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RM-4071-ARPA
JUNE 1964

AIR RADIATION FROM
NONEQUILIBRIUM WAKES OF
BLUNT HYPERSOONIC REENTRY VEHICLES

R. O. Hundley

PREPARED FOR:
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The RAND Corporation
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An understanding of the radiation signatures of reentry vehicles is of fundamental importance for both offensive and defensive missile systems. This Memorandum represents an extension of our knowledge of reentry radiation to a new area, that of nonequilibrium wake radiation. This work is part of a continuing theoretical study of reentry physics supported by the Advanced Research Projects Agency.
SUMMARY

The wakes of reentry vehicles at altitudes of 100,000 ft or above will not be in chemical equilibrium. Under these conditions the wake radiation cannot be obtained from the ordinary equilibrium calculations, but a detailed study must be made of the chemical kinetics of the nonequilibrium wake. This Memorandum presents the results of such a study for the case of a blunt, nonablating vehicle.

In this nonequilibrium regime the radiation (UV, visible, and near IR) will be produced primarily by various chemiluminescent reactions. For the case of a pure air wake the most important of these will be the Lewis-Rayleigh nitrogen afterglow, the blue nitric oxide afterglow, and the NO$_2$ "continuum." The amount of radiation produced by these reactions will depend on the species concentrations in the wake, which will in turn be affected by the chemical reactions and the amount of turbulent mixing occurring in the wake.

Knowledge of the important chemical reactions for a pure air wake is fairly complete. However, large uncertainties exist with respect to hypersonic turbulence. For that reason, two recently proposed models of the turbulent mixing process are used in this Memorandum: the inviscid random convection model, and the homogeneous mixing model. These two models give radiant intensities for the various chemiluminescent reactions that differ as much as several orders of magnitude. This large difference is another illustration of the importance of increasing our knowledge of hypersonic turbulence.

The radiation estimates presented here show that the nonequilibrium wake radiation in the UV, visible, and near IR should dominate the gas cap and surface radiation at altitudes above about 100,000 ft.
ACKNOWLEDGMENTS

The author would like to acknowledge innumerable and extremely helpful discussions with F.R. Gilmore and M. Romig of The RAND Corporation. He is also indebted to W.M. Hamilton of the Institute of Science and Technology, University of Michigan, for bringing this problem to his attention; to A.Q. Eschenroeder of the General Motors Defense Research Laboratories, for discussions of chemical kinetics; and to W.H. Wurster of the Cornell Aeronautical Laboratory, for providing unpublished research results.
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I. INTRODUCTION

During the past several years there has been a great deal of interest in the radiation emitted during the reentry of hypersonic vehicles. There has been extensive work on the various sources of radiation, with particular emphasis on the radiation from the shock layer at the front of the reentry body, and from the body surface itself. More recently, it has been realized that for most reentry bodies the dominant source of radiation, over large portions of the reentry path, will be the wake. This realization has stimulated extensive work on wake radiation. To date, however, the bulk of this work has been concerned with the radiation from a wake in chemical equilibrium.

Recent studies of the chemical kinetics of air and of chemically reacting hypersonic flow fields have shown that for altitudes above about 50,000 ft the flow field behind a reentry vehicle will not be in chemical equilibrium;\(^{(1)}\) as the flow expands and cools behind the body, the atomic recombination processes will lag behind the flow, resulting in a nonequilibrium chemical composition. This nonequilibrium chemistry will affect the temperature and density properties of the flow field. It will also affect the nature of the radiation. For slender bodies, where the initial degree of dissociation in the flow is small, the difference between the equilibrium and nonequilibrium chemistry will not be very large, and the resulting effects on the flow field and on the radiation will be slight. For blunt bodies however, where the air behind the bow shock is almost completely dissociated, the difference between equilibrium and nonequilibrium
chemistry will be very large; it will greatly affect the temperature of the flow field, and it will completely change the nature of the radiation.

The purpose of this Memorandum is to begin the study of radiation from nonequilibrium wakes. For a complete study of this problem, one should include the effects of ablation products in the wake, since they can be a very important source of radiation. In this Memorandum ablation products will be omitted and only pure air wakes will be considered. The results of this study, besides being directly applicable to nonablating vehicles, are also a necessary preliminary to a study of ablation radiation, since before one can hope to understand the chemistry of the ablation product--air mixture, one must understand the pure air problem.

The primary aim of this Memorandum will be to describe the important aerodynamic and chemical processes affecting radiation in the nonequilibrium regime, to discuss the connection between the flow field properties and the radiation, and to describe the characteristics of the radiation that is emitted. Numerical estimates of the intensity of the nonequilibrium radiation will be made for various values of the flight parameters. The only case to be considered will be that of blunt bodies, where the nonequilibrium effects will be the most pronounced, and only radiation in the UV, visible, and near IR, will be considered, since this is the spectral region containing the bulk of the radiant energy.

Before beginning the detailed discussion, it is perhaps useful to summarize the main differences between the equilibrium and nonequilibrium situations. In the case of an equilibrium wake the expansion of the flow behind the body is accompanied by atomic
recombination with its attendant release of energy. Because of this energy release, the far wake (i.e., the wake downstream of the $P \rightarrow P_\infty$ point) will be fairly hot, with temperatures initially of the order of $3000^\circ\text{K}$ to $4000^\circ\text{K}$. The far wake will also be primarily molecular; the only atomic species present in significant quantities will be oxygen. Because the wake is assumed to be in equilibrium, the excited states of the air molecules will have an equilibrium population distribution. The resulting radiation will be the extensively studied equilibrium air radiation. This equilibrium air radiation, in the temperature range of interest, has a very steep temperature dependence, so that the radiation intensity will be strongly coupled to the wake temperature. Because of this, the decay of radiant intensity in the wake will be directly related to the temperature decay in wake, which in turn is directly related to the decay of wake enthalpy. This means that the wake radiation will be directly related to the overall aerodynamic properties of the wake.

In the case of a nonequilibrium wake there is very little atomic recombination during the expansion behind the body. The energy of molecular dissociation is not liberated during this expansion, so that the temperature drop is much greater than for the equilibrium case; by the time the pressure has decayed to the ambient value, the temperature has decreased to a value of the order of $1000^\circ\text{K}$. Thus, the nonequilibrium wake will be colder than the equilibrium wake. However, it will also be much longer than the equilibrium wake; the length of the nonequilibrium wake will be determined, not by the time required for the temperature or wake enthalpy to decay, but by the time required for the atomic species to recombine. (At high altitudes, 200,000 ft for example, this
recombination time can be several seconds.) Because of the low
temperature, collisional excitation will be of negligible importance
as a source of radiation. However, the nonequilibrium wake is a highly
reactive mixture of atoms and molecules, and there are numerous
chemiluminescent reactions which can be a significant source of
radiation. Since these reactions are exothermic, their rate constants
are relatively insensitive to temperature. Because of this, the intensity
of the wake radiation will also be relatively insensitive to the wake
temperature. However, since the radiation is a chemical phenomenon, it
will be greatly affected by changes in the chemical composition of the
wake. This chemical composition will in turn be affected by the various
chemical reactions that occur in the wake, and by the extent to which
the outer, molecular-rich inviscid flow is mixed into the inner, atomic
part of the wake. Thus, in the nonequilibrium case the wake radiation
will be directly affected by the mixing processes in the wake, but it
will not be affected significantly by the temperature decay in the
wake or the enthalpy decay.

The above, very brief summary of the properties of the radiation
from nonequilibrium wakes will be expanded in the following sections.
Section II presents a discussion of the various chemiluminescent
reactions of importance in the wake, and Section III discusses the
various wake phenomena important in determining the amount of
radiation produced by the chemiluminescent reactions. In Section IV
numerical estimates are presented of the amount of radiation emitted
from the wake, and Section V is a summary and discussion of the
results.
II. CHEMILUMINESCENT REACTIONS

As indicated in the Introduction, the wake of a blunt body at altitudes above about 50,000 ft will be relatively cool, of the order of 1000°K, but will contain large numbers of unrecombined atoms (N and O). As we shall see in the next section, there will also be significant quantities of NO present. For a mixture of this temperature and chemical composition, the most important source of radiation will be chemiluminescence. The most significant chemiluminescent reactions will be

\[ \text{N} + \text{N} + (\text{M}) \rightarrow \text{N}_2 + (\text{M}) + h\nu \]  
(1)

\[ \text{N} + \text{O} + (\text{M}) \rightarrow \text{NO} + (\text{M}) + h\nu \]  
(2)

\[ \text{NO} + \text{O} + (\text{M}) \rightarrow \text{NO}_2 + (\text{M}) + h\nu \]  
(3)

where a third body (M) may or may not participate in the reaction.

There now exists a great deal of experimental information regarding these reactions. In this section we summarize those details concerning these reactions of importance for the problem of wake radiation.

A. LEWIS-RAYLEIGH NITROGEN AFTERGLOW

Reaction (1) is the well known Lewis-Rayleigh nitrogen afterglow, the spectrum of which consists principally of the first positive bands of N\(_2\). Recent kinetic studies of this reaction show that for pressures between 1 and 10 mm Hg (10\(^{-3}\) to 10\(^{-2}\) atmospheres) the overall reaction follows a two-body rate law with a rate constant\(^{(2)}\)*

*Young and Sharpless state that this value, and the other chemiluminescent rate constants given in this section, are probably accurate to a factor of 2.\(^{(2)}\)
\[ K_1 = 3.1 \times 10^{-17} \text{ cm}^3/\text{sec} \]

For pressures outside this range it is unclear at the present time as to whether the reaction follows a two-body or three-body rate law.\(^*\) However, to assume the two-body law holds is as reasonable as any other assumption.\(^{(2)}\)

The spectrum of the Lewis-Rayleigh afterglow consists primarily of the \( \text{N}_2 \) first positive bands, with additional weak emission in the \( \text{Y} \) bands in the visible and infrared, and in the Lyman-Birge-Hopfield system in the vacuum ultraviolet. The strongest bands of the first positive system observed in the afterglow are:\(^{(3)}\)**

\[
\begin{align*}
\Delta v = 5 & \text{ sequence} & 5370 \text{ Å} & \text{to} & 5440 \text{ Å} \\
\Delta v = 4 & \text{ sequence} & 5750 \text{ Å} & \text{to} & 5870 \text{ Å} \\
(11,8) & \text{ band} & 6250 \text{ Å} \\
(6,3) & \text{ band} & 6620 \text{ Å} \\
(4,2) & \text{ band} & 7500 \text{ Å} \\
(3,1) & \text{ band} & 7630 \text{ Å} \\
(2,0) & \text{ band} & 7750 \text{ Å} \\
(2,1) & \text{ band} & 8720 \text{ Å} \\
(1,0) & \text{ band} & 8910 \text{ Å} \\
(0,0) & \text{ band} & 10420 \text{ Å}
\end{align*}
\]

\(^*\) It should be pointed out that the existence of a two-body rate law over a limited region of pressure does not necessarily imply a purely two-body reaction. Many examples are known of complicated chain reactions which, over a limited pressure region, obey a two-body rate law. It is very likely that the chemiluminescent reactions discussed in this section are of that type.

\(^{**}\) The relative intensities of the various first positive bands excited by reaction (1) are not the same as for the first positive emission from equilibrium air.
B. BLUE NITRIC OXIDE AFTERGLOW

Reaction (2) produces the blue nitric oxide afterglow, consisting of emission in the $\beta$, $\gamma$, and $\delta$ bands of NO. The reaction mechanism is complicated, and results in an overall rate constant that is partly two-body and partly three-body.\(^{(4)}\) The NO $\beta$ system is entirely three-body, with a rate constant\(^{(2)}\)

$$K_\beta = 2.4 \times 10^{-34} \text{ cm}^6 / \text{sec}$$

The NO $\gamma$ system is part two-body and part three-body. The rate constants are\(^{(2)}\)

$$K_\gamma = \begin{cases} 8.2 \times 10^{-18} \text{ cm}^3 / \text{sec} & \text{(two-body)} \\ 2.4 \times 10^{-35} \text{ cm}^6 / \text{sec} & \text{(three-body)} \end{cases}$$

The NO $\delta$ system is entirely two-body, with a rate constant\(^{(2)}\)

$$K_\delta = 1.1 \times 10^{-17} \text{ cm}^3 / \text{sec}$$

These values result in a total rate constant for reaction (2) which is part two-body and part three-body, with the values\(^{(2)}\)

$$K_2 = \begin{cases} 1.9 \times 10^{-17} \text{ cm}^3 / \text{sec} & \text{(two-body)} \\ 2.6 \times 10^{-34} \text{ cm}^6 / \text{sec} & \text{(three-body)} \end{cases}$$

The relative intensity of the emission in the various vibration-rotation bands of the $\beta$, $\gamma$, and $\delta$ systems has been measured, and is shown
in Tables 1, 2, and 3. \(^{(2)}\) With respect to ground-based or airplane-based observations of this radiation, it should be noted that all of the \(\gamma\) and \(\delta\) radiation lies at wavelengths below 3000 \(\AA\), where it will be absorbed by the ozone. However, approximately 36 per cent of the radiation from the \(\beta\) system lies above 3000 \(\AA\). Using this number, one may obtain an effective rate constant for reaction (2), for radiation above 3000 \(\AA\), of

\[
K'_2 = 0.9 \times 10^{-34} \text{ cm}^6 / \text{sec}
\]

In addition to the \(\beta, \gamma, \) and \(\delta\) bands of \(\text{NO}\), the recombination of atomic nitrogen and oxygen is known to produce emission in the \(\text{NO}\) Ogawa bands, in the region from 7800 \(\AA\) to 9700 \(\AA\), and in the \(\text{NO}(^2 \Pi - ^2 \Sigma^+ )\) band system, in the region around 12200 \(\AA\). \(^{(4)}\) Quantitative information concerning the rate constants for these two band systems is unavailable at the present time. However, based on the presently accepted mechanism for the \(\text{NO}\) afterglow radiation, the \(^2 \Pi - ^2 \Sigma^+ \) system should have an intensity of the same order of magnitude as the two body part of the \(\gamma\) system. \(^{(4)}\)

C. \(\text{NO}_2\) CONTINUUM

Reaction (3) produces the so-called \(\text{NO}_2\) "continuum." This emission is not really a continuum, but has a fine grained banded structure, due to the complicated chain mechanism producing the overall reaction. \(^{(5)}\) The emission spectrum begins at 3875 \(\AA\), peaks at about 6500 \(\AA\), and extends to at least 14000 \(\AA\). \(^{(6)}\) The reaction mechanism results in an overall rate constant that is two-body, at least for pressures above about 0.1 \(\text{mm Hg (10}^{-4}\) \text{ atmospheres}). \(^{(5, 6)}\) The value
Table 1

NO $\beta$ SYSTEM

(Data taken from Ref. 2)

<table>
<thead>
<tr>
<th>Vibration band</th>
<th>$\lambda$(Å)</th>
<th>Relative intensity</th>
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<tbody>
<tr>
<td>6-1</td>
<td>2018</td>
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<tr>
<td>3-1</td>
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</tr>
<tr>
<td>4-2</td>
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<tr>
<td>3-2</td>
<td>2232</td>
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</tr>
<tr>
<td>2-2</td>
<td>2288</td>
<td>0.98</td>
</tr>
<tr>
<td>3-3</td>
<td>2327</td>
<td>1.87</td>
</tr>
<tr>
<td>2-3</td>
<td>2382</td>
<td>0.88</td>
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<tr>
<td>3-4</td>
<td>2428</td>
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<tr>
<td>2-4</td>
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<td>1.44</td>
</tr>
<tr>
<td>1-4</td>
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<td>0-12</td>
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Table 2

**NO γ SYSTEM**

*Data taken from Ref. 2*

<table>
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<tr>
<th>Vibration band</th>
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<th>Relative intensity</th>
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<td>3-1</td>
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Table 3

NO δ SYSTEM

(Data taken from Ref. 2)

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<tr>
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<td>0-2</td>
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<td>0-4</td>
<td>2227</td>
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</tr>
<tr>
<td>0-5</td>
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of the rate constant, for emission in the wavelength region from 3875 Å to 14000 Å is \( K_3 = 6.4 \times 10^{-17} \text{ cm}^3/\text{sec} \)

D. EFFECT OF TEMPERATURE ON CHEMILUMINESCENT REACTIONS

The values of the chemiluminescent rate constants given in the above discussion have all been the result of room temperature measurements. The temperatures of interest in the wake will be of the order of 1000°K to 2000°K, however, so that the question arises of the temperature dependence of the rate constants.

At the present time there is no clear experimental information concerning this temperature dependence. However, there does exist a reasonable understanding of the detailed mechanisms for these three chemiluminescent reactions, \(3-5\) and it is well established that for each of these reactions the activation energy is zero. This means that the rate constants will not be exponentially dependent on temperature; if there is a temperature dependence, it will be slight.

Attempts have been made experimentally to determine the temperature dependence of the NO + O reaction.\(7,8\) These attempts were based on high temperature shock tube measurements of the NO2 "continuum," together with the assumption that the high temperature radiation and the low temperature afterglow were both produced by the same radiative recombination mechanism. It has recently been suggested that this assumption is not correct, and that the temperature dependence deduced from it is not justified.\(9\)
In view of the uncertain situation concerning the temperature dependence of the chemiluminescent rate constants, the only well-established fact being that the dependence on temperature will be slight, and in view of the fact that the room temperature rate constants are only known to within a factor of two, the most reasonable course at the present time seems to be to use the room temperature values, without attempting a dubious extrapolation to higher temperature. The error made in doing this will most probably be no greater than the error inherent in our knowledge of the room temperature values.

Besides any effect on the rate constants, the temperature should also affect the spectral distribution of the various afterglow reactions. No information is available on this effect, but it may turn out to be more important than the rate constant effect.

E. OTHER CHEMILUMINESCENT REACTIONS

In addition to the three reactions discussed in detail above, a number of other chemiluminescent reactions will occur in the wake. The most extensively studied of these is the oxygen recombination reaction

$$0 + O + (M) \rightarrow O_2 + (M) + h\nu$$

which produces the atmospheric and Herzberg systems of $O_2$. This reaction will, for two reasons, be of much less importance than the previous three as a source of wake radiation: (1) The rate constants are much smaller; for the atmospheric bands a three-body rate constant of the order of $10^{-37}$ cm$^6$/sec; (2) for the Herzberg bands a two-body rate constant of the order of $10^{-21}$ cm$^3$/sec; (2) The radiative transitions involved are partially forbidden with relatively long
radiative lifetimes, and appear to be strongly quenched by collisional de-excitation.\( ^2 \)

The red (6300 Å) and green (5577 Å) lines of atomic oxygen are also produced by chemiluminescent reactions. The red line, with a radiative rate of about \(10^{-2}\) sec\(^{-1}\), will certainly be collisionally de-excited at reentry altitudes. The situation with respect to the green line is less clear. Also, the absolute rate of chemiluminescent production of the green line is uncertain at the present time.\( ^2 \)

In addition to the above reactions, which produce radiation in the UV, visible, and near IR, numerous chemiluminescent reactions produce radiation in the middle IR.\( ^{10} \) The most important spectral feature in this region, for a pure air wake, will probably be the fundamental vibration-rotation band of NO at 5.3 microns, and the most important reaction producing this band will probably be

\[
N + O_2 \rightarrow NO^* + O
\]

where the superscript * denotes a vibrationally excited molecule in its ground electronic state. The amount of vibrational excitation produced by this reaction has not yet been measured. However, the similar reaction

\[
N + NO \rightarrow N_2^* + O
\]

is known to produce large amounts of vibrationally excited \(N_2\).\( ^{11} \)

Because of the low excitation energy of the NO vibration-rotation band, about 0.2 ev, and because of the long radiative lifetime, of the order of \(10^{-1}\) sec, collisional excitation will also be an important source of NO*, even at altitudes of 150,000 to 200,000 ft.
F. FACTORS AFFECTING CHEMILUMINESCENT RADIATION

As we have discussed above, the various chemiluminescent reactions expected to be of importance as sources of wake radiation are not very temperature sensitive. This is in contrast to the radiation from equilibrium air, which is exponentially dependent on temperature. The amount of chemiluminescent radiation produced will, however, be very dependent on the species concentrations in the wake. It will depend on the amount of N₂O, and NO present in the wake, and it will depend on the length of time that these species are present. In the following section we will discuss the various wake phenomena that are important in determining the species concentrations in the wake, and therefore also important in determining the amount of wake radiation.
III. WAKE PHENOMENOLOGY

As indicated in the previous section, the amount of radiation from a nonequilibrium wake will depend primarily on the species concentrations in the wake, in particular on the concentrations of N₂O and NO, since these are the reactants involved in the most important chemiluminescent reactions. The value of these species concentrations will in turn depend on a number of flow field phenomena: the conditions established in the stagnation region, the expansion of the flow around and immediately behind the body, the chemical reactions occurring in the far wake,* and the effect of turbulent mixing in the wake. In this section we will discuss the effect of these various phenomena on the species concentrations.

A. STAGNATION AND EXPANSION REGION

Upon crossing the bow shock of a blunt body, the air flow is rapidly compressed and heated to a very high temperature. Molecular dissociation occurs, and the chemical composition of the air begins to approach equilibrium. Whether or not chemical equilibrium is achieved in the stagnation region at the front of the body depends on a number of factors: the free stream velocity, the ambient density, and the nose radius of the body. For typical ICBM reentry velocities, of the order of 20,000 ft/sec, and for reentry bodies of typical size, with nose radii of the order of 1 ft, it has been found that the stagnation region will be in equilibrium for altitudes up to about 200,000 ft. (12) Assuming that equilibrium is attained, the mole fraction species concentrations in the stagnation region will be as shown in Figs. 1-3.

* By "far wake" we will mean that part of the wake at ambient pressure.
Fig. 1 — Atomic nitrogen concentration in stagnation region
Fig. 2—Molecular nitrogen concentration in stagnation region
Fig. 3—Atomic oxygen concentration in stagnation region
The data in Figs. 1-3 were obtained from the Avco gas dynamic charts,\(^{(13)}\) together with Gilmore's equilibrium air tables.\(^{(14)}\)

As the airflow leaves the stagnation region, it begins to expand around and behind the body. This expansion continues until the pressure has decayed to the free stream value, \(^*\) which occurs at a downstream distance \(X/R_n \approx \frac{M_{\infty}^2}{4.5}\), where \(R_n\) is the nose radius and \(M_{\infty}\) the free stream Mach number.\(^{(15,16)}\) During this expansion the air cools rapidly, and for chemical equilibrium to be maintained, atomic recombination should occur in this expansion region. However, for low enough ambient density (or high enough altitude) the atomic recombination processes will lag behind the temperature drop, resulting in a nonequilibrium chemical composition.

Recent studies of the chemically reacting, expanding flow behind blunt bodies show that this nonequilibrium effect will be of major importance for altitudes above about 50,000 ft.\(^{(1)}\) Earlier studies have shown that for altitudes above 100,000 ft, the mole fraction concentrations of the major species in the flow \((N, N_2, O)\) freeze at essentially their stagnation point values.\(^{(17,18)}\) However, and this will be of importance in the later discussion, those species present in the stagnation region only in small amounts \((O_2, NO)\) do not freeze during the expansion; due to the effect of certain bimolecular exchange reactions to be discussed later, the concentrations of \(O_2\) and \(NO\) are reduced by over an order of magnitude below the values that would be estimated from any sudden freezing approximation.\(^{(18)}\)

\(^*\) The so-called \(P \rightarrow P_{\infty}\) point.
Since those species carrying the bulk of the chemical enthalpy (N and O) freeze during the expansion, the temperature drop associated with the expansion is much greater than for an equilibrium wake. Because the chemical composition is essentially constant during the expansion, the expansion process will be approximately isentropic, and the value of temperature and density at the end of the expansion region (the $P \rightarrow P_\infty$ point) can be estimated from the usual isentropic relations $T \propto P^{(\gamma-1)/\gamma}$ and $p \propto P^{1/\gamma}$, where $\gamma$ is now the effective specific heat ratio appropriate to the frozen chemical composition.\(^{(19)}\)

Using this method, together with the mole fraction values from Figs. 1-3, one obtains the value of temperature and density at the $P \rightarrow P_\infty$ point shown in Figs. 4 and 5.\(^{*}\) The concentrations of the frozen species N, N\(_2\), and O at the $P \rightarrow P_\infty$ point can then be obtained; these are shown in Figs. 6-8. These results, although obtained by assuming a simple isentropic expansion, are in substantial agreement with results of more detailed numerical calculations.\(^{(1,20)}\)

Several comments are now in order concerning these results. First of all, the values of temperature shown in Fig. 4 bear out our previous remarks; the far wake will be relatively cool, with temperatures initially of the order of 1000\(^0\)K. For temperatures of this magnitude, as we have discussed previously, the important chemical reactions will be exothermic ones which are relatively insensitive to temperature. In addition, Fig. 3 shows that when the flow has reached the $P \rightarrow P_\infty$ point, the density will be within a factor of approximately four to seven of the free stream value. Downstream of the $P \rightarrow P_\infty$ point, the temperature and density will asymptotically approach the free stream values. Since

\(^{*}\) Vibration equilibrium was assumed during the expansion process, since this has been shown to be a good approximation.\(^{(18)}\)
Fig. 4—Temperature at $P\rightarrow P_\infty$ point
Fig. 5 — Density at $P ightarrow P_\infty$ point
Fig. 6—Atomic nitrogen concentration at $P\rightarrow P_\infty$ point
Fig. 7—Molecular nitrogen concentration at $P \rightarrow P_\infty$ point
Fig 8 — Atomic oxygen concentration at $P \rightarrow P_\infty$ point
they are not too far removed from these values to begin with, the
temperature and density variations in the far wake will be relatively
small, much smaller than for the case of an equilibrium wake.

The results shown in Figs. 4-8 are the values on the axis. The
off-axis behavior of these quantities will also be of importance.
Detailed numerical studies\(^{(1)}\) have shown that, for the special cases
considered, the concentrations of the atomic species N and O will
remain relatively constant for some distance off-axis, and then decrease
rapidly to zero. The temperature will experience some off-axis variation,
but will remain within the range of 1000\(^{\circ}\)K to 2000\(^{\circ}\)K for about the same
distance off-axis, and then will also drop rapidly.\(^{(1)}\) These detailed
studies suggest that a reasonable model of the nonequilibrium wake
may be obtained by assuming a cylindrical region surrounding the axis
over which the axis values of species concentration (N, N\(_2\) and O) and
temperature hold. The radial extent of this region varies with down­
stream position; at the P → P\(_\infty\) point it is of the order of 3 R\(_n\).\(^{(1)}\)

There is one exception to the above statement concerning the
off-axis behavior of the species concentrations. The NO concentration
is very low on the axis, but there is a thin shell of relatively high
NO concentration about 3 R\(_n\) off-axis.\(^{(1)}\) The significance of this NO
shell will be discussed later.

The above discussion pertains to the values attained by the various
wake parameters at the end of the expansion region. Downstream of this
point the wake will continue to cool, but since the temperature is so
low to begin with, the cooling will not greatly affect the chemical
composition. The chemical composition will be affected, however, by
the various chemical reactions that occur in the wake, and by the
mixing processes that occur. We will now discuss these effects.

B. CHEMICAL KINETICS OF FAR WAKE

As shown above, the initial concentrations of N, N\(_2\) and O in the far wake (i.e., the concentrations at the P\(_\infty\) point) are determined by the processes occurring during the expansion. The duration of the N and O in the wake, either in terms of lifetime or downstream distance, will be determined by the chemical reactions that occur in the wake. These reactions will also determine the amount of O\(_2\) and NO present in the wake.

The detailed nature of the chemical kinetics in the wake will be strongly affected by the amount of mixing that occurs between the inner wake, which is mostly atomic, and the outer, molecular part of the wake. For the moment we will neglect this mixing; later in this section we will discuss its effect.

To begin with, we consider the O\(_2\) kinetics. In the absence of mixing, the O\(_2\) in the inner part of the wake will be produced by the reaction

\[ \text{O} + \text{O} + \text{M} \rightarrow \text{O}_2 + \text{M} \]  

(4)

Because of the chemical composition of the wake, i.e., large quantities of atomic oxygen present, and because of the enhanced catalytic efficiency of O relative to N or N\(_2\), O will be the most effective catalyst for this reaction, i.e., M \(\equiv\) O, so that the rate of reaction (4) will be \(\Gamma_4 = K_4 N_0^3\), where \(N_0\) is the concentration of O, and the rate constant is(21,22)

\[ K_4 = 2.1 \times 10^{-31} T^{-1/2} \text{cm}^6/\text{sec} \]

where T is in degrees Kelvin. The O\(_2\) will be consumed primarily by
with a rate $\Gamma_5 = K_5 N_0 N_0$, and a rate constant \(^{21}\)

$$K_5 = 2.2 \times 10^{-14} T^{-3540/T} \text{ cm}^{-3} / \text{sec}$$

A steady state is very quickly reached between reaction (4) producing $O_2$ and reaction (5) consuming it. The steady-state concentration of $O_2$ will be

$$N_{O_2} = \frac{K_4 N_0^3}{K_5 N_N}$$

This steady state is achieved in a time $\tau_{O_2} \sim 1/K_5 N_N$, which, for the temperature and nitrogen atom density initially present in the wake, is of the order of $10^{-3}$ seconds.

The NO in the inner wake, in the absence of mixing, will be produced by reaction (5), and by

$$N + O + M \rightarrow NO + M$$

which has a rate $\Gamma_6 = K_6 N_0 N_0 M$, and a rate constant \(^{23}\)

$$K_6 = 2.8 \times 10^{-28} T^{-3/2} \text{ cm}^6 / \text{sec}$$

for all catalysts except NO. (Since the NO concentration will be very low, it will not be an important catalyst.) The NO will be consumed by

$$N + NO \rightarrow N_2 + O$$

which has a rate $\Gamma_7 = K_7 N_N NO$, and a rate constant \(^{21,22}\)
These reactions rapidly produce an NO steady-state concentration given by

$$N_{NO} = \frac{K_5 N_O^2 + K_6 N_N N_M}{K_7}$$

This steady-state concentration is attained in a time $\tau_{NO} \sim 1/K_{NN}$, which for the cases of interest here is of the order of $10^{-5}$ seconds. If the O$_2$ steady state is also attained, we will have

$$N_{NO} = \frac{K_4 N_O^3 + K_5 N_N N_O N_M}{K_{NN}}$$

Because the O$_2$ and NO chemical relaxation times, $\tau_{O_2}$ and $\tau_{NO}$, are so short, the O$_2$ and NO concentrations are not frozen in the wake, but remain in a quasi-equilibrium with respect to the N and O concentrations. These short relaxation times are due to the fast bimolecular exchange reactions, reactions (5) and (7). The importance of these reactions depends on the presence of atomic nitrogen, and they will maintain the quasi-equilibrium of O$_2$ and NO as long as N is present in significant quantities. These same bimolecular exchange reactions are responsible for the fact mentioned previously, that the O$_2$ and NO do not freeze in the expansion region.

We can now consider the N and O kinetics, which will involve much longer characteristic times than the O$_2$ or NO kinetics. As the N and O concentrations change, the O$_2$ and NO concentrations will quickly adjust to follow the change, as long as N is present.
Let us now discuss the N kinetics. Atomic nitrogen is not being produced by any reactions. It is being consumed by reactions (5), (6), and (7), and by

$$N + N + M \rightarrow N_2 + M$$

(8)

As long as significant quantities of atomic nitrogen are present, N will be the most important catalyst for this reaction, so that

$$\Gamma_{8} = K_{8} N_2^3.$$  

The rate constant for this reaction, for temperatures of the order of $1000^\circ K$, is rather uncertain at the present time. Wray\textsuperscript{21} gives

$$K_8 = 6.4 \times 10^{-27} T^{-3/2} \text{ cm}^6/\text{sec}$$

which for $T \approx 1000^\circ K$ gives $K_8 = 2 \times 10^{-32} \text{ cm}^6/\text{sec}$. Bortner\textsuperscript{22} gives

$$K_8 = 2 \times 10^{-29} T^{-1} \text{ cm}^6/\text{sec}$$

which for $T \approx 1000^\circ K$ gives $K_8 = 2 \times 10^{-32} \text{ cm}^6/\text{sec}$. There is thus a factor of ten uncertainty. Both of these values are obtained from high temperature shock tube experiments, the results of which are extrapolated to lower temperature. There is only one direct experimental measurement available in the temperature region of interest here, that of Back et al., at $700^\circ K$.\textsuperscript{24} They found that $K_8$ was of the order of $(1-3) \times 10^{-31} \text{ cm}^6/\text{sec}$. Based on this measurement, we will here use Wray's value, although the situation is still somewhat uncertain.

For the range of values of wake parameters of interest here, all four of the reactions consuming N can be of importance, so that the atomic nitrogen recombination time will be given by

$$\tau_N \sim N_2/(\Gamma_5 \Gamma_6 + \Gamma_7 + 2 \Gamma_8),$$

or
This is approximately the length of time that atomic nitrogen will persist in the wake. While it is present, there will be negligible oxygen recombination, because reactions (5) and (7) quickly liberate any O removed by reactions (4) and (6). Once the N is removed, reaction (4) will begin to consume O, and the O₂ concentration will rise. The atomic oxygen recombination time will be approximately

\[ \tau_0 = \frac{1}{2K_4N_0^2} \]

As later numerical results will show, for all cases of interest we will have \( \tau_N \ll \tau_0 \), so that there will be a period of time during which the wake will consist mostly of N₂ and O, i.e., very little N. During this intermediate period (after N is gone, before O is gone) the NO concentration will be the same as before, since both the NO production and destruction reactions depend on \( N_N \) in the same manner. The only difference will be that the NO relaxation time, \( \tau_{NO} = 1/K_7N_N \), will become much larger, i.e., the NO concentration will "freeze" at the value established while N was present. * This frozen NO concentration will persist for a time much greater than the oxygen recombination time, and the ultimate fate of the NO need not concern us here.

*The reaction \( NO + O + M \rightarrow NO_2 + M \) will not affect the NO concentration, since it will be immediately followed by \( NO_2 + O \rightarrow NO + O_2 \) which recreates the NO molecule. (25) For the NO concentrations achieved in the wake, these two reactions will also have a negligible effect on the oxygen recombination rate.
For the problem of determining the amount of wake radiation the important quantities determined by these various chemical reactions are the concentration of NO in the wake, $N_{NO}$, and the duration of N and O in the wake, $\tau_N$ and $\tau_O$. In Figs. 9 and 10 we plot these quantities, using the axis values of the concentrations of N, $N_2$, and O, i.e., the values shown in Figs. 6-8, since these axis values are appropriate for the inner wake. In determining the rate constants, we used a temperature of 1000°K, which is roughly appropriate for the cases of interest here. The values shown in Figs. 9 and 10 apply to the inner wake, i.e., the high-enthalpy, atomic-rich part of the wake. As we said before, they are correct in the absence of mixing between the inner and outer wake.

C. EFFECT OF TURBULENT MIXING

We must now consider the effect of mixing between the inner and outer wake on the chemical kinetics and species concentrations in the wake. One effect of this mixing will be to cool the inner wake. This effect has been extensively studied for both laminar and turbulent mixing.\(^{26,27}\) However, since the radiation-producing reactions of interest to us are relatively insensitive to temperature, and the temperature is low to begin with, this cooling effect will be of no great significance as far as the nonequilibrium radiation is concerned. What will be important is the extent to which the mixing process changes the chemical composition of the wake, or more specifically, the extent to which it affects the NO concentration in the wake, and the nitrogen and oxygen recombination times.

* Since the rate constants are not very sensitive to temperature changes, using one value of temperature does not introduce a significant error.
Fig. 9—NO concentration in inner wake (no mixing)
Fig. 10—Nitrogen and oxygen recombination times (no mixing)
The steady-state NO concentration in the wake is established during the initial, atomic-nitrogen-rich part of the wake. The chemical reactions occurring during this period also determine the nitrogen recombination time. The chemical kinetics of this atomic nitrogen-rich region are strongly affected by the low O\textsubscript{2} concentration, which limits the rate at which reaction (5) produces NO, and which also helps to limit the rate at which reactions (5) and (7) remove N. As pointed out by Lin and Hayes, the mixing of additional O\textsubscript{2} from the outer, molecular-rich part of the wake into the inner, atomic-nitrogen-rich wake can have a profound effect on the wake kinetics, increasing the NO concentration, and decreasing the nitrogen recombination time. The magnitude of the effect will depend on the rapidity of the mixing process.

If the wake were to remain laminar down to low altitudes, so that mixing occurred only by molecular diffusion, the effect of mixing on the wake kinetics would not be very large. This is because, for reentry bodies of typical size (R\textsubscript{n} \sim 1 ft), the characteristic laminar diffusion time is \sim 1 sec for altitudes below about 200,000 ft. For typical ICBM velocities (7 to 8 km/sec) this is greater than the nitrogen recombination time (cf. Fig. 10), so that by the time the O\textsubscript{2} diffuses into the inner wake the N has already recombined.

However, the wake will not remain laminar. Recent data indicate that for blunt bodies the entire wake downstream of the neck will be turbulent for freestream Reynolds numbers above about 5 \times 10\textsuperscript{6}. 

*The O\textsubscript{2} to 0 ratio in the absence of mixing is K\textsubscript{4}N\textsubscript{2}/K\textsubscript{5}N\textsubscript{2}, which is of the order of 10\textsuperscript{-4} to 10\textsuperscript{-6}.

**Mixing of the NO shell into the inner wake can also affect the kinetics. However, numerical studies have shown this to be of lesser importance than the O\textsubscript{2} mixing, and we will not consider it further here.
For a 1-ft body at Mach 20 this corresponds to altitudes up to about 200,000 ft, so that over the altitude region of interest here, the entire wake will be turbulent. Turbulent diffusion processes are known to be much more rapid than laminar ones, so that in the presence of turbulence it is possible that sufficient $O_2$ could be mixed into the inner wake to affect the kinetics.

Unfortunately, at the present time there are large uncertainties in our knowledge of the turbulent diffusion process. In particular, it is not known how rapidly molecular mixing will occur within the turbulent region. This means that we cannot specify with any assurance the rate at which $O_2$ from the outer wake will be microscopically mixed into the inner wake.

In view of this uncertainty, Lin and Hayes have proposed two extreme models for the turbulent mixing process, which it is hoped will bracket the actual physical situation. In the first of these models, termed "inviscid random convection," the turbulent eddy motion is assumed to mix cold lumps of air from the outer wake into the hotter inner wake, so that the turbulent region consists of a mixture of hot and cold lumps. In this inviscid random convection model the molecular diffusion between hot and cold lumps is assumed negligible, so that a large-scale mixing occurs between the inner and outer parts of the wake, but no microscopic mixing.

*As an example of this uncertainty, Ref. 30 gives four turbulent diffusivity models, all of which are plausible on the basis of existing information, but which produce orders of magnitude difference in the values of the various wake parameters.
Since there is no microscopic mixing, the chemical kinetics of the fluid elements will be the same as for the non-turbulent case, so that the quantities of particular interest to us, the NO concentration and the nitrogen and oxygen recombination times, will be unaffected by the turbulence, i.e., they will still have the values shown in Figs. 9 and 10.

The second model of turbulence proposed by Lin and Hayes, the homogeneous mixing model, is intended to represent an upper limit on the effect of turbulence. In this model the entire turbulent region is assumed to be homogeneous, and microscopic mixing of the new fluid crossing the turbulent boundary with the old fluid already within the boundary is assumed to occur instantaneously. The entrainment of O₂-rich air by the turbulent boundary is therefore equivalent, in this model, to a uniform volume production rate of O₂ within the inner wake.

We will use this homogeneous mixing model of Lin and Hayes to obtain an upper limit on the effect of turbulence on the wake parameters of interest here. To begin, we must estimate the amount of O₂ injected into the inner, turbulent wake. The amount of O₂ crossing the turbulent entrainment boundary per unit length of wake will be given by

\[ \frac{dY}{2\pi Y_f} \frac{f}{g} u_e \]

where \( X \) is the axial distance, \( Y_f(X) \) is the radius of the turbulent front, and \( N_{O_2}^e \) and \( u_e \) are the \( O_2 \) concentration and stream velocity in the inviscid flow just outside the turbulent core. The volume of the turbulent core per unit length of wake is \( \pi Y_f^2 \), so that,
assuming instantaneous homogeneous mixing, the equivalent volume production rate of O\(_2\) is

\[ P_{O_2} = \frac{2}{Y_f} \left( \frac{dY_f}{dx} \right) N_{O_2}^e u_e \]

For \(X/R_n \gtrsim 10^2\), where \(R_n\) is the nose radius, the radius of the turbulent front is approximately\(^{(1)}\)

\[ \frac{Y_f}{R_n} = 0.8 \left( \frac{X}{R_n} \right)^{1/3} \]

This gives \(dY_f/dX = Y_f/3X\), so that

\[ P_{O_2} = \frac{2 N_{O_2}^e u_e}{3X} \]

We are interested in the entrainment of the O\(_2\)-rich part of the inviscid flow. This will begin to occur once all of the dissociated inviscid flow has been engulfed by the turbulent front. According to calculations of Lin and Hayes,\(^{(1)}\) this point is reached somewhere between 100 and 600 nose radii downstream, depending on the body size and altitude. The O\(_2\)-rich part of the inviscid flow will be cool, with temperatures of the order of 1000°K or less, and will be at ambient pressure, so that \(N_{O_2}^e \approx N_{O_2}^\infty\), the ambient value of O\(_2\) concentration.\(^*\) The flow in this region will be at essentially free stream velocity, i.e., \(u_e \approx u_\infty\).\(^{(1)}\) Finally, for bodies of about 1-ft size we are interested in values of \(X \sim 10^2\) ft. Since \(u_\infty \sim 10^4\) ft/sec, we obtain as an order of magnitude estimate of the O\(_2\) injection rate

\[ *\text{This may slightly overestimate the O}_2\text{ concentration.} \]
This $O_2$ injection into the inner wake will now modify the wake kinetics. It will provide a source of $O_2$ in addition to reaction (4), so that the steady state $O_2$ concentration (in the atomic nitrogen part of the wake) will now be

$$N_{O_2} = \frac{K_4 N_0^2 + P_{O_2}}{K_5 N} \approx \frac{P_{O_2}}{K_5 N}$$

since for the cases of interest here, the turbulent injection will dominate over reaction (4). The steady state NO concentration will be

$$N_{NO} = \frac{K_5 N_{O_2} + K_6 N_0 N_M}{K_7} \approx \frac{P_{O_2}}{K_7 N}$$

and the nitrogen recombination time will become

$$\tau_N = \frac{1}{K_5 N_{O_2} + K_6 N_0 N_M + K_7 N_{NO} + 2K_8 N^2} \approx \frac{N}{2P_{O_2}}$$

The oxygen recombination time will be left essentially unchanged by the turbulence; the atomic oxygen will be spread out over a larger region, but it will still recombine by reaction (4).

In Figs. 11 and 12 we present the values of $N_{NO}$ and $\tau_N$ for this instantaneous homogeneous mixing model. By comparison with Figs. 9 and 10, we see that the presence of turbulence has increased the NO concentration in the inner wake by a factor of between 10 and $10^4$. 
Fig. 11—NO concentration in wake (homogeneous mixing)
Fig. 12—Nitrogen recombination time (homogeneous mixing)
depending on the altitude and velocity. Except for the case of lowest altitude (100,000 ft) and highest velocity (24,000 ft/sec), the turbulent mixing has also decreased the nitrogen recombination time. At the higher altitudes (200,000 ft) this decrease can be as much as four orders of magnitude. The large magnitude of this effect is due to the fact that, compared to the three-body recombination of atomic oxygen, the turbulent mixing of \( \text{O}_2 \) into the inner wake is a very efficient source of \( \text{O}_2 \), especially at the higher altitudes.

The values of \( \tau_N \) shown in Fig. 12 represent the length of time that atomic nitrogen will remain in the wake once the turbulent front has begun to engulf the undissociated flow. This will begin to occur a few hundred nose radii downstream. For typical ICBM velocities and a nose radius of the order of 1 ft, this will correspond to a time of the order of \( 10^{-2} \) sec. This is larger than the values shown in Fig. 12, so that for the case of homogeneous mixing the total length of the atomic nitrogen wake will be determined primarily by the time required for the turbulent core to expand, and not by \( \tau_N \).

As mentioned previously, the inviscid random convection model and the homogeneous mixing model are thought to represent the two possible extremes for the turbulent mixing in the wake, with the actual physical situation lying somewhere in between. The results shown in Figs. 9 and 10 for no mixing, and in Figs. 11 and 12 for homogeneous mixing therefore represent the possible range of values for \( N_{\text{NO}} \) and \( \tau_N \). Because of the uncertainties concerning the nature of the turbulent process, it is not possible at present to delimit these quantities with any greater accuracy.
There is one further effect of turbulence which should be commented on. As pointed out by Lin and Hayes, (1) the enhancement of the exothermic reactions (5) and (7) when additional O₂ molecules are entrained into the wake from the external flow results in a reheating of the wake. Their results show that for some altitudes the magnitude of this reheating may be as large as 1000°K. This effect will increase the wake temperature above the value used here, but not by a large enough amount to affect our conclusions. The rate constants of the important chemical reactions will not be altered by a large amount, and chemiluminescent reactions will still be the principal source of radiation.
IV. RADIATION ESTIMATES

Having discussed the important chemiluminescent reactions, and the important chemical and aerodynamic phenomena affecting the chemical composition of the wake, we are now in a position to calculate the amount of radiation emitted by the nonequilibrium wake of a reentry vehicle. In carrying out this calculation, we will be primarily concerned with obtaining an order of magnitude estimate of the radiant intensity, determining the altitude and velocity dependence, and determining the relative contributions of the various chemiluminescent reactions.

As shown in Section II, the amount of radiation emitted per unit volume in the wake by the three principal chemiluminescent reactions, reactions (1), (2), and (3), will be given by

\[ J_1 = 3.1 \times 10^{-17} N_N^2 \text{ photons/cm}^3 \text{sec} \]

\[ J_2 = 1.9 \times 10^{-17} N_N N_O + 2.6 \times 10^{-34} N_N N_O N_M \text{ photons/cm}^3 \text{sec} \]

\[ J_3 = 6.4 \times 10^{-17} N_{NO} N_O \text{ photons/cm}^3 \text{sec} \]

where \( J_1 \) refers to reaction (1), and the concentrations are to be expressed in particles/cm\(^3\). The amount of radiation from reaction (2) emitted above 3000 Å will also be of interest. As discussed in Section II, this is given by

\[ J'_2 = 0.9 \times 10^{-34} N_N N_O N_M \text{ photons/cm}^3 \text{sec} \]
The total amount of wake radiation emitted by these three reactions will depend on the species concentrations and on the effective radiating volume of the wake. In order to bring out the important features of the problem without unduly complicating the calculation, we will assume that the axis values of the species concentrations hold over a cylindrical region surrounding the axis. The total amount of wake radiation from reaction (i) will then be

$$I_i = \pi R^2 L_i L_i$$

where $\pi R^2$ is the effective cross sectional area of the radiating wake, and $L_i$ the wake length appropriate to reaction (i).

As shown by the results of Lin and Hayes,(1) for reactions involving N or O, $R \sim 3 R_n$. The effective wake length will depend on the reactants producing the radiation. For reaction (3), $\text{NO} + \text{O} \rightarrow \text{NO}_2 + h\nu$, the effective wake length will be the length of the atomic oxygen wake. This will be given by $L = u_\infty \tau_0$, since the wake downstream of the point is moving at very nearly the freestream velocity. For reaction (1), $\text{N} + \text{N} \rightarrow \text{N}_2 + h\nu$, and reaction (2), $\text{N} + \text{O} \rightarrow \text{NO} + h\nu$, the effective wake length will be determined by the length of the atomic nitrogen wake. As shown in Section III, this will depend on the amount of turbulent mixing. In the no mixing limit, it will be given by $L = u_\infty \tau_N$, where we use the $\tau_N$ values of Fig. 10. In the limit of homogeneous mixing, the atomic nitrogen wake length will be determined by the downstream distance required for the turbulent core to expand into the undissociated flow. This will be approximately $L \sim 10^2 R_n\,(1)$.
Using this model of the radiating wake, in Figs. 13-18 we present the results for a nose radius of 1 ft, using the two extreme models for the turbulent mixing process. In Fig. 13 we show the radiation from the Lewis-Rayleigh nitrogen afterglow ($N_2$ 1st positive bands), in Fig. 14 the nitric oxide afterglow (NO $\beta, \gamma$, and $\delta$ bands), and in Fig. 15 the $NO_2$ continuum. Figure 16 shows the radiation from the nitric oxide afterglow above 3000 Å. In Fig. 17 we present the total nonequilibrium wake radiation from all three chemiluminescent reactions, and in Fig. 18 the total radiation above 3000 Å.

The altitude and velocity dependence are shown in the graphs. The scaling for body size will be fairly simple: In the no mixing limit the radiation will scale as $R_n^2$; in the homogeneous mixing limit, the $NO_2$ radiation will scale as $R_n^2$, the $N_2$ and NO radiation as $R_n^3$.

We see from these results that the nature of the wake radiation depends very strongly on the details of the turbulent mixing process. In the limit of no microscopic mixing (inviscid random convection) the $N_2$ 1st positive bands, and the NO $\beta, \gamma, \delta$ bands are the dominant source of radiation at all altitudes, with the $NO_2$ continuum being several orders of magnitude less intense. In the limit of instantaneous homogeneous mixing, however, the $NO_2$ continuum is greatly enhanced, and the $N_2$ and NO radiation reduced, particularly at higher altitudes, so that above about 150,000 ft the $NO_2$ radiation becomes dominant. The total amount of radiation is also affected by the turbulence; at the higher altitudes the difference between the two turbulent models can be

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*In changing units from photons/cm$^2$·sec to watts/cm$^2$, we have assumed average photon energies of: $N_2$ 1st positive system, 2 ev; NO $\beta, \gamma, \delta$ systems, 5 ev; $NO_2$ continuum, 2 ev.*
Fig. 13—Wake radiation in N₂ first positive bands
Fig. 14—Wake radiation in NO $\beta$, $\gamma$, and $\delta$ bands
Fig. 15—Wake radiation in NO₂ continuum
Fig. 16—Wake radiation in NO β bands above 3000 Å
Fig. 17—Total wake radiation
Fig. 18 — Total wake radiation above 3000 Å
be almost an order of magnitude.

As was mentioned in Section III, because of uncertainties concerning the detailed nature of the turbulent process it is not possible at the present time to specify quantities affected by turbulent mixing with any greater accuracy than that represented by the two turbulent models used here. For that reason, the differences in the radiation predictions of the two turbulent models, as shown in Figs. 13-18, represent a real uncertainty in our ability to predict the amount of nonequilibrium wake radiation. Because of this uncertainty, it would be inappropriate at present to attempt a more detailed estimate of the amount of wake radiation than was attempted here. It would have been possible, for example, to construct detailed profiles of temperature and species concentration in the wake, obtain detailed profiles of radiation emission from these, and then numerically integrate over the entire wake to obtain the total emission. However, in the absence of new knowledge concerning the turbulent mixing process, the apparent high accuracy of such a calculation would be spurious.

In Fig. 19 we compare the wake radiation with the gas cap and surface radiation. For the gas cap radiation we have used Kivel's tables, (31) together with an effective radiating volume of \( \frac{R^3}{10} \). For the surface radiation we have assumed an emissivity of unity, and an effective radiating area of \( \pi R^2 \). We see from these results that the gas cap radiation will be negligible compared to the wake radiation for all altitudes considered here. The surface radiation will become important once the surface temperature approaches 3000°K. However, this will most probably not happen until relatively low altitudes, 100,000 ft or less, so that the wake radiation will be

*The values shown are for a velocity of 20,000 ft/sec. The results for other velocities are similar.
Fig. 19—Comparison of wake, gas cap, and surface radiation
dominant at the higher altitudes.

In using the results obtained in this Memorandum it should be remembered that we have been considering a pure air wake. For wakes containing ablation products, which are much more effective radiators than air, the wake radiation would probably be greatly enhanced over the values shown here.
V. DISCUSSION

In this Memorandum we have considered the chemiluminescent reactions that will be important sources of wake radiation under nonequilibrium conditions, we have discussed the various aerodynamic and chemical phenomena that will affect the radiation, and we have made estimates of the amount of radiation emitted. The most striking fact to emerge from this study has been the sensitivity of the nonequilibrium radiation to details of the turbulent mixing process. The present study has been limited to pure air wakes, but it is almost certain that the nonequilibrium radiation from contaminated wakes will be equally sensitive to turbulence. This dependence of the wake radiation upon presently uncertain details of the turbulent process provides another reason, if any were needed, for the importance of increasing our knowledge of hypersonic turbulence.

Considering the sensitivity of the nonequilibrium wake radiation to changes in the amount of turbulent mixing, it is possible that radiation measurements could be used to study the turbulent process. For example, if reentry wakes of nonablating vehicles could be observed with sufficient spectral resolution to measure the relative intensities of the $N_2$ first positive system, the NO bands, and the $NO_2$ continuum, it is conceivable that the results could be used to choose between the inviscid random convection model and the homogeneous mixing model, or to establish a new, intermediate model for the turbulent process. Observations of the wake length of the $N_2$ or NO bands could also be used for this purpose. Similar observations of the wakes of ablating vehicles could also be used to study the
turbulent process, providing that the necessary theoretical study of nonequilibrium radiation from contaminated wakes had first been carried out.
REFERENCES


