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ABSTRACT

Very high precision measurements of capacitance have been achieved by recent developments of transformer bridge techniques. Reference capacitors for use with these bridges must be very stable with time and temperature, as well as having other superior characteristics, all of which are very difficult to achieve with the usual mechanical assembly of conductors.

This new capacitor uses fused quartz or pyrex glass tubing simply as a support for deposited metallic electrodes, and results in capacitance values of 1 pF which are stable to within one part per million (p.p.m.) for at least six months. They have a temperature coefficient of less than 3 p.p.m./°C, a voltage coefficient of less than 1 p.p.m. for applied voltages from 10 volts to 100 volts, and stability of 1 p.p.m. after free-fall drops from 10-cm heights.

It is anticipated that a capacitor of this style will prove to be very useful as a transportable standard of capacitance, and could be used in international comparisons of capacitance values.

1. INTRODUCTION

Very high precision measurements of capacitance have been achieved by recent developments of transformer bridge techniques. Reference capacitors for this use must have very high stability with time and temperature as well as other superior characteristics.

The usual mechanically assembled capacitors often show a temperature-hysteresis characteristic even when temperature-compensating techniques are applied. This is due to a stress developed between parts of different thermal expansion coefficient, which may develop into a permanent shift of the relative position of the parts during a temperature cycle.

The author had earlier developed a three-terminal capacitor which used a fused quartz column both as a dielectric and as a base for the electrodes plated on the three surfaces of the column (Kanno and Tanaka 1959; Kanno 1961). This capacitor, however, exhibited a temperature coefficient of about 20 parts per million (p.p.m.)/°C, which is relatively large for the expansion coefficient of fused quartz. It also showed an appreciable voltage coefficient, which is not desirable for work of the highest precision. Both these defects are probably caused by the dielectric behavior of the fused quartz.

The new capacitor which is presented here uses fused quartz or pyrex tubing simply as a support for the electrodes. The defects have been almost completely removed.

2. PRINCIPLE OF THE CAPACITOR

The capacitor is constructed on a piece of fused quartz tubing. Two active electrodes are plated on opposite sides of the central portion of the inner surface.

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of the tube, and a guard electrode covers the remaining portion of the inner surface, with two narrow sections between the two active electrodes. Together they comprise a three-terminal capacitor, as shown in Fig. 1.

![Diagram of three-terminal tubular capacitor](image)

**Fig. 1.** Principle of three-terminal tubular capacitor: (a) general view, (b) cross section at H–L, (c) cross section at G.

Although both ends of the tube are open to permit the plating of the electrodes, the inner fields decrease rapidly toward the end of the tube, and when the end sections of the guard electrode are long enough, the electrical effect is the same as if the ends were closed. Lead wires are attached to each electrode through small holes drilled through the wall of the tubing in the center of the electrode area.

### 3. INTERNAL END FIELD

For this style of capacitor it is essential to establish how the field decreases toward the end of the tube. Instead of measuring the field directly, the capacitance change was measured for travel of a screw which was inserted from one end of the tube toward the center along the axis, as shown in Fig. 2. As the screw is connected to the guard electrode, it acts as a shield between the two active electrodes. Large capacitance changes will be observed where the field is strong, while small changes are expected where the field is weak, such as at the end of the tube.

An example of the capacitance change with the position of the end of the screw is shown in Fig. 3. The capacitance value increases as the screw is withdrawn from the main field between the active electrodes, and the change is approximately linear while the screw is in the main field. After the end of the
screw passes the edge of the active electrodes the change in capacitance is hardly detectable on this scale, which is an indication of the rapid decrease of the field. Another proof is given in Fig. 4, where the capacitance change for one turn of the screw is plotted on a logarithmic scale against the position of the screw expressed in units of the internal diameter $D$ of the tube. This shows that the effect decreases by a factor of approximately ten for every shift of $D/4$ away from the edge of the active electrodes. Hence, the field is roughly one millionth smaller than the main field at a point $1.5D$ away from the edge of the electrodes.
As the capacitance change for one turn of the screw decreases to less than 1 aF \((10^{-14} \text{ pF})\) at a distance \(D\) away, then for a 1-pF capacitor constructed with long active electrodes, the change now corresponds to less than 1 p.p.m. of the capacitance. This means that the effect of the open ends of the tube is almost completely negligible when considering 1 p.p.m. accuracy. This moving screw method could be used also for final fine adjustment to a specific value, or to provide a variable three-terminal capacitor.

4. SMALL CAPACITOR

Smaller capacitance values are obtained, of course, with smaller sizes of the active electrodes. However, this approach reaches a limit, and it is very difficult to achieve values less than 0.001 pF even with the smallest electrodes on a tube of the dimensions shown in Fig. 2.

Very small capacitance values were easily obtained, however, with the shifted electrode method. As the field decreases rapidly in the region of the guard electrode, a longitudinal shift of one electrode with respect to the other will produce a very small effective capacitance value, while still using electrodes of reasonable size. The decrease of capacitance value with increasing displacement is shown in Fig. 5 for a given size of the active electrodes. A reasonably good logarithmic variation is shown for capacitance change with displacement of the electrodes, in the region where there is no overlap of electrodes.

5. VARIABLE CAPACITOR

A variable capacitor was made using the shifted electrode method described in the previous section. It is made up of two insulated glass plates and a case as
shown in Fig. 6. A conducting surface is deposited on each plate to form a rectangular active electrode insulated from the guard electrode which covers the remainder of the plate. The bottom plate is fixed in the metallic case, and the longer top plate extends through the ends of the case to permit variation of the relative position of the two active electrodes.

The change of capacitance between the active electrodes with change of relative position is shown in Fig. 7 for two lengths \( a \) of the electrodes and two separations \( d \). Here good linearity on a logarithmic scale is also shown. In Fig. 8 these curves are normalized to one curve on the basis of near-edge distance \( S_n \) between active electrodes, measured in units of separation distance \( d \) between electrodes. The capacitance change remains linear for several decades in the logarithmic scale, with a ten-fold decrease for every \( d/2 \) change of \( S_n \).

6. FIXED CAPACITOR

The original purpose of this type of capacitor was to obtain a very stable fixed standard capacitor. Many fixed capacitors were made and tested using fused quartz or pyrex glass tubes, with the electrodes deposited by evaporated metallic films or conductive paints. Using tubes of 21 mm inside diameter and
Fig. 6. Variable capacitor using displaced electrodes.

Fig. 7. Capacitance change with position of movable electrode (dimensions in mm).
175 mm length, capacitance values varying from 1 pF to 0.1 pF were obtained.

The effect of resistance of the thin-film electrodes and poor insulation between electrodes arising from incorrect forming of the electrodes is considered in Appendix I. It was found that these effects could be made negligible, and the design conditions necessary are outlined in Appendix I. This explanation also accounts for the negative conductance which was occasionally observed.

Although the original style of capacitor has the electrodes plated on the inside surface of the tube as shown in Fig. 9(a), a modified type with the electrodes plated on the outside as in Fig. 9(b) was also tried. In this case the material of the tube is now placed in the main field between the two active electrodes, but it was found that when the wall thickness of the tube is relatively small compared to the distance between the electrodes, the effects due to the tube material are reduced by approximately \( \frac{1}{\epsilon} \) from the effects which would be found if the whole space were filled by the tube material, where \( \epsilon \) is the specific dielectric constant of the material and \( t \) is the relative thickness of the walls with respect to the distance between the electrodes, as considered in Appendix II. This outside electrode system simplifies the problem of depositing the electrodes, and connecting the lead wires.

After a few attempts, it became quite simple to select the appropriate electrode area, at least to within about 1%. The shape and size of the electrodes
were defined by a mask of narrow strips of cellulose adhesive tape applied to the tube prior to painting or evaporating the conductive coating.

Fine adjustment of the capacitance value can be achieved by two methods. The first is to use a screw inserted from one end of the tube as described before; if screws are used in both ends of the tube, the deeply inserted screw gives coarse adjustment and the shallow one will bring the capacitance to the desired value within 1 p.p.m. or less.

For better stability, adjustment without moving parts is preferred and is achieved by scraping the electrode. When a small part of the active electrode is removed the capacitance value decreases, and when the guard electrode is scraped away the value increases. By this method, adjustment to a few parts in $10^4$ was easily obtained for a 1-pF capacitor, from an initial value which was 1% smaller than the nominal value. Incidentally, it must be remembered that capacitance decreases some 550 p.p.m. when a capacitor is evacuated to a pressure of 100 microns or less. If the capacitor is to be sealed in vacuum for better stability, this change must be taken into consideration on the adjustment.

The capacitors were mounted in metal cases, which could be evacuated as desired, with the leads taken out through glass-to-metal seals. Special precautions must be taken in mounting the tube in the case so as to minimize stresses which tend to cause instability. As far as mounting stresses are concerned, thicker tubes appear to be preferable.

Great care must also be taken with the shielding of one active electrode from another. As the smaller capacitors have wider guard electrodes between the active electrodes, shielding by outside metallic plates (as shown in Fig. 10(a)) can be applied easily and satisfactorily. For the larger capacitance values using the same size of tubing, the guard electrode becomes so narrow that it is difficult to obtain complete shielding by the outside metallic shield plates. For a capacitor with electrodes on the inside surface of the tubing, the outside plated shield system shown in Fig. 10(b) was applied with satisfactory results, and this system undoubtedly improves the shielding effect at the narrow part of the guard electrode.
Fig. 10. Shielding techniques for three-terminal tubular capacitor; (a) small capacitance value, (b) large capacitance value.

When constructed in this manner, the capacitors showed excellent characteristics, with temperature coefficients ranging from 0.3 to 3 p.p.m./°C and voltage coefficients less than 10 p.p.m. for applied voltages from 10 to 100 volts at a frequency of 1592 c.p.s. Some of the best are listed in Table I, where it

<table>
<thead>
<tr>
<th>No. 1</th>
<th>No. 2 (4 samples)</th>
<th>No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Tubing</td>
<td>Electrodes</td>
</tr>
<tr>
<td>Cap. (pF)</td>
<td>Conductance (μmho)</td>
<td>Temperature coefficient (p.p.m./°C)</td>
</tr>
<tr>
<td>1</td>
<td>$+0 \times 10^{-7}$</td>
<td>$+0.6$</td>
</tr>
<tr>
<td>$-2 \times 10^{-7}$</td>
<td>$+0.8$</td>
<td>$&lt; 5$</td>
</tr>
</tbody>
</table>
will be seen that capacitor No. 2 with its outside electrode system shows no significant differences from capacitor No. 1. For No. 2, the calculated dielectric reduction factor (Appendix II) was 1/29.

7. RECTANGULAR TUBE CAPACITOR

Some capacitors on a circular tube with capacitance values of 1 pF had poorer characteristics than others. This is probably caused by the need for larger active electrodes and consequently narrower guard electrodes between them, resulting in a very complex and concentrated electric field around the edges of the electrodes. The behavior of the capacitor is thus very dependent on the finished condition of the edges of the film making up the electrodes.

To obtain greater freedom from such an effect, capacitors were made from rectangular pyrex tubes as shown in Fig. 11. With this style, much wider guard electrodes separating the active electrodes were obtained, and the edge effect was reduced considerably. All four capacitors made from the rectangular pyrex tubes showed desirable characteristics, as listed in Table I.

Although the long-term stability has not been fully assessed, an evacuated rectangular tube capacitor has maintained its value of 1 pF stable to within ±1 p.p.m. for a period of six months. This includes a two-week period of temperature cycling, when the results are reduced to a reference temperature of 20° C. This stability was checked against other standard capacitors maintained at a constant temperature of 25.0° ± 0.05° C which are themselves based on the computable cross capacitor constructed in the National Research Council (Dunn 1964).
Other capacitors of the same rectangular type were filled with argon gas at atmospheric pressure and have shown stabilities which are almost as good, although one does show a drift of some 5 p.p.m./month. Another capacitor which was initially filled with argon and showed a relatively high drift rate became stabilized to within 1 p.p.m. for three months after it was evacuated. It has since been refilled with argon and now appears to be stable.

These rectangular-tube capacitors have also been subjected to various shock tests, the most severe being a free fall from a height of 10 cm to a wooden table surface, and no changes in capacitance value greater than 1 p.p.m. have been detected. Additional experiments are being performed to further assess the stability of these units.

8. CONCLUSIONS

Capacitors constructed by depositing a metallic coating on the surfaces of rectangular pyrex glass tubing have shown stabilities of ±1 p.p.m. for months at a time, satisfactory freedom from capacitance changes due to minor physical shocks, as well as low temperature and voltage coefficients. It is anticipated that a capacitor of this style will prove to be very useful as a transportable standard of capacitance, and could be used in international comparisons of capacitance values maintained by different national laboratories.

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APPENDIX I. CAPACITOR WITH RESISTIVE ELECTRODES

A three-terminal capacitor with resistive electrodes which have capacitance and conductance between them is represented by the equivalent circuit shown in Fig. 12(a), where

- $C$ = main capacitance between two active electrodes,
- $C_1$ and $C_2$ = capacitances between active electrodes and guard electrode,
- $r_1$, $r_2$, and $r_3$ = resistances which represent the resistance of the electrode films,
- $g_1$ and $g_2$ = conductances between active electrodes and guard electrode.
The equivalent effective capacitance $C_e$ and conductance $G_e$ (Fig. 12(b)) measured between terminals 1 and 2 are found to be:

$$C_e = C[1 - r_3 g_1 - r_3 g_2 + r_3 (C_1 g_2 + C_2 g_1)/C],$$

$$G_e = g_1 g_2 r_3 + \omega^2 [- C_1 C_2 r_3 + C^2 (r_1 + r_3) + C C_1 r_1 + C C_2 r_2].$$

For capacitors of small capacitance $C$, the ground capacitances $C_1$ and $C_2$ are much larger than $C$ and

$$G_e = g_1 g_2 r_3 - \omega^2 C_1 C_2 r_3.$$

Values found for a typical 1-pF capacitor are

- $C = 1$ pF,
- $C_1 = C_2 = 100$ pF,
- $r_1 = r_2 = r_3 = 10$ ohms,
- $g_1 = g_2 = 1/100$ megohms $= 10^{-8}$ mho,
- $\omega = 10^4$,

for which the equivalent values become

$$C_e = C[1 - 0.2 \times 10^{-8} + 20 \times 10^{-6}] \text{ pF},$$

$$G_e = (10^{-9} - 10^{-4}) \mu\text{mho}.$$
The correction of 20 p.p.m. in capacitance is appreciable, and indicates that
$C_1$, $C_2$, $g_1$, $g_2$, and $r_s$ must be reduced from the values quoted, or at least main-
tained with a stability of approximately 1%, in order to achieve a stability of
1 p.p.m. in equivalent capacitance.

It will be noted that the predominant term in the equivalent conductance is
the frequency-dependent term $(-\omega^2 C_1 C_2 r_s)$.

**APPENDIX II. CAPACITOR WITH TWO DIELECTRIC LAYERS**

A capacitor which has two layers of dielectric materials between its electrodes
(as shown in Fig. 13) has a complex capacitance per unit area of

$$\hat{C} = \frac{\hat{\varepsilon}_1 \hat{\varepsilon}_2}{\hat{\varepsilon}_1 \hat{x}_2 + \hat{\varepsilon}_2 \hat{x}_1},$$

where $\hat{\varepsilon}_1$ = complex dielectric constant of dielectric No. 1,

$\hat{\varepsilon}_2$ = complex dielectric constant of dielectric No. 2,

$\hat{x}_1$ = thickness of the layer of dielectric No. 1,

$\hat{x}_2$ = thickness of the layer of dielectric No. 2.

When $\hat{\varepsilon}_2$ changes by an amount $\Delta \hat{\varepsilon}_2$, the resultant capacitance change $\Delta \hat{C}$ is

$$\frac{\Delta \hat{C}}{\hat{C}} = \frac{\hat{\varepsilon}_1 \hat{x}_2 \cdot \Delta \hat{\varepsilon}_2}{\hat{\varepsilon}_1 \hat{x}_2 + \hat{\varepsilon}_2 \hat{x}_1}.$$

If $\hat{\varepsilon}_1 = \varepsilon_1(1 - j\theta_1),$

$\hat{\varepsilon}_2 = \varepsilon_2(1 - j\theta_2),$

where $\theta_1$ and $\theta_2$ represent the loss angles of each material, then

$$\frac{\Delta \hat{C}}{\hat{C}} = \frac{\varepsilon_1 \varepsilon_2 x_2}{\varepsilon_1 \varepsilon_2 + \varepsilon_2 \varepsilon_1} \left\{ 1 - j \frac{\varepsilon_2 x_1 (\theta_1 - \theta_2)}{\varepsilon_1 x_2 + \varepsilon_2 x_1} \right\} \frac{\Delta \varepsilon}{\varepsilon}$$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{capacitor_diagram.png}
\caption{Capacitor with two dielectric layers.}
\end{figure}
and if $\theta_1 - \theta_2 \ll 1$, then

$$\frac{\Delta C}{C} = \frac{e_1 x_1}{e_1 x_1 + e_2 x_2} \frac{\Delta x_2}{\Delta x_2}$$

and

$$\frac{\Delta C}{C} = \frac{e_1 x_1}{e_1 x_1 + e_2 x_2} \frac{\Delta x_2}{e_2}.$$