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Contract Nonr 266/041
Task 047-005

Department of Industrial and Management Engineering
Columbia University
New York, N.Y. 10027

FINAL REPORT

Sebastian B. Littauer
Project Director

15 May, 1965
The following is a final summary report on Contract Nonr 266(04), Task No. 047-005, as of the date of termination of this contract June 30, 1964. This report covers some eleven years during the incumbency of the present project director. It includes in the first part a statement of the scientific accomplishment, in the second part a review of the various honors accorded scientists associated with this contract. Reference is made to a number of the scientific contributors.

Scientific Accomplishments

Some 30 scientists were associated with this contract, a number of them quite distinguished. Principal among these were Professors J. Wolfowitz, A Dvoretzky and J. Kiefer. Among other well known scientists who worked on the contract for longer and shorter periods were

L. Hurwicz \hspace{1cm} B. Mandelbrot
S. Kakutani \hspace{1cm} J. Marschak
G. Kallianpur \hspace{1cm} J. Tiago de Oliveira
T. Koopmans \hspace{1cm} L. Weiss

A number of graduate students had the opportunity to develop while working on this contract. Some of them did
Excellent work and produced fine dissertations. Outstanding among these is Lawrence D. Brown.

Many directions were pursued contributing to the main aims of the contract. Specific directions receiving considerable attention are: 1) inventory policy, 2) information theory, 3) optimal design of experiments, and 4) statistical control principles. We are summarizing essential highlights of the scientific achievements during the life of the contract.

One of the principal and most effective contributions to the contract was the work done by A. Dworetsky, J. Kiefer and J. Wolfowitz in 1952 and 1953 on optimal inventory policy. This work was published in two parts in Econometrica Vol. 20, 1952 pages 187-222 and 250-466. The impact of these papers was so great that they are quoted in almost all texts on operations research, management science, and inventory policy. These papers also listed in some works on economics. Besides that, they are referred to extensively in the literature on inventory policy and mathematical statistics. The importance of these two papers was so great that the scientific officer asked the project director to have a paper more accessible to practitioners and users of inventory policy, prepared so that it could be published for wide circulation. This paper was to have a more applied perspective, simplification of expositions, and numerical examples. Towards this end Dr. Laderman and Professor Weiss joined with the project director in preparing a paper entitled, "The Inventory Problem" which was published
in the Journal of the American Statistical Association, December 1953, pages 717 to 732. This paper was reprinted and has very wide circulation; it is also referred in texts and periodical literature to a considerable extent. Another very important paper was published by Dworetsky, Kiefer and Wolfowitz, namely, "On the Optimal Character of the \( (s,S) \) Policy in Inventory Theory," in Econometrics Vol. 21, 1953, pages 596-596. One may without exaggeration, say that the foregoing represents a very important and stimulating phase of the scientific work which has great potential application.

An outstanding achievement was made by Professor Wolfowitz in the publication of his book entitled, *Coding Theorems of Information Theory*. This book contains mostly original contributions by Professor Wolfowitz, some of them solutions of outstanding problems, one of which is represented in his paper, "Strong Converse of the Coding Theorem for Finite Memory Channels." This work represents a marked advance in information theory.

Kiefer and Wolfowitz did some very interesting and original work culminating in two papers: "Optimum Extrapolation and Interpolation Designs." This work appeared in the *Annals of Mathematical Statistics* following the termination of this contract. It does, however, represent important original scientific work done during the tenure of this contract.

Professor Kiefer did fundamental work in optimum experimental designs. He comprehensively investigated the field

This does not exhaust the work done by either Professors Wolfowitz or Kiefer individually or jointly. For example, Professor Wolfowitz contributed some very interesting analytical methods called the "minimum distance method." Professors Kiefer and Wolfowitz worked on certain sequential decision procedures. Professor Kiefer joined with Professor Bechhofer or Cornell and Sobel of Minnesota in preparing of a book entitled, Sequential Multiple-Decisions Identification and Ranking Procedures prepared for the Chicago University Press.

The preceding resume indicates adequately the quality and the quantity of scientific research performed by Professors Kiefer and Wolfowitz.

Professor Lionel Weiss in addition to the joint work he has done has contributed to a number of directions of research, namely, among others, limiting distributions of homogenous sample spaces and minimum tests of goodness of fit.

Among the variety of other work that has been performed on the contract, one might single out the work of J. Marschack
and H.D. Block on random ordering of choice among alternatives, of L. Hurwicz and H.D. Block on stability of competitive equilibrium and of Mandelbrot on stable probability distributions with applications to Pareto type laws of distribution, because of their bearing on economics.

Another direction of application has been to statistical control and acceptance sampling in life testing. Professors de Oliveira and Littauer have established a rigorous determination of double control limits which give a whole range of flexibility to the user in the choice of statistical control limits. In addition they have applied these results to determining sample size and the intersample spacing interval, based on certain cost considerations in industrial production. They have also developed a run criterion which includes an outer limit c and a parameter R representing the run length above or below the mean value of the sample's selected. The double control limit has been tabulated and the run criterion is still in the process of tabulation. Two reports have been submitted, the first one of which has been reformulated and translated into French for forthcoming publication in Revue de Statistique Applique. A second paper, "The Economic Choice of Control Chart Parameters," together with tables and graphs is being prepared for the same journal at the invitation of the editor. Under the sponsorship of Professors de Oliveira and Littauer a doctoral dissertation was completed by I. Gibra in pursuit of work deriving from these researches.
Professor de Oliveira has also presented papers on checking statistical assumptions and on fundamentals of inductive inference.

Professor J.H.K. Kao completed a paper on "Variables Inspection for the Weibull Distribution," a basis for acceptance sampling in life testing where the items are distributed according to Weibull type distributions.

The proceeding material does not completely represent all the work done on this contract. It does, however, give a reasonable sampling of both the quantity of the work and its quality in basic aspects of research as well as in certain applications. It can be said that the research on this contract is quite alive and is being continued not only in the work of the successor contract but in the work of many others who have followed the publications which have resulted from this research. These researches have been widely referred to in the literature.

Honors and Other Recognitions

Many of the scientists associated with this contract have received various forms of recognition and honors. These have been specified in the various periodic progress reports. Herein we will list the more recent ones with respect to the scientists most closely associated with the contract.
Professor J. Wolfowitz:

Has been president of the Institute of Mathematical Statistics;

Addressed (by invitation) the International Statistical Institute Conference held in Tokyo, June 1960, on decision functions;

Addressed (by invitation) the April meeting of the American Mathematical Society, 1961;

Invited to address the academies of science of Czechoslovakia and Hungary, 1961;

Invited to lecture on information theory before the conference on brain functions held at the University of California Medical School in Los Angeles;

Invited to address the seventh all Soviet Congress on Probability and Mathematical Statistics in Georgia, USSR, October 1963;

Invited to address the Conference on Information Theory to be held in France 1965;

Professor J. Kiefer

Chairman Joint Statistical Societies Committee of Visiting Lecturers;

Invited to give the Wald Memorial Lectures at the Institute of Mathematical Statistics Annual Meeting in Minneapolis, 1962;

Invited to address the Econometrics Society at its Annual Meeting in 1961;
Invited by the Royal Statistical Society of England to give lectures on comprehensive summarization and critical evaluation of the field of Design of Experiment at its May 1959 meeting.

Professor S.B. Littauer

Invited by the European Society for Quality Control to present the lead off paper on Scientific Aspects of Integrated Quality Control at their Fifth International Meeting held in Torino September 1961;

Elected Fellow of the American Statistical Association;

Invited to present a paper at the International Meeting of the Institute of Management Sciences held in Brussels, September 1960;

Member of the staff of the NATO Seminar on Theory and Applications of Extreme Values, held in Rolighed, Denmark June 1964;

Appointed Editor-in-Chief of MANAGEMENT SCIENCE, Series B a new periodical published jointly with the MANAGEMENT SCIENCE, Series A, by the Institute of Management Science.

Invited to address the Italian Quality Control Society in Torino, October 1965.