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INVESTIGATION OF MAGNETOHYDRODYNAMIC WAVES

VOLUME I SUMMARY

FINAL REPORT

COPY NO. 4/6

SUBMITTED UNDER CONTRACT NO. AF19(628)-239

Prepared for:

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FOREWORD

This report, Volume I, is one of a series of three volumes reporting the work accomplished under Contract AF19(625)-239 initiated in December 1961.

The purpose of this program was to investigate by theory and experiment the pertinent factors influencing the creation, propagation and detection of a type of magnetohydrodynamic wave, the Alfvén wave, in a partially ionized, low density medium.

The theoretical aspects of MHD waves generated under ionospheric conditions, and their corresponding excitation in a simulated environment are investigated in Volume II. In Volume III is reported the experimental design and subsequent test results of wave propagation in three plasma facilities, a hypervelocity impulse tunnel, an electromagnetic shock tube and an arc discharge tube. This report summarizes the results of the experimental program and outlines the theoretical work performed in support of the experiments.

This program required the cooperation and assistance of many persons. The chief contributors, in addition to C. D. Joerger, Project Leader, were: J. L. Hickerson and E. W. Hobbs for plasma wave theory; T. R. McPherron, E. S. Thompson, G. L. Elder, and J. L. Walker for experimental design and execution; and H. J. Fivel and Dr. J. N. Holsen for thermodynamic analyses.
# MHD Wave Investigation

## I- Summary

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l. INTRODUCTION

This report is submitted in fulfillment of Contract AF19(268)-239 for the investigation of Magnetohydrodynamic (MHD) waves. Under this contract McDonnell has successfully generated Alfvén waves in an arc discharge tube and further, has evidence that this phenomena was also detected in the ionized flow of a Hypervelocity Impulse Tunnel (HIT).

The goal of this program was to determine, by means of theory and experiment, the pertinent factors influencing the creation, propagation, and detection of the Alfvén wave in a partially ionized, low density atmosphere. Physically, the Alfvén wave is simultaneously an electromagnetic oscillation and a fluid oscillation. It represents an energy exchange between the electromagnetic field and the motion of an electrically conductive fluid. An initial disturbance, which causes a volume of electrically conducting gas to move across the lines of a constant permeating magnetic field, induces an electric potential across the moving volume. This electric potential causes a current that links adjacent gas volumes. The current in turn is acted upon by the magnetic field, which produces a force in the direction opposite to the motion of the original gas volume. In this manner the original disturbance propagates in opposite directions from the point of excitation along a magnetic field line. The two primary conditions for Alfvén wave motion are that:

a. The initial disturbance must have an oscillation frequency well below the ion cyclotron frequency.

b. The resistivity of the fluid must be small at this wave frequency.
The investigation of MHD waves in this program could be considered a research study. In this light the program as first envisioned was changed in emphasis and direction as knowledge was gained, theory was developed and new experimental data was obtained. The application of the HIT for plasma generation in support of MHD wave studies was the first of its kind; its selection promised a high probability of supporting Alfvén waves because of its large size.

The investigation, as originally conceived, was divided into two parts, (1) verification that the gas conditions of the HIT were able to support Alfvén wave propagation, and the design of adequate detection experimentation instrumentation, and (2) analysis of the relation between the experimental environment and the ionosphere. Specifically this entailed:

a. Definition of the parameters that would exist in the HIT.

b. Theoretical establishment of those tunnel parameters, which would be sufficient to support MHD wave propagation.

c. Instrumentation of the tunnel, to provide data on shock velocity and position, density, homogeneity, equilibrium of the plasma, and electron temperature in the region of interest.

d. Design of instrumentation to detect the MHD wave.

e. Performance of 5 tunnel shots, to calibrate the instrumentation and demonstrate the validity of its use in providing appropriate parameter values to detect MHD waves.

f. Establishment of some correlation between the experimental (simulated), theoretical and ionospheric environments.

After the program had started, on the basis of preliminary results, an additional 10 HIT test shots were authorized to augment measurements of the
properties of Alfvén waves as initially envisioned. Studies were directed
toward investigating the threshold of MHD generation, and the effects of electron
density, temperature, pressure and gas velocity.

After completion of the first 15 HIT shots and analysis of data so obtained,
it was believed that Alfvén waves were actually observed. In order to confirm
this, two small plasma facilities, an electromagnetic shock tube and an arc
discharge tube, were utilized to improve instrumentation and confirm theory.
The smaller simulation experiments enabled a parametric approach to the investi-
gation. Over 800 test shots were fired at a cost, several orders of magnitude
lower than HIT experiment costs. The validity of ionospheric scaling was also
reviewed.
2. CONCLUSIONS

This program made a positive contribution to the theoretical and experimental knowledge of Alfvén wave characteristics and extended the experimental capability of high velocity ionized gas generation and simulation. In addition, this program was a first step in developing firmly based experimental methods for studying wave generation and propagation in the ionosphere.

The theoretical work resulted in a quantitative description of Alfvén wave behavior in a partially ionized gas. This description included a parametric study of the influence of magnetic field strength, ionization concentration, gas temperature, molecular specie and wave frequency on phase velocity and attenuation. It is valid for plane infinite waves, those propagated in cylindrical wave guides, and for disturbances ranging from an extremely low frequency to one above the ion cyclotron frequency. This range includes both the resonance point at the ion cyclotron frequency and the band of frequencies where ion neutral coupling is breaking down. These quantitative solutions agree with predicted characteristics of the Alfvén wave at the weak and strong ion neutral coupling conditions reported by other authors. The analysis assumed that gravitational forces and gradients of parameters were negligible, and that all propagation occurred along magnetic field lines. Ideal gas laws, thermal equilibrium, and only small wave perturbations were assumed. Damping terms were introduced in the equations expressing collision between ions and other particles, but like particle interaction was not included. The plane wave solutions are valid for the ionosphere and valid for describing the ion neutral damping in the plasma of the HIT, while the waveguide solutions are valid for
the arc discharge tube experiments. The complete propagation description for waves in the ionosphere must include; the gradients of ion concentration and percentage ionization, the curvature of the field lines, the spreading of the wave across magnetic field lines and the effects exhibited at the ionospheric boundaries.

The theoretical analysis of the Hypervelocity Impulse Tunnel for plasma generation indicated that two regions of highly ionized flow existed. The first region was generated by the shock, formed by the rupture of the diaphragm separating the arc chamber from the expansion nozzle. Theories presently exist for describing the conditions behind a high velocity shock, which predict a short duration of ionized flow.

The measured ionization level compares well with predicted levels, based on the measured shock velocity; however, the period of ionization as measured was considerably longer than that predicted. The longer ionization period was the result of multiple shocks and very turbulent flow, which is not subject to easy theoretical analysis. The usual assumed physical constants, such as the ratio of specific heats, change in these high velocity flows, further complicating any analysis. In conclusion, the theoretical analysis of the shock excited flow indicated only order of magnitude effects of parameter variation. The more accurate description of the gas conditions evolved from experimental measurements.

The second ionized flow region examined in detail in the HIT was the blow-down flow. Theory states that ionization in the flow occurs when the transit time between the arc chamber and the test section is shorter than the gas recombination time. The equilibrium ion concentration in the arc chamber, at peak temperature, is principally dissociated nitrogen, which when expanded through a
MHD Wave Investigation

1. Summary

Nozzle remains ionized. The computer solution of the rate equations showed that the ionization concentration was effectively frozen, and ionization levels were verified by microwave measurements. Ion-neutral collision frequency in the test section was computed by solving the rate equations, however, the uncertainty in the ratio of specific heats introduced error in the calculation. The same uncertainty limited the accuracy by which the flow core size could be estimated from free stream velocity and stagnation pressure measurements.

The HIT application to ionospheric wave studies is limited only by the relatively high density and ion-neutral collision frequency. Reducing the free stream density would make this facility an ideal ionospheric simulator. Nevertheless, it satisfactorily provided a highly ionized, high velocity, large volume plasma for the measurement program.

The experiments investigating Alfvén wave generation and propagation employed the HIT, the electromagnetic shock tube and the arc discharge tube.

Under the plasma conditions provided by the arc discharge tube, the following conclusions were reached:

a. Torsional Alfvén waves can be easily generated and their characteristics determined.

b. The theory for guided waves in a fully ionized gas and the experimental results in the arc discharge tube were in general agreement.

c. Characteristics of Alfvén waves can be used to investigate the conductivity, the electron temperature, and the ionization concentration in a low density arc.

d. From Alfvén velocity measurements, it was determined that the gas was not fully ionized.
MHD Wave Investigation

I - summary

e. The Alfvén wave was not dependent on the kind of gas but on the ionized mass density.

f. The Alfvén wave is not detected if the exciter is triggered in the first 30 microseconds in which the arc is burning, nor after the ionization begins to decay.

The arc discharge tube provided an excellent test facility for the investigation of Alfvén waves in a fully ionized gas. This facility is simple, inexpensive to operate, and has a wide range of operating conditions.

Under the plasma conditions provided by the Hypervelocity Impulse Tunnel, the following conclusions were reached:

a. Disturbances having characteristics of Alfvén waves were detected using a magnetic exciter to perturb the plasma.

b. The tunnel can theoretically provide plasma conditions conducive to the generation and propagation of Alfvén waves.

c. The precise prediction of critical parameters, such as phase velocity and attenuation, requires a more thorough knowledge of the tunnel gas conditions and the exciter plasma coupling than is currently available.

d. The measurement of critical parameters, phase velocity and attenuation, is complicated by the rapid damping caused by the high ion neutral collision frequency. If the tunnel free stream density could be reduced, while still maintaining a high ion concentration, more conclusive data could be obtained.

e. Microwave measurements of electron concentration compared favorably with the theoretical calculations.
f. Of the two ionized flow regions during tunnel operation, the blow-down region provided the more desirable testing conditions.
g. Oscilloscope data acquisition is required to adequately provide the accuracy for measuring the key parameters.
h. Ionization in the test section is primarily dependent upon arc chamber temperature.
i. The plasma exists in a highly non-equilibrium condition with a degree of ionization of 0.1 percent and a cold flow temperature.
j. Alfvén type disturbances can be detected by simply designed devices.
k. The Alfvén wave attenuation is considerably higher than predicted.
l. The simulation of the ionosphere is accurate in ionization percentage and large volume but is a continuum gas rather than a free molecular gas.

The HIT is a valuable research tool that can be adapted to investigating plasma wave phenomena, but must be supported by other facilities in which exploratory experiments and instrumentation development can be carried on with large numbers of experiments.

Under the plasma conditions provided by the electromagnetic shock tube, the following conclusions were reached:

a. Alfvén waves were not detected in this series of experiments. Failure of the TM mode to exist in the shock tube indicated that more favorable gas conditions were required.
b. The gas conditions cannot be theoretically predicted but they can be easily measured.
c. No plasma steady state condition existed for a sufficient time to support Alfvén waves.
MHD Wave Investigation

I. Summary

The electromagnetic shock tube is a simple, easy to operate facility capable of high ionization concentrations. It proved to be very valuable for instrumentation development.
3. RECOMMENDATIONS

The investigation of magnetohydrodynamic waves under this program has contributed to the basic understanding of waves in plasma, and has shown the possibility of generating the Alfvén wave in a simulated environment. As a consequence, additional areas exist in which meaningful theoretical, laboratory and full scale studies can be performed.

Immediate benefits could be obtained by extending the present theoretical and experimental studies. A number of recommended investigations are described below:

a. The computer program for the Alfvén wave dispersion equation should be expanded to include spreading of the wave energy across the magnetic field lines for a non-infinite plane wave. This spreading introduces an added attenuation term which is referred to as spatial attenuation in these reports.

b. A description of the reflection of the Alfvén wave at an interface, with particular attention focused on the energy exiting the plasma, is particularly important in determining whether a reflected Alfvén wave could be detected by observing the ionosphere. The conclusions from such a study should be verified in the arc discharge tube.

c. A computer program of the dispersion equation, to solve for Alfvén waves traveling at arbitrary angles to the magnetic field, should be formulated to determine trapping of the wave by magnetic field lines. If HIT conditions could be generated to maintain an adequate ionization percentage, while reducing the free stream density, the Alfvén wave
trapping could be experimentally measured.

d. The excitation of waves in the free molecular flow should be experimentally investigated in a low density (\( \sim 10^{-2} \) mm of Hg) cesium plasma facility presently under test at McDonnell. The wave modes set up in the arc discharge tube could also be repeated in the cesium facility, verifying that Alfvén modes can be excited. Then a combined theoretical and experimental program should be undertaken to determine whether a disturbance, such as a dipole interaction within the magnetic field, could excite an Alfvén wave. In other words the ionospheric coupling phenomena should be theoretically and experimentally investigated.

Many other studies could be envisioned, based upon information already gained during the course of this program. A listing of these is given below:

Theoretical Studies

1. The computer program for the solution of the Alfvén wave characteristics used in this study assumed a number of terms to be constants. These terms can be incorporated into the present program to give a more realistic description of the wave's characteristics. They include:
   a. Gradients of magnetic field and curved magnetic lines of force.
   b. Non equal temperature of electrons, ions, and neutrals.
   c. Ion-ion and electron-electron interactions.
   d. Wave effects on the plasma energy distribution function.
   e. Arbitrary boundary conditions.

2. The present computer program provides a dispersion relationship. It is important to determine the resulting waves generated by typical disturbing functions that may exist in a laboratory or ionospheric environment.
3. Pressure terms have been eliminated from consideration up to this point. By including them, MHD acoustic and shock wave conditions would be introduced into the dispersion equations.

4. The resulting MHD wave from a known disturbing function may be calculated from the dispersion equation. Studies should be performed in which the characteristics of the disturbing function are determined for various magnitudes and velocities of the forces, such as:
   a. A dipole or multipole
   b. A cloud of gas
   c. A plasma column
   d. A rarefaction wave
   e. A shock wave, plane or spherical
   f. A rocket exhaust

**Experimental Studies**

1. The arc discharge tube has already supplied a considerable amount of data on Alfvén waves. Additional data can be obtained in this facility on:
   a. Wave propagation in a non-homogeneous magnetic field.
   b. Wave propagation at various angles to the magnetic field lines.
   c. Wave characteristics at interfaces, the energy transmitted in the direction of propagation and the energy reflected.
   d. Wave characteristics of TE, as well as TM modes.
   e. The use of different exciters, including a spherical pressure wave and a high speed projectile.
   f. Wave characteristics at minimum pressure arc discharge.
   g. Various detection methods, such as microwave cross modulation.
2. The previously employed facilities have studied wave motion in continuum flow. A low density plasma source has been constructed which extends the range of the small facilities into the free molecule flow, when used with the 13,600 gauss magnetic field. Such a facility should be used to study ion-neutral damping and excitation of waves by rapidly moving objects, as well as to perform experiments previously suggested for the arc discharge tube.

3. The Hypervelocity Impulse Tunnel provides the best ionospheric simulation of Alfvén wave trapping. Prior to performing more shots in this facility, a study should be conducted to find methods of reducing the particle density in the free stream flow, and of determining the variance of the ratio of specific heats along the axis of the tunnel flow.

Field Measurements

1. Certain measurements are more easily performed in the field than in the laboratory, and others taken in the field, will help direct laboratory experimentation. Measurements that would supply important information for MHD phenomena in the ionosphere include:

   a. Noise measurements in the ionosphere and on the ground in the frequency band of MHD disturbances.

   b. Measurements of magnetic field, electric field, acoustical disturbances and ionospheric perturbations at conjugate points of known disturbances.

   c. Measurements of charge, dipole moment, and flow fields of a satellite or a booster within the ionosphere.
4. EXPERIMENTAL DESIGN AND TEST RESULTS

4.1 Experimental Design. - The experimental program, as evolved over the period of this contract, is summarized in Figure 4-1. Only the first three series of experiments were planned and contractually covered at the beginning of the study. To the knowledge of this contractor, an HIT had not been previously used for MHD wave generation. Recognizing this, McDonnell's supporting research program contributed five tunnel shots in addition to those authorized and piggybacked on others not directly associated with this program, to be used for research of tunnel gas conditions. The electromagnetic shock tube and arc discharge tube experiments were added to the program, after the results obtained from the early HIT experiments showed a need for small facility tests.

The Hypervelocity Impulse Tunnel (Figure 4-2) is basically a large nozzle, whose flow is generated by arc heating of a precharged volume of gas, nitrogen in this case. Two ionized regions are found: the shock excited region, created by the initial shock bursting the diaphragm at the time of arc discharge, and the initial phase of blow-down flow, which begins just behind the shock excited region. Both of these regions were investigated during the program. Knowledge of tunnel operation at these special conditions, was then used for the design and testing of the required instrumentation, to (1) monitor the tunnel operational parameters, (2) detect any magnetic or electrical disturbances, and (3) provide for the synchronization and timing requirements for proper event sequencing. In order to understand both of the flow regions, the conditions in the arc chamber were investigated. The discharge is completed in five milliseconds, ringing with a period of about two milliseconds. Most of
SUMMARY OF EXPERIMENTS

- 5 HIT* CALIBRATION SHOTS
- 15 HIT FULLY INSTRUMENTED ALFVEN WAVE SHOTS INCLUDING MAGNETIC AND ELECTRICAL EXCITERS
- 400 SMALL SHOCK TUBE TESTS FOR INSTRUMENTATION IMPROVEMENT
- 300 ARC DISCHARGE TUBE SHOTS FOR ALFVEN WAVE GENERATION
- 8 HIT SHOTS FOR ALFVEN WAVE GENERATION USING TORSIONAL EXCITER

* HYPERVELOCITY IMPULSE TUNNEL

FIGURE 4-1

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MHD Wave Investigation

1. Summary

MCDONNELL AIRCRAFT HYPERVELOCITY IMPULSE TUNNEL

ARC CHAMBER

10° CONICAL NOZZLE

SIMPLE CONICAL DIFUSOR

50 INCH TEST SECTION

COAXIAL CONDUCTOR

TURNTABLE

MCDONNELL

FIGURE 4-2
The megajoule of stored energy is absorbed in the first quarter cycle of the discharge. The maximum temperature reached in the chamber is computed from an empirical efficiency factor, the capacitor stored energy, and the pressure decay rate which is extrapolated to give peak pressure. Peak conditions in the arc chamber are temperatures of 12,000°K and pressures of 3000 psi. The conditions in the chamber exit (throat) are slightly lower. The throat expands to the adjoining conical nozzle which constitutes the test section of the tunnel.

The shock excited flow is generated within a fraction of a millisecond after the initiation of the arc discharge. As the shock travels through the divergent nozzle, it loses strength. Three analyses have been performed and correlated, to predict the magnitude of this shock strength attenuation. Test times have been predicted, and all indicate durations of ionized flow, between the shock and the gases from the arc chamber, of fractions of a millisecond; however, experiments have shown test times up to 2 milliseconds. The gas behind this shock is highly ionized at a high temperature and at a low density.

The blow-down flow, providing a test time of at least 4 milliseconds, provides the most useful conditions for Alfvén wave measurements. At the 12,000°K arc chamber temperature, nitrogen is highly dissociated and highly ionized. Thus, the ionized constituent at the throat is principally monatomic nitrogen. In the rapid expansion through the nozzle to the test section, recombination of this specie is not realized, and thus, the ion concentration remains essentially constant. As the arc chamber temperature drops, the number of ions created rapidly decreases, and hence, ionization in the test section ceases. Two analyses were performed to verify this picture, one, a simplified frozen flow analysis, and two, a non-equilibrium flow analysis using an IBM 7094.
computer. Theoretically, concentrations of greater than $10^{11}$ ions per cubic centimeter should exist in the test section. Other important characteristics of the HIT plasma such as temperature, pressure and degree of contamination were also examined over the usable test time.

The two smaller plasma sources used for experimentation and instrumentation development were an electromagnetic shock tube and an arc discharge tube. The electromagnetic shock tube creates a moving plasma by discharging a short high current pulse between coaxial electrodes. The best model for the flow in an electromagnetic shock tube is a piston of fully ionized hot gas, propelled down the tube, compressing and heating the gas in front, while itself losing strength. Extensive research has been done to explain the processes in the discharge and in the acceleration of the gases, but no clear cut theoretical description has so far been achieved. However, gas conditions can be easily measured and the gas was found to be highly ionized, at high temperature.

The second facility employed in the experiments, an arc discharge tube, was similar in construction to the electromagnetic shock tube. Again an arc was employed, and again the ionization processes are not well understood. The arc is struck along the axis of the evacuated tube, with experiments being performed while the arc is still burning. The plasma in this facility was more highly ionized than that in the shock tube.

Instrumentation as shown in Figures 4-3, 4-4, 4-5 and 4-6 was developed and tested during the calibration and early exciter shots. The primary flow field instrumentation included:

a. Pressure transducers, to measure the dynamic flow pressure as a function of time.
MHD Wave Investigation
1. Summary

INSTRUMENTATION RAKE ON HIT

CONDUCTIVITY PROBES
DETECTION COIL MOUNTS
VERTICAL AND LATERAL ACCELEROMETERS
PRESSURE TRANSDUCERS
LANGMUIR PROBES
IONIZATION PROBE
CONTAMINATION MONITOR

FIGURE 4-3

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b. Ionization probes, to measure the velocity of the shock front.
c. Conductivity probes, to measure the core size of the ionized gas.
d. A smear camera, to determine the velocity of the shock front and the gases.
e. A framing camera, to view the gas as it appeared in the test section.
f. A contamination wheel, to monitor the particles in the flow caused by erosion of the tunnel throat.
g. Microwave interferometers, to measure electron concentration of the plasma.

The magnetic field required for wave generation was supplied by a 36-inch diameter coil, inserted into the tunnel. The magnetic field was generated by a capacitor discharge into this coil, providing up to a maximum of 1500 gauss. Electronic timing was used to insure that the peak magnetic field strength occurred at a time when the ionization of the gas flow through the test section was a maximum. The field coils, Figure 4-4, were housed in plexiglass toroids, evacuated to a pressure of 0.1 micron Hg, to prevent arc over when up to 10,500 volts were applied.

Wave excitation was produced by electrical and magnetic exciters, operating at frequencies up to fifty kilocycles per second. The resulting disturbances were monitored by low pass magnetic search coil detectors. Exciter magnetic field strengths of 70 gauss and electrical currents up to 15,000 amperes were used in the tests. The magnetic wave component was detected by coils which measured the rate of change of the magnetic field. These detectors were placed at various positions within the magnetic field, in order that wave velocity and attenuation of a disturbance could be determined, and were oriented vertically.
as well as horizontally with respect to the magnetic field lines to monitor the polarization.

The outputs from the detectors were recorded on tape recorders and on dual beam oscilloscopes, depending upon the frequency response required. The flow field data was recorded on a high speed oscillograph. In addition, the magnetic field, microwave interferometer and exciter outputs were monitored by means of tape recorders, to determine proper magnitude and time sequencing. It was necessary that maximum magnetic field, ionized flow and exciter excitation occur simultaneously in the test section, requiring timing to an accuracy of 2 milliseconds.

4.2 Test Results. - The initial tunnel shots confirmed earlier evidence that a high velocity, low pressure, partially ionized flow was produced and would provide favorable conditions for Alfvén wave generation. Therefore, the initial shots were used to optimize tunnel conditions for shock generation and to enable perfection of measurement techniques. An ionization of $10^{12}$ electrons per cc for a period of one millisecond was produced by a helium driven shock wave, traveling at 20,000 feet per second. However, optical data indicated this region to be highly turbulent and revealed multiple shocks in the flow stream. Since this would have prevented correlation of test results, the blow-down flow period was investigated as an alternate test region. Here ionization concentrations greater than $10^{13}$ electrons per cc were produced for four to six milliseconds of test time, about three milliseconds after passage of the shock front. The flow velocity for this region reached 13,000 feet per second with the stagnation pressure equaling 0.58 psi. There was little turbulence.
The first four fully instrumented shots utilized an electric plane wave exciter. Even though this exciter was not able to maintain the desired electric field strength during ionized tunnel flow, perturbations were produced and recorded by the wave detectors. These perturbations, however, propagated without the presence of a magnetic field and hence were not characteristic of Alfvén waves.

The next eleven shots used a magnetic plane wave exciter generating a magnetic field of seventy gauss at frequencies from 1500 to 7500 cps. Figure 4-7 shows the data of the wave disturbances that were obtained. These disturbances occurred during C- and K-band blackout and during the period of maximum magnetic field strength, indicating the typical characteristics of Alfvén waves. The signal measured by the detector, aligned horizontally at the instrumentation post, is of greater amplitude and clarity than that detected one foot further downstream, by a detector mounted on the rake. The measured wave propagation velocity was greater than 30,000 feet per second, not in disagreement with a calculated value of 100,000 feet per second. The attenuation distance was 0.15 meter against a theoretical prediction of 5 meters. However, gas condition uncertainties and the neglect of spatial attenuation terms make the theoretical calculations of questionable accuracy. Additional shots indicated that with heavy ionization, but with the magnetic field and the exciter inoperative, disturbances occurred primarily in the shock excited flow, with only a very small harmonic component being measured at the exciter frequency.

These tunnel shots indicated that Alfvén waves were produced and could be studied in the HIT but that a large number of shots would be required for a full investigation of all wave characteristics. The electromagnetically driven...
MHD Wave Investigation

Report No. A219
30 November 1963

Magnetic Field on/Off

Shock Waves in Test Section

Magnetic Field Turned On

"K" Band Received Power

Blackout

"C" Band Received Power

Post Vertical On

Post Vertical 6" Off

13 MV

10 MV

Figure 4-7

McDonnell
shock tube shown in Figure 4-8 was then instrumented for magnetohydrodynamics testing, and approximately 400 shots were fired in it. The facility was found valuable in the development of high speed switching circuits and data recording, but Alfvén waves were not propagated. Attempts were made to excite the wave in both the transverse magnetic and transverse electric modes, but the tube geometry precluded establishment of the required plasma conditions and energy inputs seemed too low to enable wave excitation. Experiments were then successfully performed in the Arc Discharge Tube and Alfvén wave characteristics were measured under a wide range of conditions. The arc discharge tube is shown in Figure 4-9. Plasma was created by charging a 45 microfarad capacitor to 10K volts, and then discharging it along the tube axis. A transverse magnetic mode wave was generated by the discharge of a 5 microfarad capacitor at 10K volts, between coaxial electrodes at one end of the tube. The wave was detected by both electric probes and magnetic detectors. Figure 4-10 shows a sample of the data obtained. The existence of the Alfvén wave is shown by the increase in phase velocity with magnetic field (shown by decreasing phase shift), and by the decrease in attenuation. Alfvén wave characteristics were measured for magnetic field strengths from 6800 to 13,600 gauss in test gases of air, nitrogen, argon, and helium at pressures from 10 to 500 microns Hg.

The last series of tunnel shots was performed using an exciter identical to that of the arc tube. These shots were intended to extend the data obtained from the arc tube to a larger, less fully ionized medium with a lower magnetic field, so that scaling parameters could be obtained. Disturbances were generated and propagated by the experiment, but these did not have sufficient characteristics to be classified as Alfvén waves.
MHD Wave Investigation
Summary

FIGURE 4-9

MCDONNELL
MHD Wave Investigation
1 - summary

ALFVÉN WAVES GENERATED IN AN ARC DISCHARGE TUBE IN ARGON
PRESSURE EQUALS 250 MICRONS Hg

6.8 KILOGAUSS FIELD
EXCITER REFERENCE
MID-TUBE DETECTOR
END-TUBE DETECTOR

10.2 KILOGAUSS FIELD
EXCITER REFERENCE
MID-TUBE DETECTOR
END-TUBE DETECTOR

13.6 KILOGAUSS FIELD
EXCITER REFERENCE
MID-TUBE DETECTOR
END-TUBE DETECTOR

FIGURE 4–10

--- 5 MICROSECONDS

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5. THEORETICAL INVESTIGATION OF PLASMA WAVES

The program objectives of generating and propagating Alfvén waves required primarily an experimental investigation; however, an extensive theoretical study into wave propagation was necessary, in order to adequately prepare for the very complex program envisioned for the Hypervelocity Impulse Tunnel. This analysis, described in Volume II, involved a general derivation of the wave propagation equation, which was then specialized to enumerate the properties of transverse and longitudinal waves traveling along and across magnetic field lines. This specialization resulted in a general mathematical relationship capable of describing the Alfvén wave.

5.1 Development of Wave Theory. - The general wave propagation equation, or dispersion equation as it is usually called, was derived from the basic plasma force equations, charge and mass conservation equations, perfect gas law and Maxwell's equations.

Having once formulated the general dispersion relationship, it was then analyzed for three important physical cases: (1) Alfvén wave propagation, (2) wave propagation perpendicular to magnetic field lines, and (3) acoustic type wave propagation parallel to a magnetic field.

For case (1) the dispersion relationship determines the existence or non-existence of an Alfvén wave under the conditions of over-all gas neutrality, negligible gravitational forces and gas turbulence, simple gas pressure and low amplitude disturbances of the gas. Actually, this equation predicts that there are two different propagating entities possible in an unbounded gas, left and right circularly polarized waves.
The general parametric analysis of the Alfvén dispersion relationship was accomplished by means of a computer program for the IBM 7094. The parametric analysis consisted of an examination of the solution to the equation for each parameter, varied one at a time. This type of a solution was found necessary, since the number and range of parameter values to be considered was large. Previous investigators were forced to consider only limiting cases and were thus unable to achieve a complete description of the dispersion equation. This parametric analysis of the dispersion relation was the first known attempt to mechanize the complete dispersion equation, and provided the first opportunity to compare the qualitative results of theory with quantitative calculations.

From the solutions to the dispersion equation, quantitative information as to the propagation velocity, attenuation and allowed disturbance frequencies was obtained. The computer results were applied directly to experiments in the HIT and to the analysis of actual disturbances in the F-region of the ionosphere. A separate analysis was necessary for McDonnell's two small plasma facilities, since the size and shape of the confined plasma volume determined the modes of oscillation. In each case criteria for the existence of Alfvén waves and their propagation characteristics were derived, such as the effects of varying the magnetic field strength, the degree of ionization, gas density, gas temperature and molecular weight on the propagation velocity and attenuation.

Two models of the ionosphere were assumed, an unbounded medium and an array of wave channels, or ducts. The results of the computer investigations can be applied to either model, provided the distance between the point of generation and the point of detection is not greater than the dimensions of the assumed wave channel. The assumption of a wave channel model was deemed reasonable,
since there are only two wave restraining phenomena, the magnetic forces and the ionized gas density. A plot of attenuation distance, defined as the distance the wave travels before its strength falls to about 37 percent of its initial value, as a function of disturbance frequency is shown in Figure 5-1. It can be seen that the polarization of the wave greatly influences its propagation. The ion cyclotron frequency determines, for given conditions, the frequency range where Alfvén waves are transformed to the "whistler" mode of magneto-ionic theory. The lower frequencies, actually extending far below the range depicted in the figure, are the ones important for Alfvén waves. To briefly summarize the computer results: An Alfvén wave in the F-2 region of the ionosphere travels at about 1/10 percent of the speed of light, attenuates very slightly from source to reflection point and travels in a channel along a magnetic field line.

Whereas two types of Alfvén waves may exist in the ionosphere, a plane wave traveling along a magnetic field line and a guided wave traveling between dielectric boundaries of ionospheric layers, the latter type is the most commonly simulated in the laboratory, because of the confined nature of laboratory plasmas. The physical shape and size of the plasma not only directly determines the mode of wave propagation, but also its minimum frequency and attenuation distance. Ion concentration is an important parameter in determining not only the speed of propagation of the wave, but also its attenuation. This influence is shown in Figure 5-2. Another very important parameter is the intensity of magnetic field permeating the plasma. Figure 5-3 shows the transition region beginning near the ion cyclotron frequency. In order to provide the best conditions for Alfvén wave generation in a controlled experiment, compromises between facility dimensions, ionization level, magnetic field strength and
PLASMA WAVE ATTENUATION IN IONOSPHERE

(185 MILES ALTITUDE)

\[ \nu_{in} = \text{ion-neutral collision frequency} \]

\[ \omega_{ci} = \text{ion cyclotron frequency} \]

FIGURE 5-1

MCDONNELL
EFFECT OF IONIZATION RATIO ON ATTENUATION DISTANCE

$B = 1000$ GAUSS

$R = \%$ OF IONIZATION

$\omega_{pi} =$ ION PLASMA FREQUENCY

DISTURBANCE FREQUENCY (RADIANS PER SECOND)

ATTENUATION DISTANCE (METERS)

FIGURE 5-2

MCDONNELL
EFFECT OF MAGNETIC FIELD ON ATTENUATION DISTANCE
FOR 0.1% IONIZATION

\[ B = \text{MAGNETIC FIELD STRENGTH} \]
\[ \omega_{ci} = \text{ION CYCLOTRON FREQUENCY} \]

\[
\begin{align*}
\text{ATTENUATION DISTANCE (METERS)} & \quad \text{10^{-3}} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \quad 10^1 \quad 10^2 \\
\text{DISTURBANCE FREQUENCY (RADIANS PER SECOND)} & \quad 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \quad 10^9 
\end{align*}
\]

\[ B = 5000 \text{ GAUSS} \]
\[ B = 1000 \]
\[ B = 500 \]

FIGURE 5-3
disturbance frequency had to be made. Figure 5-4 gives the Alfvén wave propagation velocity (denoted as phase velocity) for the partially ionized gas of the HIT in terms of the classical Alfvén velocity for a fully ionized gas.

In laboratory devices such as McDonnell's electromagnetic shock tube and arc discharge tube, the Alfvén waves are quite different from waves observed in the HIT. The dimensions of the tube and conditions at its boundary enter directly into the theory. Before Alfvén wave experiments could be sensibly performed in the HIT, the shock and arc discharge tubes, quantitative results from the hydromagnetic wave guide theory were required. While no attempt at ionospheric simulation was made in these devices, they were used to verify Alfvén wave theoretical predictions.

The description of wave propagation perpendicular to magnetic field line is complicated by the fact that the oscillations are mixed acoustic and modified Alfvén type waves. At low frequencies coupled waves involving the entire gas occur. As the frequency is increased, the oscillations eventually go from the form of plasma acoustic waves to ion acoustic waves. Such plasma disturbances do not readily lend themselves to theoretical or experimental analysis. Detection of these waves would require devices capable of responding to both major types of waves for electrons, as well as ions.

The acoustic wave mode is distinguished by oscillations in the direction of propagation, which, for the third case above, is along the magnetic field. This disturbance travels at the speed of sound corresponding to the kind of particles involved in the oscillation. An examination of the associated dispersion equation reveals that at low frequencies, the constituent gases unify, and an acoustic wave with velocity characteristic of the entire gas is propagated. When the coupling factor becomes quite small for low degrees of
RATIO OF PHASE VELOCITY TO ALFVEN VELOCITY FOR HIT CONDITIONS

\[ \omega_{ci} = \text{ION CYCLOTRON FREQUENCY} \]
\[ \omega_{pi} = \text{ION PLASMA FREQUENCY} \]

DISTURBANCE FREQUENCY, \( \omega \) - RADIANS PER SECOND

FIGURE 5-4

MCDONNELL
ionization, two acoustic waves, one associated with the plasma and another with the neutral gas, are disseminated. As the disturbance frequency approaches the ion cyclotron frequency, the plasma breaks up. At this point an ion acoustic wave and an electron acoustic wave propagate with their separate phase velocities going to their appropriate acoustic velocities. The acoustic wave theory has been extensively treated in the literature and is mathematically manageable. Such waves have been observed in the ionosphere. Since acoustic waves have low propagation velocities and a tendency to couple with the more complex MHD waves, their theoretical analysis was not extended beyond qualitative description.

5.2 Analysis of the Laboratory Plasma Environment. - Prior to and concurrent with the theoretical study of waves in plasmas were theoretical and experimental investigations of plasma environments produced in McDonnell's laboratory facilities. Consideration of the HIT flow field properties originally pointed out the possibility of simulating ionospheric conditions and conducting plasma wave experiments. A preparatory knowledge of the gas conditions in terms of constituents, total gas density, temperature, ionization and flow velocity and an estimate of the gas turbulence was essential to the selection of the previously outlined wave theory. Since numerical estimates for the important parameters involved in the wave theory were not originally available, preliminary experimentation was performed in each apparatus and a literature survey conducted in order to establish suitable quantitative models. A theoretical analysis of the proposed simulation environment was undertaken in order to optimize conditions for Alfvén wave generation and to determine the relationship between the experiments and actual ionospheric disturbances. This was accomplished by studying the published characteristics of the ionosphere, and thereby establishing the
important simulation parameters. Since the HIT was to be employed as a plasma generator, a thorough understanding of its operation in this mode was necessary. In addition, investigation into the environment provided by an electronic shock tube and an arc discharge tube was undertaken, in order to develop instrumentation and to verify the wave mode portion of the theory.

The ionosphere is a fairly noisy medium, consisting of drifting ionized formations moving between 10 and 1200 meters per second. The ionized regions are caused by solar radiation, which is absorbed through photo ionization as it passes through the atmosphere. This causes various levels of ionization which can range between $2 \times 10^5$ electrons per cubic centimeter in the E layer (140 km) to $1.5 \times 10^6$ electrons per cubic centimeter in the F layer (450 km). It would be necessary to generate plasma densities much greater than these in the HIT if the wave attenuation distance and disturbance frequency were to be commensurate with that of the ionosphere.

Section 3 of Volume II discusses the characteristics of the ionosphere, the theoretical study of the ionized gas existing in the initial phases of the HIT flow and the properties of two smaller plasma facilities, an electromagnetic shock tube and an arc discharge tube.