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**A COMPARISON OF METHODS FOR COMPUTING
THE VELOCITY DISTRIBUTION AND THE BOUNDARY LAYER
ON BODIES OF REVOLUTION**

Task 1D121401A14203
(Formerly Task 9R38-11-009-03)
Contract DA 44-177-AMC-892(T)

November 1963

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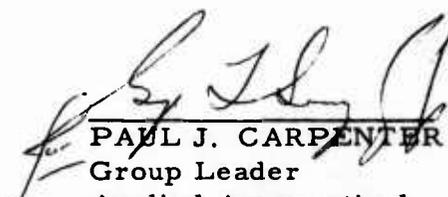
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HEADQUARTERS
U S ARMY TRANSPORTATION RESEARCH COMMAND
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A Government review of this report was conducted by the U. S. Army Transportation Research Command. The work was performed by the Aerophysics Department of Mississippi State University.

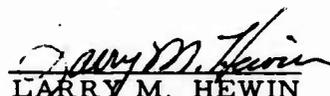
This report represents an analytical and experimental study into the solution of flow around a body of revolution. The major consideration in obtaining the solution of flow around a body of revolution is the problem of obtaining the general solutions of the equations of motion. This investigation approached the problem in two parts. The flow outside the boundary layer was assumed to be a perfect fluid and was explored by potential flow methods. The flow inside the boundary layer was explored by boundary layer methods admitting the predominance of viscosity.


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Prepared by
The Aerophysics Department
Mississippi State University

for
U. S. ARMY TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA

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LIST OF SYMBOLS

| | |
|-----------------------|-------------------------------------------------------------------------------|
| a | One-fourth of a distance between foci of the body |
| c | Chord length |
| q_i | Strength of the source of the i -th interval on the axis of the body |
| r | Radial distance measured from axis of the body |
| u | Velocity outside boundary layer obtained by uniform flow parallel to the axis |
| u' | Velocity outside boundary layer obtained by uniform flow normal to the axis |
| u'' | Velocity outside boundary layer obtained by uniform flow with angle of attack |
| (x, y, z) | Cartesian coordinate system |
| z | Distance measured along axis of the body |
| P_n | Legendre polynomial of first kind |
| Q_n | Total strength of source on i -th interval of axis |
| Q''_n | Legendre polynomial of second kind |
| t | Maximum thickness of the body |
| U_x | Component of uniform velocity parallel to the axis |
| U_y | Component of uniform velocity normal to the axis |
| U_α | Uniform velocity with angle of attack |
| α | Angle of attack |
| θ | Angle between local tangent on the surface and axis of the body |
| δ | Momentum thickness |
| (λ, μ, ν) | Elliptic cylindrical coordinate |

ϕ

Velocity potential

ψ

Stokes stream function

INTRODUCTION

The problem of obtaining the flow around a body of revolution is one of the most important examples of fluid mechanics. This subject covers quite a wide range of flow speeds, from airships to missiles. However, it is nonetheless important to treat the subject under the condition that the fluid is incompressible.

Solution of the flow around a body of revolution may be solved if, like many other problems of fluid mechanics, the general solutions of the equations of motion are obtained. Unfortunately, it is very difficult to obtain the general solution of the equations of motion. Therefore, assumptions were made to make the treatment of the problem possible. As one extreme, attempts were made to treat the problem without consideration of the effect of viscosity; that is, by means of the potential flow theory. As another extreme, attempts to obtain the solution under the assumption that the viscosity was predominant, encountered mathematical difficulties. Therefore, a compromise was made and the whole flow was divided into two parts: the one which is called flow inside the boundary layer, in which effect of the viscosity is not negligible, and the other flow outside of the boundary layer, where flow is considered as a perfect fluid. Research has been made in both regions and many results have been obtained. Treatment of the flow around the body of revolution is quite similar to that of the two-dimensional case, and it was pointed out that the flow around the body of revolution was more accurately described by the incompressible potential flow than that of the two-dimensional case.

It is planned at the Aerophysics Department, Mississippi State University, to do extensive research about a body of revolution. For a thorough study of the body of revolution, the boundary layer problems must be solved clearly. Importance of the boundary layer in this field may easily be understood from the fact that in the case of an airship, because of its low speed and large scale, over several feet of boundary layer have been observed.

It was reported that for the case of a body of revolution, the boundary layer has a more stable character than the corresponding two-dimensional case. This means that the boundary layer study for a body of revolution will give a more accurate description of the actual boundary layer than solutions of the two-dimensional boundary layer.

For an extensive research, improvement of the situation, in addition to the collection of the knowledge of the given situation, is necessary. In the case of a torpedo, separation of the boundary layer formed along its surface may cause a problem. Irregular separation of the boundary

layer from its surface may cause, in addition to the increase of the drag, a change of its course in the long run. Such a condition may be improved by means of the boundary layer control method, which was initiated at the Aerophysics Department, Mississippi State University, and which has proved to be very effective when applied to sailplanes, power planes, and axial compressors.

THE POTENTIAL FLOW METHODS

Many methods have been proposed for the solution of the flow around a body of revolution by assuming that the fluid is an incompressible potential flow. Roughly speaking, these methods are divided into two groups: the one which uses a singularity distribution along the axis or the contour of the body, and the other which is free from the assumption of a singularity distribution along the axis or the contour of the body of revolution. In both cases, computations are very laborious, and heaviness of the computations increases rapidly with the accuracy of the process.

The purpose of the treatment of the body of revolution by means of the potential flow theory in this paper is not in obtaining or using the most accurate methods possible. These methods must be closely connected with the boundary layer calculation methods used here. The boundary layer calculation methods require the velocity distribution around the body surface, and this velocity distribution is obtained by means of the potential flow theory. Therefore, more accurate values of velocity distribution around the body obtained by means of the potential flow theory mean more accurate evaluation of the boundary layer. However, there should be some limitation about the choice of the potential flow theory. First, the computational means, and secondly, the heaviness of the computation must be taken into consideration. On this basis, some of the methods that would otherwise be treated are not used here.

In this paper, von Karman's method is used as a representative of the singularity distribution method. The second method used is Kaplan's, which is regarded as being one of the more accurate methods. In connection with Kaplan's method, Young's method, which is a modification of Kaplan's method, is used here. In addition to these and for comparison, the two-dimensional case is also chosen.

For ease of calculation, spheroids of various fineness ratios are used. Especial emphasis is put on the spheroid of fineness ratio 0.3 as the representative of the spheroid. For future research, cases of flow around the body with angles of attack will be treated.

Karman's Method

In this method, shape of the body of revolution is replaced by the uniform flow in addition to the continuous sink and source distribution on the axis of the body. The axis of the body is divided into small intervals, and at each interval, strength of sink and source is assumed to be constant.

The velocity potential caused by the distribution of a source (sink) from ξ_1 to ξ_2 on the z-axis of the body is given by

$$\phi_c = \frac{1}{4\pi} \int_{\xi_1}^{\xi_2} \frac{\rho_c(\xi) d\xi}{\sqrt{(z-\xi)^2 + r^2}} = \frac{\rho_c}{4\pi} \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(z-\xi)^2 + r^2}} \quad (1)$$

where $\rho_c(\xi)$ is a strength of the source on the $d\xi$ interval of the axis of the body. Using the relationship between Stokes' stream function and the velocity potential, and after simple calculation, Stokes' stream function is given by

$$\psi = \frac{U r^2}{2} - \left(\sum_{i=1}^n \frac{\rho_i}{2\pi} \left(r_1 - \frac{r_1^2 - r_i^2}{r} \right) \right) \quad (2)$$

where G_i is the total strength of the singularity of the $d\xi$ interval and r_1 and r_2 are the distances from the end points of the $d\xi$ interval to the point on the surface of the body of revolution.

G_i is determined by solving the simultaneous linear equations which are obtained by putting $\psi = 0$ on the surface of the body.

$$\left(\frac{r}{r_i}\right)^2 - \sum_{i=1}^n \frac{\rho_i}{2\pi U r_i^2} \left(1 + \frac{r_i^2 - r^2}{r}\right) = 0 \quad (3)$$

Then the velocity components are given by

$$u_z = \frac{\partial \phi}{\partial z} \quad \text{and} \quad u_r = -\frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad (4)$$

The pressure distribution is given by

$$p = p_\infty - \rho \left\{ \left(\frac{u_z}{\rho}\right)^2 + \left(\frac{u_r}{\rho}\right)^2 \right\} \quad (5)$$

Kaplan's Method

This method is roughly divided into two parts, one for the flow parallel to the axis of symmetry and the other for the flow normal to the axis of symmetry. By combining these two solutions, a general solution of the flow around the body of revolution with an angle of attack is obtained.

Coordinates of the body surface are given by coordinates where ξ is taken along the axis of the body and r is the radius of the body from the axis to the surface. (λ, μ, ξ) constitute elliptic cylindrical coordinates and are related to the cartesian coordinates by

$$\begin{aligned}x &= 2a(\lambda^2 - 1)^{\frac{1}{2}}(1 - \mu^2)^{\frac{1}{2}} \cos \xi \\y &= 2a(\lambda^2 - 1)^{\frac{1}{2}}(1 - \mu^2)^{\frac{1}{2}} \sin \xi \\z &= 2a\lambda\mu\end{aligned}$$

where a is a constant determined from the body shape only. λ is assumed as a function of μ and is represented by

$$\lambda = a + \sum_{n=1}^{\infty} \frac{a_n \mu^{2n}}{1 - \mu^2}$$

By solving the Laplace equation, the velocity potential function is obtained. Laplace's equation is given, taking the axisymmetric character of the problem into consideration, by means of the elliptic cylindrical coordinates

$$\frac{\partial}{\partial \lambda} \left[(1 - \mu^2) \frac{\partial \phi}{\partial \lambda} \right] + \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \phi}{\partial \mu} \right] = 0$$

After simple calculations and arrangements, the velocity potential is given by

$$\phi = \sum_{n=1}^{\infty} A_n P_n(\mu^2) Q_n(\lambda)$$

where A_n is a constant and $P_n(\mu)$ and $Q_n(\lambda)$ are the Legendre polynomials of the first and second kind respectively.

Stokes' stream function is calculated through the relationship between the stream function and velocity potential. Contour of the body is given by the condition that the stream function is zero. Thus

$$\sum_{n=1}^{\infty} \frac{A_n}{n(n+1)} \frac{dP_n(\mu)}{d\mu} \cdot \frac{dQ_n(\lambda)}{d\lambda} = a U_0 = C \quad (10)$$

Velocity components are given by

$$\frac{u}{U_0} = \left(\frac{\lambda^2 - 1}{\lambda^2 - \mu^2} \right)^{1/2} \left\{ \sum_{n=1}^{\infty} \frac{A_n}{2n+1} \frac{P_n(\mu)}{P_n(\lambda)} \frac{dP_n(\mu)}{d\mu} - \mu \right\} \quad \text{and} \quad (11)$$

$$\frac{v}{U_0} = \left(\frac{\mu}{\lambda^2 - \mu^2} \right)^{1/2} \left\{ \sum_{n=1}^{\infty} \frac{A_n}{2n+1} \frac{dP_n(\mu)}{d\mu} Q_n(\lambda) - \lambda \right\}$$

When fluid flows normal to the axis of symmetry, the problem is treated quite the same way as in the case of the flow parallel to the axis of symmetry except, here, the velocity potential is a function of λ also.

$$\frac{u}{U_0} = \left(\frac{1 - \mu^2}{\lambda^2 - \mu^2} \right)^{1/2} \left[\sum_{n=1}^{\infty} C_n \frac{dP_n(\mu)}{d\mu} \left(P_n(\lambda) - \frac{P_n(\lambda)}{\lambda} \right) \right] \quad (12)$$

$$\frac{v}{U_0} = \left(\frac{\mu}{\lambda^2 - \mu^2} \right)^{1/2} \left[\sum_{n=1}^{\infty} C_n \frac{dP_n(\mu)}{d\mu} \left(\frac{dQ_n(\lambda)}{d\lambda} - n Q_n(\lambda) \right) - \lambda \right] \quad (13)$$

$$\frac{w}{U_0} = \left[\sum_{n=1}^{\infty} C_n \frac{dP_n(\mu)}{d\mu} \frac{dQ_n(\lambda)}{d\lambda} - 1 \right] \quad \text{and } C$$

From these results, velocity components at an angle of attack α are given.

$$\begin{aligned} \frac{w\lambda}{W_0} &= \frac{u\lambda}{W_0} + \frac{v\lambda}{W_0} = \frac{u\lambda}{U_0} \cos\alpha + \frac{v\lambda}{V_0} \sin\alpha \\ \frac{w\mu}{W_0} &= \frac{u\mu}{W_0} + \frac{v\mu}{W_0} = \frac{u\mu}{U_0} \cos\alpha + \frac{v\mu}{V_0} \sin\alpha \\ \frac{w\epsilon}{W_0} &= \frac{v\epsilon}{W_0} = \frac{v\epsilon}{V_0} \sin\alpha \end{aligned} \quad (13)$$

The pressure distribution is defined as

$$C_p = 1 - \left(\frac{w}{W_0}\right)^2 = 1 - \left\{ \left(\frac{w\lambda}{W_0}\right)^2 + \left(\frac{w\mu}{W_0}\right)^2 + \left(\frac{w\epsilon}{W_0}\right)^2 \right\}. \quad (14)$$

Young's Method

The usefulness of this method is in simplifying Kaplan's method, which otherwise contains heavy computations. By means of applying suitable assumptions, the calculation becomes easy and still preserves a practical amount of accuracy.

The velocity potential which is given by equation (9) is simplified to

$$\phi = \frac{cU_0}{2} \left\{ \sum_{n=1}^{\infty} A_n P_n(\mu) Q_n(\lambda) \cdot \lambda^{-n} \right\} \quad (15)$$

where c is a chord length of the body. The second term of the equation represents the main flow.

Assumptions are made that disturbance velocity caused by the body is small compared to the main flow velocity, and λ is assumed to be constant and given by $\sqrt{(\gamma/\delta)^2 + 1}$, where γ is the maximum thickness of the body.

The velocity component u_μ is thought to be negligibly small compared to u_λ . The velocity distribution in this case is given by

$$\frac{u}{U_0} = \left[\frac{1-u^2}{\lambda^2-u^2} \right]^{\frac{1}{2}} \left\{ \lambda \sum_{n=1}^{\infty} A_n \frac{dP_n(\mu)}{d\mu} Q_n(\lambda) \right\}. \quad (16)$$

If the body is placed in a uniform flow with angle of attack, from equation (14), the velocity distribution is given by

$$\left(\frac{w}{W_0} \right)^2 = \left(\frac{u_\lambda}{W_0} \right)^2 + \left(\frac{u_\mu}{W_0} \right)^2 + \left(\frac{u_v}{W_0} \right)^2, \quad (17)$$

Assuming $u_\lambda \gg u_v$ and $u_\mu \gg u_v$, the equation is further simplified to

$$\begin{aligned} \left(\frac{w}{W_0} \right)^2 &= \left(\frac{u_\lambda}{W_0} \right)^2 + \left(\frac{u_\mu}{W_0} \right)^2 + \cos \theta \frac{u_\lambda u_\mu + u_\mu u_\lambda}{W_0^2} \\ &\approx \frac{u^2}{W_0^2} \cos^2 \alpha + \cos \theta \sin 2\alpha \cdot \sin 2\epsilon, \end{aligned} \quad (18)$$

where ϵ is an angle between the local tangent on the surface of the body and the axis of the body.

Pressure distribution is given by

$$C_p = 1 - \left(\frac{w}{W_0} \right)^2. \quad (19)$$

For Karman's method, calculations were carried out at 20 points on the axis of the body. The axis of the body was divided into 20 intervals of the same length, and these intervals were represented by midpoints of intervals.

Both for Kaplan's and Young's methods, 43 points were chosen to represent the profile of the body. Calculations were carried out comparatively easily because of the fact that the body was given by spheroid which represented the least labor for the computations.

THE BOUNDARY LAYER METHODS

The flow around a body of revolution is accurately described by the potential flow theory. It has been shown that the flow around the body is more precisely described by the potential flow than that of the two-dimensional case. However, for the calculation of drag, potential flow theory does not give a reasonable estimation.

For estimation of the shear force, it is necessary to treat the subject by means of viscous flow theory. It is very difficult to solve the entire flow by means of the viscous flow theory; therefore, the assumption is made that viscous effects are confined in the thin layer next to the body surface. Outside of this thin layer, which is called the boundary layer, flow is described by means of potential flow theory. Like the case of potential flow around the body of revolution, it has been shown that the boundary layer around the body of revolution has a more stable character than the corresponding one of the two-dimensional body.

Boundary layers are divided into two groups: laminar and turbulent. Starting from the leading edge of the body, a laminar boundary layer is formed. This laminar boundary layer tends to become unstable and transition takes place. A turbulent boundary layer is then formed along the surface of the body. Since little is known about the transition problem, it is assumed that the boundary layer is made up of only one kind of flow status, either entirely laminar or turbulent.

Because of its simplicity of the flow state, methods for the calculation of the laminar boundary layer as opposed to the turbulent boundary layer are well established. For example, using the Mangler transformation which connects the boundary layer over an axially symmetric body to the two-dimensional boundary layer, Thwaites' and Tani's methods which give momentum thicknesses for the laminar boundary layer are transformed to exactly the same equation which is given by Truckenbrodt as

$$\frac{\theta}{c} = \frac{c}{10^2 \left(\frac{c}{c_0}\right)^5 \left(\frac{c}{c_0}\right)} \left[\int \frac{10 \left(\frac{c}{c_0}\right)^5 \left(\frac{c}{c_0}\right)^2 + \left(\frac{3}{c_0}\right)}{6 \left(\frac{3}{c_0}\right)} \right]^2 \quad (2)$$

except that the constant c is given a slightly different value by each method. Since main interest is in the turbulent boundary layer, here only Truckenbrodt's method is used for the calculation of the laminar boundary layer. It is shown that in the laminar case, there exists a non-zero value for the momentum thickness at the leading edge stagnation point. In this paper, however, this value is disregarded.

For the purpose of the present paper, many conditions have to be taken into consideration for the selection of the method which will be used here. Although the mechanism of the transition from laminar to the turbulent is not yet fully explained, it is desirable to choose a method which will give a calculation method for both the laminar and turbulent flow. It will be convenient if the method can principally be applied for the body with a transpiration surface. For these reasons, Truckenbrodt's method is chosen as the representative method for calculation of the turbulent boundary layer. For comparison, other methods are given here. They are given for the calculation of momentum thickness around the two-dimensional body but, by means of the Mangler transformation, are transformed to that of the body of revolution. For applying the Mangler transformation to the turbulent boundary layer calculation, Truckenbrodt's method is first written in the form of the two-dimensional case. Then, by means of the Mangler transformation, it is transformed to the case of the boundary layer over the body of revolution. Comparing these two equations, the transformation relation for the turbulent case is determined. With this transformation relationship, two-dimensional results are transformed to the case of the body of revolution.

The equation given by Truckenbrodt is

$$\delta \frac{d\delta}{ds} \left[1 + \frac{2}{3} \left(\frac{d\delta}{ds} \right)^2 \right] = \frac{0.37}{\sqrt{Re}} \quad (2)$$

where s is measured along the surface and δ is the length of the curve measured along the surface of the body. This is rewritten to

$$\delta \frac{d\delta}{d\bar{s}} \left[1 + \frac{2}{3} \left(\frac{d\delta}{d\bar{s}} \right)^2 \right] = \frac{0.37}{\sqrt{Re}} \quad (2')$$

where \bar{s} is measured along the chord line and $\bar{\delta}$ is the chord length of the body. $\frac{d\delta}{d\bar{s}}$ is given by

$$\frac{d\delta}{d\bar{s}} = \frac{1}{1 + \left(\frac{r(\bar{s})}{r(\delta)} \right)^2} \quad (2'')$$

1/6 is given for n , and

$$Cf = .0306 \times \left(\frac{U_0 C}{V} \right)^{-1/4} \quad (24)$$

Other methods used here are written by a single equation,

$$\frac{\theta}{C} = \frac{1}{\left(\frac{U_0 C}{V} \right)^{3/4} \left(\frac{r}{C} \right)^{1+n}} \left[\alpha Re^{-n} \frac{\left(\frac{U_0 C}{V} \right)^{3/4} \left(\frac{r}{C} \right)^{1+n}}{B \left(\frac{r}{C} \right)} \right]^{1/n} \quad (25)$$

except constants used are different from one to the other. These are given below:

| | n | α | B |
|---------|-------|----------|------|
| Maskell | .2155 | .01173 | 4.2 |
| Schuh | .268 | .0185 | 4.27 |
| Spence | .200 | .0106 | 4.0 |

RESULTS AND DISCUSSION

It will be convenient to think about the results from two directions: first, results from the potential flow around the body of revolution, and second, results from the calculations of the boundary layer. General notation for the calculation of boundary layer methods, as well as potential flow methods, is shown in Figure 1.

At a small fineness ratio, the pressure distribution curve is well represented by a flat curve except in the vicinity of the leading and trailing edges. As the fineness ratio of the body increases, the pressure distribution curve becomes a more rounded one. It is found that the difference between Kaplan's method and Young's method increases as the thickness of the body increases. As is shown in Figure 2, at a fineness ratio of 0.3, Kaplan's method gives larger values for the velocity distribution around the body than does Young's method. For comparison, the result of the two-dimensional case is also included. It is shown in Figure 3 that Karman's method gives a pressure distribution close to the curve given by Kaplan's method. The slightly wavy character of the pressure distribution curve by Karman's method will be due to the comparatively small number of divisions used for the calculations.

Some results are obtained for the case of flow at an angle of attack to the body. In Figure 4 and Figure 5, pressure distributions given by Kaplan's and Young's methods are compared at angles of attack $\alpha = 5$ degrees and $\alpha = 10$ degrees. In contrast to the effect of the thickness of the body, the pressure difference between a certain angle of attack and the zero angle of attack by means of Young's method is larger than that of Kaplan's method. However, an increase of angle of attack has the effect of giving closer agreement between the pressure distributions given by the two methods. At a large angle of attack, $\alpha = 10$ degrees, the pressure distribution by Young's method gives better coincidence to that by Kaplan's method than at a small angle of attack, $\alpha = 5$ degrees.

For the laminar boundary layer, Truckenbrodt's method is used, and in Figure 6, momentum thicknesses for two- and three-dimensional cases are compared.

For the turbulent boundary layer, Truckenbrodt's method is taken as a representative method for the estimation of the momentum thickness. In addition, three other methods are chosen for comparison of results. Originally, these methods were for the two-dimensional case, but here they are transformed to the three-dimensional case by means of the Mangler transformation. These are compared in Figure 7. In Figure 8, momentum thicknesses for two- and three-dimensional cases by means of Truckenbrodt's method are compared.

The effect of the velocity distribution around the body obtained by potential flow theory on the calculation of the momentum thickness is given in the table. Momentum thicknesses are calculated from three different velocity distributions; one is the velocity distribution given by Kaplan's method, and others are ten per cent less and more than the value obtained by Kaplan's method. It will be noticed that these differences do not give more than two per cent difference for the momentum thicknesses. This means that the results are not so much affected by the velocity distribution on which the momentum thickness calculation is based. Therefore, it can be said that from the easiness of the calculation, Young's method may be chosen as a representative method for the calculation of the velocity distribution with a considerable amount of accuracy.

Comparison of the momentum thickness for the laminar and the turbulent boundary layers is shown in Figure 9. As Reynolds number increases, the difference between the two methods increases.

All of the above results are obtained for the case of a spheroid. If the body is not a spheroid, velocity distribution difference between Kaplan's and Young's methods is large compared to the case of the spheroid. However, it is assumed that the difference does not have a large effect on the calculation of the momentum thickness.

REFERENCES

1. von Karman, T., "Calculation of Pressure Distribution on Airship Hulls," NACA TM 574, 1930.
2. Millikan, C. B., "The Boundary Layer and Skin Friction for a Figure of Revolution," Trans. A.S.M.E. Vol. 54, No. 2, 1932.
3. Kaplan, C., "Potential Flow about Elongated Bodies of Revolution," NACA Rep. 516, 1935.
4. Jahnke, E., & Emde, F., "Tables of Functions with Formulae and Curves," Dover, 1943.
5. Kaplan, C., "On a New Method for Calculating the Potential Flow Past a Body of Revolution," NACA Rep. 752, 1943.
6. Young, A. D., & Owen, P. R., "A Simplified Theory for Streamline Bodies of Revolution, and its Application to the Development of High-Speed Low-Drag Shapes," British R & M, No. 2071, 1943.
7. Mangler, W., "Boundary Layers on Bodies of Revolution in Symmetrical Flow," M.A.P. Volkenrode, 1946.
8. Thwaites, B., "Approximate Calculation of the Laminar Boundary Layer," Aeronaut Quart. Vol. 1, Part 1, 1949.
9. Landweber, L., "The Axially Symmetric Potential Flow about Elongated Bodies of Revolution," The David W. Taylor Model Basin Rep. 761, 1951.
10. Maskell, E. C., "Approximate Calculation of the Turbulent Boundary Layer in Two-dimensional Incompressible Flow," Rep. Aero. Res. Coun. London, 1951.
11. Granville, P. S., "The Calculation of the Viscous Drag of Bodies of Revolution," The David W. Taylor Model Basin Rep., 849, 1953.
12. Schuh, H., "On Calculating Incompressible Turbulent Boundary Layers with Arbitrary Pressure Distribution," Aero. Teck. Notes K. Tek. Hogsh., Stockholm, 1954.
13. Tani, I., "On the Approximate Solution of the Laminar Boundary Layer Equations," Jour. Aero. Sci., Vol. 21, No. 7, 1954.

14. Truckenbrodt, E. , "A Method of Quadrature for Calculation of the Laminar and Turbulent Boundary Layer in Case of Plane and Rotationally Symmetrical Flow," NACA TM, 1379, 1955.
15. Raspet, A. , Cornish, J. J. , III, and Bryant, G. D. , "Delay of the Stall by Suction Through Distributed Perforations," Aerophysics Department, Mississippi State University, 1956.
16. Spence, D. A. , "The Development of Turbulent Boundary Layers," Jour. Aero.Sci., Vol. 23, 1956.
17. Smith, A. M. O. , & Pierce, J. , "Exact Solution of the Neuman Problem, Calculation of Non-Circulatory Plane and Axially Symmetric Flows about or within Arbitrary Boundaries," Douglas Aircraft Co. Rep., ES 26988, 1958.
18. Miles, E. G. , "An Application of Boundary Layer Control to Rotating Blades," Research Report 23, Aerophysics Department, Mississippi State University, 1959.
19. Cornish, J. J. , III, and Boatwright, D. W. , "Application of Full Scale Boundary Layer Measurements to Drag Reduction of Airships," Research Report No. 28, Aerophysics Department, Mississippi State University, 1960.
20. Stewart, Charles C. , "A Comparison of Turbulent Boundary Layer Theories," Gas Turbine Laboratory, Report No. 57, M.I.T., 1960.
21. Thwaites, B. , "Incompressible Aerodynamics," Oxford University Press, 1960.
22. Smith, A. M. O. , & Clutter, D. W. , "Solution of the Incompressible Laminar Boundary Layer Equations," Douglas Aircraft Co. Rep., ES 40446, 1961.

APPENDIX

| z/c | θ/c | | |
|-------|----------------------------|----------------------------|----------------------------|
| | $0.9 \times \frac{u}{U_0}$ | $1.0 \times \frac{u}{U_0}$ | $1.1 \times \frac{u}{U_0}$ |
| 0.00 | 0.000000 | 0.000000 | 0.000000 |
| 0.05 | 0.000116 | 0.000112 | 0.000110 |
| 0.10 | 0.000204 | 0.000201 | 0.000199 |
| 0.15 | 0.000293 | 0.000290 | 0.000286 |
| 0.20 | 0.000384 | 0.000377 | 0.000373 |
| 0.25 | 0.000475 | 0.000468 | 0.000462 |
| 0.30 | 0.000567 | 0.000558 | 0.000551 |
| 0.35 | 0.000662 | 0.000650 | 0.000644 |
| 0.40 | 0.000761 | 0.000748 | 0.000740 |
| 0.45 | 0.000866 | 0.000850 | 0.000842 |
| 0.50 | 0.000978 | 0.000961 | 0.000951 |
| 0.55 | 0.001097 | 0.001082 | 0.001071 |
| 0.60 | 0.001238 | 0.001216 | 0.001202 |
| 0.65 | 0.001394 | 0.001369 | 0.001356 |
| 0.70 | 0.001582 | 0.001554 | 0.001538 |
| 0.75 | 0.001829 | 0.001787 | 0.001767 |
| 0.80 | 0.002142 | 0.002102 | 0.002081 |
| 0.85 | 0.002640 | 0.002589 | 0.002562 |
| 0.90 | 0.003582 | 0.003519 | 0.003481 |
| 0.95 | 0.006562 | 0.006449 | 0.006316 |

MOMENTUM THICKNESS CALCULATED FOR TURBULENT
BOUNDARY LAYER AROUND SPHEROID OF FINENESS RATIO 0.3
FOR DIFFERENT VELOCITY DISTRIBUTIONS (TRUCKENBRODT'S METHOD)

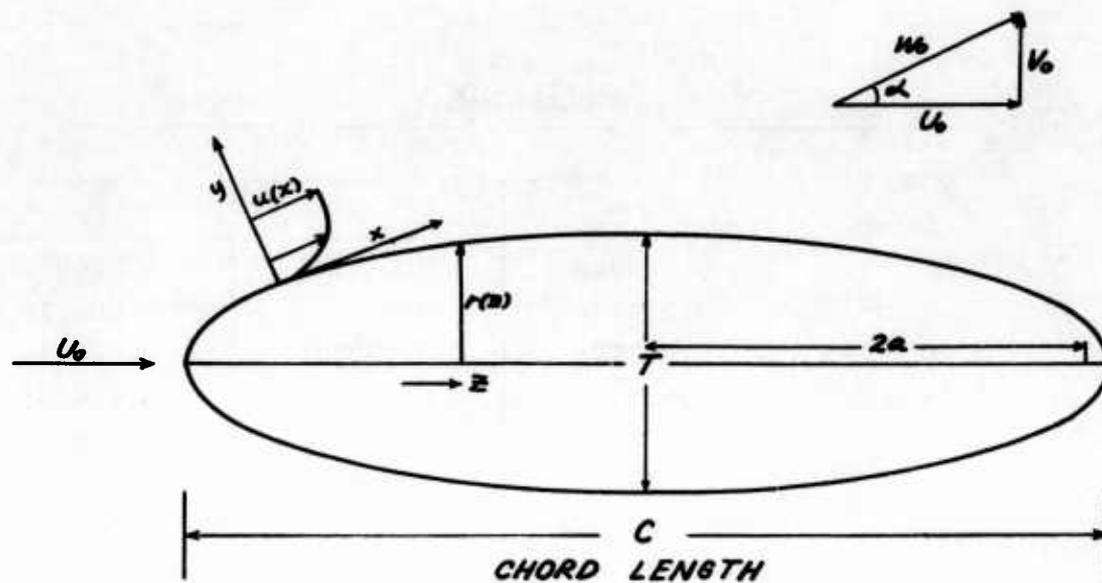


Figure 1. General Notation for a Body of Revolution

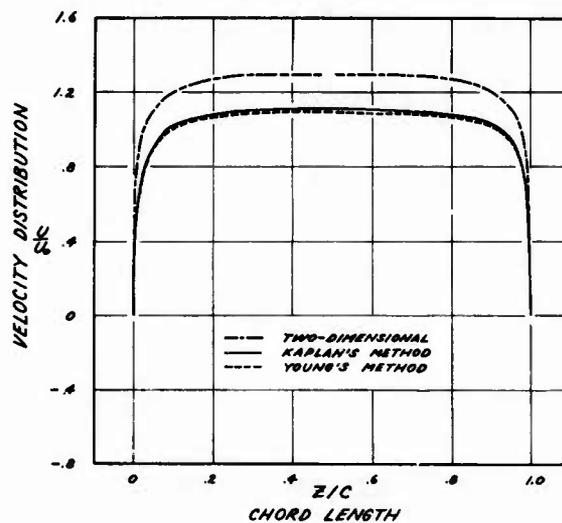


Figure 2. Comparison of Velocity Distributions Around Spheroid at Zero Angle of Attack

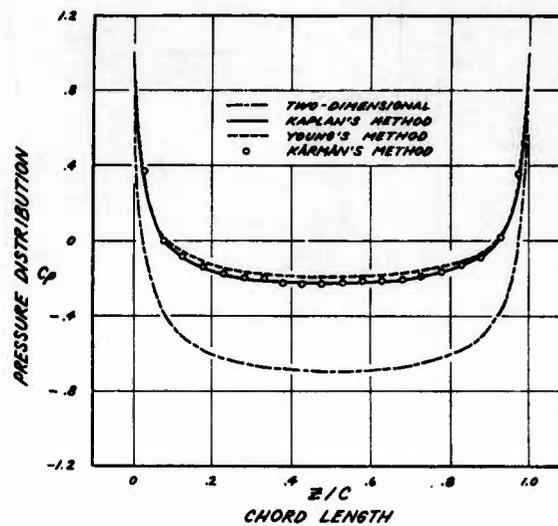


Figure 3. Comparison of Pressure Distributions Around Spheroid at Zero Angle of Attack

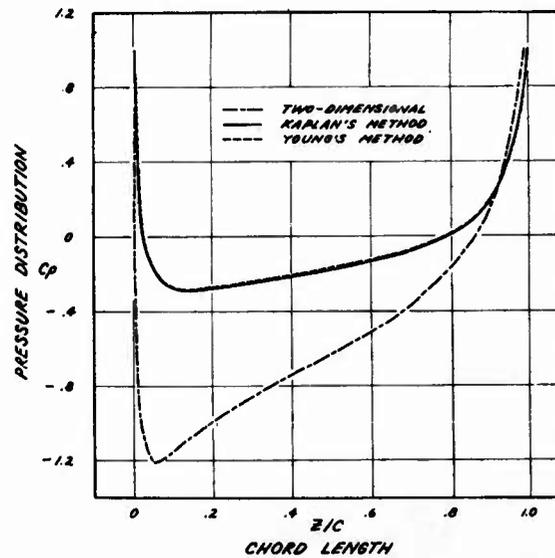


Figure 4. Comparison of Pressure Distributions Around Spheroid at Angle of Attack = 5 Degrees

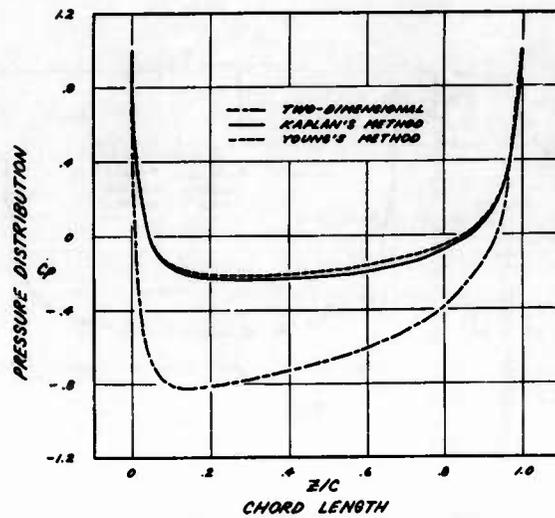


Figure 5. Comparison of Pressure Distributions Around Spheroid at Angle of Attack = 10 Degrees

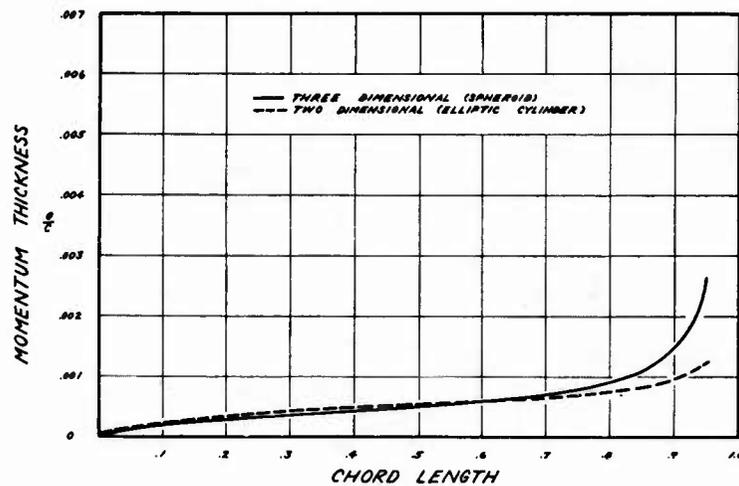


Figure 6. Comparison of Two-dimensional and Three-dimensional Momentum Thickness for Laminar Boundary Layer

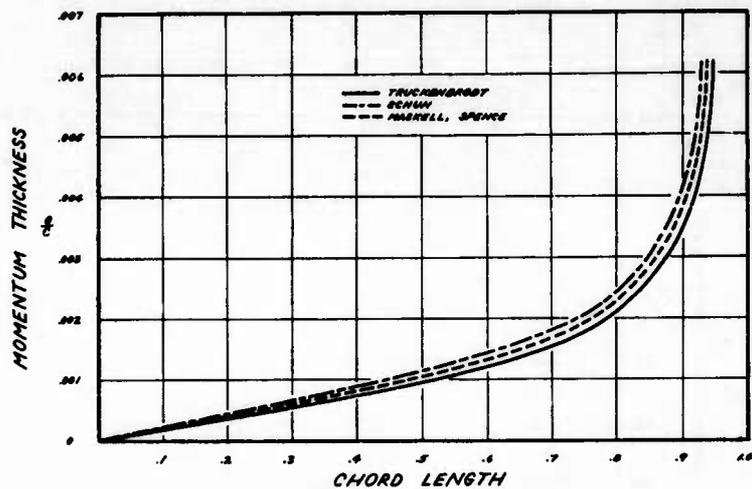


Figure 7. Comparison of Momentum Thickness for Turbulent Boundary Layer

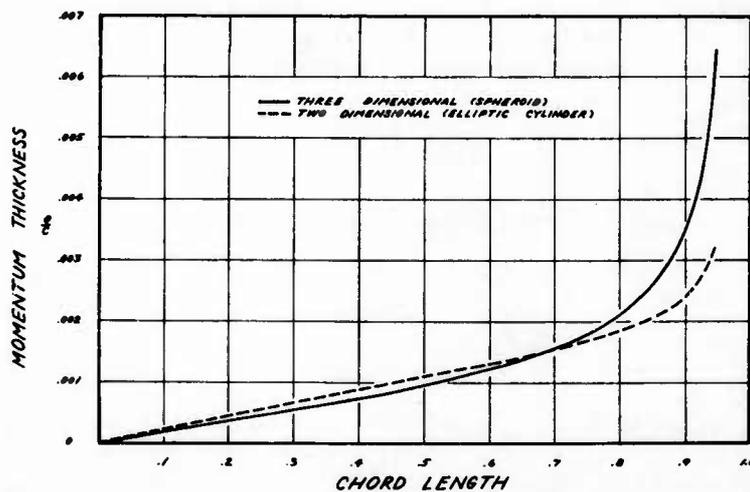


Figure 8. Comparison of Two- and Three-dimensional Momentum Thickness for Turbulent Boundary Layer (Truckenbrodt's Method)

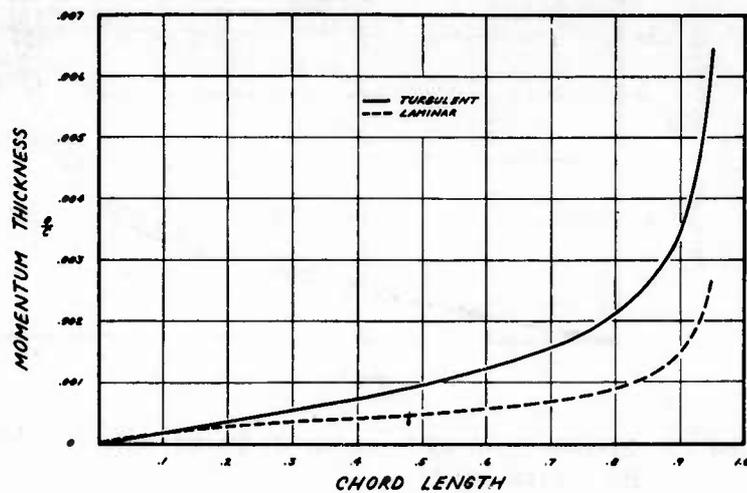


Figure 9.

Figure 9. Comparison of Momentum Thickness for Laminar and Turbulent Boundary Layers for Spheroid (Truckenbrodt's Method)

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