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THIRD QUARTERLY TECHNICAL PROGRESS REPORT

DIRECT AND INDIRECT ELECTRON CONTROL
BY PHOTOCONDUCTIVE ELEMENTS

(20 December 1964 through 20 March 1965)

DA 44 009-AMC-579(T)

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The negative resistance amplification method is discussed and the electrical requirements for an ideal Esaki diode for this application are given.

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THIRD QUARTERLY TECHNICAL PROGRESS REPORT

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DIRECT AND INDIRECT ELECTRON CONTROL
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SUMMARY

This report contains further work on the two methods of controlling electron space currents previously reported. * The temperature dependent electron energy distribution function for electric tunnel effect emitters is developed and curves corresponding to three temperatures are presented. The cumulative probability distribution function for electrons is derived and is shown to be a measure of energy spread of the tunnel electrons. Curves are given for two useful temperatures.

The theoretical basis for control of electron space current by energy filtering (selection) is derived and its application to photoconductor control is shown. Gain requirements and means of achieving adequate gain are discussed. The negative resistance amplifier method is further developed and electrical requirements for the semiconductor element are detailed.

*Ref. 1
PREFACE

The following people contributed to this study during the report period:

J. P. DeBarber
R. D. Laughlin
R. S. Norris

The project director was R. D. Laughlin
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I. TUNNEL EFFECT ELECTRON EMITTERS

A. INTRODUCTION

In the second quarterly Technical Progress Report a new method for obtaining a modulated electron space current was shown to be feasible. Elsewhere in the present report this method has been discussed in further detail. One of the more important features of this analysis is that large gain is possible through two factors; a large voltage stress across the photoconductor and a very narrow energy spread of the electron source.

Since one is limited in the magnitude of the voltage across a semi-conducting photoelement by power dissipation and breakdown considerations, the other option is the only one susceptible to manipulation.

In the Second Progress Report we analyzed electric tunnel effect cold cathode devices to determine their use as narrow spread electron emitters. We showed that, without regard to the present availability of such emitters, they had promise of being able to produce the desired spread of electron energies. Again let us point out the distinction between the true ETF emitter and the various kinds of internal field emission processes which also emit electrons in cold cathode devices but with large electron energy spreads.

We derived the zero temperature and the moderate temperature electron energy distribution function which gave the relative tunnel current (number of electrons) as a function of electron energy. These results were put in terms of the generalized potential barrier of arbitrary shape due to Simmons.

To show how the effect of increasing temperature affects the energy distribution function, we will continue the analysis and plot the results for various temperatures. The cumulative probability distribution function for elevated (nonzero) temperatures will also be derived. This is a new result.

These results apply only to the tunnel electrons as they enter the second metal electrode which serves as the conductor which applies the field to the dielectric spacer. In many practical cases this metal is gold (Au) applied over the dielectric spacer to a thickness of about 200 A. The actual energy distribution of the emitted electrons is the product of the P(E) versus E curve
which will be given in this section and a transmission \( T(E) \) characteristic for
the gold film. Within the assumptions and simplifications made, the \( N(E) \)
derivation given here is accurate. On the other hand both theory and experi-
ment for range-energy relationships which can give a \( T(E) \) vs \( E \) curve are
not at present extremely accurate. However, it is known that hot electrons
in gold exhibit a transmission such that lower energy electrons are more
easily transmitted than fast electrons. Due to this phenomenon, the distribu-
tion function should be narrowed rather than broadened since the gold effect-
ively acts as a low pass "filter" for electrons. This is in the correct direction
for use in an electron space current control scheme for image intensifiers.
Thus there are reasons to believe that properly prepared ETE emitting diodes
should have a very narrow spread of electron energies.

B. THE TEMPERATURE DEPENDENT EQUATION FOR \( P(E) \)

In the progress Report #2, Eq. 44, was derived to show how the number of
electrons impinging upon electrode #2 varies with electron energy. To cal-
culate the points for the curves at various temperatures, we will use the
following equation (rather than Eq. 44) since it does not involve the approxi-
mation \( \ln (1 + \exp Z) \approx \exp Z \). The curves are therefore somewhat more
accurate than if Eq. 44 had been used. The correct equation is

\[
P(E)_T = \frac{4 \pi m k T}{h^3} \ln \left\{ 1 + \exp \left( \frac{E - \eta}{k T} \right) \right\} \exp \left[ -A \phi \left( \frac{h}{2} \right) + \left( E - \eta \right) A / 2 \phi \left( \frac{h}{2} \right) \right] \tag{1}
\]

To be specific, the following parameters have been chosen to approximate
closely an actual physical case.

\[
\phi = 1 \text{eV} \quad \Delta S = 50 \text{Å} \quad B = 1 \quad A = 1.02 B \Delta S \approx \Delta S \tag{2}
\]

The significance of these symbols is detailed in the previous Progress
Report #2. For the plots three temperatures; 0\(^{\circ}\)K, 77\(^{\circ}\)K and 300\(^{\circ}\)K have
been chosen while keeping the physical dimensions of the hypothetical tunnel emitter fixed. At zero degrees Kelvin the functional part of the equation becomes, after the insertion of the values given in Equation 2,

\[ P(E) = (E - \eta) \exp 25 (E - \eta - 2) \]  

(3)

At liquid nitrogen temperature (77°K)

\[ P(E)_{770} = \ln \left( 1 + \exp -150 (E - \eta) \right) \exp 25 (E - \eta - 2) \]  

(4)

At room temperature (\( \approx 300^\circ K \) \( kT = 1/40 \) ev)

\[ P(E)_{300} = \ln \left( 1 + \exp -40 (E - \eta) \right) \exp (E - \eta - 2) \]  

(5)

Equations 3, 4, and 5 have been plotted as a function of \( E - \eta \) and are shown in Figure 1. All the curves have been normalized to a peak value of unity.

The position of the peak value on the \( (E - \eta) \) scale for the zero degrees K case may be found analytically from

\[ E - \eta = -2\frac{\phi}{2A} \]  

(6)

Using the values given in equation 2, the value of \( E - \eta \) is 0.04 ev. and this is verified by Figure 1 as may be seen by inspection of the curve.

Inspection of Figure 1 shows that at zero degrees there are no electrons having energies greater than the Fermi energy (an expected result) and that the high energy tail is quite sharp, falling rapidly from unity at \( (E - \eta) = 0.04 \) ev to zero at \( E - \eta = 0 \). For the 77°K curve, the high energy portion of the curve shows only a small added "tail" that is relatively sharp and the lower part is indistinguishable from the zero degree curve. At room temperature (\( kT = 0.025 \) ev), the high energy part is very slow in falling to zero and the energy spread is large. Unless the transmission coefficient \( T(E) \) for the upper metal film is a very rapidly declining function of electron energy, room
FIG. 1 - TUNNEL ELECTRON NUMBER DISTRIBUTION (NORMALIZED) AS A FUNCTION OF ELECTRON ENERGY MINUS FERMI ENERGY

FIG. 2 - CUMULATIVE NUMBER DISTRIBUTION FUNCTION (ENERGY SPREAD) PLOTTED VS (E-\eta)
temperature operation of an ETE would not give the desired narrow energy spread of emitted electrons. Cooling to nitrogen temperatures or perhaps thermoelectric cooling by Peltier refrigeration would reduce the spread substantially.

C. THE CUMULATIVE PROBABILITY DISTRIBUTION FUNCTION $P(X)$

Since we have developed the temperature dependent equation for $P(E)$, it is now possible to find the cumulative distribution function $P(X)$. This function was found for the zero temperature case in Progress Report #2 (Eq. 37) and it shows the percentage of the total electrons incident upon the top electrode which lie below any given energy. In effect, $P(X)$ is a measure of the spread of energies of the electrons. This concept is believed to be of more use than determining the spread of energies as taken at, for example, the 50 percent points or some other arbitrary point.

To find the cumulative distribution function $P(X)$, we integrate equation 1 and normalize the results to unity. In Progress Report #2 we had let $X = \eta - E$ but we now find that the negative of this parameter is more useful and we shall let $X = E - \eta$ for the rest of this development. Making the substitution $X = E - \eta$ in equation 1 gives

$$p(X) = \frac{4\pi m kT}{h^3} \exp \left(-A \phi^2 \frac{X}{2}\right) \ln \left(1 + \exp \left(-\frac{X}{kT}\right)\right) \exp \left(\frac{AX}{2\phi^2}\right)$$

(7)

This is of the form (without constants)

$$p(\lambda) = \ln \left(1 + \exp (-a\lambda)\right) \exp b \lambda$$

(8)

Therefore, the desired cumulative distribution function $P(X)$ is

$$P(X) = \int_{X}^{\infty} \ln \left(1 + \exp (-a\lambda)\right) \exp b \lambda d\lambda$$

(9)
A change of variable helps solve this integral. Make the following substitutions.

\[ Z = e^{-a\lambda} \quad \text{so} \quad dZ = -aZd\lambda \]  \hspace{1cm} (10)

\[ Z^n = e^{b\lambda} \quad \text{so} \quad n = b/a \]

Thus, when \( \lambda = \infty \) \( Z = 0 \) and when \( \lambda = \chi \) \( Z = e^{-a\chi} \)

This transforms equation 9 into

\[ P(\chi) = -\frac{1}{a} \int_0^\infty \frac{\ln(1+Z)Z^{n-1}dZ}{\exp-a\chi} = -\frac{1}{a} \int_0^\infty \frac{\ln(1+Z)Z^m dZ}{\exp-a\chi} \]

where \( m = n-1 \).

Integrating by parts yields

\[ P(\chi) = -\frac{1}{an} \left\{ \left[ Z^n \ln(1+Z) \right]_0^\infty - \int_0^\infty \frac{Z^n dZ}{\exp-a\chi (1+Z)} \right\} \]  \hspace{1cm} (11)

or

\[ P(\chi) = -\frac{1}{an} \left\{ -e^{b\chi} \ln(1+\exp-a\chi) - \int_0^\infty \frac{Z^n dZ}{\exp-a\chi (1+Z)} \right\} \]  \hspace{1cm} (12)

The integral in equation 12 may be further reduced by a change of variable as follows

\[ \int_0^\infty \frac{Z^n dZ}{\exp-a\chi (1+Z)} = -\int_0^\infty \frac{dy}{1+\exp-a\chi y (y+1)^{n+1}} = F(\chi) \]  \hspace{1cm} (13)
where \( y = \frac{1 + Z}{Z} \). Note the change in the limits of integration due to the change of variable. In general, \( F(\chi) \) is not tabulated for all values of \( n \) but it is tabulated for \( n = 1/2 \). This case has a real physical meaning with regard to the tunnel emitter. The general result is given by

\[
P(\chi) = -\frac{1}{\alpha n} \left\{ -e^{-b\chi} \ln(1 - \exp -a\chi) + \int_{1 + \exp a\chi}^{\infty} \frac{dy}{y(y+1)^{n+1}} \right\}
\]  

(14)

In Eq. 14 the values of \( a \) and \( b \) have been identified by comparison with Eq. 1 as

\[
a = \frac{1}{kT} \quad b = A/2e^{-1/2}
\]

(15)

\[
n = -\frac{b}{a} = \frac{-AkT}{2e^{-1/2}}
\]

Substituting gives

\[
P(\chi) = \frac{2e^{-1/2}}{A} \left\{ -\exp \left( \frac{A\chi}{2e^{-1/2}} \right) \ln(1 + \exp \frac{-\chi}{kT}) + \int_{1 + \exp \frac{\chi}{kT}}^{\infty} \frac{dy}{y(y-1)^{1/2} - AkT} \right\}
\]

(16)

If the dielectric spacer has a width of 40 A and \( kT = 0.025 \) ev (room temperature), the exponent \( n + 1 \) in the integral of equation 16 is very close to one half. Therefore, for this special, but useful, case we get the final result (normalized)

\[
P(\chi) = \frac{1}{\pi} \left\{ -\exp b\chi \ln(1 + \exp -a\chi) + 2\left( \frac{\pi}{2} - \tan^{-1} \exp \frac{a\chi}{p} \right) \right\}
\]

(17)
The limits of equation 17 are seen to behave properly since at \( X = -\infty \), 
\( P(X) = 1 \) and at \( X = +\infty \), \( P(X) = 0 \). Plots of this equation for room temperature and for zero temperature are shown for comparison in Figure 2.

For the zero temperature case, it may be seen that 90% of the total number of electrons lie within a band of width 0.16 ev, situated immediately below the Fermi level.

For the case where the temperature of the tunnel emitter is 300°K, the curve is not so easily analyzed since the high energy end projects into the region of positive values of \( X \). Inspection of the curve shows that 90% of the total number of electrons falls within a band of width \( \approx 0.24 \) ev. This band is situated such that 25% of the total electrons is within 0.1 ev above the Fermi level and 65% is within 0.14 ev below the Fermi level. The cumulative distribution curve \( P(X) \) was not drawn for the case of the temperature of 77°K (liquid nitrogen) because of the difficulty in evaluating the integral. However, in this case the exponent \( n + 1 \) is approximately unity since \( n \) is small and the curve would lie close to the curve already drawn for the zero temperature case.
FIG. 3 - BASIC CONTROL STRUCTURE

FIG. 4 - GRAPHICAL REPRESENTATION OF ELECTRON CONTROL PROCESS
II. SPACE CURRENT CONTROL BY ENERGY FILTERING

A. THE GENERAL GAIN EQUATION

The triode control scheme and the method of Auphan, as described in the Second Quarterly Technical Progress Report, are essentially the same. Both effect control by filtering electrons on the basis of energy. Thus, it is possible to develop a general theory and to establish a general performance criterion. In this section these matters are discussed.

Consider a source which emits electrons having a velocity distribution such that the probability density function of the energy, \( V \), associated with any one of the velocity components is \( p(V) \) where \( V \) is given in electron volts. Now suppose that a fine mesh grid is placed between the source and a collector as shown in Figure 3. The grid is at a potential \( V_1 \) volts negative with respect to the source and the collector is held at a slightly positive potential. The collector current, \( i \), is comprised of electrons having \( X \)-directed energies in excess of \( V_1 \) and which do not collide with the grid structure. Thus,

\[
i = I \int_{V_1}^{\infty} a(V) p(V) \, dV \quad (18)
\]

where \( I \) is the emission current and \( a(V) \) is the transmittance of the grid. The latter is taken as a function of \( V \). If \( V_1 \) is altered by a small amount, \( \Delta V_1 \), the corresponding change in collector current, \( \Delta i \), is given by

\[
\Delta i = I a(V_1) p(V_1) \Delta V_1 \quad (19)
\]

The significance of Eq. 18 and Eq. 19 is shown graphically in Figure 4. The fractional change in collector current, \( \Delta i / i \), that results from a fractional change in grid voltage, \( \Delta V_1 / V_1 \), is given by
\[
\frac{\Delta i}{i} = \frac{V_1 a(V_1) p(V_1)}{\int_{V_1}^\infty a(V) p(V) dV} \left( \frac{\Delta V_1}{V_1} \right) \tag{20}
\]

We have previously shown in the Second Quarterly Technical Progress Report that

\[
\frac{\Delta V_1}{V_1} = \frac{\Delta R}{R} \tag{21}
\]

where \( R \) is the resistance of the photoconductive element. Eq. 21 represents an optimum value and, under this condition, \( V_1 \) represents the bias voltage appearing across the photoconductive element. Therefore,

\[
\frac{\Delta i}{i} = \frac{V_1 a(V_1) p(V_1)}{\int_{V_1}^\infty a(V) p(V) dV} \left( \frac{\Delta R}{R} \right) \tag{22}
\]

Now define

\[
F(V_1) \equiv \frac{V_1 a(V_1) p(V_1)}{\int_{V_1}^\infty a(V) p(V) dV} \tag{23}
\]

\( F(V_1) \) may be regarded as a contrast enhancement factor and as such it represents a figure of merit for the control process. In the Second Quarterly Technical Progress Report it was estimated that \( F(V_1) = 10^4 \) is required for marginal feasibility of a night sky intensification scheme.
B. ENERGY FILTERING BY TRANSMISSION GRIDS

One of the primary questions to be answered, then, in considering this kind of gain mechanism, is whether $F(V_1)$ can be made to be as large as $10^4$. It must be noted that this treatment is general and therefore a control method based on modulating an electron stream using grid-like structures will have the limitations placed on it as shown above.

Further, it is not adequate to produce a grid structure and emitter that will yield an $F_1(V)$ factor to say $10^2$ and then further amplify this variation with high gain devices such as a channel multipliers. This is true since there exists along with the signal variation (as enhanced by $F(V_1)$), the steady electron stream composed of those electrons corresponding to the cross-hatched area shown in Figure 4. This steady flow is analogous to the quiescent dc plate current in an ordinary vacuum tube amplifier. In that case the steady current may effectively be removed by the simple expedient of using a capacitor as a coupling means to a subsequent stage of amplification. Only the alternating signal variations are passed and further amplification may be effected to almost any desired level. At present we have no simple analog to use in the electron beam case under discussion. If such an energy transmission filter could be found, a large post amplification could be effected by channel or other electron multiplying means and adequate contrast gain would be assured.

It is instructive to examine Eq. 6 for some assumed functions of emitted electron energy probability density and grid transmittance. A Maxwellian distribution of velocity of emission results in an exponential energy probability density,

$$\rho(V) = \frac{1}{V_0} \exp \left( -\frac{V}{V_0} \right)$$

where $V_0$ is the most probable energy of an emitted electron. It is convenient to represent the grid transmittance as an exponential,
\[ a(V) = K \exp\left(-\frac{V}{V_c}\right) \]  \hspace{1cm} (25)

where \( K \) is a constant and \( V_c \) will be referred to as the cutoff energy. With these assumed functions the contrast enhancement factor, \( F(V_1) \), becomes

\[ F(V_1) = \frac{V_1}{V_0} \left(\frac{V_0 + V_c}{V_c}\right) \] \hspace{1cm} (26)

The first factor, \( \frac{V_1}{V_0} \), in this equation is the same as for the case in which the transmittance of the grid is independent of \( V \) (\( V_c \rightarrow \infty \)) and the second factor represents an improvement resulting from the energy dependence of the transmittance of the grid. It is clear that the transmittance must favor low energy electrons and that a significant improvement is achieved only if \( V_c \ll V_0 \). While this exercise is based upon simplifying and perhaps unwarranted assumptions, some useful conclusions can be drawn. The value of the ratio, \( \frac{V_1}{V_0} \), is probably limited to 1000 as demonstrated in the Second Quarterly Technical Progress Report. Thus, the transmittance of the grid must display a cutoff energy less than 0.1 \( V_0 \) if an overall enhancement factor of \( 10^4 \) is to be realized.

The exponential transmittance assumed in Eq. 25 cannot be realized by a simple wire mesh but perhaps another loss mechanism may be found which favors the transmission of only the signal electrons. Such a transmission is exhibited by certain thin metallic films.
III. NEGATIVE RESISTANCE AMPLIFICATION

In the Second Quarterly Technical Progress Report, there was advanced the idea of employing a negative resistance element to enhance the photoconductor's control of a space current. The scheme is shown in Figure 5. Here $R$ represents the resistance of the photoconductor and $i$ is the space current. It is shown readily that,

$$\frac{\Delta i}{i} = - \frac{1}{1 + \frac{f_1'(i) + f_2'(i)}{R}} \frac{\Delta R}{R}, \quad (27)$$

where $f_1(i)$ and $f_2(i)$ are respectively the volt-ampere characteristics of the diode and the negative resistance element and $f_1'$ and $f_2'(i)$ are the associated dynamic resistances. In the SQTPR the conditions for stable operation with gain are discussed. There it is shown that useful and stable operation of the circuit requires that the composite characteristic, $f_1(i) + f_2(i)$, displays negative dynamic resistance, that the bias is adjusted so that operation takes place in this negative resistance region, and that

$$2R > |f_1'(i) + f_2'(i)| > R. \quad (28)$$
The more closely \( |f'_1(i) + f'_2(i)| \) approaches R, the larger the magnitude of the gain factor. When operated in this way, the circuit responds to a decrease in photoconductor resistance (an increase in infrared radiation) with a decrease in current so that if a mosaic of such elemental circuits is employed as an image intensifier, there would result a negative image.

It is necessary to investigate further into the requirements of the elements involved. First, the fact that the composite characteristic, \( f_1(i) + f_2(i) \), must display a region of negative resistance requires that \( f_2(i) \) also displays a region of negative resistance since the diode characteristic, \( f_1(i) \), is a monotonically increasing function of \( i \). In the light of Eq. 2 this may be written as,

\[
-2R - f'_1(i) < f'_2(i) < - R - f'_1(i). \tag{29}
\]

It is clear that the diode must not be emission limited, for in that case \( f'_1(i) \longrightarrow \infty \) and \( i \) cannot be controlled by \( R \). In the more favorable situation in which \( f'_1(i) \ll R \), we can write

\[
-2R < f'_2(i) < - R. \tag{30}
\]

For the sake of simplicity we shall neglect the diode drop and use Eq. 30.

In order to achieve high gain it is necessary that conditions be adjusted so that \( f'_2(i) \) be very nearly equal to the negative of \( R \). The negative resistance magnitude for Esaki diodes is of the order of 100 ohms while the resistance of a lead sulfide photoconductor cell is about 100,000 ohms. The disparity prevents any hope of securing stable operation, let alone securing any gain. However, it is possible and instructive to specify the Esaki diode characteristics required. For this purpose it is convenient to idealize the Esaki diode characteristic as shown in Figure 6. This idealization shows that the characteristic displays a negative resistance for \( E_p < v < E_{V_1} \). The voltages \( E_p \) and \( E_{V_1} \) are primarily dependent upon the semiconductor material and are largely fixed. For germanium
these voltages are respectively and approximately 55 mV and 200 mV. The magnitude of the negative resistance, \( R \), is given by,

\[
R = \frac{E_{V_1} - E_p}{I_p - I_v} \text{ ohms.} \tag{31}
\]

For \( R = 10^5 \) and assuming that germanium is employed

\[
I_p - I_v = 1.45 \text{ microamperes.}
\]

Since we require a dynamic range of at least 10 percent, a peak point current, \( I_p \), of the order of 14.5 microamperes is implied. Although we have used a grossly oversimplified model of the tunnel diode, it has served to define the general requirements of the element. In brief, we require a tunnel diode displaying a peak point current of the order of 14.5 microamperes with a peak point current to valley current ratio of 1:1. For materials other than germanium the numbers involved will be somewhat different but not significantly so.
IV. CONCLUSIONS AND FUTURE PLANS

The conclusions which may be drawn as a result of the reported work are as follows: As a result of the general analysis given under the sections dealing with space current control by energy filtering, we now have an equation which relates fundamental quantities such as the energy distribution of the emitted electron beam, the transmittance of the selecting grid and the voltage at which the selection is accomplished, to the contrast gain of such devices when used for image intensifier purposes. The ways in which this gain may be varied and thus made large is shown to depend upon two factors: The width of the distribution function and the voltage stress across the photoconductor. If a transmittance element can be found that has the proper energy cutoff dependence, gains may be greatly enhanced over that available from the previously described methods. A search for such an element will be commenced.

The tunnel electron (ETE) source has been found to exhibit some of the basic properties necessary for a narrow spread source and the tunnel current's distribution as a function of operating temperature has been shown to narrow sharply with refrigeration to LN$_2$ temperatures.

Negative resistance amplification is, in principle, possible but as shown in the report, Esaki diodes of the correct operating characteristics are not at present available and the requirements are very stringent and perhaps not physically realizable.

As a result of this work it appears desirable to do several things. First, the Auphan method for controlling electron beams should be exploited to find the maximum gain that can be achieved. This would require a device which would emit a very narrow energy spread electron beam to achieve meaningful experimental work. Second, a means of energy filtering of the beam issuing from an Auphan type device followed by post electron gain seems attractive and possible. In this way very large gains should be available. This second method of contrast amplification will be investigated during the last quarter of the present contract.
REFERENCES
