Report No. 3
Third Quarterly Report  
Covering the Period
1 January 1964 to 31 March 1964

Investigation of
MICROWAVE DIELECTRIC-RESONATOR FILTERS

Prepared for:
U. S. ARMY ELECTRONICS RESEARCH AND DEVELOPMENT LABORATORY
FORT MONMOUTH, NEW JERSEY

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TASK NO. 5544-PM-63-91

By: S. B. Cohn and K. C. Kelly
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Rantec Project No. 31635

Approved:

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SECTION I
PURPOSE

This program is intended to study the feasibility of high-dielectric-
constant materials as resonators in microwave filters, and to obtain de-
sign information for such filters. Resonator materials shall be selected
that have loss tangents capable of yielding unloaded $Q$ values comparable
to that of waveguide cavities. The materials shall have dielectric con-
stants of at least 75 in order that substantial size reductions can be
achieved compared to the dimensions of waveguide filters having the
same electrical performance.
SECTION II
ABSTRACT

The coupling-coefficient formula derived in the Second Quarterly Report is discussed. Its discrepancy from experimental data for close spacing is attributed to the use of only the $TE_{10}$ mode in the analysis. The derivation is then extended to include all higher $TE$ and $TA$ modes excited by the dielectric resonators. The solution is particularized for dielectric disks spaced along the centerline of a rectangular waveguide below cutoff, with the axes of the disks parallel to the transverse coordinate. Coupling-coefficient curves computed from the new multiple-mode formula are shown to provide much better agreement with experimental data than the previous single-mode curves.

Sources of error are evaluated in the measurement technique for high-dielectric-constant samples. It is shown that the circular-waveguide dielectrometer has a typical maximum error of ±0.6 percent and probable error of ±0.25 percent, while the radial-waveguide dielectrometer has typically ±0.42 percent maximum error and ±0.17 percent probable error. An order-of-magnitude improvement in accuracy appears feasible, but is not needed in this program. The radial-waveguide dielectrometer has been used to obtain data on the new USAERDL cold pressed $TiO_2$ ceramic samples and on several samples cut from a single tile of Eccoceram Hi-K90 material. Dielectric constant temperature was also measured for four different groups of $TiO_2$ materials. The temperature coefficients ranged from $-3.5 \times 10^{-5}$ to $-10^{-5}$ ppm/°C. These values are higher than the nominal value of $-800$ for $TiO_2$ ceramic.

The previous unloaded-$Q$ measurement technique was improved in several minor details. Data was taken on three groups of $TiO_2$ ceramic disks in propagating WR-284 waveguide. The best values were
obtained with the USAERDL cold-pressed high-purity samples, with $Q_u$ between 10,000 and 12,000 in the majority of pieces tested. One of these pieces was then used to determine the effect of metal-wall losses by repeating the $Q_u$ measurements in a series of "cut-off" square waveguides having internal dimensions ranging from 1.25 by 1.25 inch to 0.430 by 0.430 inch. In the latter case, the 0.430-inch-diameter disk was tangent to the top and bottom walls of the waveguide. Despite the extremely close proximity, the unloaded $Q$ was diminished only to 5250 in the presence of the silver-plated walls. Resonant-f frequency data versus the dimension of the square waveguide are also given.

Frequency-response data are shown for several two-resonator band-pass filters adjusted for maximally flat performance. The bandwidth and center-frequency dissipation loss agree quite well with previously obtained coupling-coefficient and unloaded-$Q$ values.
SECTION III
CONFERENCES

On 20 May 1964, a conference was held to discuss third-quarter progress and plans for the fourth quarter. The location of the conference was the International Hotel at Idlewild, L. I., N. Y., where the 1964 PTGMTT Symposium was being held. The conference was attended by Messrs. J. Agrios, N. Lipez, and E. A. Mariani of USAERDL, and Dr. S. B. Cohn of Rantec Corp. Draft material for the Third Quarterly Report was reviewed and suggestions were made for further items of investigation.
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SECTION IV

FACTUAL DATA

1. Introduction

The First Quarterly Report discusses the nature of dielectric resonators and describes how such resonators may be used in microwave filters. The introduction to that report should be consulted for background information, and for a discussion of problems to be solved before dielectric resonators can be used in practice.

In the Second Quarterly Report, an analysis was given of coupling between dielectric resonators spaced along the centerline of rectangular waveguide below cutoff. The resulting formula for coupling coefficient was found to agree with experimental data within about ±10 percent when the center-to-center spacings exceeded about three quarters of the larger of the two waveguide cross-section dimensions. The deviation for smaller spacings was attributed to the use of only the $TE_{10}$ mode in the analysis. In the present report, the theory is extended to all TE and TM modes excited by the resonators. The new multimode formula is shown to yield theoretical curves agreeing very well with the previous experimental points even when the resonant disks are in contact with each other. Based on this excellent verification, the new formula is concluded to have sufficient accuracy for ordinary design purposes.

Two techniques were described in the Second Quarterly Report for measurements on high dielectric-constant materials. Both techniques utilize geometries such that minor airgaps between the surfaces of the cylindrical samples and adjacent metal walls have negligible effect on measured dielectric-constant values. In order to complete the treatment of these new methods, computations of typical errors
have been made and are discussed in this report. Accuracies on the order of one-half percent are shown to be easily obtained. Measured data on several groups of TiO₂ samples are given.

The method of unloaded-Q measurement presented in the First Quarterly Report is susceptible to major error if the input signal has appreciable frequency modulation. Because the signal source was internally square-wave modulated at 1000 cps, it was suspected that incidental frequency modulation might have affected the accuracy of the measured values. In order to eliminate this possible source of error, an isolator and diode modulator were placed between the signal source and the input of the test setup. This permitted the signal source to be operated with CW output, ensuring absence of any significant frequency deviation. Experimental results were found to agree quite well with the data included in the First Quarterly Report, thus showing that frequency modulation had not appreciably affected the previous measurements. Additional measurements were then made on several groups of samples and on a single sample in a series of waveguides of varying cross sections. The latter data reveals the effect of metal-wall proximity on both Q_u and I_0.

In order to determine the validity of the coupling coefficient and Q_u data, several two-resonator band-pass filters were made having maximally flat response. Reasonably good agreement was obtained between actual and predicted bandwidth and insertion loss.

2. Coupling Between Dielectric Resonators - Higher Modes

In the Second Quarterly Report, a formula was derived for the coupling coefficient between a pair of dielectric-disk resonators spaced along the centerline of a rectangular waveguide below cutoff (Figure 2-1). Good agreement was found between computed curves and
experimental data when the center-to-center spacing, $s$, is at least three-fourths of the larger of the two waveguide dimensions, $a$ and $b$. The discrepancy for closer spacing was attributed to the fact that only the $TE_{10}$ mode was used in the analysis. During the past quarter the treatment was extended to include higher modes.

Considerable improvement in accuracy for close spacing was achieved.

Equation 3-36 of the Second Quarterly Report gives the coupling coefficient, $k$, in generalized form for a pair of identical magnetic dipole resonators whose axes are either parallel or colinear:

$$k = \frac{\mu_0 m_1 \cdot H_2}{2W m_1}$$

where $\mu_0$ is the permeability of free space, $m_1$ and $W m_1$ are the magnetic dipole moment and stored energy of the first resonator, while $H_2$ is the magnetic field at the center of the second resonator $m_1$.

In the Second Quarterly Report, $H_2$ was computed from $m_1$ by first determining the amplitude of the $TE_{10}$ mode excited by $m_1$, then evaluating the magnetic-field amplitude of this mode at a distance $s$, from $m_1$. The configuration is as shown in Figure 2-1. Because the waveguide is used below its cut-off frequency, the field amplitude is proportional to $e^{-\alpha s}$, where $\alpha$ is the attenuation constant of the $TE_{10}$ mode. Therefore, $k$ is also proportional to $e^{-\alpha s}$, and the theoretical coupling-coefficient curve is a straight line on a semi-log graph. The experimental data showed this to be a good approximation for $s$ relatively large, but not for $s$ relatively small.
latter case may be expected from the addition of higher modes to the coupling analysis.

Consider Figure 2-1. Equation 2-1 indicates that all modes having \( H_x \) not zero at the center of the cross section will contribute to the total field \( H_2 \). The significant modes are as follows:

\[
\begin{array}{ccc}
TE_{10} & TE_{12} & TM_{12} \\
TE_{30} & TE_{32} & TM_{32} \\
TE_{50} & TE_{52} & TM_{52} \\
\text{etc.} & \text{etc.} & \text{etc.}
\end{array}
\]

Note that for both \( TE_{mn} \) and \( TM_{mn} \) the admissible \( m \) and \( n \) values are \( m = \text{odd integers} \) and \( n = \text{even integers} \).

In the Second Quarterly Report, the waveguide-mode notation of Collin was introduced. The following formulas from pp. 28-30 of the Second Quarterly Report are needed. The MKS system of units are used throughout the analysis.

\[
H_2 = H_x^+ \text{ at } z = \infty \quad (2-2)
\]

\[
H_x^+ = \sum_{m,n} a_{mn} h_{xmn} e^{-j\omega t} \quad (2-3)
\]

\[
a_{mn} = -j \frac{\omega c_0}{2} h_{xmn} m_1 \quad (2-4)
\]

where \( m_1 \) is the moment of the magnetic dipole located at \( z = 0 \) and directed along the \( x \)-axis. The mode field functions \( e_{mn} \) and \( h_{mn} \) are normalized according to the following power-flow relationship:

\[
\int_S e_{mn} \times h_{mn} \cdot dS = 1 \quad (2-5)
\]
where the integration is performed over the waveguide cross-sectional area. The attenuation constant $\alpha_{mn}$ is given by

$$\alpha_{mn} = \frac{2\pi}{\lambda_{cmn}} \sqrt{1 - \left(\frac{\lambda_{cmn}}{\lambda}\right)^2} \text{ nepers per meter} \quad (2-6)$$

and the cut-off wavelength $\lambda_{cmn}$ is as follows for rectangular waveguide.

$$\lambda_{cmn} = \frac{i}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}} \quad (2-7)$$

Equations 2-2 through 2-7 apply to both TE and TM modes, and the summation in Equation 2-3 is carried out over TE and TM modes of all admissible orders $m$ and $n$.

When Eqs. 2-2, 2-3, and 2-4 are combined, the following formula for $H_2$ is obtained.

$$H_2 = -\frac{j\omega \mu_0 m_1}{2} \sum_{m,n} h_{xmn} Z e^{-\alpha_{mn}s} \quad (2-8)$$

The normalized field component, $h_{xmn}$, will now be computed. Equation 2-5 can be rewritten with the aid of the relation $\mathbf{e} \times \mathbf{h} \cdot d\mathbf{S} = Z h_{t}^{2} \, d\mathbf{S} = Z (h_{x}^{2} + h_{y}^{2}) \, d\mathbf{S}$:

$$Z_{mn} \left[ \iint_{S} h_{xmn}^{2} \, d\mathbf{S} + \iint_{S} h_{ymn}^{2} \, d\mathbf{S} \right] = 1 \quad (2-9)$$

where $h_{xmn}$ and $h_{ymn}$ are the transverse components of $h_{mn}$, and the characteristic wave impedance $Z_{mn}$ is as follows:
\[ Z_{mn} = \frac{\eta \lambda}{\lambda} = \frac{j2\pi\eta}{\lambda_{mn}} \text{ for TE modes} \quad (2-10) \]

\[ Z_{mn} = \frac{\eta \lambda}{\lambda} = \frac{\alpha_{mn} \lambda\eta}{j2\pi} \text{ for TM modes} \quad (2-11) \]

The free-space wave impedance, \( \eta \), is equal to \( \sqrt{\mu_0/\varepsilon_0} = 377 \) ohms in air.

The \( h_x \) and \( h_y \) field components are

\[ h_{xmn} = \frac{m}{i} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \]

\[ h_{ymn} = \frac{n}{i} B_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (2-12) \]

\[ h_{xmn} = \frac{n}{a} B_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \]

\[ h_{ymn} = -\frac{m}{a} B_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (2-13) \]

and

\[ h_{xmn} = \frac{n}{b} B_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \]

\[ h_{ymn} = -\frac{m}{b} B_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (2-14) \]

where \( A_{mn} \) and \( B_{mn} \) are constants to be evaluated through application of Eq. 2-9. The coordinate system is as shown in Figure 2-2. When Eqs. 2-12 to 2-15 are substituted in Eq. 2-9, the following integrals must be evaluated:
\[
\int_0^a \int_0^b \sin^2 \left( \frac{m \pi x}{a} \right) \cos^2 \left( \frac{n \pi y}{b} \right) \, dx \, dy = \int_0^a \int_0^b \sin^2 \left( \frac{m \pi x}{a} \right) \cos^2 \left( \frac{n \pi y}{b} \right) \, dy \\
0 \quad 0 \\
= \frac{ab}{4} \text{ for } m \geq 1, \, n \geq 1 \\
\frac{ab}{c} \text{ for } m \geq 1, \, n = 0
\]

and

\[
\int_0^a \int_0^b \cos^2 \left( \frac{m \pi x}{a} \right) \sin^2 \left( \frac{n \pi y}{b} \right) \, dx \, dy = \int_0^a \int_0^b \cos^2 \left( \frac{m \pi x}{a} \right) \sin^2 \left( \frac{n \pi y}{b} \right) \, dy \\
0 \quad 0 \\
= \frac{ab}{4} \text{ for } m \geq 1, \, n \geq 1 \\
= 0 \text{ for } m \geq 1, \, n = 0
\]

Equations 2-9, 2-10, 2-12, 2-13, 2-16, and 2-17 yield the following for TE_{m0} modes, \( m \geq 1 \):

\[
\frac{mA_{m0}}{a^2} = \left( \frac{2}{ab} \right) \left( \frac{a_{m0}^\lambda}{j2\pi \eta} \right)
\]

\[
h_{xm0} = \frac{a_{m0}^\lambda}{jab\pi \eta} \sin^2 \left( \frac{m \pi x}{a} \right)
\]

and for TE_{mn} modes, \( m, n \geq 1 \).

\[
\left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right] A_{mn}^2 = \left( \frac{4}{ab} \right) \left( \frac{a_{mn}^\lambda}{j2\pi \eta} \right)
\]

\[
h_{xmn} = \left( \frac{2(m/a)^2}{(m/a)^2 + (n/b)^2} \right) \left( \frac{a_{mn}^\lambda}{j2\pi \eta} \right) \sin^2 \left( \frac{m \pi x}{a} \right) \cos^2 \left( \frac{n \pi y}{b} \right)
\]
Similarly, Eqs. 2-9, 2-11, 2-14, 2-15, 2-16, and 2-17, give the following for TM\(_{mn}\) modes, \(m, n \geq 1\):

\[
\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right) B_{mn} = \frac{1}{ab} \left( \frac{i2\pi}{a}\right) \left( \frac{a}{\alpha_{mn}} \lambda \eta \right)
\] (2-22)

\[
\eta_{xtrn} = \left( \frac{z(n/b)^2}{(m/a)^2 + (n/b)^2} \right) \left( \frac{i4\pi/a}{\alpha_{mn}} \right) \sin^2 \left( \frac{\lambda \eta \pi}{a} \right) \cos^2 \left( \frac{\lambda \eta \pi}{b} \right)
\] (2-23)

Note that for TM\(_{mn}\) modes, neither \(m\) nor \(n\) can be zero in the rectangular-waveguide case.

Examination of Eqs. 2-19, 2-21, and 2-23 shows that at the center of the cross section \(\eta_{xtrn} = 0\) if either \(m\) is even integer or \(n\) is odd integer. Thus only modes for which \(m\) = odd integer and \(n\) = even integer will contribute to the coupling when the \(x\)-directed magnetic dipoles are placed along the central longitudinal axis of the waveguide.

When Eqs. 2-19, 2-21, and 2-23 are substituted in Eq. 2-8, we obtain

\[
H_2 = -\frac{m}{ab} \left[ \sum_m \alpha_{m0} e^{-\alpha_{mn} z} + 2 \sum_{m,n} \frac{(m/a)^2 + (n/b)^2}{(m/a)^2 + (n/b)^2} \frac{\alpha_{mn} e^{-\alpha_{mn} z}}{\alpha_{mn} \lambda \eta} \right]
\]

\[
-\frac{8\pi^2}{\lambda^2} \sum_{m,n} \left[ \frac{(n/b)^2 e^{-\alpha_{mn} z}}{(m/a)^2 + (n/b)^2} \alpha_{mn} \lambda \eta \right]
\] (2-24)

where the summations are performed for \(m = 1, 3, 5, \ldots\) and \(n = 2, 4, 6, \ldots\) Use was made of the identity \(\omega_{\xi, \eta} = 2\pi \eta\). The three terms in the brackets are due to the TE\(_{01}\), TE\(_{21}\), and TM\(_{mn}\) modes, respectively. A simplification may be made by observing that Eqs. 2-6 and 2-7 lead to
\[ \left( \frac{m}{a} \right)^2 \alpha_{mn} - \left( \frac{2\pi}{\lambda} \right)^2 \left( \frac{n}{b} \right)^2 \left( \frac{\lambda}{\alpha_{mn}} \right) = \left[ \left( \frac{m}{a} \right)^2 \alpha_{mn} \right] \left( \frac{\lambda}{\alpha_{mn}} \right) \]

thus,

\[ H_2 = -\frac{m_1}{ab} \left[ \sum_m \alpha_{m0} e^{-\alpha_{m0}^2} + 2 \sum_{m,n} \alpha_{m,n} \alpha_{mn} \right] \]

where \( m = 1, 3, 5, \ldots \) and \( n = 2, 4, 6, \ldots \)

A further simplification gives the following compact result.

\[ H_2 = -\frac{m_1}{ab} \left[ \sum_{m,n} \alpha_{m,n} \alpha_{mn} \right] \]

In this particular case, the double summation is carried out over

\[ m = 1, 3, 5, \ldots, n, \quad n = -\omega, \ldots, -6, -4, -2, 0, 2, 4, 6, \ldots, \infty \]

However, Eq. 2-26 in terms of positive integers is more convenient for computation.

The coupling coefficient between a pair of identical \( x \)-directed

magnetic dipoles on the central longitudinal axis of a rectangular wave-

guide below cutoff is obtained from Eqs. 2-1 and 2-26.

\[ k = \frac{1}{ab} \left( \frac{\lambda_0 m_1 \alpha_{m0}^2}{2 \lambda} \right) \left[ \sum_m \alpha_{m0} e^{-\alpha_{m0}^2} + 2 \sum_{m,n} \alpha_{m,n} \alpha_{mn} \right] \]

The factor \( \mu_0 \frac{m_1 \alpha_{m0}^2}{2 \lambda} \) depends only on the dimensions \( D \) and \( L \), dielectric constant, \( \varepsilon_r \), and resonant wavelength. \( \lambda_0 \) of the electric resonator. In the Second Quarterly Report, Eq. 3-52, this factor was evaluated to be

\[ \frac{\mu_0 m_1 \alpha_{m0}^2}{2 \lambda} = \frac{0.927 D^4 \varepsilon_r}{\lambda_0^2}, \quad 0.25 \leq L/D \leq 0.7 \]

-13-
Equation 2-29 was simplified from a more complicated expression given in Eq. 3-51 of the Second Quarterly Report. In the range 0.25 ≤ L/D ≤ 0.7 the two formulae agree within ±2%. A dielectric resonator of practical proportions will generally lie in this range. Thus,

\[ k = \frac{927D^2L e_r}{ab\lambda_0^2} \left[ \sum_{m} a_m e^{-a_m \phi^2} + 2 \sum_{m,n} \frac{a_m^2}{\alpha_{mn}} e^{-a_{mn} \phi^2} \right] \quad (2-30) \]

where

1.25 ≤ L/D ≤ 0.7

\( m = 1, 3, 5, \ldots \)

\( n = 2, 4, 6, \ldots \)

If L/D falls outside of the range 0.25 to 0.7, the curve plotted in Figure 3-5 of the Second Quarterly Report may be applied as a correction factor to Eq. 2-30.

An interesting property of Eq. 2-24 is that the third summation term, arising from the TE\(_{mn}\) modes, is opposite in sign to the TE\(_{m0}\) and TE\(_{mn}\) terms. The explanation for this difference in sign is as follows. An x-directed magnetic dipole in free space produces an electric field having only \( v \) and \( z \) components. Thus \( E_x \) is everywhere zero. An x-directed magnetic dipole in a rectangular waveguide is equivalent to a two-dimensional infinite array of x-directed dipoles in free space. Since \( E_x = 0 \) for each dipole, \( E_x \) is zero for the superposition of fields of all the dipoles in the array. Hence, only \( E_y \) and \( E_z \) components exist in the waveguide and \( E_x \) is everywhere zero. The field produced by each individual TE\(_{mn}\) and TM\(_{mn}\) mode (\( m \) and \( n \geq 1 \)) contains finite \( E_x \) components. However, when the TE\(_{mn}\) and TM\(_{mn}\) mode fields have the relative amplitudes \( A_{mn} \) and \( B_{mn} \) determined from Eqs. 2-20 and 2-22, the \( E_x \) components of these modes are equal in amplitude and opposite in sign for each set of integers \( m \) and \( n \geq 1 \).
Consequently, the above solution conforms to the necessary condition that $E_x = 0$ at all points in the waveguide.

When only the $m = 1$ term of the first summation is used and the second summation is omitted entirely, the single-mode formula in the Second Quarterly Report is obtained as follows:

$$k = \frac{0.927D^4 \pi a_{10} e^{-\lambda_0 L}}{a b \lambda_0}$$

(2-31)

The correction factor

$$K = \frac{1}{a_{10} e^{-\lambda_0 L}} \left\{ \sum_{m} a_{m0} e^{-\lambda_m L} + 2 \sum_{m,n} \frac{a_{m0}^2}{a_{mn}} e^{-\lambda_{mn} L} \right\}$$

(2-32)

$$m = 1, 3, 5, \ldots$$

$$n = 2, 4, 6, \ldots$$

converts the single mode formula Eq. 2-31 into the multi-mode formula Eq. 2-30. The factor, $K$, may be used to correct the computed curves in the Second Quarterly Report for the effects of higher modes. This has been done, and the results are described below.

Application of Correction Factor to Previous Data

In the Second Quarterly Report, three waveguide sizes were used in obtaining experimental coupling data. The sizes were:

$a = b = 0.750$ in., $a = b = 0.995$ in., and $a = b = 0.625$ in., $b = 1.374$ in.

In each waveguide, two different sizes of dielectric disks were used. However, Eq. 2.32 is a function only of $f_0$ of the disk, and not of the other disk parameters. Since $f_0$ is approximately the same for both disks, and $K$ is quite insensitive to changes in $f_0$, the $f_0$ values for
the $D = 0.393$ in. and $L = 0.160$ in. disk will be used, but the resulting $K$ factor will be used for the other disk as well.

For the square waveguides, calculations show that significant contributions to the coupling coefficient are made only by the modes with subscripts $mn = 10, 30, 50, 12, 14, 32, 34$. For the rectangular waveguide the significant subscripts are $10, 30, 12, 14, 16, 32, 34, 36$. A graph of the correction factor $K$ versus $s$ appears in Figure 2-3 for the three different waveguides.

Figure 2-4 shows the six sets of coupling data from the Second Quarterly Report. The solid curves are theoretical for single-mode coupling, while the dotted curves include the higher-mode contribution, as given by Eqs. 2-30 and 2-32. A substantial improvement in accuracy is evident when the higher-mode terms are added to the single-mode formula.

3. Dielectric-Constant Measurements

a. Measurement Error with the Circular-Waveguide Dielectrometer

The circular waveguide dielectrometer was described in the Second Quarterly Report. The device was shown to offer good precision in determining the dielectric constant of materials. An especially important feature of this technique is that it is unaffected by small air gaps between the sample holder and the adjacent
Figure 2-4. Coupling-Coefficient Data for Configuration of Figure 2-1. Solid Curves from Single-Mode Theory, Dotted Curve from Multi-Mode Theory, and Circled Points are Experimental.
metal wall. As a result, the technique is particularly useful in measurements on high dielectric-constant materials, where small air gaps are ordinarily a major source of error. The dielectric constant is calculable from the TE₀₁-mode resonance of a right-circular cylinder of the material closely fitted in a below-cutoff metal tube. As indicated in the discussion in the Second Quarterly Report, the computation of dielectric constant, ε₀, requires knowledge of the resonant frequency, the inside diameter of the tube, and the length of the sample.

The resonant frequency can easily be measured to 1 percent and linear dimensions of the sample to ±0.0005 inch. As a typical example, a sample was assumed with a dielectric constant of 90.000, a diameter of 0.36000 in., and a length of 0.11800 in. The calculated resonant wavelength of this sample in an ideal circular waveguide dielectrometer is 2.2598 in. When the wavelength, diameter, and height are deliberately offset by −0.1 percent, +0.0005 in. and +0.0005 in., respectively, the dielectric constant is calculated to be ε₀ = 89.43, which is 0.6 percent low. Note that the errors in the three input quantities were taken with a "worst case" combination of algebraic signs to give the maximum error in ε₀. With more precise instruments, the frequency and dimensions can be determined with an order-of-magnitude improvement, so that an accuracy of ε₀ of better than 0.06 percent appears feasible.

The range of dielectric constants and sample sizes being measured in this program generally conform to the hypothetical case considered above. Thus, in the measurements made with the circular-waveguide dielectrometer, the maximum error in the determination of ε₀ is less than ±0.6 percent, and the probable error is about ±0.25 percent.
b. Measurement Error with the Radial-Waveguide Dielectrometer

The radial-waveguide dielectrometer described in the Second Quarterly Report was also examined from the standpoint of maximum-possible error. The sample employed in the analysis was equivalent, essentially, to the smallest-diameter samples (such as Sample A₂) of the Second Quarterly Report. These smaller samples are nominal 1/4-in. in diameter by 1/10-in. long, with εᵣ = 86.5.

A numerical error computation was performed assuming the same wavelength and dimension errors as above. The maximum possible error in εᵣ was found to be ±0.42 percent while the probable error would be about ±0.17 percent. Again, order of magnitude improvements on accuracy in frequency and dimensions, and hence in εᵣ, are feasible.

c. Anomalous Behavior of Sample A₂

It has been found that compressed polycrystalline TiO₂ ceramics from a given batch produce an εᵣ versus density curve that is well behaved (smooth and monotonic). Sample A₂ of the group of samples tested and reported in the Second Quarterly Report appeared to violate this concept. Numerous remeasurements of this sample reinforces the reported result that its data points fell off the curves in opposite directions for the two methods of measurement. These measurements included such refinements as use of a more careful count to give better than 0.001 percent accuracy in the measurement of resonant frequency.

The only reasonable explanation found for the anomaly depends on the assumption that a marked inhomogeneity exists in the
density of the material within the sample. Note that the electric field is zero on the cylindrical walls of the test sample when it is tested in the circular-waveguide dielectrometer. In the radial-waveguide dielectrometer, the electric field is zero on the parallel faces of the sample. In both cases the resonant mode is designated \( TE_{01} \), but the differences in boundary conditions are such that the inhomogeneities within the sample can have markedly different effects on the resonant frequency for the two field distributions of interest.

Support for the idea of inhomogeneities within a small volume comes from test results on a specimen of Eccoceram \( 95 \% \) \( TiO_2 \) to be reported below. A further test of the inhomogeneity theory will be the careful measurement of the unloaded \( Q \) of sample \( A_2 \) as compared to the \( Q \) of a similarly sized sample whose indicated \( \varepsilon_r \) fits the \( \varepsilon_r \)-versus-density curve. Marked inhomogeneities would be expected to lower the unloaded \( Q \) since asymmetric inhomogeneities should cause energy transfer from the \( TE_{01} \) mode into other modes.

d. The Dielectric Constant of Cold Pressed Polycrystalline \( TiO_2 \) Pellets from USAERDL

The hot-pressed pellet of first polycrystalline \( TiO_2 \) identified as \( S_1 \) was reported to have a dielectric constant of 97.6 - 98.7 in the Second Quarterly Report. New samples of high-purity polycrystalline \( TiO_2 \) have been received from USAERDL. These samples are marked "cold pressed," and various press pressures are indicated. In general, these samples displayed lower dielectric constants than the earlier samples. The relation between dielectric constant and density is evident though no assurance was given that all pellets were formed from the same batch. The dielectric constants were determined with the aid of the radial waveguide dielectrometer. The results are given in Figure 3-1. The density and stated pressing pressures used in the fabrication of the pellets tracked in a general way.
The Dielectric Constant of Eccoceram Hi-K90

The firm of Emerson and Cuming, Inc. produces and markets a material known as Eccoceram Hi-K. The material is said to be polycrystalline TiO₂, but the company's Technical Bulletin 9-2-5 does not identify the composition. Dissipation factor is given as less than 0.001. The material is available in 12 values of εᵣ. The Bulletin statement that the material is "usable continuously over the temperature range -70°F to +150°F" followed by a listing of dielectric constants implies that εᵣ was stabilized against temperature. Inquiries failed to produce information on the temperature coefficient associated with εᵣ.

A single tile, 2-1/2" by 2-1/2" by 3/8", of Eccoceram Hi-K90 was obtained for tests. Pellets of 0.340 inch diameter by 0.1180 length were ground from various locator. The density of the pellets was found to differ by as much as 2 percent. There is no reason to believe that the two-percent field would not be exceeded if more pellets were made from the remaining portions of the tile. This inhomogeneity within one tile further suggests that the explanation given above for the anomaly observed in Sample A₂ has merit.
One sample, with a density of 63.81 grams/cubic inch, was measured to have a dielectric constant of 91.75. The dielectric constants of the other samples were not measured at this time.

f. Temperature and Dielectric Constant

Four groups of TiO₂ have been measured at a minimum of two temperature, 76°F and 176°F. The results are summarized in Figure 3-2. None of the materials tested show evidence of temperature-compensating ingredients. Because of the high sensitivity of dielectric constant to temperature, it would be generally impractical to construct narrow-band filters for field use out of any of these materials.

![Figure 3-2. Dielectric Constant Vs. Temperature (Temperature Coefficient in Brackets)](image)

4. Unloaded-Q and Center-Frequency Measurements

a. Unloaded Q of TiO₂ Samples

During the third quarter, a series of measurements were made on the unloaded Q, Q_u, of various TiO₂ samples. These samples, which were in the form of cylindrical disks approximately 0.4 inch diameter by 0.2 inch long, were placed at the center of a WR-284 waveguide cross section. As indicated by previous data shown in Table 3-3 of the First Quarterly Report, the metal wall losses in this case would be expected to have negligible effect on Q_u.
The method of $Q_u$ measurement utilized in these tests is essentially as described in the First Quarterly Report. The resonant disk produces a band-rejection response in the operating range of the WR-284 waveguide. By means of a reflectometer setup the rejection bandwidth is determined from reflection-coefficient data. An additional parameter measured is the peak insertion loss of the rejection response. The First Quarterly Report explains how $Q_u$ is computed from bandwidth, center frequency, and peak insertion loss.

Several refinements in technique were introduced in the original equipment shown in the First Quarterly Report. One refinement was the use of an external square-wave diode modulator instead of the internal 1000-cps modulation of the generator. A ferrite isolator was connected between the generator and the diode modulator. This system modification permitted CW operation of the generator, thus eliminating the possibility of frequency modulation as a source of error. A second refinement was the use of a transfer oscillator and counter for precise measurement of frequency. Measurements were repeated on several of the same TiO$_2$ samples tested during the first quarter. The agreement with the earlier data was very good, indicating that frequency modulation and frequency error had not had a serious effect. However, the use of the isolated external modulator and of the transfer oscillator and counter has been retained because of the greater inherent reliability and precision they afford.

Unloaded Q measurements were made on three sets of TiO$_2$ samples: (1) hot-pressed samples supplied by the USAERDL ceramics laboratory during the first quarter of this program; (2) cold-pressed samples supplied by the same source during the second quarter; and (3) Eccoceram Hi-K90 material purchased from Emerson and Cuming, Inc. The resulting $Q_u$ values are shown in Table 4-1 with dimensions, center frequencies, and measured dielectric constants.
TABLE 4-1
UNLOADED Q OF VARIOUS TiO₂ CERAMIC DISKS

<table>
<thead>
<tr>
<th>Material</th>
<th>Identification of Sample</th>
<th>D (inch)</th>
<th>L (inch)</th>
<th>$f₀$ (Mc)</th>
<th>$\varepsilon_r$</th>
<th>$Q_u$</th>
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<tr>
<td>USAERDL</td>
<td>No. 1</td>
<td>0.393</td>
<td>0.250</td>
<td>2980</td>
<td>98</td>
<td>7300</td>
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<tr>
<td></td>
<td>No. 2</td>
<td>0.393</td>
<td>0.250</td>
<td>2961</td>
<td>98</td>
<td>8550</td>
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<tr>
<td></td>
<td>No. 3</td>
<td>0.353</td>
<td>0.160</td>
<td>3217</td>
<td>98</td>
<td>7160</td>
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<tr>
<td></td>
<td>No. 4</td>
<td>0.293</td>
<td>0.160</td>
<td>3317</td>
<td>98</td>
<td>6390</td>
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<tr>
<td>USAERDL</td>
<td>1500 psi</td>
<td>0.430</td>
<td>0.220</td>
<td>3051</td>
<td>85.6</td>
<td>12,000</td>
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<tr>
<td></td>
<td>2000 psi</td>
<td>0.430</td>
<td>0.220</td>
<td>3029</td>
<td>87.1</td>
<td>11,650</td>
</tr>
<tr>
<td></td>
<td>2500 psi</td>
<td>0.440</td>
<td>0.240</td>
<td>2507</td>
<td>87.2</td>
<td>11,700</td>
</tr>
<tr>
<td></td>
<td>3000 psi</td>
<td>0.400</td>
<td>0.200</td>
<td>3289</td>
<td>87.6</td>
<td>7,350</td>
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<td></td>
<td>3750 psi</td>
<td>0.540</td>
<td>0.240</td>
<td>2503</td>
<td>88.2</td>
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<td>0.244</td>
<td>2675</td>
<td>-</td>
<td>7,350</td>
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<td>0.220</td>
<td>2904</td>
<td>92</td>
<td>7,163</td>
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<tr>
<td>Hi-K90</td>
<td>No. 2</td>
<td>0.430</td>
<td>0.220</td>
<td>2922</td>
<td>61</td>
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</table>

b. $Q_u$ Versus Waveguide Dimensions

Figure 4-1 shows the effect of waveguide-wall proximity on the unloaded $Q$ of a dielectric resonator. The resonator used in this experiment is the previously tested USAERDL, hot-pressed 2000 psi sample. As shown in Table 4-1, the $Q_u$ value in 6.35 by 6.35 inch waveguide was 11,650. The plotted points in Figure 4-1 are for this resonator in square waveguides ranging from 0.430 by 0.430 to 1.25 by 1.25 inch ID. The resonator was supported at the center of each waveguide cross section by polyfoam, with the axis of the disk in the transverse x direction. Because the square waveguides are non-propagating at the resonant frequency of the disk, the configuration was measured as a single-resonator band-pass filter. Coupling loops of closely fitted
sliding blocks were used to obtain coaxial input and output connections. The loops were adjusted to give a center-frequency insertion loss of at least 3 db. Symmetry of coupling was ensured by setting the loops such that the VSWR at the two ports were the same. The following relationship then holds between loaded Q, Q_L, unloaded Q, Q_u, and center-frequency insertion loss, L_o:

\[ L_o = 20 \log_{10} \left( \frac{Q_u}{Q_L} \right) \text{ db} \] (4-1)

or

\[ Q_u = \frac{Q_L}{1 - 10^{-L_o/20}} \] (4-2)

The loaded Q, Q_L, of the single-resonator filter is related to bandwidth by:

\[ Q_L = \frac{f_o}{(BW)_{3\text{db}}} \] (4-3)

\[ Q_L = \sqrt{3} \frac{f_o}{(BW)_{6\text{db}}} \] (4-4)

\[ Q_L = 3 \frac{f_o}{(BW)_{10\text{db}}} \] (4-5)

where (BW)_{3\text{db}}, (BW)_{6\text{db}}, and (BW)_{10\text{db}} are bandwidths determined at levels of L_o + 3 db, L_o + 6 db, and L_o + 10 db, respectively. These three bandwidths were measured in all cases, and were found to yield sets of Q_L values within about 2 percent of each other. The average
the \( Q_L \) values computed from Eqs. 4-3, 4-4, and 4-5 was used in Eq. 4-2 to obtain \( Q_u \).

The plotted points in Figure 4-1 are for a number of different tubing materials. After the three brass tubes were tested, they were silver plated, thus yielding additional points. The curve in Figure 4-1 applies to aluminum; for cross sections smaller than \( ^{\frac{3}{4}} \) inch the curve is interpolated between the points for silver-plated and 70-30-brass surfaces.

The \( Q_u \) curve in Figure 4-1 is seen to be consistent with an asymptotic value of 11 650 for a "isolated" resonator, as previously measured in 2.84 by 1.34 inch ID waveguide. Although \( Q_u \) has dropped to 5250 in silver-plated 0.430 by 0.430 inch tubing, this is nevertheless a highly respectable value. For example, a strip-line or coaxial resonator occupying the same volume would have a \( Q_u \) value of about 1250.

The 0.430 by 0.430 inch tubing size is especially interesting, since the 0.430-inch diameter disk is tangent to the top and bottom walls. In this case, a convenient structural technique would be to fasten the disk in place by means of small amounts of epoxy cement at the points of tangency. Another piece of brass tubing was constructed to explore further this possibility. The height was maintained at 0.430 inch to retain the tangency feature, while the width was made 0.860 inch. The unloaded \( Q \) in this 70-30-brass tubing was measured to be 5100. After silver plating, \( Q_u \) increased to 6760. These values are considerable improvements over the 0.430-inch-square case.

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c. $f_0$ Versus Waveguide Dimensions

The effect of waveguide-wall proximity on resonant frequency of the USAERDL 2000 psi cold-pressed sample is shown in Figure 4-2. The measured points are for the same series of square tubings as in the $Q_u$ measurements of Figure 4-1. Table 4-1 shows the resonant frequency of the disk to be 3029 Mc in 2.84 by 1.34 inch waveguide. This asymptotic value is consistent with the curve in Figure 4-2.

In the case of the 0.430 by 0.860 inch tubing, the resonant frequency was measured to be 3234 Mc.

Figure 4-2. $f_0$ of Dielectric Resonator Vs. Dimension of Square Waveguide

d. Theoretical Analysis of $f_0$ and $Q_u$

An approximate theoretical analysis of the effect of metal-wall proximity on $f_0$ and $Q_u$ was carried out during the latter part of the third quarter. Initial results of the theory indicate good confirmation of the $f_0$ data in Figure 4-2, and $Q_u$ data in Figure 4-1. The theory will be further evaluated, and included in the next report.

Experiments with Two-Resonator Band-Pass Filters

In the Second Quarterly Report, coupling data were given for two different pairs of resonators in the different sizes of waveguides.
below cutoff. The graphs of experimental points are repeated in the present report as Figure 2-4, where they may be compared with theoretical curves. As an experiment to check the correlation of the data with actual filter performance, the end loops were moved closer to the coupled-resonator pairs in order to produce maximally flat response curves. The results for two cases are shown in Figures 5-1 and 5-2. The pertinent dimensions are indicated in the figures.

![Figure 5-1. Maximally Flat Dielectric-Resonator Filter, $a = b = 0.750''$](image1)

![Figure 5-2. Maximally Flat Dielectric-Resonator Filter, $a - b = 0.995''$](image2)

The 3-dB bandwidth of a two-resonator, maximally flat filter is related to the coupling coefficient $k_{12}$ by

$$\text{(BW)}_{3\text{db}} = \sqrt{2f_0 k_{12}}$$

or,

$$k_{12} = 0.707 \frac{(\text{BW})_{3\text{db}}}{f_0} \quad (5-2)$$

From the experimental data in Figure 5-1, the coupling coefficient was computed and compared with the previously measured value from
Figure 2-4d. Thus, in Figure 5-1, \((BW)_{3db} = 7.69 \text{ Mc}, f_o = 3040 \text{ Mc}\), and hence,

\[
k_{12} = \frac{0.707 \times 7.69}{3040} = 0.00179
\]

This is very close to the value shown in Figure 2-4d, where at 
\(s = 1.266 \text{ inch}\), \(k_{12} = 0.0017\) from the measured points and \(k_{12} = 0.0016\) from the theoretical curve.

In Figure 5-2, \((BW)_{3db} = 18.3 \text{ Mc} \text{ and } f_o = 3030 \text{ Mc}\). Therefore,

\[
k_{12} = \frac{0.707 \times 18.3}{3030} = 0.00427
\]

while Figure 2-4e gives \(k_{12} = 0.0046\) from the measured points and 0.0050 from the theoretical curve.

A third case that was not plotted had the same parameters as 
Figure 5-1, except that \(s = 1.000 \text{ inch}\). The coupling coefficient determined from the 3-db bandwidth is 0.00482, while the measured points in Figure 2-4d give 0.0048, and the theoretical curve gives 0.0052.

The resonator unloaded \(Q\) values were deduced from the bandwidth and center-frequency dissipation-loss measurements. The relationship for a maximally flat two-resonator filter is as follows:

\[
Q_u = \frac{4.34 \times 2.83 \ f_o}{(BW)_{3db} f_o}
\]

where \(f_o\) and \((BW)_{3db}\) are in Mc, and \(L_o\) is in db. For the three cases discussed above, the \(Q_u\) values were found to be 6060, 7260, and 5140, respectively. Note that the second value applies to 0.995 by 0.995 inch tubing, while the other values are for 0.75 by 0.75 inch tubing. In all cases the walls are aluminum.
Both Figure 5-1 and 5-2 indicate a dissymmetrical response. This is especially pronounced in Figure 5-2, where the bandwidth is approximately twice that of Figure 5-1. The possible causes of the observed dissymmetry will be investigated during the next quarter. Techniques of eliminating or reducing this dissymmetry will be explored.
SECTION V
CONCLUSIONS

The new multiple-mode coupling-coefficient formula is vastly superior to the earlier single-mode formula when the dielectric disks are close together. Comparison with experimental points shows good agreement even when the disks are touching each other.

Calculations of errors in the two techniques for dielectric-constant measurement indicate typical precisions of about $\pm 0.5\%$ maximum error and $\pm 0.2\%$ probable error. These values are based on dimensional measurements to $\pm 0.0005$ inch and frequency measurements to $\pm 0.1$ percent. An improvement of accuracy by at least an order of magnitude appears feasible, but is not necessary for the purposes of this program.

Density variations in ceramic samples were previously suspected as the cause of minor anomalies in measured dielectric-constant values. Further confirmation of this possibility was found when various samples cut from a large single piece of TiO$_2$ ceramic were found to have densities varying by as much as 2 percent. The effect of temperature on dielectric constant was also found to exceed the nominal sensitivity of $-800$ ppm/$^\circ$C by amounts ranging from about 5 to 37 percent in four different groups of samples.

The setup for $Q_u$ measurements described in the previous quarter report was improved through the use of an isolated external modulator and more precise frequency-measurement equipment. Several of the dielectric pieces previously tested were rechecked and found to agree reasonably well with the earlier data. Therefore, the setup improvements were not found to be essential. Nevertheless they have been retained because of the greater reliability and precision they afford.
Measurements made with this setup show the USAERDL high-purity cold-pressed samples to have considerably higher $Q_u$ than previously obtained $\text{TiO}_2$ batches. Other measurements on a dielectric disk in a series of square waveguides confirm that $Q_u$ is drastically affected when the waveguide dimension is made less than twice the diameter of the disk. However, even when the waveguide dimension was equal to the diameter of the disk, $Q_u$ was approximately 5000 for silver-plated waveguide-walls. This is about four times the $Q_u$ value obtainable in a practical coaxial or strip-line resonator having the same external volume of approximately $0.5 \times 0.5 \times 0.5$ inch.

Experiments on several two-resonator maximally flat filters confirm the validity of the coupling-coefficient and unloaded-$Q$ data previously obtained. A dissymmetry of the stop-band response was observed. This dissymmetry becomes more severe as the filter bandwidth is widened. Since a dissymmetrical response is usually a disadvantage in a bandpass filter, methods of reducing this effect should be found.
SECTION VI

PROGRAM FOR NEXT INTERVAL

The analysis of resonant frequency and coupling will be further developed. The theoretical treatment of metal-wall proximity and its effect on $f_0$ and $Q_u$ will be completed.

Experimental studies will be continued on $c_r$ and $Q_u$ of dielectric samples. In cooperation with the USAERDL Ceramics Laboratory, an effort will be made to obtain a suitable material or combination of materials having relatively low temperature sensitivity.

An investigation will be made of the stop-band-dissymmetry effect observed in relatively wide-band filters. Techniques for reducing this effect will be examined.

Parameters for deliverable filter models will be selected. These filters will then be designed, constructed and tested.
SECTION VII
LIST OF REFERENCES


SECTION VIII
IDENTIFICATION OF KEY TECHNICAL PERSONNEL

<table>
<thead>
<tr>
<th>Name</th>
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<tr>
<td>Dr. Seymour B. Cohn</td>
<td>152</td>
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<td>Specialist</td>
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</tr>
<tr>
<td>Mr. Kenneth C. Kelly</td>
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<td>Senior Engineer</td>
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<td>Mr. Richard V. Reed</td>
<td>237</td>
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<td>Engineer</td>
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Rantec Corporation, Calabasas, California

MICROWAVE DIELECTRIC-RESONATOR FILTERS, by S. B. Cohn and K. C. Kelly, an investigation. Third Quarterly Report, 1 January to 31 March 1964, 36p. incl. illus. tables, 7 refs. (rept. no. 3, proj. 3162). (Contract DA-36-039-AMC-02267(E) uncl.)

The coupling analysis is extended to include higher modes. Curves from the new formula provide greatly improved agreement with measurements.

Sources of error are evaluated in the two measurement techniques for high-ε_s samples. Maximum errors are about 0.5.

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The coupling analysis is extended to include higher modes. Curves from the new formula provide greatly improved agreement with measurements.

Sources of error are evaluated in the two measurement techniques for high-ε_s samples. Maximum errors are about 0.5.
percent for $\varepsilon_r$ near 100. Effect of temperature on $\varepsilon_r$ was measured for four groups of TiO$_2$ samples. $Q_u$ data were taken on three groups of TiO$_2$ disks. The best values were obtained with USAERDL cold-pressed high-purity samples, with $Q_u$ between 10,000 and 12,000. One piece was measured in "cut-off" square waveguides of various size. When the disk was tangent to the top and bottom walls, $Q_u$ was diminished to 5250. Resonant-frequency data versus the dimension of the square waveguide are also given.

Response curves are shown for several band-pass filters. Bandwidth and center-frequency loss agree quite well with coupling coefficient and $Q_u$ data.

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