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Diplexer Using Side-Wall Couplers in One-Half Height Large X-Guide

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DIPLEXER USING SIDE-WALL COUPLERS
IN ONE-HALF HEIGHT LARGE X-GUIDE

J. A. KOSTRIZA

Group 61

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ABSTRACT

A diplexer in one-half height large X-guide uses side-wall couplers. The analysis is based on the scattering matrix approach.

A compact unit is made possible because of an abrupt 180° E-plane bend.
I. SCATTERING MATRIX OF TWO HYBRIDS IN CASCADE

A schematic of a side-wall coupler is shown in Fig. 1.

![Side-Wall Coupler Schematic](image)

Fig. 1. Side-Wall Coupler Schematic, showing location of reference planes.

The reference planes are labeled 1, 2, 3 and 4. The scattering matrix is given by:

\[
S = \frac{1}{\sqrt{2}} \begin{bmatrix}
0 & 0 & 1 & j \\
0 & 0 & j & 1 \\
1 & j & 0 & 0 \\
j & 1 & 0 & 0 \\
\end{bmatrix}
\]  \( (1) \)

Now terminal 3 is moved to the right, through \( \theta_o \), to 3' and terminal 4 is moved through \( \theta_e \), to 4'. The scattering matrix for terminals 1, 2, 3', 4' becomes:
Equation (1) holds for hybrid A, whereas Eq. (2) holds for hybrid A'. If the output of A' is joined to a hybrid B whose scattering matrix is the same as that of A, a new four-port device results with terminal planes 1, 2, 3, 4 as shown in Fig. 2.

![Diagram of four-port device](image)

**Fig. 2.** Four-Port 1, 2, 3, 4 consisting of hybrid A' in cascade with hybrid B.

To join ports 4' with 2, and 3' with 1, the scattering equation $b = Sa$ ($b =$ reflected wave amplitudes, $a =$ incident wave amplitudes) is written for both hybrids:

\[
S' = \frac{1}{\sqrt{2}} \begin{bmatrix}
0 & 0 & -j\theta_o & -j\theta_e \\
0 & 0 & -j\theta_e & -j\theta_o \\
-j\theta_o & -j\theta_o & e & 0 \\
je & je & 0 & 0
\end{bmatrix}
\]
\[
\sqrt{2} b_1 = e^{-j\theta} a_3 + je^{j\theta} a_4,
\]

\[
\sqrt{2} b_2 = je^{-j\theta} a_3 + je^{j\theta} a_4,
\]

\[
\sqrt{2} b_3 = e^{-j\theta} a_1 + je^{j\theta} a_2.
\]

\[
\sqrt{2} b_4 = je^{-j\theta} a_1 + e^{j\theta} a_2
\]

Hybrid A', \hspace{1cm} (3)

\[
\sqrt{2} B_1 = A_3 + jA_4
\]

\[
\sqrt{2} B_2 = jA_3 + A_4
\]

Hybrid B. \hspace{1cm} (4)

To "connect" Eqs. (3) and (4), the wave reflected from port 4' of hybrid A' must equal the wave incident on port 2 of hybrid B, etc., so that the following must hold:

\[
b_{4'} = A_2 \quad \text{and} \quad b_{3'} = A_1
\]

\[
a_{4'} = B_2 \quad \quad a_{3'} = B_1
\]

(5)

Using Eqs. (5), (3) and (4), the composite structure of Fig. 2 may be characterized by Eq. (6) where small case letters are used throughout for reflected and incident wave amplitudes.

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II. DIPLEXER REQUIREMENTS

To achieve diplexer action, the following conditions must be met:

1. A signal of frequency $f_1$, incident at port 1, emerges out of port 3 (or port 4) with negligible coupling to port 4 (or port 3), and

2. A signal of frequency $f_2$, incident at port 2, emerges out of port 3 (or port 4) with negligible coupling to port 4 (or port 3), and

3. Zero or small coupling between ports 1 and 2.

If the signal $f_1$ is applied at port 1 (i.e., $a_2 = 0$), then:

\[
\begin{align*}
\text{b}_3 \text{ (at } f_1) &= \frac{a_1}{2} \begin{bmatrix} -j\varnothing_0 & -j\varnothing_e \\ e^{-j\varnothing_0} & e^{-j\varnothing_e} \end{bmatrix}, \\
\text{b}_4 \text{ (at } f_1) &= j \frac{a_1}{2} \begin{bmatrix} -j\varnothing_0 & -j\varnothing_e \\ e^{-j\varnothing_0} & e^{-j\varnothing_e} \end{bmatrix}.
\end{align*}
\]

From the above:

when $\varnothing_0 - \varnothing_e = \pm 2\pi n \ (n = 0, 1, 2---)$, then $\text{b}_3 = 0$, and

when $\varnothing_0 - \varnothing_e = \pm 2\pi \left[m + \frac{1}{2}\right] \ (m = 0, 1, 2---)$, then $\text{b}_4 = 0$. 

4
If the signal \( f_2 \) is applied at port 2 (i.e., \( a_1 = 0 \)), then:

\[
b_3 \text{ (at } f_2) = j \frac{a_2}{2} \left[ e^{-j\theta_o} + e^{-j\theta_e} \right],
\]

\[
b_4 \text{ (at } f_2) = \frac{a_2}{2} \left[ -e^{-j\theta_o} + e^{-j\theta_e} \right]. \tag{10}
\]

From Eq. (10) it follows that:

when \( \theta_o - \theta_e = \pm 2\pi \left[ m + \frac{1}{2} \right] (m = 0, 1, 2-\ldots) \), then \( b_3 = 0 \), and \( \tag{11} \)

when \( \theta_o - \theta_e = \pm 2\pi n (n = 0, 1, 2-\ldots) \), then \( b_4 = 0 \). \( \tag{12} \)

III. COMMON OUTPUT IS PORT 4

Assume that \( b_4 \) is of interest at both frequencies \( f_1 \) and \( f_2 \). Then Eqs. (8) and (11) must be satisfied.

\[
P_{b_4} (f_1) = \frac{1}{4} (1 + \cos\theta_1),
\]

\[
P_{b_3} (f_1) = \frac{1}{8} \left( 1 - \cos\theta_1 \right), \text{ where } \theta_1 = \theta_o - \theta_e.
\]

\[
b_3^* = b_3 \text{ conjugate, and } P_{b_3} = \text{ power reflected at port 3}.
\]

\[
P_{b_4} (f_1) = \frac{1}{4} (1 + \cos\theta_1),
\]

\[
P_{\text{in } 1} (f_1) = \frac{1}{2} a_1 a_1^* = \frac{1}{2}; \ 
\]

\[
P_{b_3} + P_{b_4} = \frac{1}{2}.
\]

\[
\text{I.L. (} f_1 \text{)} = 10 \log \frac{P_{\text{in } 1}}{P_{b_4}} = 10 \log \frac{2}{1 + \cos\theta_1}.
\]

If \( \theta_1 = 2\pi n \pm \delta_1 \), \( \cos\theta_1 = \cos\delta_1 \), and

\[ I.L. (f_1) = 10 \log \frac{2}{1 + \cos \delta_1}. \]  \hspace{1cm} (13)

\[ Pb_3 (f_2) = \frac{1}{4} (1 + \cos \theta_2), \text{ where } \theta_2 = \theta_r - \theta_e, \]

\[ Pb_4 (f_2) = \frac{1}{4} (1 - \cos \theta_2), \]

\[ P_{in}(f_2) = \frac{1}{2} a_2 \cdot a_2^* = \frac{1}{2}; \text{ } Pb_3 + Pb_4 = \frac{1}{2}. \]

\[ I.L. (f_2) = 10 \log \frac{P_{in}(f_2)}{P_{b_4}} = 10 \log \frac{2}{1 - \cos \theta_2}. \]

If \( \theta_2 = 2\pi \pm \pi \pm \delta_2, \cos \theta_2 = -\cos \delta_2, \text{ and} \)

\[ I.L. (f_2) = 10 \log \frac{2}{1 + \cos \delta_2}. \]  \hspace{1cm} (14)

\text{Input mismatch at } f_1, f_2.

At \( f_1 \): \( a_2 = 0, a_3 = \Gamma_3 b_3, a_4 = \Gamma_4 b_4 \) where \( \Gamma \) is the voltage reflection factor,

From Eq. (6):

\[ b_1 = \frac{a_1}{4} \left[ \Gamma_3 \left( e^{-j\theta_r} - e^{-j\theta_e} \right)^2 - \Gamma_4 \left( e^{-j\theta_r} + e^{-j\theta_e} \right)^2 \right], \]

\[ \Gamma_{in}(Q) = \frac{b_1}{a_1} = \frac{e^{-j2\theta_r}}{4} \left[ \Gamma_3 (1 - e^{j\theta_r})^2 - \Gamma_4 (1 + e^{j\theta_r})^2 \right], \]

with \( \theta = \theta_r - \theta_e \).
But \( \theta_1 = 2\pi n + \delta_1 \), \( e^{j\theta_1} = e^{j\delta_1} \),

\[
\begin{align*}
\therefore \quad \Gamma_{in} \Theta &= \frac{-e^{j2\theta_0}}{4} \left[ \Gamma_3 \left( 1 - e^{j\delta_1} \right) \right]^2 - \Gamma_4 \left( 1 + e^{j\delta_1} \right)^2.
\end{align*}
\]

At \( f_2 \): \( a_1 = 0 \), \( a_3 = \Gamma_3 b_3 \), \( a_4 = \Gamma_4 b_4 \).

From Eq. (6):

\[
\begin{align*}
b_2 &= \frac{a_2}{4} \left[ -\Gamma_3 e^{j2\theta_0} \left( 1 + e^{j\theta} \right) \right]^2 + \Gamma_4 e^{j2\theta_0} \left( -1 + e^{j\theta} \right)^2,
\end{align*}
\]

with \( \theta = \theta_0 - \theta_e \):

\[
\begin{align*}
\Gamma_{in} \Theta &= \frac{b_2}{a_2} e^{j2\theta_0} \left[ -\Gamma_3 \left( 1 + e^{j\theta} \right)^2 + \Gamma_4 \left( -1 + e^{j\theta} \right)^2 \right].
\end{align*}
\]

But \( \theta_2 = 2\pi n + \pi + \delta_2 \), \( e^{j\theta_2} = -e^{j\delta_2} \),

\[
\begin{align*}
\therefore \quad \Gamma_{in} \Theta &= \frac{e^{j2\theta_0}}{4} \left[ -\Gamma_3 \left( 1 - e^{j\delta_2} \right)^2 + \Gamma_4 \left( -1 - e^{j\delta_2} \right)^2 \right].
\end{align*}
\]

IV. COMMON OUTPUT IS PORT 3

At \( f_1 \), from Eq. (9), \( \theta_1 = \theta_0 - \theta_e = 2\pi \left[ m + \frac{1}{2} \right] + \delta_1 \), \( \cos \theta_1 = -\cos \delta_1 \).

I.L. \( f_1 \) = 10 \log \frac{P_{in} \Theta}{P_{b_3}} = 10 \log \frac{2}{1 - \cos \theta_1} = 10 \log \frac{2}{1 + \cos \delta_1}. \hspace{1cm} (15)

At \( f_2 \), from Eq. (12), \( \theta_2 = \theta_0 - \theta_e = 2\pi n + \delta_2 \), \( \cos \theta_2 = \cos \delta_2 \).
\[
I.L. (f_2) = 10 \log \frac{P_{in}(2)}{P_{in}(3)} = 10 \log \frac{2}{1 - \cos \theta_2} = 10 \log \frac{2}{1 + \cos \delta_2}.
\]  

(16)

V. SAMPLE DESIGNS

A. Design for \( f_1 = 7.75 \text{ Kmc}, f_2 = 8.35 \text{ Kmc}, \) power out of port 4.

\[\theta_1 = \frac{L}{\lambda g_1} , \quad 2\pi = 2\pi n \quad \text{or} \quad L = n\lambda g_1.\]

\[\theta_2 = \frac{L}{\lambda g_2} , \quad 2\pi = 2\pi \left( m + \frac{1}{2} \right) + \delta_2.\]

In large X-guide, the minimum L comes out 8.26", with I.L. \( (f_1) = 0 \) and I.L. \( (f_2) = .12 \text{ db}. \)

B. Design for \( f_1 = 7.75 \text{ Kmc}, f_2 = 8.35 \text{ Kmc}, \) power out of port 3.

\[L = \left( m + \frac{1}{2} \right) \lambda g_1.\]

\[\therefore \theta_2 = \theta_0 - \delta = \frac{L}{\lambda g_2} \quad 2\pi = 2\pi n + \delta_2.\]

In large X-guide, the minimum L is 7.245", with I.L. \( (f_1) = 0 \text{ db}, \)
I.L. \( (f_2) = .02 \text{ db}. \)

VI. DIPLEXER IN \( \frac{1}{2} \) HEIGHT LARGE X-GUIDE

For satellite applications, to limit weight, it was decided to fabricate the \( L = 7.245" \) design. Also, the large X-guide was decreased to \( \frac{1}{2} \) height. An MDL large X-guide side-wall coupler was decreased to half height and the capacitive dimple was replaced by a \#4-40 screw. With a screw penetration of
approximately 0.120", $P_{13}$ and $P_{14}$ were within 0.1 db at 7750, $P_{12}$ was greater than 30 db, and VSWR was 1.05. At 8350, the corresponding values were 0.1 db, 23 db and 1.12 VSWR.

A second hybrid gave somewhat worse results:

\[
\begin{align*}
7750 & : 0.1 \text{ db}, > 30 \text{ db}, 1.05 \text{ VSWR}; \\
8350 & : 0.4 \text{ db}, 20 \text{ db and 1.09 VSWR}.
\end{align*}
\]

The screw sensitivity was about 0.5 db/turn w/r $P_{13}$ and $P_{14}$, with a measurable but small effect on input VSWR's.

In Fig. 2, imagine that hybrid B, being pivoted at ports 1, 2 is lifted upward through 180° and then is slid over until it lies exactly over hybrid A. Ports 4' and 2, and 3' and 1 are now connected by an abrupt 180° bend as illustrated in Fig. 3.

![Diagram](image)

Fig. 3: 180° E-Plane Bend in $\frac{1}{2}$ Height Large X-Guide.

The VSWR characteristics of the bend in Fig. 3 are:

\[
\begin{align*}
7750 & : 1.05 \\
8050 & : 1.02 \\
8350 & : 1.02
\end{align*}
\]

The complete diplexer appears in Figs. 4 and 5. Because of soldering difficulties, it was not possible to align each $\frac{1}{2}$ height hybrid for optimum behavior prior to joining. Therefore, each hybrid has a tuning screw for power-split trimming (where the dimple had been) and a "shorting" screw in the line
Fig. 4: Photograph of Completed Diplexer
Fig. 5: Photograph of Diplexer with Inside View of 180° Bend
past ports 3 and 4. It was hoped that with the shorting screw in position, it
would be possible to set the power-split screw for minimum VSWR and so balance
each hybrid.

Circuit A was found to be somewhat worse than that experienced on the two
preliminary 1/2 height hybrids, giving VSWR's of 1.12 at 7750 and 1.21 at 8350.
Circuit B gave VSWR's of 1.38. The reason for such inferior performance of B
is not known.

In view of the poor performance of hybrid B, all four shorting screws were
used as tuning elements, in addition to the two power-split screws. By a con-
verging process, the diplexer was tuned to the following characteristics:

<table>
<thead>
<tr>
<th>7750</th>
<th>8350</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.L.</td>
<td>I.L.</td>
</tr>
<tr>
<td>0.3 db</td>
<td>0.5 db</td>
</tr>
<tr>
<td></td>
<td>(= P_{13}) (= P_{23})</td>
</tr>
<tr>
<td>P_{12}</td>
<td>P_{21}</td>
</tr>
<tr>
<td>32 db</td>
<td>27 db</td>
</tr>
<tr>
<td>P_{14}</td>
<td>P_{24}</td>
</tr>
<tr>
<td>30 db</td>
<td>22 db</td>
</tr>
<tr>
<td>VSWR 1</td>
<td>VSWR 2</td>
</tr>
<tr>
<td>1.3</td>
<td>1.28</td>
</tr>
</tbody>
</table>
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