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Memorandum of Project MICHIGAN

HALL MEASUREMENTS OF THIN LAYERS OF SEMICONDUCTORS (U)

ALFRED E. ATTARD

INFRARED AND OPTICAL SENSOR LABORATORY
INSTITUTE OF SCIENCE AND TECHNOLOGY
THE UNIVERSITY OF MICHIGAN

June 1965

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NOTICES

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PREFACE

Project MICHIGAN is a continuing, long-range research and development program for advancing the Army's combat-surveillance and target-acquisition capabilities. The research and development effort is oriented toward achieving new and improved techniques which will lead to new or greatly improved combat-surveillance and target-acquisition equipment that will meet the long-range operational requirements of the Army in the field. Sponsored by the U. S. Army Electronics Laboratories, of the U. S. Army Electronics Command, of the U. S. Army Materiel Command, this Project is carried out by a full-time Institute of Science and Technology staff of specialists in physics, engineering, mathematics, and related fields, by members of the teaching faculty, by graduate students, and by other research groups and laboratories of The University of Michigan.

The Project's emphasis is on the subjects of imaging radar, MTI radar, infrared-optical imaging and signal correlation techniques, image interpretation, and data transmission beyond the line of sight. Project MICHIGAN was established at The University of Michigan in 1953 and has received continuing support from the U. S. Army. The Project constitutes a major portion of the diversified program of research conducted by the Institute of Science and Technology. The function of the Institute of Science and Technology is to make available to government and industry the resources of The University of Michigan and to broaden the educational opportunities of students in the scientific and engineering disciplines.

Documents issued in this series of Technical Memorandums are published by the Institute of Science and Technology in order to disseminate scientific and engineering information as speedily and as widely as possible. The work reported may be incomplete, but it is considered to be useful, interesting, or suggestive enough to warrant this early publication. Any conclusions are tentative, of course. Also included in this series are reports of work in progress which will later be combined with other materials to form a more comprehensive contribution in the field.

Progress and results described in reports are continually reassessed by Project MICHIGAN. Comments and suggestions from readers are invited.

Robert L. Hess
Director
Project MICHIGAN
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HALL MEASUREMENTS OF THIN LAYERS OF SEMICONDUCTORS

ABSTRACT

Hall measurements on thin films via a four-point probe method are described. Basic formulas are derived for an infinite sheet. Modifications of the infinite-sheet formula to correct for boundary conditions imposed by real, finite samples are discussed. Experimental evidence is presented which verifies the theory.

1 INTRODUCTION

Present trends in semiconductor technology reflect an increased interest in epitaxial layers of semiconducting materials on various kinds of substrates, as in surface-diffused layers in semiconducting materials. The accurate determination of the true carrier concentration in these thin layers is the subject of this paper.

The theory of measurement of sheet and bulk resistivities of semiconductors via four-point probe methods has been discussed by various authors [1, 2, 3]. In this paper, the theory of Hall measurements on thin samples by means of a four-point probe technique is developed for the case of an infinite sheet. Further, it is assumed that there is no conductivity modulation caused by the injection of carriers. The effects of sample thickness and geometry are discussed in relation to measurements on real, finite samples. (Since the completion of the work described here, two papers have been published on Hall measurements in thin samples [4, 5].)

2 INFINITE-SHEET FORMULA

Consider a current dipole (consisting of a current source and a current sink, each of current strength I) in an infinite sheet of thickness t. The potential at a point in the plane of the sheet can be demonstrated to be (see Appendix A)

\[ \psi - \psi_0 = \frac{\rho}{2\pi t} \ln \left( \frac{r_1}{r_2} \right) \]  \hspace{1cm} (1)
where \( r_1 \) and \( r_2 \) are the distances from the point to the source and the sink, respectively, and \( \rho \) is the volume resistivity. Let the source and sink be located at \( \left( \frac{s}{2}, 0 \right) \) and \( \left( -\frac{s}{2}, 0 \right) \), respectively, in the \( x, y \) plane. The electric field components at a point \((x, y)\) are then,

\[
E_x = K \left[ \frac{s + x}{(s + x)^2 + y^2} + \frac{s - x}{(s - x)^2 + y^2} \right] 
\]

\[
E_y = Ky \left[ \frac{1}{(s + x)^2 + y^2} - \frac{1}{(s - x)^2 + y^2} \right] 
\]

\[ K = \frac{1}{2\pi\rho} \]

On the median plane, where \( x = 0 \), the field components become

\[
E_x = \frac{Ks}{s^2 + y^2} 
\]

\[
E_y = 0 
\]

Consider now the application of a magnetic field \( H \) in a direction perpendicular to the plane. The electric-field components under steady-state conditions can be shown to be (see Appendix B)

\[
E_x = E_x^0 + \mu HE_y^0 
\]

\[
E_y = E_y^0 - \mu HE_x^0 
\]

where \( E_x^0 \) and \( E_y^0 \) are the electric-field components for \( H = 0 \)

\( \mu \) = the mobility

On the median plane, where \( x = 0 \), these field components become

\[
E_x = \frac{Ks}{s^2 + y^2} 
\]

\[
E_y = \frac{-\mu Hks}{s^2 + y^2} 
\]

The Hall probes are located on the median plane at \( \left( 0, \pm \frac{d}{2} \right) \). The Hall voltage is given by (see Appendix C)
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\[ V_H = \frac{2 \mu H \rho}{\pi t} \arctan \left( \frac{d}{s} \right) = \frac{2H I}{\pi e N} \arctan \left( \frac{d}{s} \right) \]  

(11)

where \( N \) is the carrier concentration.

Equation (11) represents the case of a single carrier for which no allowance was made for the difference between Hall and drift mobilities. The corresponding equation, for a single carrier, which takes into account this difference is (see Appendix D)

\[ V_H = \frac{2HI}{\pi e t} \frac{1}{N < \tau^2 >} \arctan \left( \frac{d}{s} \right) \]  

(12)

The corresponding equations for a two-carrier model is (see Appendix E)

\[ V_H = \frac{2HI}{\pi e t} \left[ \frac{-N \tau^2 > n^2 + \frac{P \tau^2 > p^2}{m_e n^2 + m_h p^2} \right] \arctan \left( \frac{d}{s} \right) \]  

(13)

For \( \tau_n = \tau_p = \tau \), this equation reduces to

\[ V_H = \frac{2HI}{\pi e t} \frac{1}{< \tau^2 >} \left[ \frac{-N \mu_n^2 + P \mu_p^2}{(N \mu_n + P \mu_p)^2} \right] \arctan \left( \frac{d}{s} \right) \]  

(14)

3

BOUNDARY-EDGE CORRECTION

The modification of the potentials due to boundary edges of a semi-infinite sheet is now considered. Analysis is by the method of images. The image-current sources must be of the proper sign, magnitude, and position so that the boundary conditions are satisfied. For a nonconducting edge, the image-current source must be of the same sign as the object-current source in order to satisfy the condition that the current at the boundary (and hence the electric field) vanish in a direction normal to the boundary. For a conducting edge, the sign of the image-current source must be opposite to satisfy the condition that the electric field at the boundary vanish in a direction parallel to the boundary. Consider the boundary edge to be at a distance \( \xi \) from the origin. Let \( \theta \) be the angle between the edge and the x-axis measured clockwise from the negative x-axis (see Fig. 1). The object-current sources are located at \( \left( \frac{s}{2}, 0 \right) \), and the Hall probes are located at \( \left( 0, \frac{d}{2} \right) \).
The perturbation of the Hall voltage due to the image-charges is determined for two cases: 

First, for the contribution to the Hall voltage due to the additional electrostatic potentials produced by the image charges for zero magnetic field, and second, for the contribution due to the magnetic coupling of these additional field components.

Case 1: Zero Field Terms. The net contribution to the potential difference measured between the Hall probes as a result of the boundary edge is easily shown to be (see Appendix F)

$$\Delta V_H = \pm \frac{1}{2\pi t} \ln \frac{AB}{CD}$$  (15)

where

$$A = \left( -\frac{s}{2} \cos 2\theta - 2t \sin \theta \right)^2 + \left( \frac{d}{2} + \frac{s}{4} \sin 2\theta - 2t \cos \theta \right)^2$$  (16)

$$B = \left( \frac{s}{2} \cos 2\theta - 2t \sin \theta \right)^2 + \left( -\frac{d}{2} - \frac{s}{4} \sin 2\theta - 2t \cos \theta \right)^2$$  (17)

$$C = \left( \frac{s}{2} \cos 2\theta - 2t \sin \theta \right)^2 + \left( \frac{d}{2} - \frac{s}{4} \sin 2\theta - 2t \cos \theta \right)^2$$  (18)

$$D = \left( -\frac{s}{2} \cos 2\theta - 2t \sin \theta \right)^2 + \left( \frac{d}{2} + \frac{s}{4} \sin 2\theta - 2t \cos \theta \right)^2$$  (19)

Note that the perturbing potential vanishes when $\theta = 0, \frac{\pi}{2}, \pi, \text{ or } \frac{3\pi}{2}$. For these symmetry cases, the perturbing effect of the edge vanishes for any potential function of the central-force.
type. The use of the plus or minus sign depends upon whether the edge is conducting (+) or insulating (-).

Case 2: Magnetic-Coupling Terms. The magnetic-field-induced contribution to the Hall voltage can be shown to be of the form
\[ \Delta V_H = -\mu_H \int_{-d/2}^{d/2} \Delta E_x \, dy \] (see Appendix G):

\[ \Delta V_H = \frac{\pm \mu H \rho}{2\pi} \left[ \arctan \frac{A_1}{B_1} - \arctan \frac{A_2}{B_1} - \arctan \frac{A_3}{B_2} + \arctan \frac{A_4}{B_2} \right] \] (20)

\[ A_1 = \frac{s}{2} \sin 2\theta - 2\ell \cos \theta + \frac{d}{2} \] (21)
\[ A_2 = \frac{s}{2} \sin 2\theta - 2\ell \cos \theta - \frac{d}{2} \] (22)
\[ A_3 = -\frac{s}{2} \sin 2\theta - 2\ell \cos 2\theta + \frac{d}{2} \] (23)
\[ A_4 = -\frac{s}{2} \sin 2\theta - 2\ell \cos 2\theta - \frac{d}{2} \] (24)
\[ B_1 = -\frac{s}{2} \cos 2\theta - 2\ell \sin \theta \] (25)
\[ B_2 = \frac{s}{2} \cos 2\theta - 2\ell \sin \theta \] (26)

For the case where \( \theta = 0 \), the perturbing potential difference is:

\[ \Delta V_H(\theta = 0) = \frac{2\mu H \rho}{\pi \ell} \frac{1}{2} \arctan \frac{1 - \frac{d}{4\ell}} {\frac{s}{4\ell}} - \arctan \frac{1 + \frac{d}{4\ell}} {\frac{s}{4\ell}} \] (27)

For the case where \( \theta = \frac{\pi}{2} \), the perturbing potential difference becomes:

\[ \Delta V_H(\theta = \frac{\pi}{2}) = \frac{2\mu H \rho}{\pi \ell} \frac{1}{2} \arctan \frac{\frac{d}{4\ell}} {1 + \frac{s}{4\ell}} - \arctan \frac{\frac{d}{4\ell}} {1 - \frac{s}{4\ell}} \] (28)

Expansion of the arctangent terms into infinite series permits evaluation of the coefficient of \( \frac{2\mu H \rho}{\pi \ell} \) for the two cases. For \( \theta = 0 \), the series expansion is

\[ \pm \frac{1}{2} \sum_{r=1}^{\infty} \frac{(-1)^r (s/4\ell)^{2r-1}}{(2r-1)(4\ell)^{2r-1}} \left[ \frac{1 + \frac{d}{4\ell}} {1 - \frac{d^2}{16\ell^2}} \right] \] (29)
For $\theta = \frac{\pi}{2}$, the series expansion is

$$\frac{1}{2} \sum_{r=1}^{\infty} \frac{(-1)^r (d)}{(2r-1)(4r)} \left( \frac{1 + \frac{s}{2r-1}}{4r} - \frac{1 - \frac{s}{2r-1}}{4r} \right)^{2r-1} \left( \frac{1 - \frac{s^2}{16r^2}}{16r^2} \right)^{2r-1}$$

The two forms are symmetric with respect to the interchange of $s$ and $d$ (as could be expected from the geometric symmetry). To facilitate the calculation of the boundary edge effect, we will let $s = d$.

The correction factor in terms of the percentage of variation in measurement is displayed in Fig. 2 as a function of $t/s$. For $t/s > 3$, the correction amounts to less than 1\%. (Note that $t/s = \frac{1}{2}$ corresponds to the boundary edge being at the probes, and that this "worst case" corresponds to an error of 23\%). Note that the magnitude of the correction is independent of the conductivity of the boundary edge.

4 CORRECTIONS FOR BOTTOM SURFACE

The perturbation of the Hall voltage due to the bottom surface is determined for the electrostatic zero-field case. Remarks about the magnetic coupling terms follow.

Case 1: Zero-Field Terms. The modifications of the potentials due to the effect of the bottom surface can be conveniently calculated with the use of Uhlir's M-functions [2]. However, it should be noted that the conclusions are, again, independent of the detailed form of the potential function, provided that it is of the central-force type.

For a nonconducting bottom surface, the potential at some point $(x, y)$ in the plane of the sheet will be modified by

$$\Delta \varphi = \frac{-1\rho}{4\pi t} \left[ M \left( \frac{F_3}{2t} \right) - M \left( \frac{F_4}{2t} \right) \right]$$

For a conducting bottom surface, the potential will be modified by

$$\Delta \varphi = \frac{-1\rho}{4\pi t} \left[ M \left( \frac{F_3}{2t} \right) - M \left( \frac{F_4}{2t} \right) - M \left( \frac{F_3}{4t} \right) + M \left( \frac{F_4}{4t} \right) \right]$$
FIGURE 2. PERCENTAGE ERROR DUE TO MAGNETIC COUPLING. Edge Boundary: 
\[ s = d; \theta = 0, \pi/2. \]

where

\[ r_3 = \sqrt{\left(x + \frac{a}{2}\right)^2 + y^2} \]  \hspace{1cm} (33)
\[ r_4 = \sqrt{(x - \frac{s}{2})^2 + y^2} \]  

(34)

\[ M(x) = 2 \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{\sqrt{n^2 + x^2}} \right) \]  

(35)

For both of these cases, the perturbing potential will vanish for a point on the median plane where \( x = 0 \). Hence, the measured Hall voltage will not depend on whether the bottom surface is conducting or insulating.

Case 2: Magnetic Coupling Terms. Magnetic coupling terms were calculated for the case of an edge boundary because the presence of such a boundary does indeed change the current flow from that assumed for the original thin infinite sheet model. However, a correction for the effect of sample thickness is not necessary, because in the original model the current flow was assumed to be bounded by the upper and lower surfaces. Thus, a "correction factor" of the magnetic-coupling type was already built into the theory.

5 CONCLUSIONS AND EXPERIMENTAL VERIFICATION

Corrections to the Hall voltage necessitated by the effects of edge boundaries can be made small. The magnitude of such an effect is independent of whether the edge is conducting or insulating. Consequently, corrections to the Hall voltage are zero and also are independent of whether the bottom surface is conducting or insulating.

Measurements have been made to verify this latter conclusion. The Hall measurements were made on pieces of a thin slice of GaAs which had been doped with Te to an atomic concentration of approximately \( 1 \times 10^{17} / \text{cm}^3 \). The pieces had irregular boundary edges, and no attempt was made to orient the Hall probes with respect to any edge. The results are tabulated in Table I. The numbers refer to the individual pieces of the same slice. The agreement is good, except for sample 3. This particular sample was almost too small to fit between the current and the Hall probes.
### TABLE I. HALL-MEASUREMENT DETERMINATION OF CARRIER CONCENTRATION (77°C)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Backing</th>
<th>( n(\text{exp}) )</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None</td>
<td>( 1.53 \times 10^{17} ) /cm(^3)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Gold (1)</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>None</td>
<td>2.31</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Gold (1)</td>
<td>6.57</td>
<td>sample barely touching probes</td>
</tr>
<tr>
<td>4</td>
<td>None</td>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gold (2)</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>None</td>
<td>1.66</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Indium (1)</td>
<td>1.71</td>
<td>sample reoriented between measurements</td>
</tr>
<tr>
<td>5</td>
<td>Indium (3)</td>
<td>(1.12)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.57)</td>
<td></td>
</tr>
</tbody>
</table>

(1) evaporated  
(2) conductive paint  
(3) soldered

### SUMMARY

Equations for the four-point Hall measurements of thin films were derived and experimentally verified, and the effects of boundaries were determined. It was found that the perturbing effects of boundary edges can be made small and that the perturbing effect of the conducting state of the bottom surface is zero. It must be emphasized that a boundary must exist on the bottom surface. Indeed, this is true for the case of semiconductors deposited on metal or insulating substrates, for the case of a surface-diffuse p-n junction where the junction itself forms a high-resistance boundary layer, and for the case of heterojunction layers. The method of analysis described here should therefore be useful in evaluating epitaxial layers of semiconductors on various kinds of substrates.
Appendix A
THE POTENTIAL FUNCTION IN A THIN SHEET

Consider the potential \( \phi \) at a distance \( r \) from a current source of magnitude \( I \). For an infinite sheet, when \( r \gg t \), the thickness, we may assume cylindrical symmetry for the current density \( J \):

\[
I = 2\pi rt J
\]

The potential at point \( r \) can be determined with reference to some standard point at \( r_0 \):

\[
\phi(r) - \phi(r_0) = \int_{r_0}^{r} dR = \sum_{K} I_{K} R_{K}
\]

where \( dR = \rho \frac{dr}{A} \) = incremental resistance
\( I' \) = current along path of integration = \( JA \)
\( \rho \) = volume resistivity
\( A \) = area of "tube" of current flow

Therefore,

\[
\phi(r) - \phi(r_0) = \int_{r_0}^{r} \rho \frac{dr}{2\pi} = \frac{\rho}{2\pi} \ln\left(\frac{r}{r_0}\right)
\]

For a dipole where \( r_1 \) is the distance of the point from the source \( I \), and \( r_2 \) is the distance from the sink \(-I\), the potential is

\[
\phi - \phi_0 = \frac{\rho}{2\pi} \ln\left(\frac{r_1}{r_2}\right)
\]

Appendix B
ELECTRIC-FIELD COMPONENTS IN THE PRESENCE OF A MAGNETIC FIELD

The Lorentz-force equation for a charged particle of charge \( e \) moving with velocity \( \vec{V} \) in an electric field \( \vec{E} \) and a magnetic field \( \vec{H} \) is

\[
\vec{F} = m \ddot{\vec{V}} = e(\vec{E} + \vec{V} \times \vec{H})
\]

The current density \( \vec{J} \) given in terms of the free carrier concentration \( n \) is

\[
\vec{J} = ne\vec{V}
\]
The change in this current \( d_1 \mathbf{J} \) in the direction of \( \mathbf{J} \) due to collisions in time \( dt \) is given by

\[
d_1 \mathbf{J} = -\mathbf{J} \frac{dt}{\tau}
\]  

(42)

where \( \tau \) is the relaxation time for collisions.

In steady-state conditions, the net change in current \( d\mathbf{J} \) is zero; by definition, therefore,

\[
d\mathbf{J} = ne \dot{\mathbf{V}} dt - \mathbf{J} \frac{dt}{\tau} = \left[ \frac{ne^2}{m} \mathbf{E} + \frac{ne}{m} \mathbf{J} \times \mathbf{H} \right] dt - \mathbf{J} \frac{dt}{\tau} = 0
\]

(43)

Whence,

\[
\mathbf{J} = \frac{ne^2}{m} \mathbf{E} + \frac{e}{m} \mathbf{J} \times \mathbf{H} = ne \mu \mathbf{E} + \mu \mathbf{J} \times \mathbf{H}
\]

(44)

where

\[
\mu = e\tau / m
\]

The current components, then, are

\[
\begin{align*}
J_x &= \frac{ne\mu}{1 + \mu H^2} \left[ E_x 0 + \mu H E_y 0 \right] = ne\mu' E_x \\
J_y &= \frac{ne\mu}{1 + \mu H^2} \left[ E_y 0 - \mu H E_x 0 \right] = ne\mu' E_y
\end{align*}
\]

(45)

(46)

where

\[
\mu' = \frac{\mu}{1 + \mu H^2} = \text{magnetic mobility}
\]

(47)

\[
\mathbf{E} = E_x 0 + E_y 0
\]

(48)

The magnetoresistivity is, then,

\[
\rho = \rho_0 \left( 1 + \mu^2 H^2 \right)
\]

(49)

This is for the case of a single and constant relaxation time \( \tau \). If \( \tau \) is not the same for all carriers, or if \( \tau \) is dependent upon other parameters, the magnetoresistivity term will be expressed by a more complicated equation.
Appendix C

HALL-VOLTAGE EVALUATIONS

\[ V_H = -\int_{-d/2}^{d/2} (E_y) \, dy \]

\[ = -\frac{\mu H I \rho S}{2\pi} \int_{-d/2}^{d/2} \frac{dy}{\sqrt{\frac{S}{2} + y^2}} \]

\[ = -\frac{\mu H I \rho S}{2\pi} \left[ \arctan \left( \frac{y}{\sqrt{\frac{S}{2}}} \right) \right]_{-d/2}^{d/2} \]

\[ = -\frac{\mu H I \rho}{\pi} \left[ \arctan \left( \frac{d}{\sqrt{S}} \right) \right. \]

\[ = -\frac{2\mu H I \rho}{\pi} \arctan \left( \frac{d}{S} \right) \cdots (52) \]

Appendix D

RELAXATION TIME FOR A SINGLE-CARRIER CHARGE

Let

\[ \mu_i = \frac{e\tau_i}{m} \]

be the microscopic mobility of the i-th type of carrier. The corresponding microscopic conductivity \( \sigma_i \) is

\[ \sigma_i = \frac{n_i e^2 \tau_i}{m} \cdots (54) \]

where \( n_i \) is the carrier density for the i-th type of carrier.

The ensemble average of the K-th power of the macroscopic relaxation time is given by

\[ \langle \tau^K \rangle = \frac{\sum n_i \tau_i^K}{n} \]

where \( n = \sum n_i \cdot 12 \).
Equation (10) must then be modified to take into account the variation of the relaxation time. To do this, we consider the microscopic current density $J_i$ for the $i$-th type of carrier characterized by a velocity $V_i$:

$$E_{y_i} = \mu_1 \frac{H S I \rho}{2 \pi t} \frac{1}{\frac{S}{2} + y^2} = \mu_1 Q$$

(56)

where $I$ and $\rho$ are gross macroscopic quantities, $\mu_1$ refers to the microscopic mobility, and $E_{y_i}$ refers to the microscopic fields.

Now,

$$J_y = \sigma E_y = \sum_i \sigma_i E_y = \sum_i \sigma_i E_{y_i} = e^2 \frac{m}{e} \sum_i n_i \tau_i = \frac{3}{2} \sum_i n_i \tau_i^2$$

(57)

Therefore,

$$\frac{e^2}{m} E_{y} <T> = \frac{3}{2} Q <T>$$

(58)

and

$$E_y = \frac{e^2}{m} <T> \frac{H I \rho}{2 \pi t} \frac{S}{2} + y^2$$

(59)

$$= \frac{e^2}{m} <T> \frac{S H I}{2 \pi t} \frac{S}{2} + y^2$$

$$= \frac{1}{n e \pi t} <T> \frac{S}{2} + y^2$$

(60)

The Hall voltage is then,

$$V_H = -\int_{-d/2}^{d/2} E_y |_{y=0}^{x=0} dy$$

(61)
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$$\frac{-H_1 <r^2>}{\pi \text{tn} e <r^2>^2} \int \frac{S}{2} \frac{d}{y^2} dy$$

$$= \frac{2H_1 <r^2>}{\pi \text{nt} n <r^2>^2} \arctan \left( \frac{d}{S} \right)$$

(62)

Appendix E

TWO-CARRIER MODEL

For electrons and holes, the $y$ components of the current densities for electrons $J_{ny}$ and for holes $J_{py}$ are given as

$$J_{ny} = \frac{n e \mu_n}{1 + \mu_n \frac{2}{H}} [E_y + \mu_n H E_x]$$

(63)

$$J_{py} = \frac{p e \mu_p}{1 + \mu_p \frac{2}{H}} [E_y - \mu_p H E_x]$$

(64)

The $y$ component of the total current density is

$$J_y = \sigma E_y = \sum_1 \sigma_{n_i} E_{y_i} + \sum_j \sigma_{p_j} E_{y_j} = (\sigma_n + \sigma_p) E_y$$

(65)

$$\left[ \sum \sigma_{n_i} + \sum \sigma_{p_j} \right] E_y = \left[ \sum \sigma_{n_i} \mu_{n_i} + \sum \sigma_{p_j} \mu_{p_j} \right] Q$$

$$= [\sum \sigma_{n_i} \mu_{n_i} + \sum \sigma_{p_j} \mu_{p_j} \frac{H}{2} \pi t \frac{1}{\sum \sigma_{n_i} + \sum \sigma_{p_j} (\frac{S}{2})^2 + y^2}]$$

(66)

Whence,

$$E_y = \frac{\sum \sigma_{n_i} \mu_{n_i} + \sum \sigma_{p_j} \mu_{p_j} \frac{H}{2} \pi t \frac{S}{2} (\frac{S}{2})^2 + y^2}{\left[ \sum \sigma_{n_i} + \sum \sigma_{p_j} \right]}$$

(67)
\[ E_y = \frac{n e^3 \tau^2}{m_e^2} + \frac{p e^3 \tau^2}{m_h^2} \]

The Hall voltage is evaluated as before:

\[ V_H = \frac{2HI}{\pi} \frac{\arctan \left( \frac{d}{s} \right)}{\left( \frac{S^2}{2} + y^2 \right)} \]

Appendix F

BOUNDARY-EDGE CORRECTIONS: ELECTROSTATIC TERMS

F.1. LOCATION OF THE IMAGE CHARGES

Let the source be located at \( X_1, Y_1 \) and the image at \( X_2, Y_2 \). Let \( X_0, Y_0 \) be a point on the intersection of the mirror plane and the line joining the image and object points. Let \( X, Y \) be any point on the image plane.

\[ Y = -\frac{B}{A} X + B \]

Let \( X', Y' \) be any point on the line joining image and object points.

\[ Y' = \frac{A}{B} X' + K \]

Obviously, when

\[ Y = Y' \]

and

\[ X = X' \]

then,

\[ Y = Y' = Y_0 \]

\[ X = X' = X_0 \]

15
Further,

\[ X_2 - X_1 = 2(X_0 - X_1) \]
\[ Y_2 - Y_1 = 2(Y_0 - Y_1) \]  

(72)

Let \( \theta \) be the angle between the nearer plane and the \( X \) axis measured in counterclockwise direction from the mirror plane (see Fig. 1). Then,

\[ \tan \theta = B/A \]
\[ \sin \theta = \ell/A \]
\[ \cos \theta = \ell/B \]  

(73)

where \( \ell \) is the normal distance to the mirror plane measured from the origin.

Using the above relations, we obtain

\[ X_2 = \frac{(A^2 - B^2)X_1 - 2ABY_1 + 2AB^2}{A^2 + B^2} \]
\[ Y_2 = \frac{-2ABX_1 - (A^2 - B^2)Y_1 + 2A^2B}{A^2 + B^2} \]  

(74)

The coordinates of the image point \( X_2, Y_2 \) are then obtained in terms of the mirror distance to the origin \( \ell \) and the mirror angle \( \theta \) (Fig. 1), and the object point coordinates \( X_1, Y_1 \):

\[ X_2 = X_1 \cos 2\theta - Y_1 \sin 2\theta + 2\ell \sin \theta \]
\[ Y_2 = -X_1 \sin 2\theta - Y_1 \cos 2\theta + 2\ell \sin \theta \]  

(75)

F.2. ELECTROSTATIC CONTRIBUTION TO THE HALL VOLTAGE

The change in electrostatic potential caused by the two image charges, when the object charges are at \((S/2, 0)\) is, then,

\[ \Delta \phi(X, Y) = \frac{IP}{2\ell} \ln \left[ \frac{\left[ (X - S/2 \cos 2\theta - 2\ell \sin \theta)^2 + (Y + S/2 \sin 2\theta - 2\ell \cos 2\theta)^2 \right]^{1/2}}{\left[ (X + S/2 \cos 2\theta - 2\ell \sin \theta)^2 + (Y - S/2 \sin 2\theta - 2\ell \cos 2\theta)^2 \right]^{1/2}} \right] \]  

(76)

The contribution to the voltage difference measured between the Hall probes at \((0, \pm \ell/2)\) is, therefore,
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\[ \Delta V_H = \Delta \phi(0, d/2) - \Delta \phi(0, -d/2) \]  

(77)

and Eq. (15) in the text is obtained.

Appendix G
BOUNDARY-EDGE CORRECTIONS: MAGNETIC-COUPLING TERMS

The variation of the X component of the field is

\[ \Delta E_X = \frac{\partial}{\partial X} \Delta \phi(X, Y) \]  

(78)

where \( \Delta \phi(X, Y) \) is derived as in Appendix F. The contribution to the Hall voltage is

\[ \Delta V_H = -\mu H \int_{-d/2}^{d/2} \Delta E_X \bigg|_{X=0} \, dy \]  

(79)

Equation (20) in the text is then obtained by simple but laborious algebra.
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