NEW LIMITATION CHANGE

TO
Approved for public release, distribution unlimited

FROM
Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; APR 1965. Other requests shall be referred to Office of Naval Research, 800 North Quincy Street, Arlington, VA 22217-5660.

AUTHORITY
ONR ltr, 4 May 1977
HYDRONAUTICS, incorporated
research in hydrodynamics

Research, consulting, and advanced engineering in the fields of NAVAL and INDUSTRIAL HYDRODYNAMICS. Offices and Laboratory in the Washington, D. C., area: Pindell School Road, Howard County, Laurel, Md.
NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
ON THE SURFACE WAVE PATTERN OF
SUBMERGED BODIES
STARTED FROM REST

By

C. C. Hsu

April 1965

Prepared Under

Office of Naval Research
Department of the Navy
Contract No. Nonr 3688(00)
NR 220-016
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>2</td>
</tr>
<tr>
<td>GENERAL FORMULATION</td>
<td>2</td>
</tr>
<tr>
<td>SURFACE WAVE PROFILES</td>
<td>7</td>
</tr>
<tr>
<td>NUMERICAL RESULTS AND DISCUSSION</td>
<td>12</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>17</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1 - Definition Sketch
Figure 2 - Surface Wave Pattern of a Submerged Body Started Impulsively From Rest
Figure 3 - Amplitude of the Initial Unsteady Disturbance With t as a Varying Parameter
Figure 4 - Amplitude of the Initial Unsteady Disturbance With x as a Varying Parameter
Figure 5 - Comparison of Theoretical and Experimental Centerline Wave Profiles Due to a Submerged 9 Feet Long, 7 to 1, Rankine Ovoid Started from Rest
NOTATION

-\( f \) depth of submergence measured from the centerline of the body
-\( g \) gravitational acceleration
-\( M \) strength of point source
-\( \mathbf{q} \) velocity vector
-\( R \) radial distance from source or sink
-\( t \) time
-\( U \) forward speed
-\( x, y, z \) rectangular cartesian coordinates
-\( \phi \) velocity potential
-\( \zeta \) wave height
-\( \zeta_r \) regular wave height
-\( \zeta_l \) local disturbance
-\( \zeta_s \) steady wave height
-\( \zeta_1 \) unsteady disturbance
-\( \overline{\zeta}_s, \overline{\zeta}_1 \) amplitude of the steady wave and unsteady disturbance, respectively
ABSTRACT

The initial surface wave pattern due to a submerged body started from rest with uniform speed in a perfect fluid is studied. The mean surface elevations may, in general, be separated into three parts: (i) A steady local disturbance which travels both upstream and downstream and diminishes as the distance from the body increases, (ii) a group of steady regular waves which travel downstream from the origin \( x = x - x_0 = 0 \) to \( x = 1/2 \, U \) with group velocity \( 1/2 \, U \), and (iii) a time-dependent cylindrical disturbance which travels both upstream and downstream and diminishes rapidly as time increases. For sufficiently long time \( t \), the general expression for the unsteady disturbance can be obtained by the method of stationary phase. It is found that for a given finite value of \( \mu \), the effect of the unsteady disturbance becomes more serious as the downstream distance from the body increases. It is also shown that the initial unsteady effect is, at lower Froude numbers, more persistent and therefore takes a longer time to subside. A comparison of the predicted centerline wave profile due to a submerged 9 feet long, 7 to 1, Rankine ovoid started from rest with DTMB data has been made; the results are in very good agreement.
INTRODUCTION

The problem of submerged bodies or ships in accelerated motion is primarily of interest in connection with towing-tank experiments. No matter how quickly the final desired speed is attained by the model, the question always arises as to how long it takes before the effect of the starting conditions becomes inappreciable. This question has been discussed by Havelock (1), (2), Maruo (3) and Wehausen (4) in dealing with the wave resistance of submerged bodies or of thin ships. The effect of the initial acceleration upon the wave pattern of submerged bodies has received, however, very little attention. It is the purpose of this report to investigate the surface wave pattern of simple submerged bodies started from rest. The result is of practical importance in connection with the experimental determination in towing tanks of the surface waves made by submarine models. The theoretical results for the particular case of a Rankine body are compared with experiments conducted at the David Taylor Model Basin.

GENERAL FORMULATION

The governing equation for submerged bodies moving through an incompressible, inviscid fluid in irrotational motion may be shown to be

\[ \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \]  

[1]
where

\begin{align*}
x, y, z & \text{ are the cartesian coordinates and the origin is taken on the mean free surface with } O-z \text{ vertically upwards,} \\
\phi & \text{ is the perturbation velocity potential and is assumed to be small.}
\end{align*}

The perturbed fluid velocity \( \vec{q}(u, v, w) \) is given by

\[ \vec{q} = -\nabla \phi = -\text{grad} \phi \]  \[ \text{[2]} \]

The linearized boundary conditions at the free surface may be expressed as

\[ \begin{aligned}
\frac{\partial \phi}{\partial t} - g \zeta & = \text{const.} \quad \text{(dynamical condition)} \\
\frac{\partial \phi}{\partial z} + \frac{\partial \zeta}{\partial t} & = 0 \quad \text{(kinematic condition)}
\end{aligned} \]  \[ \text{at } z = 0 \]  \[ \text{[3]} \]

where \( \zeta \) is the free surface elevation,
\( g \) the gravitational acceleration, and
\( t \) the time variable.

At the rigid submerged body surface it is evident that the condition

\[ \frac{\partial (\phi + Ux)}{\partial n} = 0 \]  \[ \text{[4]} \]
must hold, in which \( \frac{\partial}{\partial n} \) represents differentiation along the normal to the boundary surface and where \( U \) is the free stream velocity.

Boundary condition \([4]\) can generally be satisfied by distributing appropriate singularities inside or on the surface of the body. In order to satisfy boundary conditions \([3]\) and \([4]\) simultaneously, a complementary function, in addition to the prescribed singularities, is needed. It can be shown, Reference 3, that the total velocity potential must have the form

\[
\phi = \phi_0 + \phi_1
\]

\[
= \phi_0 - \frac{1}{\pi} \int_0^{\pi} \int_{-\infty}^{\infty} G(\kappa, \theta, \tau) e^{i(\kappa - 1\tilde{\omega})} \sin \left( \sqrt{\kappa} (t - \tau) \right) \sqrt{\kappa} \, d\kappa \, \, \text{d}x \tag{5}
\]

where

- \( \phi_0 \) is the velocity potential due to the prescribed singularities and depends mainly on the body shape.
- \( \tilde{\phi}_0 \) is the image of \( \phi_0 \).
- \( G(\kappa, \theta, \tau) \) is an arbitrary surface enclosing the singularities.

\[
G(\kappa, \theta, \tau) = -\frac{1}{4\pi} \int_{S'} \left( \frac{\partial \phi_0}{\partial n} - \phi_0 \frac{\partial}{\partial n} \right) e^{i(\kappa + 1\tilde{\omega})} \, dS
\]

\( \tilde{\omega} = x \cos \theta + y \sin \theta \).
Consider now the special case of a Rankine body (ovoid) submerged in water at a given depth, as shown in Figure 1, suddenly started from rest with a given velocity U and maintaining that speed. For a first order approximation, the fluid motion may be taken to be that due to a point source and a point sink of strength |M| distributed at the points \((0,0,-f)\) and \((C,t,-f)\) respectively. The velocity potential \(\varphi_o\) due to an impulsive point source or sink at \((x_n,y_n,z_n)\) is given by

\[
\varphi_o = \frac{M_n}{R_n}
\]  

where

\[
M_n(t) = 0 \quad t \leq 0
\]

\[
M_n(t) = M_n = \text{constant} \quad t > 0
\]

\[
R_n = \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}
\]

The function \(G\) of Equation [5] in this case is simply

\[
G = M_n e^{i(z_n + 1(x_n \cos \theta + y_n \sin \theta))} \quad [7]
\]

HYDRONAUTICS, Incorporated

\[ \phi = \frac{M_n}{R_n} - \frac{M_n}{\bar{R}_n} + \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{M_n(\tau)}{\bar{R}_n} \int_{0}^{t} e^{-\kappa(x+z_n)} \cos \tilde{\omega}'_n \sin \left\{ \sqrt{\kappa(t-\tau)} \right\} \sqrt{\kappa} \, dx \]

where

\[ \bar{R}_n = \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z+z_n)^2} \]

\[ \tilde{\omega}'_n = (x-x_n) \cos \theta + (y-y_n) \sin \theta \]

The total velocity potential for a Rankine ovoid started impulsively from rest with uniform velocity can then be shown to be

\[ \phi = \phi_{\text{source}} + \phi_{\text{sink}} \]

\[ = M \left( \frac{1}{\sqrt{x^2 + y^2 + (z+f)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z-f)^2}} + \frac{1}{\sqrt{(x+l)^2 + y^2 + (z+f)^2}} - \frac{1}{\sqrt{(x+l)^2 + y^2 + (z-f)^2}} \right) \]

\[ + \frac{1}{\pi} \int_{-\pi}^{\pi} M(\tau) d\tau \int_{0}^{t} e^{-\kappa f + \kappa z} \left[ \cos \tilde{\omega}'_2 - \cos \tilde{\omega}'_1 \right] \sin \left\{ \sqrt{\kappa(t-\tau)} \right\} \sqrt{\kappa} \, dx \]

with

\[ \tilde{\omega}'_1 = x \cos \theta + y \sin \theta \quad \tilde{\omega}'_2 = (x+l) \cos \theta + y \sin \theta \]

\[ l = \text{distance between the point source and sink} \]
It is seen from Equation [3] that the mean free surface elevations can be expressed as

\[ \zeta = \frac{1}{g} \left. \frac{\partial \varphi}{\partial t} \right|_{z=0} \]

\[ = \frac{1}{g} \frac{\partial}{\partial t} (\varphi - \varphi_0) + \frac{1}{\pi} \int_{0}^{t} \int_{-\pi}^{\pi} M(k, \theta, \tau) e^{ik(z-i\bar{w})} \cos\left\{ \sqrt{gk}(t-\tau) \right\} \, dk \]

at \( z=0 \)

Consider first the case of an impulsive point source started from rest with velocity \( U \) which is maintained constant; the mean surface wave height can then be expressed as

\[ \zeta = \frac{1}{\pi} \text{Re} \left[ \int_{0}^{t} \int_{-\pi}^{\pi} M(\tau) \, d\tau \int_{0}^{\infty} \int_{-\pi}^{\pi} -x e^{-ikx} \cos\sqrt{gk}(t-\tau) \, dx \right] \]

[11]

with

\[ \bar{w} = \left[ (x-x_0) + U(t-\tau) \right] \cos \theta + y \sin \theta \]

\text{Re} denotes the real part of

Integrating with respect to \( \tau \), Equation [11] becomes
The limiting value of the regular wave $\zeta_r$ as $t$ becomes infinite may be derived from the principle values of the integrals in Equation [12]. The steady local disturbance $\zeta_s$ arises from the first integral in Equation [12]. The steady surface wave pattern ($\zeta_s = \zeta_r + \zeta_1$) due to a point source has been discussed in detail in a previous report by Yim (5). The primary interest here is to evaluate approximately the effect the initial time-dependent disturbances have on the general wave profiles.

For sufficiently large positive values of $t$, the surface wave heights due to the initial disturbances may be evaluated from the following double integral by applying the method of stationary phase twice in succession:
HYDRONAUTICS, Incorporated

\[
\zeta_1 = \frac{1}{\pi} \frac{M}{U} R_t \left\{ \int_{-\pi/2}^{\pi/2} d\theta \int_0^\infty \frac{x e^{-\kappa x}}{1} \left[ e^{\frac{1}{8}\left(\kappa \bar{w}_1 - \sqrt{gnt} \right)} + e^{\frac{1}{8}\left(\kappa \bar{w}_1 + \sqrt{gnt} \right)} \right] \right\}
\]

with

\[
\bar{w}_1 = \left[ (x-x_0) + Ut \right] \cos \theta + y \sin \theta = R_1 \cos(\theta - \delta) > 0
\]

\[
R_1 = \sqrt{(x-x_0 + Ut)^2 + y^2}
\]

\[
\varepsilon_1 = \tan^{-1} \left( \frac{y}{x-x_0 + Ut} \right)
\]

Evaluating \( \bar{w}_1 \) with respect to \( x \), Equation [13] has the form

\[
\zeta_1 \propto \frac{1}{\pi} \frac{M}{U} R_t \left\{ \int_{-\pi/2}^{\pi/2} \exp \left[ \frac{g t^3}{4 R_1^3 \cos^3(\theta - \delta)} - 1 \left( \frac{g t^3}{4 R_1 \cos(\theta - \delta)} - \frac{\pi}{4} \right) \right] d\theta \right\}
\]

\[
-\pi/2 \quad \sqrt{R_1^3 \cos^3(\theta - \delta)} \left[ \cos \theta - \frac{2 R_1 \cos(\theta - \delta)}{Ut} \right]
\]

[14]

The initial wave heights \( \zeta_1 \) for sufficiently large time \( t \), may then be shown to be
This formula however is not valid in the neighborhood of $R_1 = 0$ and $R_0 = Ut/2 \cos \delta_1$ since in these regions the non-circular part of the integrand in Equation [14] varies rather rapidly and the method of stationary phase therefore fails to apply. The correct evaluation of $\zeta_1$ around these points, which is generally very tedious and has little bearing on the outcome, will not be pursued further.

The mean water free surface elevation due to a Rankine ovoid started impulsively from rest with constant velocity $U$ can be obtained by superposing the values due to a point source and sink with distance $t$ apart. The resulting disturbance may be separated into three parts as Havelock did in (1) for the two-dimensional case (see Figure 2).

(1) A steady local disturbance ($\zeta_4$) which traveled both upstream and downstream and diminishes as the distance from the body increases.
(i) A group of steady regular waves ($\zeta_r$) which travel downstream from the origin $x-x_0=x=0$ only to $x=-1/2 U t$ with group velocity $1/2 U$, and behave like $x^{-3/2}$ far downstream ($1 \ll x/\lambda < U t/2\lambda$).

(ii) A time-dependent cylindrical disturbance ($\zeta_t$) which travels both upstream and downstream and diminishes rapidly as time increases.

The first two parts have been studied in (5). The last part, for sufficiently long time $t$, is simply

$$\zeta_t \approx -2/2 M U \left\{ \frac{-gt^3}{4R_1} f e \sin \frac{gt^2}{4R_1} \cos \delta_1 - \frac{-gt^3}{4R_2} f e \sin \frac{gt^2}{4R_2} \cos \delta_2 \right\}$$

[16]

where

$$R_1 = \sqrt{(x+U t)^2 + y^2}, \quad R_2 = \sqrt{(x+U t)^2 + y^2}$$

$$\delta_1 = \tan^{-1} \left( \frac{y}{x+U t} \right), \quad \delta_2 = \tan^{-1} \left( \frac{y}{x+U t} \right)$$

respectively.

In order to assess quantitatively the initial unsteady disturbance on the overall wave motion, simple numerical calculations are made in the following section.
HYDRONAUTICS, Incorporated

-12-

NUMERICAL RESULTS AND DISCUSSION

For simplicity only the waves on the centerline of the mean water free surface due to a Rankine ovoid started impulsively from rest are to be discussed in detail here. From Equation [16], taking $y = 0$, the unsteady cylindrical wave pattern $\zeta_1$ for sufficiently long time takes the form

$$
\zeta_1 = +2\varepsilon \left( \frac{M}{U} \right) \frac{1}{U_1} \left[ -\frac{gT}{4\rho^2 \left( 1 + \frac{x}{U_1} \right)^2} e^{\frac{-\rho^2}{U_1} \left( \frac{1 + x}{U_1} \right)} \sin \left( \frac{gt}{4U_1 \left( 1 + \frac{x}{U_1} \right)} \right) - \frac{gT}{4U^2 \left( 1 + \frac{x}{U_1} \right)^2} e^{\frac{-\rho^2}{U_1} \left( \frac{1 + x}{U_1} \right)} \sin \left( \frac{gt}{4U \left( 1 + \frac{x}{U_1} \right)} \right) \right]
$$

where

$$
\zeta_1 = 2 \sqrt{2} \left( \frac{M}{U} \right) \left( \frac{1}{U_1} \right) e^{\frac{-\rho^2}{U_1} \left( \frac{1 + x}{U_1} \right)} \left( \frac{1 + x}{U_1} \right) \left( \frac{1 + 2x}{U_1} \right)
$$

is defined as the amplitude of the cylindrical unsteady disturbance,
\[ A_1 = \frac{(1 + \frac{x}{Ut})(1 + \frac{2x}{Ut})}{(1 + \frac{x+t}{Ut})[1 + \frac{2(x+t)}{Ut}]} \exp \left\{ -\frac{1}{4F^2} \left[ \left( \frac{1}{1 + \frac{x+t}{Ut}} \right)^{-\frac{3}{2}} - \left( \frac{1}{1 + \frac{x}{Ut}} \right)^{-\frac{3}{2}} \right] \right\} \]

in general, of \( O(1) \).

\[ F = \frac{U}{\sqrt{g}f} \]

is the Froude number based on depth.

It can be seen that the amplitude of the unsteady cylindrical wave, \( \zeta_1 \), for a fixed value of \( x \), diminishes rapidly like \( 1/t \) with increasing \( t \). At a given time \( t \), the value of \( \zeta_1 \) decreases upstream but increases downstream as the distance from the body increases. It is, therefore, to be expected, that the effect of the unsteady cylindrical disturbance on the general steady wave pattern would be more serious far downstream. To illustrate, it is assumed that the distance from the body downstream is sufficiently large so that the steady local disturbance is negligibly small and the regular wave can be evaluated by the method of stationary phase, i.e.,

\[ \zeta_8 = \zeta_{\zeta_1} + \zeta_{\zeta_8} \approx \zeta_8 \]

\[ \zeta_8 = \zeta_{\zeta_1} + \frac{1}{4T} \left[ \frac{1}{|x|^\frac{3}{2}} \cos \left( \frac{g|x|}{U^2} + \frac{\pi}{4} \right) - \frac{1}{|x+t|^\frac{3}{2}} \cos \left( \frac{g|x+t|}{U^2} + \frac{\pi}{4} \right) \right] \]

\[ = \zeta_8 \left[ \cos \left( \frac{g|x|}{U^2} + \frac{\pi}{4} \right) - A_8 \cos \left( \frac{g|x+t|}{U^2} + \frac{\pi}{4} \right) \right] \quad [18] \]
where

$$\zeta_s = \pi \frac{M}{U} \left| \frac{x}{\lambda x} \right|^{1/2} e^{-\frac{1}{P^2}}$$

is the amplitude of the steady wave profile far downstream

$$1 < x/\lambda < Ut/2$$

$$A_s = \left| \frac{x}{x+U} \right|^{1/2} \sim \gamma(1)$$

$$\lambda = \frac{2\pi U^3}{g}$$

is the centerline wave length.

From Equations [17] and [18], the amplitude ratio of the unsteady disturbance and the steady wave is found to be

$$\frac{\zeta_1}{\zeta_s} = \frac{1}{2\sqrt{2\pi}} \frac{|\lambda x|}{Ut} e^{-\frac{1}{P^2}} \left[ \frac{1}{4 \left( 1 + \frac{x}{Ut} \right)^2} - 1 \right]$$

$$= \frac{1}{2\sqrt{2\pi}} \frac{|\lambda x|}{x \lambda} \frac{x}{|U|} e^{-\frac{1}{P^2}} \left[ \frac{1}{4 \left( 1 + \frac{x}{Ut} \right)^2} - 1 \right]$$

$$= \frac{1}{2\sqrt{2\pi}} \frac{|\lambda x|}{x \lambda} \frac{x}{|U|} e^{-\frac{1}{P^2}} \left[ \frac{1}{4 \left( 1 + \frac{x}{Ut} \right)^2} - 1 \right]$$

[19]

The non-dimensional value of $$\frac{\zeta_1}{\zeta_s}$$, at $$x/\lambda = 4$$, versus $$Ut/\lambda$$ for various Froude numbers $$F$$ is shown in Figure 3. It can be seen that the value of $$\zeta_1$$ diminishes very rapidly with increase in $$t$$. 
In Figure 4 the amplitude ratio $\tilde{C}_1/\tilde{C}_s$, at $Ut/\lambda = 10$, is plotted against $x/\lambda$ for various $F$. It is evident that the effect of the unsteady disturbance on the steady wave profile becomes more prominent (since the value of $\tilde{C}_1/\tilde{C}_s$ gets bigger) as the wave moves further downstream. It is also of interest to note from Figures 3 and 4, that the value of $\tilde{C}_1/\tilde{C}_s$ at a given $x$ and $t$ increases with decreasing $F$. This would seem to suggest the fact that the unsteady disturbance takes a much longer time to subside at the lower Froude numbers.

A preliminary experimental study of the Kelvin wake produced by submerged bodies has been made at the David Taylor Model Basin, (6). In those tests, a 9 feet long, 7 to 1, Rankine ovoid was towed at several submergence depths, and wave height measurements were made at various points in the basin. In Figure 5 the centerline wave pattern in a test run, at $U = 10$ ft/sec, $t = 13.22$ sec., and $F = 1.016$, together with the corresponding calculated values are shown. Since at distances very far downstream ($> 1/2 Ut$) the wave profile is entirely contributed by the unsteady disturbance as discussed in the previous section and is of no practical interest, the comparison between the theoretical calculations and experimental data is, therefore, made only from $x = 0$ (near the nose of the ovoid) extending downstream to the neighborhood of $1/2 Ut$. It is seen that the theory and experiment are in quite good agreement. The value of $F$, for the particular case studied, is rather large and the effect of unsteady disturbance on the steady surface wave profile is, in general, small as can be seen from Figure 3 or 4; the measured and the steady surface profiles
are quite similar. At lower Froude numbers, say $F = 1/2$, however, the unsteady effect is considerably larger; it is to be expected then that the true steady surface wave pattern may have quite a departure from the measured values in this latter case.
REFERENCES


FIGURE 1 - DEFINITION-SKETCH
FIGURE 2—SURFACE WAVE PATTERN OF A SUBMERGED BODY STARTED IMPULSIVELY FROM REST
FIGURE 3 - AMPLITUDE OF THE INITIAL UNSTEADY DISTURBANCE WITH $t$ AS A VARYING PARAMETER
FIGURE 4 - AMPLITUDE OF THE INITIAL UNSTEADY DISTURBANCE WITH X AS A VARYING PARAMETER
FIGURE 5 - COMPARISON OF THEORETICAL AND EXPERIMENTAL CENTERLINE WAVE PROFILES DUE TO A SUBMERGED 9 FEET LONG, 7 TO 1, RANKINE OVOID STARTED FROM REST
ON THE SURFACE WAVE PATTERN OF SUBMERGED BODIES STARTED FROM REST

Technical Report

Hsu, C. C.

April 1965

26

Technical Report 231-7

Qualified requesters may obtain copies of this report from DDC.

Office of Naval Research
Department of the Navy

The initial surface wave pattern due to a submerged body started from rest with uniform speed in a perfect fluid is studied. The mean surface elevations may, in general, be separated into three parts: (i) A steady local disturbance which travels both upstream and downstream and diminishes as the distance from the body increases, (ii) a group of steady regular waves which travel downstream from the origin x=0 to x=1/2 Ut with group velocity 1/2 U, and (iii) a time-dependent cylindrical disturbance which travels both upstream and downstream and diminishes rapidly as time increases. For sufficiently long time t, the general expression for the unsteady disturbance can be obtained by the method of stationary phase. It is found that for a given finite value of t, the effect of the unsteady disturbance becomes more serious as the downstream distance from the body increases. It is also shown that the initial unsteady effect is, at lower Froude numbers, more persistent and therefore takes a longer time to subside. A comparison of the predicted centerline wave profile due to a submerged 9 feet long, 7 to 1, Rankine ovoid started from rest with DTMB data has been made; the results are in very good agreement.
1. Submerged bcly
2. Velocity potential
3. Impulsive motion
4. Surface waves
5. Unsteady disturbance

UNCLASSIFIED

Security Classification

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the acceptable number of the contract or grant under which the report was written.

8b. & 8c. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through...

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through...

(5) "All distribution of this report is controlled. Qualified DDC users shall request through...

INSTRUCTIONS

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (Tl), (T2), (TS), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.