CALIBRATION OF DOPPLER SATELLITE OBSERVATIONS
BY SHORT ARC SOLUTION FOR STATION COORDINATES

by

E. H. Nott

Computation and Analysis Laboratory

U. S. NAVAL WEAPONS LABORATORY
DAHLGREN, VIRGINIA
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by Short Arc Solution for Station Coordinates

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18 March 1964
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This report presents the results obtained by using the short arc method to determine station coordinates for the purpose of calibrating the Doppler instrumentation used in determining geodetic parameters from satellite observations. The coordinates obtained were within three standard deviations of surveyed station coordinates in the majority of cases. Since this represented differences of five to ten meters, depending on the relative positions of the stations, when 24 arcs were combined, it is concluded that Doppler data processed by the short arc method give a reliable estimate of station positions.
FOREWORD

This work was sponsored by the Astronautics Division, Bureau of Naval Weapons, under WEPTASK RT8801001/2101/S4390001.

Appreciation is expressed here for the assistance in computation given by Mrs. J. L. Earley and Mr. W. D. Earley.

APPROVED FOR RELEASE:

/s/ RALPH A. NIEMANN
Acting Technical Director
I. INTRODUCTION

Project ANNA was established with the threefold objective of determining, by the use of range, optical, and Doppler observations on a satellite, (1) the gravitational field about the earth, (2) the locations of survey positions relative to the center of mass of the earth, and (3) the relative distances between stations on the surface of the earth. (The test plan for this project is given in reference 1.) In order to achieve this objective, it was necessary to calibrate the instrumentation used. The calibration phase of the project took place during the first three months after the launch of the ANNA 1B satellite, during which time the optical and Doppler observing stations were located at essentially the same sites. Thus, the distance between stations obtained with Doppler instrumentation could be compared with that obtained with the optical system and with ground survey data; by means of these comparisons, any biases in the instrument systems could be determined. As the range instrumentation operated for only 16 hours after launch, no use could be made of range data.

The short arc method is one of the techniques which was used to determine the rectangular coordinates of an "unknown" station, using Doppler data from this station and three other base stations for which the coordinates were known. The stations had to be so located that the time span of observation for each station overlapped that of the other three stations. Each arc (one pass of the satellite observed by the four stations) was integrated from an independent set of orbit parameters. A variable number of these independent arcs were then combined to obtain solutions for a set of parameters which consisted of the corrections to the three surveyed coordinates of the unknown station, corrections to the six orbit parameters applicable to each of the passes over the station set, and corrections to satellite oscillator frequency and frequency drift for each pass. Only the station coordinate part of each solution, however, is discussed below, since the results obtained for the remaining parameters do not provide information concerning the existence of biases in the instrumentation.

Since the orbit parameters for each arc are held fixed for less than one revolution of the satellite, the effects of biases due to errors in atmospheric drag are reduced to a level which is believed to be negligible. The sensitivity to errors in origin of the geodetic datum is practically negligible. For example, a 200 meter displacement of the center of the earth resulted in a 10 meter change in station position.
II. PROCEDURE

In general, short arc station coordinate solutions are obtained by means of the following procedure:

A. Computation of Arcs: The orbit of the satellite is numerically integrated from an independent set of orbit parameters (obtained from preliminary data processing) for each pass of the satellite over the station set; the integration is performed with a tenth-order Cowell's routine. Aggregated, filtered data points obtained from preliminary data-processing (see Appendix A for method of filtering) are used in the computation of the arcs. For each time of observation, the theoretical counterpart of the observation is computed from the integrated satellite position and the preliminary estimate of the station position. The differences between the observed and computed observations are the residuals which must be reduced by least squares adjustment of parameters.

B. Parameters of Solution: The parameters of the solution include the six orbital constants for each arc used and the three components of the position vector of the unknown station.

C. Differentials: In order to form the normal equations necessary to improve the parameters of the solution, partial derivatives with respect to each parameter are required. The partials of the satellite position with respect to orbit parameters are computed by numerical integration of the perturbation equations for these partials. They are then multiplied by the partial of the observation with respect to satellite position, which is computed in closed form, to make the partials of the observation with respect to the orbit parameters. The partials of the observations with respect to station position, satellite frequency and frequency drift are computed in closed form.

D. Normal Equations: The complete normal equations include as unknowns the three components of the position vector of the unknown station, six orbit constants for each of an unlimited number of arcs and a frequency and frequency drift parameter for each pass of the satellite over each station. Partitioning of the matrices involved and some elimination steps reduce the rank of the largest matrix which must be stored.

E. Miscellany: The short arc method requires that the best available values for the coefficients of the gravity harmonics be used. The NWL-3B (reference 2) set of gravity coefficients consisting of the first five zonal harmonics plus four pairs of
tesserals \((A_2^2, B_2^2, A_4^2, B_4^2, A_4^1, B_4^1\) and \(A_4^3, B_4^3\)) was used. The geocentric coordinates of the stations were obtained by transforming the latest ground survey data to the world geodetic system described in reference 3.

The station configuration consisted of four North American stations, Stations 2, 3, 710 and 711 (see Figure 1). In order to demonstrate the effect of geometry, separate solutions were made for Station 710 and Station 711. That is, when solving for Station 710, the coordinates of Stations 2, 3, and 711 were held fixed; when solving for Station 711, Stations 2, 3 and 710 were held fixed.

First order ionospheric refraction effects were removed at the tracking stations by analog combination of coherent frequencies. The error due to tropospheric refraction effects was corrected in the orbit improvement program. No corrections were made for the perturbing effects of the sun, moon, drag and radiation pressure because of the short data span.

A set of twenty-four independent arcs, having data spans of approximately 1500 seconds, was selected arbitrarily. By aggregating various combinations of the normal equations obtained from these twenty-four arcs, six different solutions were found for both Station 710 and 711. Solution 1 contained all twenty-four arcs; Solutions 2, 3 and 4 contained eight independent arcs each; and Solutions 5 and 6, twelve independent arcs each.

III. RESULTS

The six solutions found for each station are given in Tables 1 and 2, together with the standard deviations of the coordinate corrections and the ratio of each correction to its standard deviation.

It can be seen that, with the exception of Solution 3 for both stations, all coordinate differences were less than or equal to three times their standard deviations. For the twenty-four arc solutions, the differences compare favorably with the estimated survey accuracy, which is about 10 meters for the distance between the two stations furthest apart. The coordinate differences and standard deviations of Station 711 were approximately one-half those of Station 710, due to the improved geometry in the positions of the stations with
### TABLE 1  
SHORT ARC SOLUTIONS FOR COORDINATES OF STATION 710

<table>
<thead>
<tr>
<th>Solution Number</th>
<th>1</th>
<th>2</th>
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<th>4</th>
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<td></td>
<td>24 arcs</td>
<td>8 arcs</td>
<td>8 arcs</td>
<td>8 arcs</td>
<td>12 arcs</td>
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<tr>
<td>ΔX(M)</td>
<td>8</td>
<td>-1</td>
<td>23</td>
<td>17</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>ΔY(M)</td>
<td>4</td>
<td>15</td>
<td>-19</td>
<td>13</td>
<td>11</td>
<td>-19</td>
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<tr>
<td>ΔZ(M)</td>
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<td>-12</td>
<td>-16</td>
<td>10</td>
<td>-14</td>
<td>11</td>
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#### Difference\(^{(1)}\) in Station Coordinates

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<td>σY(M)</td>
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<tr>
<td>σZ(M)</td>
</tr>
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#### Ratio of Difference to Standard Deviation in Coordinates\(^{(2)}\)

| | 3 | 0 | 4 | 2 | 2 | 2 |
| ΔX/σX | 1 | 2 | 2 | 1 | 2 | 2 |
| ΔY/σY | 2 | 2 | 2 | 1 | 3 | 2 |

(1) Between satellite solution and surveyed position.  
(2) Computed before rounding.
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<td>12 arcs</td>
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<td>-6</td>
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<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>ΔY(M)</td>
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<tr>
<td>σZ(M)</td>
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<td>6</td>
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<tr>
<td>ΔX/σX</td>
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<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ΔY/σY</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>ΔZ/σZ</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
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(1) Between satellite solution and surveyed position.
(2) Computed before rounding.
respect to the path of the satellite. The large standard deviations of Solution 4 illustrate the fact that the solutions are dependent on the geometry and distribution of the satellite passes relative to the station net. The elevations at time of closest approach of the passes used in Solution 4 averaged ten degrees less than those used in Solutions 2 and 3. Figures 1, 2 and 3 show the direction of passes with respect to Station 710 for Solutions 2, 3 and 4, respectively. It is important that arcs having different directions be used in the solutions.

IV. CONCLUSIONS

The short arc solution for station coordinates has revealed no significant biases in the Doppler satellite observations. The solutions found by this series of tests were, for the most part, no larger than three times the standard deviations of the corrections, and approached the 10 meter accuracy of ground survey data.

If the "unknown" station is centrally located in the four-station net, the results are about twice as good as those for an isolated station.

REFERENCES

(1) R. J. Anderle, Project ANNA Data Processing Plan, 1 October 1962, U. S. Naval Weapons Laboratory, Dahlgren, Virginia.
FILTERING AND AGGREGATING DATA POINTS

In order to filter out bad observations and passes, a preliminary orbit is extrapolated from the orbit computed the previous day. For each pass of Doppler data, the frequencies corresponding to this nominal orbit are computed at each time of observation. In order to facilitate the filtering, the differences between the observed and computed frequencies are used to form a least squares solution for a fictitious station position which would reduce the residuals. (The station is constrained to lie in the plane containing the velocity vector and the sight line in order to avoid a near-singularity.) A least squares straight line is then fitted to the residuals based on the nominal orbit and the fictitious station position; observations corresponding to residuals which depart from the straight line by more than 2.5 times the rms of the differences from the straight line are rejected. The station coordinate solution and filtering is then iterated until no additional observations are deleted. Contiguous straight line fits are then made to the filtered residuals in segments nominally containing about eight data points. The fitted residuals are evaluated at the central data time and added to the computed data point at that time in order to reconstruct the filtered, aggregated data point. The rms of the residuals from the straight line fit is evaluated and divided by the square root of the number of degrees of freedom to provide the standard deviation of the aggregated data point.

FILTERING PASSES

After all passes have been filtered for bad data points, tests are made to determine if an entire pass is bad. Each component of the "correction" (i.e., the correction in slant range and the correction along the velocity vector) to the station position described in the previous section is compared across passes. Corrections to slant range departing from the mean correction by more than 2.5 times the standard deviation of the corrections are taken to be an indication of a bad pass. Similar departures of the component of correction parallel to the velocity vector from a straight line fit with respect to time also lead to rejection of the pass. Deviations less than a preset value of about one kilometer are permitted even if they exceed 2.5 times the standard deviation.

1Reference (1), Section 2.1.2
2Reference (1), Section 2.1.3
DIRECTION OF PASSES WITH RESPECT TO STATION 710

SOLUTION 4

FIGURE 3
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This report presents the results obtained by using the short arc method to determine station coordinates for the purpose of calibrating the Doppler instrumentation used in determining geodetic parameters from satellite observations. The coordinates obtained were within 3 standard deviations of surveyed station coordinates in the majority of cases. Since this represented differences of five to ten meters, depending on the relative positions of the stations, when 24 arcs were combined, it is concluded that Doppler data processed by the short arc method give a reliable estimate of station positions.