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SURFACE PRESSURE FLUCTUATIONS
PRODUCED BY ATTACHED AND
SEPARATED SUPersonic BOUNDARY
LAYERS

by

ALAN L. KISTLER

APRIL 1963

NORTH ATLANTIC TREATY ORGANIZATION
SURFACE PRESSURE FLUCTUATIONS PRODUCED BY ATTACHED AND SEPARATED SUPersonic BOUNDARY LAYERS

by

Alan L. Kistler

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SUMMARY

Measurements have been obtained of the pressure fluctuations on a solid surface immersed in a supersonic stream for Mach numbers up to 5.0. The pressures resulting from both the attached and separated turbulent boundary layers were investigated. The results for the attached layer show that the root mean square value of the pressure fluctuation is proportional to the mean shear stress at the wall with a proportionality constant that is only weakly dependent on Mach number. The convection velocity characterizing the space-time correlations of pressure on the wall decreases with increasing Mach number, as does the ratio of the scale of the pressure fluctuations to the geometrical boundary layer thickness.

The pressures associated with the separated flow produced by a forward facing step were significantly larger than the pressures produced by an attached boundary layer. The data can be interpreted as showing that the pressure fluctuations originate from two distinct causes, fluctuations due to changes in geometry of the separated region and fluctuations due to the disturbed motion within the separation bubble. The levels to be expected from each cause can be estimated from a simple model.

SOMMAIRE

On a obtenu des mesures des fluctuations de pression sur une surface solide immergée dans un courant hypersonique pour des nombres de Mach jusqu’à 5.0. On a étudié les pressions résultant des couches limitrophes turbulentes aussi bien liées que distinctes. Les résultats obtenus pour la couche liée montrent que la valeur de la racine de la moyenne des carrés de la fluctuation de pression est proportionnelle à l’effort de cisaillement moyen au mur avec constante de proportionnalité ne dépendant que faiblement du nombre de Mach. La vitesse de convection caractérisant les corrélations espace-temps de pression au mur décroit à mesure que croît le nombre de Mach, comme le fait le rapport échelle des fluctuations de pression-épaisseur de couche limitrophe géométrique.

Les pressions associées à l’écoulement distinct produit par un échelon orienté en avant ont été notablement supérieures aux pressions créées par une couche limitrophe liée. Les données peuvent s’interpréter comme indiquant que les fluctuations de pression proviennent de deux causes distinctes, à savoir fluctuations résultant de changements de la géométrie de la région distincte, et fluctuations dues au mouvement perturbé au sein de la bulle de séparation. Les niveaux que l’on doit attendre de chaque cause peuvent s’évaluer d’après un simple modèle.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMMARY</td>
<td>ii</td>
</tr>
<tr>
<td>SOMMAIRE</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>iv</td>
</tr>
<tr>
<td>NOTATION</td>
<td>v</td>
</tr>
<tr>
<td>SURFACE PRESSURE FLUCTUATIONS PRODUCED BY ATTACHED AND SEPARATED SUPERSO</td>
<td>1</td>
</tr>
<tr>
<td>NIC BOUNDARY LAYERS</td>
<td></td>
</tr>
<tr>
<td>REFERENCES</td>
<td>7</td>
</tr>
<tr>
<td>FIGURES</td>
<td>8</td>
</tr>
<tr>
<td>DISCUSSION</td>
<td>23</td>
</tr>
<tr>
<td>DISTRIBUTION</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Fig.1(a)</td>
<td>Reynolds number variation of $p'$</td>
</tr>
<tr>
<td>Fig.1(b)</td>
<td>Pressure fluctuation levels</td>
</tr>
<tr>
<td>Fig.2</td>
<td>Normalized power spectra of the wall pressure</td>
</tr>
<tr>
<td>Fig.3</td>
<td>Spectra of hot-wire signal near the center of the boundary layer</td>
</tr>
<tr>
<td>Fig.4</td>
<td>Space-time correlations</td>
</tr>
<tr>
<td>Fig.5</td>
<td>Peak values of the space-time correlations</td>
</tr>
<tr>
<td>Fig.6</td>
<td>The convection speed ratio</td>
</tr>
<tr>
<td>Fig.7</td>
<td>Mean pressure distribution</td>
</tr>
<tr>
<td>Fig.8</td>
<td>Mean pressure distribution</td>
</tr>
<tr>
<td>Fig.9</td>
<td>Variation of the pressure with time at several locations</td>
</tr>
<tr>
<td>Fig.10</td>
<td>Pressure fluctuation levels in the separated flow</td>
</tr>
<tr>
<td>Fig.11</td>
<td>Power spectra of the pressure</td>
</tr>
<tr>
<td>Fig.12</td>
<td>Space-time correlations and the convection speed ratio</td>
</tr>
<tr>
<td>Fig.13</td>
<td>Pressure distribution near the separation point</td>
</tr>
<tr>
<td>Fig.14</td>
<td>Pressure levels inside the separated region</td>
</tr>
</tbody>
</table>
NOTATION

f \quad \text{frequency (1/sec)}

L \quad \text{integral scale of a fluctuation}

M \quad \text{Mach number}

M' \quad \text{Mach number fluctuation}

p \quad \text{mean pressure}

p' \quad \text{rms value of pressure fluctuation}

q \quad \text{dynamic pressure \(= \frac{1}{2}U'^2\)}

Re \quad \text{Reynolds number \(= \frac{U\delta}{v}\)}

R(\xi) \quad \text{space correlation of wall pressure \(= p(x)p(x+\xi)\)}

T_w \quad \text{free-stream static temperature}

T' \quad \text{rms static temperature fluctuation}

U \quad \text{mean velocity}

U' \quad \text{rms value of the streamwise component of the velocity fluctuation}

\delta \quad \text{geometrical boundary layer thickness, i.e. \(U/U = 1\) at \(y = 1\)}

\epsilon \quad \text{fraction of time high pressure is over a point}

\rho \quad \text{local mean density}

\tau_w \quad \text{mean skin friction at wall}
SURFACE PRESSURE FLUCTUATIONS PRODUCED BY
ATTACHED AND SEPARATED SUPersonic BOUNDARY LAYERS

Alan L. Kistler*

This Report is concerned with some measurements of the fluctuating pressure field associated with both attached and separated boundary layers at supersonic speeds. The extension of the subsonic results to \( M > 1 \) is of some interest since for many modern flight trajectories the maximum '\( q \) (dynamic pressure) occurs for \( M > 1 \), and we can guess from low speed measurements that \( p' \approx q \).

We will first consider the attached boundary layer, and, in particular, try to see what flow mechanism is responsible for the change in wall pressure levels with changing Mach numbers. It is difficult to visualize the pressure field itself, even in incompressible flow, through the pressure equation. This is true since the pressure equation does not admit discrete sources, and, in general, the source term can only be zero at discrete points in the flow (except for the trivial case of rectilinear flow). Furthermore, the pressure field has no conserved features to guide our thoughts. We will try, therefore, to associate the pressure with a simpler flow pattern in order to gain a qualitative indication of what to look for at large Mach numbers. In the past few years, the following picture has emerged for the turbulence structure in supersonic boundary layers\(^1\). For \( M < 5 \) and for conditions with no heat transfer, the fluctuating field in a boundary layer consists of two main elements: a velocity field (turbulence) similar to the incompressible velocity field, and a temperature fluctuation field. The fluctuation levels for the velocity field can be summarized by the fact that \( \frac{\rho u''}{\rho} \) is approximately independent of Mach number, i.e. this distribution is the same as for an incompressible flow. The Mach number of the fluctuations is small \( (M' < 0.1 \text{ at } M = 5) \) so that this velocity field is essentially incompressible. The temperature fluctuation level, however, varies strongly with \( M \), an approximate fit to the experimental data being given by \( \frac{T'}{T_o} \approx 0.02M^2 \). The incompressible velocity field can be considered to be associated with some distribution of vorticity which is also carried along at the local mean speed [Kovasznay\(^7\)]. We can now discuss the effect of compressibility on the wall pressures associated with these two types of fluctuating fields.

If we consider a single concentrated vortex element in a mean shear flow, we see that it has a certain pressure and velocity field associated with it. We look at this case not because it is the only way in which vorticity produces pressure at the wall, but because it is easy to talk about and has a representative Mach number effect. If we keep the velocity field fixed and decrease the sound speed, then, by analogy to problems in compressible flow theory, the pressure field spreads out in the direction normal to the mean flow direction; when the relative Mach numbers get sufficiently large, the propagation of the pressure occurs along the relative Mach directions. This rough picture does not include the fact that the changing pressure field would in turn affect the vorticity itself, but the fact that \( p u'' / \rho \) remains reasonably

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independent of Mach number in a boundary layer can be taken to imply that this effect is small. On the basis of this model, the wall pressure associated with this vortex element would be expected to change with Mach number, but without a detailed calculation it is difficult to say in which direction.

The pressure associated with the temperature field comes about rather indirectly. If the dissipation process in the turbulent gas produces local hot spots, these would expand and be equivalent to sources with their velocity and pressure fields. If this process were an important source of pressure fluctuations at large Mach numbers, it would be expected that \( p' \) would increase at least as fast as \( M^2 \) as the Mach number were increased. Other types of sources are possible, and some of these are systematically discussed in the work of Chu and Kovasznay.

The important point that emerges from these considerations is that if temperature fluctuations become an important source of pressure at large Mach numbers, one would expect a strong variation of \( p' \) with \( M \), whereas if the velocity field remains the dominant mechanism, no strong effects of Mach number are indicated. We will now examine the experimental results.

The experimental work reported here was done in the JPL supersonic wind tunnel. The transducers used, the length correction and the general instrumentation are described in a JPL Report. The properties of the pressure fluctuations were obtained both from measurements of the tunnel side wall boundary and from measurements of the boundary layer on a flat mounted in the tunnel. The boundary layers could be characterized to a good approximation as adiabatic, flat-plate boundary layers.

Figure 1(a) shows the effect of the Reynolds number variation on the root mean square pressure level \( p' \) for fixed Mach number. Data are shown for several different Mach numbers. In order to isolate the effects of the Reynolds number from the effects of boundary layer thickness change, it was necessary to apply a length correction to the data. For this reason only relative values are shown. The data show that \( p' \sim Re^k \) or that \( p' \sim \tau_w^k \), a result that is consistent with our knowledge of the turbulent velocity fluctuations. Therefore, if \( p' \) is divided by the local mean shear, the effects of \( Re \) are accounted for effectively.

Figure 1(b) shows \( p'/\tau_w \) as a function of \( M \). Here we see that \( p'/\tau_w \) does change with Mach number, increasing with \( M \) up to about \( M = 2 \) and then maintaining a constant value of about \( p'/\tau_w = 5 \) for larger \( M \). Subsonic data from the work of Harrison and Willmarth are also included to show that the results obtained here are systematically higher than their results but that the trend of the data is consistent with their results. The one subsonic point is the least reliable of the data shown.

Figure 2 shows the power spectra of the pressure fluctuations plotted against \( f\delta/U \) and normalized so that the area is equal to 1(one). This particular method of plotting results in some collapse of the data, but there is a systematic change of the zero intercept with \( M \). If the length correction is small, and if the spectra are interpretable in terms of a convection speed, then the zero intercept of the spectra is related to the integral scale of the fluctuations, \( L = \int_0^\delta R(\xi) d\xi \). The data show that \( L/\delta \) decreases with Mach number at a rate close to the change of the ratio of momentum thickness to boundary layer thickness. Figure 3 shows the hot wire spectra...
in the boundary layer compared to the pressure spectra. These hot wire spectra show, mainly, the distribution of temperature energy and indicate a scale of about 0.15. The velocity spectra, according to Morkovin, are approximately the same as for incompressible flow (this spectra is taken from Klebanoff’s paper). The wall pressure spectra are thus seen to have a significantly smaller scale than the velocity spectra.

Measurements of the longitudinal space time correlation for the wall pressure were also obtained and are shown in Figure 4. These correlations are quite similar to the incompressible correlations and from them the gross convection velocity can be computed. The rate of decay of the peak value of these correlations is rapid and is shown in Figure 5. The peak is down to 0.5 within a distance of 0.2 boundary layer thickness and the rate of decrease seems to scale approximately with the boundary layer thickness with no obvious dependence on the Mach number. The convection speed associated with these correlations is shown in Figure 6. It is seen that this convection speed, at least for the transducer spacings used, decreases with increasing M. Also included in this Figure are the velocity ratio for an eddy whose speed is equal to the sonic velocity at the wall. The measured convection speed more or less follows this curve, but at the higher Mach numbers (M > 3) the convection speed is supersonic with respect to the wall.

Returning now to our original questions concerning the influence of compressibility on the wall pressures, we can ask whether the experimental results that were presented furnish any clear answers. I feel that these results can be used to lend strength to the following conjectures.

First, it seems that no new types of sources become important at the Mach numbers investigated here, i.e. the temperature fluctuations become important only at higher Mach numbers (M > 5) if at all. This conclusion is based on the fact that $\frac{p'}{T}$ shows no strong variation with M in the region between M = 2 and M = 5, a region where the temperature fluctuations increase by a factor of about 5 or 6. In addition, the pressure spectra show no qualitative change with M in this region. Therefore, the velocity fluctuations still seem to be the main contributors to the pressure field. Second, the local velocity interactions producing a pressure source are subsonic and are qualitatively the same as for the incompressible flow. Since $\rho u'^2$ is proportional to the pressure source strength, the implication is that the distribution of sources as well as their strengths do not change with Mach number. Therefore, within the framework discussed earlier, the most likely cause of the observed changes in the pressure fluctuation level with Mach number is the effect of the Mach number gradient on the distribution of pressure associated with a source. In addition to the evidence produced above, this conclusion is supported by the variation of convection speed with Mach number. The fact that the convection speed more or less follows the speed for those eddies whose influence at the wall is subsonic can be taken to indicate that the existence of regions where the relative speed is supersonic influences the pressure reaching the wall from a given source.

When we turn to the problem of the pressure fluctuations associated with a separated boundary layer the possibility of comparing the results to a well-documented incompressible flow does not exist. This is true not only because only limited subsonic data exist, but also because subsonic and supersonic separation seem to be different enough to prevent a fruitful comparison being made. Even simple properties of the separated flow pressure field are difficult to estimate from first principles.
For instance, it is impossible to make an a priori statement of whether separation increases or decreases the pressure fluctuations or whether the convection speed is in the direction of the outside stream or in the direction of the local reversed flow. The great variety of possible geometries for separated flows also makes it difficult to select one for study with the hope of obtaining results of some universality. The one selected here, the flow ahead of a forward-facing step, was selected for experimental convenience and for the fact that certain features of the mean flow had been extensively studied so that the emphasis could be placed on the non-steady features of the flow. Furthermore, some evidence existed that the flow would have large fluctuating pressures associated with it.

Figure 7 shows the commonly accepted picture of this flow. The main stream breaks away from the surface at the separation point. The streamline originating at the separation point, the dividing streamline, returns to the solid boundary at the reattachment point. This streamline divides the flow into two regions, the separated region or recirculating flow region and the external stream. The region of flow in the neighborhood of the dividing streamline is called the free shear layer. Also shown in the Figure is the mean pressure distribution associated with this flow and the special features of this distribution are named, i.e. the separation pressure and the peak pressure.

The experimentally investigated flow was produced by separating the side wall boundary layer of the JPL 20-inch supersonic wind tunnel with an obstruction two inches high and ten inches wide. Measurements were obtained at Mach numbers of 3.01 and 4.54. The thickness of the boundary layers before separation were 1.5 inches and 2.05 inches respectively. Consequently the step height was comparable to the unseparated boundary layer thickness and the 10-inch width did not preclude some three dimensional effects. Pressure measurements were obtained on the center line of the separated flow; no measurements were obtained on the step face.

Figure 8 shows the measured mean pressure for the \( M = 3.01 \) flow along with some measurements of similar flows by Chapman and Bogdenoff. The three sets of measurements have a qualitative similarity; the differences can most probably be explained by the differences in the details of the boundary conditions. Bogdenoff's step spanned the tunnel, Chapman had essentially two-dimensional flow. The important point about the comparison here is that the pressure gradient in the neighborhood of the separation point for the \( M = 3.01 \) data is as large as the gradient measured by others. Only the \( M = 3.01 \) data are shown, since no comparison data exist at the higher Mach number.

The signal produced by the fluctuating pressure at a point had different characteristics in different regions of the flow. Well within the separated region the signal appeared as a normal turbulence signal, i.e. as a finite band, white noise. In the neighborhood of the separation point, however, the signal had a distinct on-off character. A sketch of the signal shape in the different regions is shown in Figure 9. The jump from the low to the high pressure level occurred at random and at a low frequency (less than 500 cps). The shape of the signal when all the energy below 1 kc was filtered out is also shown. The significance of this on-off behavior of the signal will be discussed later.

Figure 10 gives the measured rms pressure fluctuations for the separated flows in two ways: as the ratio of \( p' \) to the value obtained for the attached boundary layer
ahead of the separated region, and in relation to the mean pressure in the flow. It
is seen that the separation causes a significant increase in the pressure fluctuation
level. The M = 3.01 data are shown both with the spectral region below 1 kc filtered
out and with this region not filtered. It can be seen that the effect of the filter
on the rms value is limited to the region near the separation point and that well
within the separated region, not much energy is contained in the region below 1 kc.

The measured pressure spectra within the separated region are shown in Figure 11,
along with the spectra for the attached boundary layer. The spectra are area
normalized; the true spectra would have an area ratio of about 400 to one. It is seen
that relatively more low frequency energy is present in the separated region and that
the spectral shape depends on the location within the region. The length scales
estimated from the spectra show that the pressure fluctuations within the separated
region are 3 or 4 times as large as in the attached boundary layer. This is still
much smaller than the scale of the separated region itself.

Correlations and convection speed measurements within the separated region are shown
in Figure 12. The correlations have the same general features as those for attached
boundary layers, but their widths are larger by a factor of 3 to 4 for the same Ax .
This is consistent with the scale change obtained from the spectra. The convection
speed obtained from the correlations is in the direction of the external stream (not
in the direction of the local recirculating flow) and decreases in magnitude as the step
face is approached.

To give some indication as to what is going on, I shall relate the measurements to
features of the mean flow. With regard to the pressures near the separation point, the
feature of the random square wave modulation leads one to suspect that the separation
point is not fixed but is instead moving about over some limited region. If the
separation in fact occurred almost discontinuously, with a sharp boundary between a
region of high pressure (the separated region) and the unseparated flow, then motion
of this boundary would produce a signal similar to the one observed. Such a model
implies a relationship between the measured mean pressure and the fluctuating signal.
This relationship is displayed in Figure 13 along with the measured and computed
results near the separation point. The number \( \epsilon \), the fraction of the time that the
high pressure region is over a point was calculated from the mean measurements, and
the rms pressure fluctuation was calculated from this value of \( \epsilon \). The agreement
between the measured and calculated results is seen to be quite good. The interesting
point in relation to the general problem of separated flows is that this result
implies that the separation is associated with a much stronger pressure gradient than
that obtained from the mean measurements, which in itself is quite large. The entire
mean pressure distribution from the first rise in pressure to the point where the
pressure starts to level off is associated with this moving discontinuity in pressure.
The measured location of the separation point is in fact the average of a position
that moves over a limited region.

The level of fluctuations inside the separated region is also large. No simple
model suggests itself for relating these fluctuations to the mean flow, although the
convection speed suggests that the major source is located near the free shear layer.
Therefore, the magnitude of \( p' \) will be compared to the relevant quantity, the mean
dynamic pressure of the recirculating low. In order to compute this, it is necessary
to use the information that the recirculating flow has a velocity near the wall of
0.3 to 0.4 of the mean velocity outside the free shear layer and that this flow is subsonic with a static temperature near the free stream stagnation temperature for adiabatic wall conditions. The method of calculation is indicated in Figure 14. The result of this calculation is that the rms pressure fluctuation within the separated region is about the same size as the mean dynamic pressure of the recirculating flow. This result implies that the recirculating flow must be sufficiently unsteady that at times it even reverses direction.

The above calculations were presented only to tie down the values of the observed pressures for the forward facing step at these Mach numbers. Whether these estimates can be extended to flows characterized by greatly different parameters can only be decided on the basis of a greater accumulation of quantitative experimental results.

An interesting problem to consider is whether it is possible to construct a dynamically consistent model of the separated flow field on the basis of the information presented or, more precisely, to explain the mechanism that causes the separation point to move. Unfortunately this problem would lead us far from the main purpose of this meeting.

In summary then, the information presented in this Report leads to the following conclusions concerning the fluctuating pressure field in a compressible flow field for attached boundary layers. The given data along with the known information about the turbulence in supersonic boundary layers imply that the phenomena occurring in such boundary layers relevant to the surface pressure fluctuations are qualitatively the same as those occurring in an incompressible boundary layer. The change brought about by compressibility can most probably be accounted for by a simple extension of incompressible theories. For supersonic separated flows no such hopeful statement can be made. The pressures associated with this flow are significantly higher than those for an attached boundary layer, and quantitative estimates of the magnitude of these fluctuations can be obtained for some regions of the flow. Primarily, I want to leave the impression that significant effects can occur with flow separation and that there is a penalty attached to allowing it to happen on a vehicle. Further study of the non-steady features of these flows will pay double dividends, yielding information not only about the important fluctuating forces associated with these flows but also contributing to our understanding of the general phenomenon of separation.
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Fig. 1(a) Reynolds number variation of $p'$

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Fig. 7 Mean pressure distribution
Fig. 10 Pressure fluctuation levels in the separated flow.
Fig. 11  Power spectra of the pressure
Fig. 12  Space-time correlations and the convection speed ratio
\[ P = \epsilon P_2 + (1 - \epsilon) P_1 \]
\[ \langle (\tilde{P} - P) \rangle = \epsilon (1 - \epsilon) (P_2 - P_1) + \epsilon P_2 + (1 - \epsilon) P_1 \]

Filtered
\[ \langle (\tilde{P} - P) \rangle = \epsilon P_2 + (1 - \epsilon) P_1 \]

---

**Fig. 13** Pressure distribution near the separation point
$U_i = C U_e = C \left( \frac{U_e}{U_i} \right) U_i$

$q_i = \left( \frac{1}{2} \right) \rho_i U_i^2 = \left( \frac{1}{2} \right) \left( \frac{T_i}{T_e} \right) \left( \frac{P_e}{P_i} \right) \rho_i C_i U_i^2 \left( \frac{U_e}{U_i} \right)^2$

$= C_i^2 \left( \frac{T_i}{T_e} \right) \left( \frac{P_e}{P_i} \right) \left( \frac{U_e}{U_i} \right)^2$

FOR CASES PRESENTED

$p_i^* = 5 C_i$ \quad $q_i^* \quad C_i \sim 10^{-3}$

IF $C = 0.375$, THEN

$q_i^* / p_i^* = 19 \quad M = 3.01$

$q_i^* / p_i^* = 12.3 \quad M = 4.54$

Fig. 14 Pressure levels inside the separated region
DISCUSSION

P. Bradshaw

Dr. Kistler finds that the pressure fluctuations near the separation point have a maximum spectral density at a frequency many times less than the typical frequencies of the turbulence in the attached layer. Might this be because the exact point of separation is moved about only by unusually large velocity excursions in the turbulent flow? Such excursions would be represented by points at the extremes of the probability distribution and would therefore occur very infrequently. This kind of low-frequency modulation of separation position can be responsible for unsteady flow in wind tunnels with poor diffusers.

Author's reply

This is, of course, one possible explanation for the observed effect, but I feel it is not a probable one. Cavity flows are also quite unsteady (Charwat), so that it is likely that the whole flow is involved in the unsteadiness, either through some instability or through the necessity of obtaining simultaneously both a force balance and a mass balance for the separation region.

Comment by J. Laufer

I, too, doubt very much that the separation phenomenon is controlled by upstream conditions. The phenomenon should rather be looked upon as a relaxation oscillator. The triggering mechanism is the separation at the corner, which changes the upstream separation point, which in turn changes conditions at the corner and so on.

L.S. Kovasznay

Would you comment upon

(a) The effect of temperature fluctuation due to high correlation.

(b) Pressure fluctuation at reattachment point.

(c) The frequency of motion of separating point; can it be calculated?

Author's reply

(a) It is true that much of the temperature signal is correlated with the velocity and therefore is not associated with local dissipation. However the spectra of the temperature fluctuation contain more high frequency energy than the velocity and this spectral region might contribute to the pressure. My aim in bringing this point up was to indicate that most temperature effects could be expected to increase at least as fast as $M^2$, and that the data seem to rule out any major temperature effect for $M < 5$.

The data presented do not contain enough information to rule out the possibility that the change in wall pressure with $M$ is caused simply by the mean density gradient rather than being an effect involving the sound speed. My belief that the sound speed or relative Mach number is important arises mainly from the theory of laminar stability, where large changes in the fluctuating wall pressure occur when the wave speed exceeds the wall sound speed.
(b) I did not measure the fluctuating component of the pressure near the reattachment point, but most people who have measured the mean pressure at this point have observed oscillations in their manometers.

(c) I have not been able to produce any simple reasoning that gives the correct order for the frequency of the motion of the separation point.

J. Sternberg

(a) You have suggested that the temperature fluctuation at supersonic speeds may be a source of pressure fluctuation at the boundary due to local dissipation and heating. It is not clear that this mechanism will be present, since the large temperature fluctuations are primarily due to the convection effect of the velocity fluctuation field.

(b) You have shown that the pressure fluctuation at the wall in the separated region is of the same order as your estimate of the dynamic pressure of the circulating flow. I would not expect a direct connection between the near circulating dynamic pressure and the fluctuating dynamic pressure near the wall that appears as a pressure fluctuation. There is a large convective velocity associated with the wall pressure fluctuation which indicates that vorticity fluctuations in the mixing region are the source of the wall fluctuation. I do not see why the mean circulation dynamic pressure is connected with the vorticity fluctuations in the mixing region.

Author's reply

(a) Much of the temperature is correlated with the velocity but the temperature fluctuations increase so rapidly as M increases that there still could be a significant effect on the pressure. The measurements show, however, that the temperature is not a significant direct cause of the pressure fluctuations.

(b) The reason I gave the relation between the wall pressures in the separated region and recirculation dynamic pressure was to illustrate their magnitude, not to indicate that they are necessarily related. Both are probably related, however, to the turbulent shear on the dividing streamline, which in turn is related to the vorticity fluctuations there, so that some relation between $p^*$ and $q_0$ is possible.

Comment on (b) above by L.S. Kovasznay

Temperature effect can be observed by measuring low Mach number ($M \approx 0.5$) heater flow.

Comment by A. Powell

The separated boundary layer apparently carries large disturbances from the separation point to the back of the step, causing the stagnation point, and presumably the shock wave also, to oscillate. The separated flow immediately behind the step, on the flat surface, must be interesting.
G.M. Lilley

(a) My calculations on wall pressure fluctuations in supersonic flow also ignore entropy fluctuations and I agree with Dr. Kistler that the wall pressure fluctuations appear to be dominated by the vorticity mode.

(b) The scale of the pressure fluctuations is changed with increase in Mach number and roughly scales like the momentum thickness.

(c) The high-frequency spectrum also scaled roughly with the incompressible spectrum and this is again a confirmation of the importance of the vorticity mode in a compressible flow as it affects the wall pressure fluctuations.

(d) Could you say anything about the disturbances in the flow in the supersonic tunnel in which you made the measurements, even if these are dominated by the sound radiated from the tunnel wall, and what effect these have on the wall pressure fluctuations? It is interesting to note that Dr. Garrick’s paper gives values of $p(0)$ from missile data which are significantly below the tunnel data and at $M = 4$ are closely in agreement with my theoretical curve.

Author’s reply

(a) Measurements of the pressure fluctuations on the flat plate could be obtained for both a laminar and a turbulent boundary layer at the same tunnel conditions. This was done by artificially tripping the boundary layer. The pressure signal under a laminar boundary layer was negligible compared to that under the turbulent boundary layer. Since it is unlikely that the free stream disturbances cause a bigger effect at the wall through a turbulent boundary layer than through a laminar one, this was taken as evidence that the effect of free stream disturbances on the measurements was negligible. A comparison of the data obtained here with Dr. Garrick’s can only be made qualitatively without knowing the effects of heat transfer and disturbed upstream conditions on his data, as well as the magnitude of the length correction for his transducer.

K. Karamcheti

With regard to the factors determining the frequency of oscillations, the stability of the separated boundary layer should play an important part. For instance, in my work on sound radiation from surface cut-outs in aerodynamic surfaces (1955, 1956), it was shown that the frequency at the onset of sound emission could be correlated with the frequency at which the shear layer (i.e. the separated boundary layer) first begins to oscillate. In any case, it was found that the natural (higher) frequencies for the cut-out did not satisfactorily explain the observed frequency behavior.

Furthermore, the shape of the back edge of the cut-out had relatively little effect on the frequency.

For details, reference may be made to ‘Sound Production by Surface Cut Outs in High Speed Flow’ by Krishnamurty Karamcheti, thesis at California Institute of Technology 1956.
Author's reply

Both cavity flow and the separated flow ahead of a step have features in common. The cavity flow has one more characteristic length that the step flow, however, and this might influence the comparison.

I.E. Garrick

(a) It is interesting to note that the idea of the random square wave mentioned by Dr. Kistler to describe the separated flow in a forward facing step seems to apply also to the fluctuating pressures or buffeting arising from oscillating shocks in transonic flows on various aerodynamic shapes. The main power of the pressure fluctuations seems to be, then, in a band of relatively low frequencies.

(b) In view of the variation of the shear at the wall \( \tau_w \) with Mach number (as well as R.N.), may it not help for engineering purposes to use \( \rho/(\rho u_d^2) \), i.e. reference to dynamic pressure \( q \) instead of \( \tau_w \) for showing his results on the Mach number effect?

Author's reply

This might be a more convenient way of plotting the data, but then of course you would get different curves for different Reynolds numbers. In the analysis of engineering problems, the dynamic pressure is certainly known at an earlier stage than the skin friction.

Comment by J.E. Ffowcs Williams

My comment is on a point mentioned by Dr. Sternberg. If I understood him correctly, he enquired how the fact that wall pressure convection velocity seemed to have a high value was consistent with the observation that the pressure level was of the same order as the fluctuating dynamic head under the dividing streamline. I would like to remark that even under the streamline there exists a considerable velocity in the downstream direction, so the apparent correlation noted by Professor Kistler may not be entirely fortuitous.
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proportional to the mean shear stress at the wall with a proportionality constant that is only weakly dependent on Mach number. The convection velocity characterizing the space-time correlations of pressure on the wall decreases with increasing Mach number as does the ratio of the scale of the pressure fluctuations to the geometrical boundary layer thickness.

The pressures associated with the separated flow produced by a forward facing step were significantly larger than the pressures produced by an attached boundary layer. The data can be interpreted as showing that the pressure fluctuations originate from two distinct causes, fluctuations due to changes in geometry of the separated region and fluctuations due to the disturbed motion within the separation bubble. The levels to be expected from each cause can be estimated from a simple model.

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Measurements have been obtained of the pressure fluctuations on a solid surface immersed in a supersonic stream for Mach numbers up to 5.0. The pressures resulting from both the attached and separated turbulent boundary layers were investigated. The results for the attached layer show that the root mean square value of the pressure fluctuation is

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