td>PA 1</td>
<td>.5906</td>
<td>15</td>
<td>1.3780</td>
<td>.23425</td>
<td>10</td>
<td>.516</td>
</tr>
<tr>
<td></td>
<td>PA 2</td>
<td>.9843</td>
<td>25</td>
<td>1.8504</td>
<td>.3125</td>
<td>12</td>
<td>.516</td>
</tr>
<tr>
<td>1640</td>
<td>PB 1</td>
<td>1.3780</td>
<td>35</td>
<td>2.8346</td>
<td>.4375</td>
<td>12</td>
<td>.516</td>
</tr>
<tr>
<td></td>
<td>PB 2</td>
<td>2.1654</td>
<td>55</td>
<td>3.9370</td>
<td>.5625</td>
<td>14</td>
<td>.516</td>
</tr>
<tr>
<td>3750</td>
<td>PC 1</td>
<td>2.9528</td>
<td>75</td>
<td>5.1181</td>
<td>.6875</td>
<td>16</td>
<td>.516</td>
</tr>
<tr>
<td></td>
<td>PC 2</td>
<td>3.750</td>
<td>58</td>
<td>4.3745</td>
<td>.6076</td>
<td>23</td>
<td>.520</td>
</tr>
<tr>
<td>7300</td>
<td>PD 1</td>
<td>4.5276</td>
<td>70</td>
<td>5.1181</td>
<td>.6875</td>
<td>16</td>
<td>.516</td>
</tr>
<tr>
<td></td>
<td>PD 2</td>
<td>5.625</td>
<td>90</td>
<td>6.2500</td>
<td>.7788</td>
<td>28</td>
<td>.520</td>
</tr>
</tbody>
</table>
### Table III  Axial Loaded Deep-Grooved Bearings

Table of approximate load values corresponding to C/P = 5 and C/P = 10 load levels.

<table>
<thead>
<tr>
<th>Bearing Symbol</th>
<th>Load (Lb)</th>
<th>C/P = 5</th>
<th>C/P = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>290</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>600</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>1100</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>2250</td>
<td>950</td>
<td></td>
</tr>
<tr>
<td>E1</td>
<td>3650</td>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>70</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>175</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>300</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>550</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>E2</td>
<td>950</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>AA1</td>
<td>380</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td>BB1</td>
<td>800</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>CC1</td>
<td>1550</td>
<td>650</td>
<td></td>
</tr>
<tr>
<td>DD1</td>
<td>3000</td>
<td>1250</td>
<td></td>
</tr>
<tr>
<td>EE1</td>
<td>5050</td>
<td>2100</td>
<td></td>
</tr>
<tr>
<td>AA2</td>
<td>95</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>BB2</td>
<td>200</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>CC2</td>
<td>400</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>DD2</td>
<td>1500</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>EE2</td>
<td>2550</td>
<td>700</td>
<td></td>
</tr>
</tbody>
</table>
PURE RADIAL LOAD
Fig. 1  Radial Stiffness for Deep Groove Ball Bearing—Pure Radial Load
Fig. 2 Radial Stiffness for Deep Groove Ball Bearings—Pure Radial Load
PULL THRUST LOAD
Fig. 5  Axial Stiffness versus Axial Load
No Radial Load
Fig. 6  Axial Deflection versus Axial Load  
No Radial Load
Fig. 7  Axial Stiffness versus Axial Load
No Radial Load
Fig. 8  Axial Deflection versus Axial Load
No Radial Load

29
Fig. 9  Axial Stiffness versus Axial Load
No Radial Load
Fig. 10 Axial Deflection versus Axial Load
No Radial Load
Fig. 11  Axial Stiffness versus Axial Load
No Radial Load
Fig. 12  Axial Deflection versus Axial Load
No Radial Load
RADIAL LOAD WITH AXIAL PRELOAD

34
Fig. 16  Radial Stiffness for Angular Contact Bearing
Pre-Load - Selected Light
$\beta = 25^\circ$
Fig. 19. Radial stiffness for 4 in. Contact Bearing.
Fig. 18 Radial Stiffness for Angular Contact Bearing

Pre-Load - Moderate

ρ = 250
Fig. 21  Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
\( \theta = 0^\circ \)
Fig. 23  Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
\[ \beta = 25^\circ \]
Fig. 24. Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
\[ \beta = 25^\circ \]
Fig. 25  Radial Stiffness for Angular Contact Bearing
Pre-Load - Selected Light
$\beta = 15^\circ$
Fig. 28 Radial Stiffness for Angular Contact Bearing
Pre-Load - Selected Light
$\beta = 15^\circ$
Fig. 29  Radial Stiffness for Angular Contact Bearing
Pre-Load - Moderate
$\beta = 15^\circ$
Fig. 30  Radial Stiffness for Angular Contact Bearing  
Pre-Load - Moderate  
$\beta = 15^\circ$
Fig. 31  Radial Stiffness for Angular Contact Bearing
Pre-Load - Moderate
\[ \beta = 15^\circ \]
Fig. 32  Radial Stiffness for Angular Contact Bearing  
Pre-Load - Moderate  
\[ \beta = 15^\circ \]
Fig. 34  Radial Stiffness for Angular Contact Bearing  
Pre-Load - Preferred Heavy  
\( \beta = 15^\circ \)
Fig. 35  Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
$\beta = 15^\circ$
IV

SAMPLE PROBLEMS TO ILLUSTRATE USE OF DESIGN CHARTS

Four particular examples are included in this section:

1. Pure Radial Loaded Bearing
2. Unidirectional Thrust Loaded Bearing
3. Double-Acting Thrust Loaded Bearing
4. Radial Loaded, Axial Preloaded Angular Contact Bearing

1. Pure Radial Loaded Bearing

What are the radial stiffness values corresponding to radial loads of 100 and 700 pounds for a deep-grooved ball bearing with a bore = .5906 inch and $f_1 = f_0 = .5707$?

From Table 1, this bearing corresponds to A2.

From Figure 2:

<table>
<thead>
<tr>
<th>Radial Load (lb)</th>
<th>Radial Stiffness (lb/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$2.88 \times 10^5$</td>
</tr>
<tr>
<td>700</td>
<td>$5.55 \times 10^5$</td>
</tr>
</tbody>
</table>

2. Unidirectional Thrust Loaded Bearing

What are the axial stiffness values corresponding to axial loads of 100 and 700 pounds for the same deep-grooved ball bearing as used in sample problem 1, above?

From Figure 7:

<table>
<thead>
<tr>
<th>Thrust Load (lb)</th>
<th>Thrust Stiffness (lb/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$9.10 \times 10^4$</td>
</tr>
<tr>
<td>700</td>
<td>$3.22 \times 10^5$</td>
</tr>
</tbody>
</table>

3. Double-Acting Thrust Loaded Bearing

What is the axial stiffness for a double-acting, deep-grooved ball bearing, thrust bearing set, preloaded to 200 pounds? The bearings are type A-2.

From Figure 8: Read off loads corresponding to equal deflections around preload of 200 pounds.
Thus,

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Deflection (in)</th>
<th>Stiffness (lb/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>$2.8 \times 10^{-3}$</td>
<td>$1.22 \times 10^5$</td>
</tr>
<tr>
<td>200</td>
<td>$3.1 \times 10^{-3}$</td>
<td>$1.4 \times 10^5$</td>
</tr>
<tr>
<td>240</td>
<td>$3.4 \times 10^{-3}$</td>
<td>$1.58 \times 10^5$</td>
</tr>
</tbody>
</table>

$S_A = (1.58 + 1.22) \times 10^5 = 2.8 \times 10^5$ lb/in

or $S_A = 2 \times 1.4 \times 10^5 = 2.8 \times 10^5$ lb/in

For light loads, i.e., loads less than the axial preload, the load-deflection characteristics are essentially linear.

4. Radial Loaded, Axial Preloaded Angular Contact Bearing

What are the radial stiffness values corresponding to radial loads of 100 and 700 pounds for an angular contact bearing with a BORE = .5906 inch and $f_i = f_o = .570$. The contact angle is 15 degrees, and it has a medium axial preload.

From Table 2, this bearing corresponds to PA2.
From Figure 30,

<table>
<thead>
<tr>
<th>Radial Load (lb)</th>
<th>Radial Stiffness (lb/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$4.45 \times 10^5$</td>
</tr>
<tr>
<td>700</td>
<td>$5.90 \times 10^5$</td>
</tr>
</tbody>
</table>

Note: One must be careful in using the charts, in particular for the angular contact bearing, that the design contact angle and axial preloading design value correspond to the values given in the Figure legend.
REFERENCES


A. Analysis

The theory used for predicting the load carrying capacity, deflections, and stresses of deep-grooved and angular contact bearings is that of Reference 1.*

The equations relating load and deflection were differentiated to obtain the equation for stiffness.

A calculational procedure was devised for predicting the maximum ball load, deflection, stiffness, and inner and outer race stresses, as a function of total applied load, preload and bearing geometry. This procedure was programmed as computer program PNO182, IBM 1620-60K.

Three separate cases are treated:

1) Pure Radial Load, deep grooved bearing (Ref. 2,3)
2) Pure Thrust Load, deep grooved bearing
3) Combined Radial Load with Axial Preload, angular contact bearing (Ref. 2,3)

Pure Radial Load

Maximum Ball Load, $P_0$

$$P_0 = 4.37 \times P/n$$  \hspace{1cm} A-1

Radial Deflection, $b_N$

$$b_N = C_0 \times P_0^{2/3}$$  \hspace{1cm} A-2

where

$$C_0 = 7.8107 \times 10^{-6} \times (C_{o_0} + C_{o_1})/d^{1/3}$$  \hspace{1cm} A-3

and

$$C_{o_0}, C_{o_1} = f(f_1, f_0, E, d, \beta_0)$$  \hspace{1cm} A-4

* See Reference: Section 63
**Radial Stiffness, \( S_R \)**
\[
S_R = \frac{3}{2} \frac{P}{\pi N}
\]

**Compressive Stresses, \( S_0, S_1 \)**
\[
S_m = \frac{f_{sm}(15079)}{d} \left( \frac{P}{d^2} \right)^{1/3}
\]
where \( f_{sm} = f(f_1, f_o, E, d, \beta_o) \)

and \( m = i, \text{ or } o \).

**Pure Thrust Load**

**Maximum Ball Load, \( P_0 \)**
\[
P_0 = \frac{T}{n \sin \beta_1}
\]

**Axial Deflection, \( \delta_H \)**
\[
\delta_H = Bd \sin \left( \beta_1 - \beta' \right) \cos \beta_1
\]
where \( B = f_1 + f_o - 1 \)

and \( \beta_1 \) is found by an iteration scheme, written below.

**Applied Thrust (Axial) Load, \( T \)**
\[
T = nd^2 K \sin \beta_1 \left( \frac{\cos \beta'}{\cos \beta_1} - 1 \right)^{3/2}
\]
where \( K = \left[ \frac{B \times 10^{+6}}{7.58107(C_0 + C_1)} \right]^{3/2} \)

and \( \beta_1 \) is found as follows:
Define the following quantities:
\[
\frac{T}{\eta d^2 K} = b \quad \cos \beta_0 = a \quad \cos \beta_1 = x
\]

Then Equation (A-11) may be written as
\[
(1 - x^2)^{1/3} \left(\frac{a}{x} - 1\right) = b
\]

Let \( y = \frac{a}{x} - 1 \) \((A-13)\)

Then \( y = b(1 - x^2)^{-1/3} \) \((A-14)\)

It is a known fact that \( b << 1 \). Therefore, as a good guess to start the iteration scheme for solving (A-14), let
\[
y = b y_1 + b^2 y_2 + \ldots.
\]

where
\[
y_1 = 1 + \frac{a^2}{3}
\]
and
\[
y_2 = \frac{2}{3} a^2 y_1
\]

\((A-15)\)

\((A-16)\)

The procedure is:

1) Calculate \( y_1, y_2 \) from (A-16) and \( y \) from (A-15), knowing \( a \) and \( b \).
2) Calculate \( x \) from (A-13) knowing \( y \).
3) Calculate \( y \) from (A-14).
4) Check \( y \) from step (3) with \( y \) from step (2).
5) If the values are equal
\[
\beta_1 = \cos^{-1} (x),
\]
otherwise use an average value for \( y \) and repeat steps (2) through (5) until agreement is obtained.
Axial Stiffness, $S_A$

$$S_A = \frac{B}{nd_B k} \left\{ \frac{3}{2} \sin^2 \beta_1 \left[ \frac{\cos \beta_o}{\cos \beta_1} - 1 \right]^{1/2} + \frac{\cos^3 \beta_o}{\cos \beta_1} \left[ \frac{\cos \beta_o}{\cos \beta_1} - 1 \right]^{3/2} \right\}$$  \hspace{1cm} \text{(A-17)}

Compressive Stresses, $S_0$, $S_1$

$$S_m = f_{sm} \left( \frac{P_0}{l^2} \right)$$  \hspace{1cm} \text{(A-6)}

where

$$f_{sm} = f(f_i, f_o, E, d, \beta_1)$$  \hspace{1cm} \text{(A-7)}

and

$$P_0$$ is calculated from (A-8)

Combined Radial Load and Axial Preload

The same procedure for finding $\beta_1$ as applied in the pure thrust load case is applied in this case in order to find $\delta_H$. A value of radial deflection $\delta_V$, is assumed, then the radial force, $\Sigma V$, is calculated as a function of $\delta_H$ and $\delta_V$:

The following definitions are written:

$$k' = \frac{\delta_V}{Bd}, \quad h' = \frac{\delta_H}{Bd}$$  \hspace{1cm} \text{and}

$$\phi' = \cos^{-1} \left[ \frac{1 - (\sin \beta_o + h')^2}{k'} - \cos \beta_o \right], \quad \cos \phi' > -1$$  \hspace{1cm} \text{(A-18)}

$$\phi' = \pi, \quad \cos \phi' < -1$$

Maximum Ball Load, $P_0$

$$P_0 = Kd \left[ \sqrt{(\sin \beta_o + h')^2 + (\cos \beta_o + k' \cos \phi)^2} - 1 \right]^{3/2}$$  \hspace{1cm} \text{(A-19)}

where $\phi = 0^\circ$
Radial Deflection, $\delta_V$: Axial Deflection, $\delta_H$

$$\delta_V = k' Bd \quad \delta_H = h' Bd$$

Axial Preload, $T$

Calculate from (A-11) and (A-12).

Radial Load, $\Sigma V$

$$\Sigma V = nd \frac{2}{\pi} \frac{1}{\phi} A d\phi$$

where

$$A = \left[ \frac{\left( \sin \beta_o' + h' \right)^2 + \left( \cos \beta_o' + k' \cos \phi \right)^2 - 1}{\left( \sin \beta_o' + h' \right)^2 + \left( \cos \beta_o' + k' \cos \phi \right)^2} \right]^{1/2} \left( \cos \beta_o' + k' \cos \phi \right) \cos \phi$$

and

$$0 \leq \phi \leq \phi'$$

Radial Stiffness, $S_R$

$$S_R = \frac{B}{nd K} \frac{1}{\pi} \int_0^\phi A \left[ \frac{\cos \phi}{\left( \cos \beta_o' + k' \cos \phi \right)} + \right.$$}

$$\left. \left[ \frac{1}{2} \left( \sin \beta_o' + h' \right)^2 + \left( \cos \beta_o' + k' \cos \phi \right)^2 \right]^{1/2} + 1 \right]^{3/2} \left( \sin \beta_o' + h' \right)^2 \left( \cos \beta_o' + k' \cos \phi \right)^2$$

Compressive Stresses, $S_o, S_1$

$$S_m = f_{sm} \left( \frac{P_o}{d} \right)^{1/3}$$

A-20

A-21

A-22

A-6
where

\[ f_{sm} = f(f_x, f_y, E, d, \beta_1) \]

\[ m = 1, 0, \text{or} P_0 \text{ is calculated from (A-19)}. \]

B. Computer Program

A Fortran II computer program listing is included in this memorandum.

The Input Format written below should be followed when using this program, PNO182 IBM 1620-60K.

Input Format

Card 1 Identification Card

Anything may be punched in columns 2-72.

Card 2 (6 F10. 6, 3I4)

Item
1. BORE, Bore diameter, in.
2. OD, Extreme outer diameter, in.
3. DB, Ball diameter, in.
4. FI, Radius of Curvature of Inner Race
5. FO, Radius of Curvature of Outer Race
6. BETA, Contact Angle, deg (\( \beta = 0^\circ \) for pure radial load)
7. N, Total number of balls
8. IND, An indicator used to specify either one of three different types of calculations.
   IND: 0 Pure Radial Load
   IND: 1 Pure Thrust Load
   IND: 2 Combined Radial Load - Axial Preload
9. LC, An indicator used to stop calculation procedure
   LC: 0 Program returns to Card 1 for more input.
   LC: 1 Program stops after computation is completed
Card 3  (3F10.6, IS)
IND = 0

Item
1. RI, Initial Radial Load, lb.
2. RD, Radial Load Increment, lb
3. RF, Final Radial Load, lb. (Not used in calculation. RF = 0.0)
4. M, Total number of radial loads

IND = 1

Item
1. TI, Initial Thrust Load, lb
2. TD, Thrust Load Increments, lb
3. CONV, A radius of convergence used in the iteration process for calculating $\beta_1$. CONV = .0005 is a typical value.
4. M, Total number of thrust loads

IND $\geq$ 2

Item
1. TI, Axial Preload, lb
2. TD, Axial Preload Increment, lb (Not used in calculation $T_D = 0.0$)
3. CONV, a radius of convergence used in the iteration process for calculating $\beta_1$.
4. M, Total number of preloads (must be set equal to one; i.e. M = 1)

Note: The total number of radial loads which will be calculated as a function of axial preload and radial deflection is equal to the value of IND. Thus, IND = 20, the calculational procedure will solve for '20' consecutive radial loads. A maximum of IND = 24 is allowed.

Output Format
The output is self explanatory. All linear dimensions are in inches. All loads are in pounds. The stiffness units are in lb/in. The stresses are measured in psi.
C BALL BEARING STIFFNESS AND STRESS CALCULATION ROUTINE FOR
C PURE RADIAL + PURE THRUST OR COMBINED LOADING INCLUDING CENTRIFUGAL
C
DIMENSION FF(12), SB(12), CC(12), D(12), CS(12), DS(12)

| FF(1) | FF(2) | FF(3) | FF(4) | FF(5) | FF(6) | FF(7) | FF(8) | FF(9) | FF(10) | FF(11) | FF(12) | BB(1) | BB(2) | BB(3) | BB(4) | BB(5) | BB(6) | BB(7) | BB(8) | BB(9) | BB(10) | BB(11) | BB(12) | CC(1) | CC(2) | CC(3) | CC(4) | CC(5) | CC(6) | CC(7) | CC(8) | CC(9) | CC(10) | CC(11) | CC(12) | D(1) | D(2) | D(3) | D(4) | D(5) | D(6) | D(7) | D(8) | D(9) | D(10) | D(11) | D(12) | BS(1) | BS(2) | BS(3) | BS(4) | BS(5) | BS(6) | BS(7) |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.506 | 0.516 | 0.540 | 0.560 | 0.700 | 0.590 | 0.600 | 0.816 | 0.928 | 1.037 | 0.927 | 1.197 | 1.275 | 1.335 | 1.385 | 1.428 | 1.465 | 1.478 | 1.525 | 0.790 | 0.955 | 1.00 | 1.05 | 1.15 | 1.22 | 1.278 | 1.321 | 1.36 | 1.395 | 1.425 | 1.445 | 0.850 | 0.968 | 1.085 | 1.145 | 1.260 | 1.341 | 1.410 | 1.462 | 1.510 | 1.555 | 1.585 | 1.662 | 0.89 | 0.96 | 1.03 | 1.075 | 1.17 | 1.24 | 1.30 |
BS(8) = 1.35
KS(4) = 1.65

BS(10) = 1.44
BS(11) = 1.47

BS(12) = 1.51
CS(1) = 70
CS(2) = 78
CS(3) = 86

CS(4) = 90
CS(5) = 98
CS(6) = 1.04
CS(7) = 1.09

CS(8) = 1.17
CS(9) = 1.18
CS(10) = 1.21
CS(11) = 1.25

CS(12) = 1.275
DS(1) = 1.15

DS(2) = 1.25
DS(3) = 1.40

DS(4) = 1.65
DS(5) = 1.58

DS(6) = 1.65
DS(7) = 1.75

DS(8) = 1.65
DS(9) = 1.65

DS(10) = 1.92
DS(11) = 1.95

DS(12) = 2.30

6 READ 100
READ 102: BORE;OD;OB;FI;FO;BETA;N;IND;LC
PUNCH 111
PUNCH 100
PUNCH 103
PUNCH 109: BORE;OD;UD;FI;FO;BETA;N
IF (IND=11) 1+2+3
1 READ 107: M1;RF;RF;
PUNCH 106
GO TO 4
2 READ 107: TI;TD;CONV;N
PUNCH 108
GO TO 4
3 READ 107: TI;TD;CONV;N
KOP=KOP+1
IF (KOP=1) 1B=1B+4

18 AK(1) = 0.02
AK(2) = 0.03
AK(3) = 0.06
AK(4) = 0.05
AK(5) = 0.06
AK(6) = 0.07
AK(7) = 0.08
AK(8) = 0.09
AK(9) = 0.10
AK(10) = 0.12
AK(11) = 0.14
AK(12) = 0.16
AK(13) = 0.18
AK(14) = 0.20
AK(15) = 0.22
AK(16) = 0.24
GO TO 4.

C

HELIX HANKY CALCULATIONS

4  SET L=0.0174533*SETA
    L=(DWC+UJ)/2.*
    L,N
    C5SB*C5S* (FSET)
    SINF*SINF (FSET) 
    Z=DU*C5S (F)
    CALL TLU (F1+BFF+6.G+1+12)
    CALL TLU (F1+BFF+B.G+12)
    C12=(L-B1)*Z+2.5+.
    CALL TLU (FU*+BFF+6.G+1+12)
    CALL TLU (FU*+BFF+C.G+12)
    C12=(L-B1)*Z+2.5+.
    CALL TLU (F0*+BFF+6.G+1+12)
    CALL TLU (F0*+BFF+C.G+12)
    F50=H2-(L-C2)*Z+2.5+.
    CALL TLU (F1*+BFF+6.G+1+12)
    CALL TLU (F1*+BFF+C.G+12)
    F51=(L-B2)*Z+2.5+.
    DO 3=0*#0*933333
    DO 3=0*#0*933333
    C=7.017E-6*(CO+C.G)/.
    #CON=15.579*V.*0.3/V.*
    IF (IND-1) 8*9+9
    R=R+1D
    DU 5 I=1+4
    R=R+2
    RDU=4.37*H/AN
    RDU=PD+4*33333
    DN=C*PO*PO
    DDDN=1*5*9/6W
    SX=PD*14V
    SMU=1*14V
    S1F=1*5+1
    5 PUNCH 151=HP#11N#DBU=5#1+5
    20 IF (LC) 6*6+15
    9 B=F1+FO-10
    AK=(B/(CUO+CD1)/(L+8107L-08)+**3
    D=AK*4*93
    AK=AK*FD
    T=TI-TD
    A=GLN3
    Y1=1*U+AK/3.0
    Y2=-666666*AK*Y1
    DU 1V I=1+4
    T=T+TD
    B1=T/AK
    B2=1*0.333333
    B2=U2*U2
    Y=U2*Y1+U2*U2*Y2
    13 A=A/(Y+1.Y)
    YY=(1.0-X*X)**U.333333
YY=B2/YY
ET=SQRTF(Y+YY+YY)
ETA=ABS(Y)-ABSF(YY)
ETA=ABSF(ETA)/ET-CNV
IF (ETA < -11.1112)
12 Y=(YY+Y)/2+6
GO TO 13
11 XS=SQRTF(1.-XX)
BET1=ATANF(XS/X)
DHI=1.0/X*SINF(BET1-BET)
DHI=1.0/DB*DHI
Z=DB/E*X
DS0=(UZ-CZ)*2+0
ZS1=(UZ-ZD)*2+0
DTA/X=1+0
DTS=SQRTF(DT)
DDM=DTX=AK/B/DB*(1.5*XS*XS*DT*X*X*X/A)
IF (IND-1) 16,16,17
16 PO=Y/AN/XS
PO3=PO**0.333333
SM1=PO*HCON
SMG=FSO*SM1
SMT=FSH*SM1
10 PUNCH 105*T*PO*DH*DDM*SMI*SM0
GO TO 25
17 S1=(SINB+DHI)**2
PUNCH '09'
17 PUNCH 105*T*PB1*+DH*DDM*UDM
PUNCH 106
DO 19 1=1,IND
FK=AKITI
PHC=(SQRTF(1.0-S1)-COSU)/F
PHCA=ABSF(PHC)
IF (PHCA=1.0) 40,41,41
41 PHI=3.1415927
GO TO 31
46 PHI=SQRTF(1.C-PHC*PHC)
PHI=ATANF(PS/PHC)
IF (PHC < 3.0) 31,31,31
30 PHI=3.1415927+PHI
31 DPHI=PHI/30.5
PHID=PHI/57.29578
X=-DPHI
SUM=0
SUMS=0+0
DO 32 J=1,31
X=X+DPHI
CP=COSF(X)
S2=COSF+FK*CP
IF (J-1) 35,35,35
35 X*X+5
GO TO 34
33 X*X=1.0
34 P=51+52**2
P=SQRTF(P)
IF (P-1.0) 53,53,54
53 PM=1.0E-06
GO TO 55
54 PM=SQRTF(P-1.0)
55 PM2=PM1**3
PHS=PM2*PH1*PHM
SST = PX + S2 * CP / P
SUM = SUM + ST * YX
S3 = CP / S2
ST = ST + SST * (u + S#P + 1) / P / PM3

1 IF (SENSE SWITCH 21) = 32
2 PUNCH 105: ST; ST; PM1; SST; S2
33 SUM = SUM + SST * YX
SUM = (SUM + SST) / DPHI / 3.1415927
SUM = (SUM + SST) / DPHI / 3.1415927

1 IF (SENSE SWITCH 21) = 30, 31

50 PUNCH 105; SUM; S2; PM1; SST
51 PO = SORTF(S1: (COSB + FK) * 2)
PO = SORTF(P0 = 1 + u)
PO = P0 * 3 / AK / AN
PO = P0 * 0.33333

S1 = PM3 * HCON
S1 = PM3 * HCON
SMO = FS0 * SMI
SMO = FS0 * SMI
R = SUM * AK
DRDDN = SUM * AK / B / DD
DN = FK * B * DB

19 PUNCH 105; R = PO; O; DD; DRDDN; SMI; SMO
GO TO 20

15 STOP
C FORMAT STATEMENTS
100 FORMAT(72HO)
11 FORMAT(6F10.6 / 93)
10 FORMAT(6F10.5 / 8)
10 FORMAT(6F10.4 / 6)
10 FORMAT(6F10.3 / 4)
10 FORMAT(6F10.2 / 2)
10 FORMAT(6F10.1 / 1)
10 FORMAT(6F10.0 / 0)

10 FORMAT(7F10.6 / 93)
10 FORMAT(7F10.5 / 8)
10 FORMAT(7F10.4 / 6)
10 FORMAT(7F10.3 / 4)
10 FORMAT(7F10.2 / 2)
10 FORMAT(7F10.1 / 1)
10 FORMAT(7F10.0 / 0)

10 FORMAT(6F10.6 / 93)
10 FORMAT(6F10.5 / 8)
10 FORMAT(6F10.4 / 6)
10 FORMAT(6F10.3 / 4)
10 FORMAT(6F10.2 / 2)
10 FORMAT(6F10.1 / 1)
10 FORMAT(6F10.0 / 0)

END
SUBROUTINE TLU(A,B,C,D,U,N)
C A INDEPENDENT VARIABLE
C B DEPENDENT VARIABLE
C C INDEPENDENT TABLE
C D DEPENDENT TABLE
C N NO OF ENTRIES IN TABLE
DIMENSION C(3),D(50)

NTX2=C
I=1
NTXI=N
M=N/2
13 IF (M-1) 39,39,42
39 B=D(1)+(A-C(1))*D(1) / (C(1)-C(2))
GO TO 99
42 IF (C(11)-C(1)) 45,43,44
43 I=I+1
GO TO 42
44 IF (C(M)=A) 6,6,7,8
45 IF (A-C(M)) 6,6,7,8
7 R=D(M)
GO TO 99
6 IF (M-NTX2-1)=19,19,15
15 NTX1=M
M=M-(M-NTX2)/2
GO TO 13
9 NTX2=M
GO TO 14
8 NTX2=M
14 M=(NTXI-M)/2+M
IF (NTX2-NTXI+1) 13,13,13,13
18 M=NTXI
10 DENO= C(4)-C(M-1)
DIFF= A-C(M)
B=DENODIFF*U(M)-D(M)
99 RETURN
END
### C. Nomenclature for Analysis

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Total Curvature, Eq. A-10</td>
<td></td>
</tr>
<tr>
<td>C°</td>
<td>Radial Deflection Constant, Eq. A-3</td>
<td></td>
</tr>
<tr>
<td>C°o</td>
<td>Deflection Constant of Outer Race, Eq. A-4</td>
<td></td>
</tr>
<tr>
<td>C°i</td>
<td>Deflection Constant of Inner Race, Eq. A-4</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Ball Diameter</td>
<td>in.</td>
</tr>
<tr>
<td>E</td>
<td>Pitch Circle Diameter</td>
<td>in.</td>
</tr>
<tr>
<td>f°i</td>
<td>Inner Race Curvature</td>
<td></td>
</tr>
<tr>
<td>f°o</td>
<td>Outer Race Curvature</td>
<td></td>
</tr>
<tr>
<td>f°si</td>
<td>Stress Factor for Inner Race, Eq. A-7</td>
<td></td>
</tr>
<tr>
<td>f°so</td>
<td>Stress Factor for Outer Race, Eq. A-7</td>
<td></td>
</tr>
<tr>
<td>h°</td>
<td>Relative Displacement of Races in Axial Direction, Eq. A-18</td>
<td>in.</td>
</tr>
<tr>
<td>K</td>
<td>Axial Deflection Constant, Eq. A-12</td>
<td></td>
</tr>
<tr>
<td>k°</td>
<td>Relative Displacement of Races in Radial Direction, Eq. A-18</td>
<td>in.</td>
</tr>
<tr>
<td>n</td>
<td>Number of Balls</td>
<td></td>
</tr>
<tr>
<td>P°</td>
<td>Maximum Ball Load</td>
<td>lb.</td>
</tr>
<tr>
<td>P</td>
<td>Magnitude of Radial Load for Deep Grooved Bearing</td>
<td>lb.</td>
</tr>
<tr>
<td>S°A</td>
<td>Stiffness/Bearing in Axial Direction Due to Load in Axial Direction</td>
<td>lb./in.</td>
</tr>
<tr>
<td>S°R</td>
<td>Stiffness in Radial Direction</td>
<td>lb./in.</td>
</tr>
<tr>
<td>S°i</td>
<td>Compressive Stress in Inner Race</td>
<td>psi</td>
</tr>
<tr>
<td>S°o</td>
<td>Compressive Stress in Outer Race</td>
<td>psi</td>
</tr>
<tr>
<td>T</td>
<td>Axial Load, or Preload</td>
<td>lb.</td>
</tr>
<tr>
<td>β°</td>
<td>Initial Contact Angle</td>
<td>deg.</td>
</tr>
<tr>
<td>β°'</td>
<td>Contact Angle after Preload</td>
<td>deg.</td>
</tr>
<tr>
<td>β°l</td>
<td>Operating Contact Angle</td>
<td>deg.</td>
</tr>
<tr>
<td>δ°</td>
<td>Deflection in Axial Direction</td>
<td>in.</td>
</tr>
<tr>
<td>δ°o</td>
<td>Deflection in Radial Direction</td>
<td>in.</td>
</tr>
<tr>
<td>δ°N</td>
<td>Deflection in Vertical or Radial Direction</td>
<td>in.</td>
</tr>
<tr>
<td>δ°v</td>
<td>Magnitude of Radial Load for Angular Contact Brg.</td>
<td>in.</td>
</tr>
<tr>
<td>δ°N</td>
<td>Angle Measured to a Load Vector within Loaded Zone of Ball</td>
<td>deg.</td>
</tr>
<tr>
<td>δ°v</td>
<td>Half Angular Extent of Loaded Zone of Ball</td>
<td>deg.</td>
</tr>
</tbody>
</table>

\[ 0 \leq \phi' \leq \pi \]
The contractor shall type in the same abstract as that used in the front of the report.
1. **ORIGINATING ACTIVITY**: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (commercial author) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION**: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP**: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE**: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES**: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHORS**: Enter the name(s) of author(s) as shown on or in the report. Each last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE**: Enter the date of the report as day, month, year, or month, year, if more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES**: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES**: Enter the total number of references cited in the report.

8. **CONTRACT OR GRANT NUMBER**: If appropriate, enter the applicable number of the contract or grant under which the report was written.

9a. **S. & R. & S. PROJECT NUMBER**: Enter the appropriate military department identification, such as project number, subproject number, system number, task number, etc.

9b. **ORIGINATOR'S REPORT NUMBER**: Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9c. **OTHER REPORT NUMBER(S)**: If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICE**: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

   1. "Qualified requesters may obtain copies of this report from DDC."
   2. "Foreign announcement and dissemination of this report by DDC is not authorized."
   3. "U. & Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ______.
   4. "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ______.
   5. "All distribution of this report is controlled. Qualified DDC users shall request through ______.

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES**: Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY**: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development, include address.

13. **ABSTRACT**: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TF), (S), (C), or (F).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS**: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical content. The assignment of links, rules, and weights is optional.