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INTERIM REPORT

Contract No. FAA/ARDS-444

TASK 2: LABORATORY EVALUATION OF BREADBOARD EQUIPMENT

EXPERIMENTAL EVALUATION OF COMPATIBLE PWI/CAS INTERROGATOR-TRANSPONDER TECHNIQUES

VOLUME 1 OF 2

September 1963

Prepared for

AVIATION RESEARCH AND DEVELOPMENT SERVICE
FEDERAL AVIATION AGENCY
WASHINGTON 25, D.C.

By

SPERRY GYROSCOPE COMPANY
DIVISION OF SPERRY RAND CORPORATION
GREAT NECK, NEW YORK

Sperry Report No. EB-5261-0341-1

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SECTION I

INTRODUCTION

This report has been prepared for the Aviation Research and Development Service of the Federal Aviation Agency by the Sperry Gyroscope Company Division of the Sperry Rand Corporation, Great Neck, New York. The work described herein is being performed under Contract Nr. FAA/ARDS-444 in accordance with the requirements of Task 2, dated November 1961.

The results of an experimental evaluation of interrogation-transponder techniques (Task 2) are presented as applied to the airborne Pilot Warning Indicator (PWI) and Collision Avoidance System (CAS) problem. Both PWI and CAS are referenced because the system is designed to provide both levels of collision prevention capability on a mutually compatible basis in terms of transmitted signals. The operating principles of the system are reviewed in Section II of this report.

Task 1 of the experimental evaluation program was a design study which resulted in

- establishing design goals for the experimental system
- synthesizing a system most likely to meet these goals
- estimating the operational environment for the anticipated operating period (1975)
- predicting the performance of the system in this environment.

The performance predictions for the CAS indicated that the following thresholds could be employed for threat discrimination with less than 0.1-percent risk of failing to detect a true threat:

- altitude separation of ± 600 feet
- impending miss distance (at 40 seconds to collision) for velocities below 300 knots: 1/3 nm, for velocities below 800 knots: 1 nm, and for velocities below 1700 knots: 2 nm.

The estimated effects of interference on the CAS, in the most severe case, were a false alarm rate of one in 60 hours, and a miss probability of one part in 500.

Task 2 of the experimental program covered the fabrication of breadboards of the coding, decoding, and computing portions of the system and the laboratory evaluation of these breadboards in a simulated real world environment. This simulation included injecting into the system, background interference anticipated for the operational period due to the transmissions of other PWI and CAS interrogators and transponders, and noise from extra-system sources. The test methods are described in Section III of this report.

Additional system analysis conducted during Task 2 indicated that several changes in system threshold levels were desirable. A study of the effect of sidelobes indicated that the minimum range limit (the range at which warnings are displayed regardless of the encounter geometry) should be increased to eliminate the possibility of a dangerous situation arising in very low closing rate encounters. In table 1-1 the previous thresholds and the modified thresholds are given.

TABLE 1-1

SYSTEM THRESHOLD LEVELS

System Class	Minimum Range Limit (nm)	
	Original	Modified
Propeller	1/3	1
Subsonic Jet	1	3
Supersonic Jet	2	6

A more comprehensive estimate of miss probabilities, in which the effect of actual antenna patterns was examined more thoroughly and impending miss distances other than zero were considered, provided more insight into the effect of threshold levels on miss probabilities (previously, threshold levels were based on accuracy considerations only). Thus, a 50-percent increase in the normal velocity thresholds is recommended. This results in the following revised values of the impending miss distance at the threshold point at 40 seconds to collision:

for velocities below 300 knots: 1/2 nm

for velocities below 800 knots: 1.5 nm

for velocities below 1700 knots: 3 nm.

These analyses are summarized in Sections II and IV of this report, respectively.

The results of the laboratory evaluation, described in Section IV (volume 2) of this report, are generally in good agreement with the predictions. Errors in the normal velocity threshold points are greater than anticipated. However, this discrepancy is attributable to excessive errors in the decoded values of the transponder velocity components, which could be corrected by known techniques. A second discrepancy has not been explained. When a transponder is properly replying to an interrogator and is far enough away from the normal velocity threshold point that predicted miss or false alarm rates (depending on the direction of the displacement) are negligible, empirical results show a residual error rate of about one per hour, corresponding to a miss probability of about 1/2000. Although this rate is acceptable, the lack of an explanation makes it impossible to extrapolate this result to a higher performance class system.

SECTION II

BREADBOARD EQUIPMENT

A. OPERATION

The compatible PWI/CAS can provide PWI or CAS data or both using the same transmitted signals. It is a cooperative system, and the threat evaluation is based on unaccelerated flight.

In a cooperative collision avoidance system the primary problem is not the detection of potentially threatening aircraft but discrimination against non-threatening aircraft and the selection of a proper evasive maneuver when required. For this system discrimination in the horizontal plane (figure 2-1) is accomplished by the following:

- resolving the relative velocity between aircraft into components normal to and parallel to the line of sight between them
- inhibiting warnings if the normal velocity component exceeds a small threshold value or if the time-to-close is greater than 40 seconds. Time-to-close is approximated by dividing the range between aircraft by the parallel velocity component (closing rate).

Discrimination in the vertical direction is based on altitude separation. In cases involving climb or descent, the altitude which the aircraft will reach a fixed time in the future is used instead of actual altitude. Relative altitude information is also employed to determine the proper vertical evasive maneuver (figure 2-2). For level flight, the proper direction is that which increases the existing separation. When climb or descent is involved, the proper maneuver is to level out. In this case, the burden to maneuver is placed on the climbing or descending craft.

The functions described are accomplished through the use of microwave interrogator-transponder techniques (figure 2-3). Relative plan position of a transponder-equipped aircraft is measured in the interrogator by beacon radar techniques. A flush-mounted airborne scanning antenna is employed for this purpose. Altitude and velocity data are communicated between aircraft on the same radio frequency link.

A complete collision avoidance system consists of an interrogator, transponder, and a collision computer (see figure 2-4). A pilot warning indicator would consist of the interrogator and transponder only, less the collision computer. The transponder only constitutes the minimum useable level of capability. Although all this equipment would normally be carried in one aircraft, the system operation will be explained based on the interrogator and collision computer being in one aircraft and the transponder in a second aircraft.

The operating sequence of the system is as follows. Interrogations are transmitted periodically by the scanning directional antenna. These interrogations are coded with the interrogator's altitude (or projected altitude). Each transponder transmits replies when the scanning beam strikes it, provided that the coded altitude information received from the interrogator is close to the transponder's altitude (or projected altitude). The transponder's reply message is coded with single-bit relative altitude information and the transponder's velocity vector.

Back at the interrogator, the transponder's range and bearing are measured based on these replies. The range is determined from round-trip propagation time. Bearing is determined from the scanning antenna's position at the time the replies are received. These data, together with received relative altitude, are sufficient for PWI purposes. For CAS, the velocity data contained in the reply message is also decoded. Relative velocity is derived from local and communicated velocity information and is resolved into components normal to and parallel to the line of sight. These components are used for threat evaluation and a maneuver is selected, when necessary, as described previously. The complete sequence of operations is accomplished in a period of 2 to 4 seconds.

The system also has the following special features not covered by this normal routine:

- Climb and descent: Since only one aircraft maneuvers in a case involving climb or descent, special communications are provided through the same link, to relay maneuver requirements from the interrogator to the transponder, to assure appropriate action. The reply message is coded to indicate the transponder is not in level flight. The next interrogation is coded with a "maneuver" signal if the time-to-close is less than 40 seconds. The "normal velocity" criterion is not employed in this case.
- Range blanking: In an airlane situation it is not unlikely that two intruders may be on a common bearing from a protected aircraft, but only the farther of the two presents a collision threat. To make the decoders available to evaluate this threat, decoding of data from the nearer craft is inhibited on each scan as soon as it is determined that his time-to-close exceeds 40 seconds.

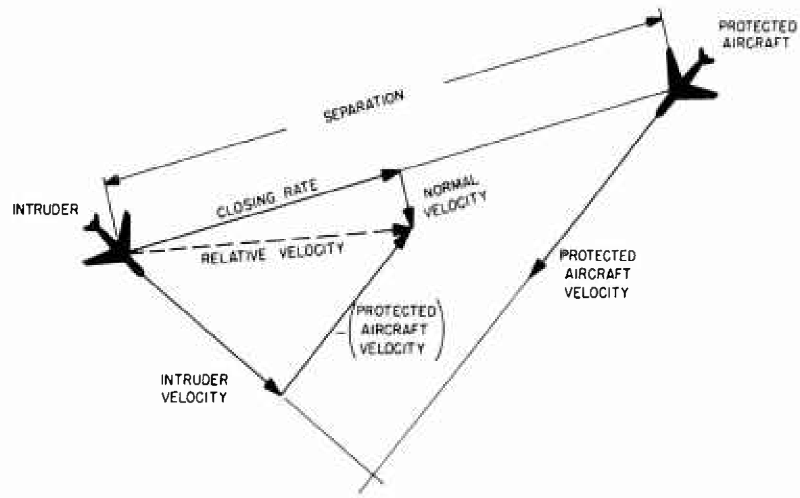

FIGURE 2-1. PLAN VIEW OF ENCOUNTER

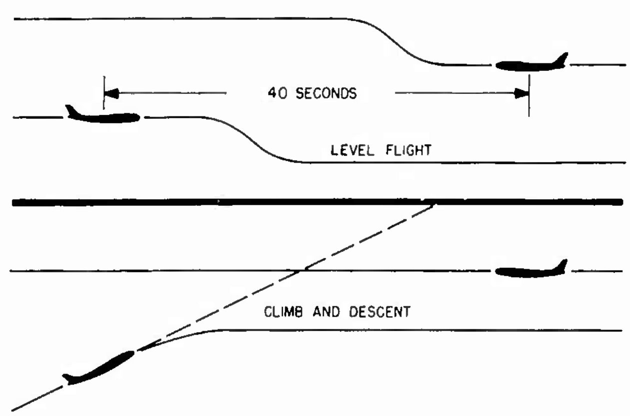

FIGURE 2-2. AVOIDANCE MANEUVERS

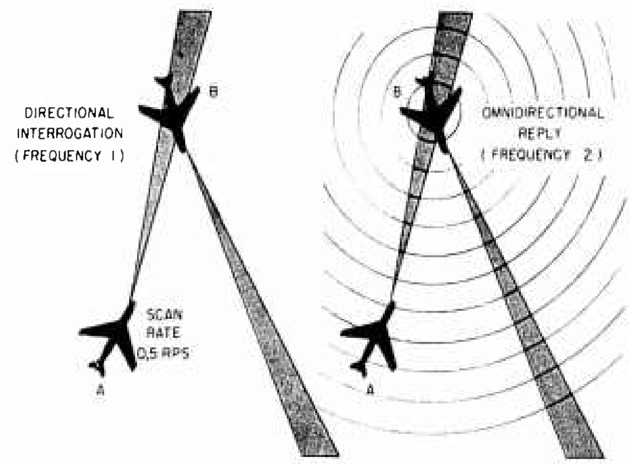

FIGURE 2-3. INTERROGATOR TRANSPONDER TECHNIQUE

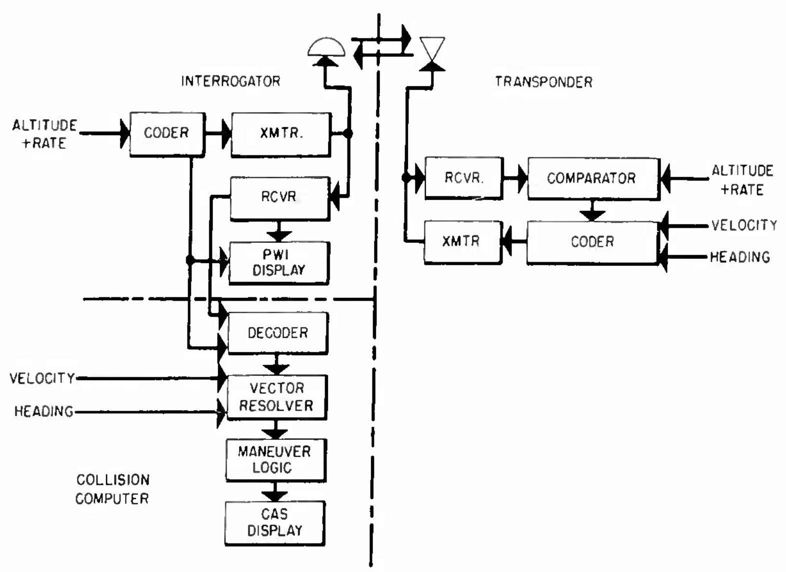

FIGURE 2-4. PWI/CAS BLOCK DIAGRAM

- Minimum range limit: For two aircraft on nearly parallel courses, where the closing rate is very low, a minimum range warning is provided regardless of time-to-close. This minimum range is set so that no warning will be given as two aircraft pass if the "normal velocity" were great enough to inhibit warnings previously.

The message structure employed in the PWI/CAS is designed to minimize the probability of interference, a potential hazard in a multi-aircraft environment. Pulse spacings are shown in microseconds at the bottom of figure 2-5. In the interrogation message, a coarse-fine altitude code is contained in the spacings between pulses 2 and 3 and 3 and 5 respectively. The spacing between pulses 1 and 2 is changed from a nominal of 8 microseconds if a wider than normal altitude guard band is required, for high rates of climb or descent. The spacing between pulses 4 and 5 is changed from a nominal of 4 microseconds to relay a maneuver signal to the transponder.

The reply message, shown in figure 2-6, is similar to the interrogation message. The spacing between pulses 2 and 3 is used to convey the relative altitude information or the fact that the transponder is not in level flight. Velocity information is conveyed as a scale factor, in the interval between pulses 1 and 3, and two velocity components coded in terms of absolute magnitude and polarity, as indicated.

In general, a number of measures is taken in the system to minimize the effects of interference. Besides normal frequency discrimination, the extraneous signal rejection of both receivers is enhanced by making them sensitive only to groups of pulses with certain spacings, while the altitude discrimination in the transponder helps measurably in limiting reply densities. In the interrogator, the following additional steps are taken:

- the scanning directional antenna provides bearing discrimination
- knowledge of the maximum round-trip propagation time allows blanking of the receiver except for a short interval after each interrogation
- knowledge that the range to a true intruder will change very little between interrogations allows rejection of non-coherent replies on a range-change basis
- the filtering provided by the collision criteria is of value in minimizing interference effects
- the principle of redundancy is employed, on both an interrogation-to-interrogation basis and a scan-to-scan basis, by requiring more than one sample of a given result before action is taken.

B. EFFECT OF SIDELOBES

The tests conducted in Task 2 of the PWI/CAS experimental program did not include the effects of sidelobes in the azimuth pattern of the directional antenna. As a result, these effects will be examined in some detail in this section.

Typical azimuth pattern data for a flush-mounted luneberg lens directional antenna are shown in figure 2-7. These data apply to one AN/APN-121 antenna employed in FAA antenna evaluation tests.[1] The antennas specified for an operational collision avoidance system would have sidelobes approximately 6 decibels lower than those of the antenna shown. This degree of improvement is considered feasible by technically qualified personnel. Also shown in figure 2-7 are values of communication range corresponding to selected power levels. The communication ranges were computed on the following basis:

- communication is between two low performance (propeller) class systems
- the transmitted power and receiver sensitivity of these systems is at the upper end of the specification tolerances
- weather attenuation is zero
- the encounter is head-on (maximum communication range).

The maximum communication range of 27.5 nautical miles compares to a required communication range (40-second warning with a 600-knot closing rate) of about 7 nautical miles. In this worst case, for the AN/APN-121 antenna, sidelobes would begin to appear at a range of 5.5 nautical miles. The significance of the 2.75 and 1.4 nautical mile range points is discussed in the examination of the effect of these sidelobes on the system, which follows.

Assuming the whole sidelobe curve is reduced by 6 decibels with respect to the main lobe for an operational antenna (lacking better data), the selected range values, other than that corresponding to the peak of the beam, would be reduced by a factor of two. These values are also shown in figure 2-7, in parenthesis. It is the latter set of figures which will be used where ranges are discussed in the material which follows.

In examining the effect of sidelobes, the following three major possibilities must be considered.

- a non-threatening intruder within sidelobe range may cause an excessive false alarm rate
- a non-threatening intruder within sidelobe range may blank replies from a threatening intruder

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[1] Flight Test Evaluation of Flush Mounted Luneberg Lens Antenna for PWI/CAS System, furnished by Sperry Gyroscope to Federal Aviation Agency under Contract No. FAA/BRD-190.

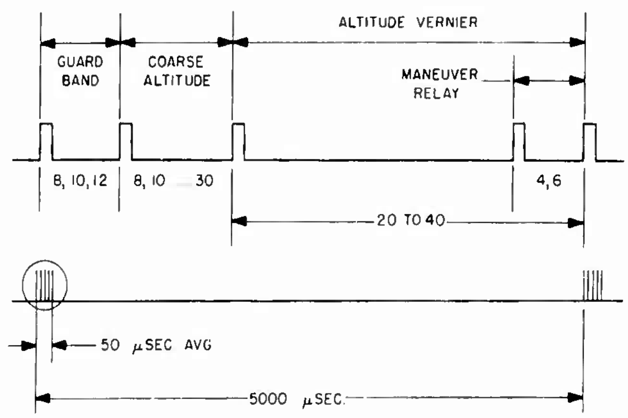

FIGURE 2-5. INTERROGATION MESSAGE

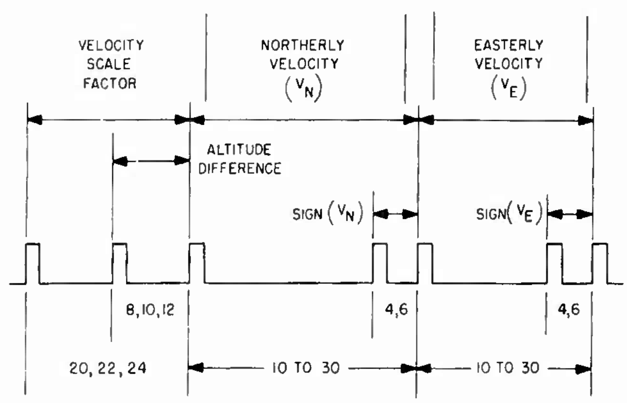

FIGURE 2-6. TRANSPONDER REPLY MESSAGE

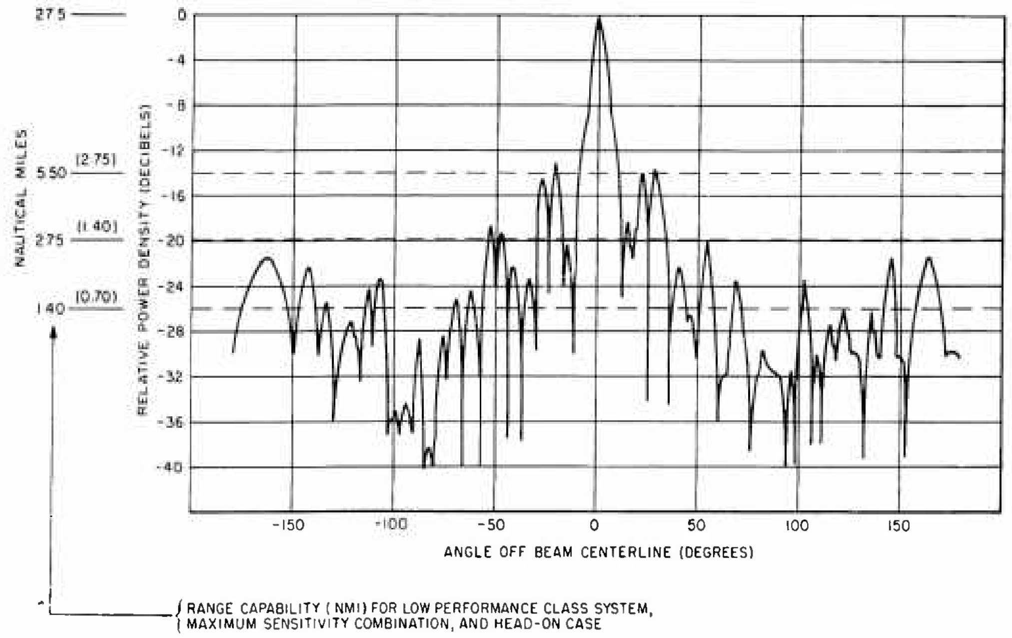

FIGURE 2-7. AZIMUTH PATTERN OF AN/APN-121 DIRECTIONAL ANTENNA, UNIT 1

- a threatening intruder within sidelobe range may be missed due to the effects of sidelobes.

These possibilities will be examined separately.

To examine the effects of sidelobes, it is useful to introduce the concept of a collision warning zone in bearing. For any encounter between two aircraft in unaccelerated flight, a fixed relative velocity vector (velocity of the intruder with respect to an observer in the protected craft) will exist, regardless of the relative position of the two craft. Thus a unique intruder bearing will exist where the "normal velocity" is zero and the closing rate is positive, (the reciprocal of the direction of the relative velocity vector). If the intruder appears at this "collision bearing", and the derived ratio of range to range rate is less than 40 seconds, a collision warning will be produced. Moreover, a warning will result if and only if the intruder appears within a "collision zone" centered at this collision bearing. This collision zone will extend on each side of the collision bearing by an angle equal to $\sin^{-1}(V_T/V)$ where V_T is the "normal velocity" threshold and V is the magnitude of the relative velocity. It is emphasized that the width of this zone is determined only by velocity data, independent of the width of the antenna beam or lobe impinging on the intruder.

1. False Alarm Rate Due to a Non-Threatening Intruder Within Sidelobe Range

Collision computations will be performed by the system on a series of sidelobe returns as well as on the main lobe series. These computations, however, will be based on erroneous bearing data because the antenna will not be trained on the intruder at the time. If the erroneous bearing falls within the previously discussed collision zone, and the derived time-to-close is less than 40 seconds, a warning will result. One such computation will be made for each lobe of the antenna which impinges on the intruder during the scan, provided that the lobe width exceeds about three degrees, neglecting minor discontinuities of the order of one degree.

As an example, an estimate of the false alarm rate due to sidelobes follows based on the following assumptions:

- a non-threatening intruder is 0.7 nautical mile from the protected aircraft (judging from figure 2-7 by eye, this range will result in about a maximum number of sidelobes intercepted)
- the relative velocity between aircraft is 150 knots (±12-degree collision zone).

Under these conditions, inspection of figure 2-7 indicates that there are about 12 lobes which will cause collision computations. Assuming any position of the collision bearing with respect to the lobe pattern is equally likely, except within ±12 degrees of the center of the main lobe, then a warning will result if that bearing falls within any of 12 zones each 24 degrees wide (neglecting overlap) or 288 degrees out of a possible 336 degrees. The resulting false alarm probability is about 85 percent.

However, this false alarm effect of sidelobes will have essentially the same end result as the minimum range limit intentionally incorporated into the system; it will tend to provide a warning based on proximity regardless of the relative velocity. Unfortunately, the effect is not sufficiently predictable to be relied upon for this function, nor is the range at which it is effective controllable. Inspection of figure 2-7 indicates that the false alarm rate might start rising significantly for an intruder as far away as one nautical mile, depending on the power and sensitivity levels of the particular equipment involved.

2. Blanking of a Threatening Intruder by a Non-Threatening Intruder Within Sidelobe Range

The interrogator decoder is capable of decoding only one set of reply data per interrogation, normally the first reply to be received. Thus it would appear that a benign intruder well within sidelobe range, as in the above example, might tie up the decoder throughout most of the scan, preventing the proper detection of a threatening intruder. However, the range blanking feature of the system will minimize this loading effect in that, if the first reply from a given lobe results in a computed time-to-close in excess of 40 seconds, the subsequent replies from the benign intruder will be blanked while in that lobe, permitting detection of the more distant intruder. In the case taken previously, this range blanking will operate except in a ±25-degree zone centered at the collision bearing, making the decoders available 310 degrees out of the 360-degree scan, or about 85 percent of the time.

3. Missing a Threatening Intruder Within Sidelobe Range

As a threatening intruder comes within sidelobe range, collision computations will be performed on the sidelobe returns as well as on the main lobe return. However, as long as the main lobe return continues to correspond to a collision situation, a warning will be displayed. This will continue to be the case until, at about 1.4 nautical miles on figure 2-7, the proximity of a sidelobe prevents the computer from resolving it from the main lobe. This inability to resolve the lobe results in an apparent asymmetry of the main beam, causing a shift in the bearing data used in the collision computation. It is at this point that the reliability of the collision prediction will be degraded.

For encounters involving high closing rates, a warning will have been displayed and evasive action taken long before the range closes to 1.4 nautical miles and thus the above performance degradation will not create operational problems. However, for closing rates below about 125 knots, the range at which degradation may occur will be reached before the time-to-close falls below 40 seconds. Thus, a dangerous situation could result in encounters involving closing rates of 125 knots or less with the system as presently configured. This deficiency could be completely eliminated, however, by increasing the proximity warning range to about one nautical mile. The one nautical mile figure will be adequate because, for non-head-on encounters associated with lower closing rates, the gain product of properly installed antennas will be at least 3db lower than in the head-on case.

The above discussion was restricted to an encounter between two propeller class systems. However, the results can be extended to other classes by proportionately scaling up all velocity and range figures. Thus appropriate modified proximity warning ranges for subsonic and supersonic jet classes would be about 3 nautical miles and 6 nautical miles, respectively.

C. MODIFICATIONS

The breadboard equipment is constructed in accordance with equipment and component characteristics presented in Task 1[2] with deviations applicable to the experimental models of the system. In addition the following changes were found to be desirable as a result of tests and were incorporated into the system:

- the level at which normal velocity samples are limited prior to integration was increased from 60 knots to 133 knots to prevent computation errors due to limiting at wide beamwidths
- the normal velocity threshold was increased from 30 knots to 44 knots prior to the interference tests. Additional interference analysis, summarized in Section IV of this report, indicated that this modification would prevent miss rates from exceeding anticipated levels at wide beamwidths.

D. SUGGESTED MODIFICATIONS

During the course of the experimental program, some variations on the techniques employed were suggested. The nature of these suggested modifications is outlined here. It is emphasized, however, that these are not necessarily recommended modifications. A conclusive examination of their desirability or feasibility was beyond the scope of the experimental program.

1. Performance Improvement

No evaluation has been made of the performance of the compatible PWI/CAS under conditions of accelerated flight. However, it seems logical to expect that the high degree of discrimination provided in preventing warnings due to non-threatening intruders in unaccelerated flight would also cause a high miss probability for a threatening intruder in accelerated flight. If this occurs the following possible improvements suggest themselves:

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[2] Design Study Report - Experimental Evaluation of Compatible PWI/CAS Interrogator-Transponder Techniques - Volume II, furnished by Sperry Gyroscope Company to Federal Aviation Agency under Contract No. FAA/ARDS-444.

- manually increase the threshold on "normal velocity", or eliminate the "normal velocity" criterion altogether, when in a turn, or
- modify heading data when in a turn so that the aircraft's apparent straight-line path intercepts the actual curved path a selected time in the future (such as 27 seconds). For a given turn rate, this heading bias would be approximately one half the angle by which the heading would change in the selected time interval due to the turn rate.

As indicated previously, vertical evasive maneuvers are implemented in the breadboard CAS. This choice was made primarily in the interest of achieving equipment simplicity. It is possible that operational studies will indicate a strong preference for horizontal maneuvers, to minimize the possibility of disrupting the flow of other traffic. If this occurs sufficient data is available in the CAS to implement horizontal maneuvers without a major increase in complexity. For horizontal turns, E. S. Calvert[3] has proposed a maneuver rule which will provide compatible maneuvers in an encounter whether the intruder applies the same rule or applies existing rules of the road. The rule is: turn right if the intruder is in the forward semicircle, turn left if in the aft semicircle. This rule could be simply implemented in the CAS based on the polarity of the cosine of relative bearing. Other provisions which would be required to assure safety whether one or both aircraft maneuvered include:

- basing urgency indications on range rather than time-to-go, to assure safe ultimate separation in cases where the "standard turn" initially decreases rather than increases miss distance
- some provisions to avoid conflicting maneuvers near the points of ambiguity in the maneuver rule. These provisions might consist of communicating intent, or of reverting to vertical maneuvers near the singular points.

Altitude discrimination in the CAS is not completely compatible with present ATC rules, in that a threat could be indicated when an IFR and a VFR aircraft pass in compliance with the rules. Nominal ATC altitude separation in this case is 500 feet at altitudes below 29,000 feet. Compatibility might be achieved by employing a non-linear altitude scale in the CAS, so that the width of the guard band varied in approximately the same manner that altitude sensor errors vary with altitude. One method of implementation would be to make the vertical scale proportional to static pressure rather than altitude. With this type of scaling, a guard band of ±0.1 inch of mercury would provide compliance with ATC rules up to an altitude of about 45,000 feet, covering the operating range of most subsonic aircraft. The width of this guard band in feet of altitude is as follows:

- - - - - -

[3] E. S. Calvert, "Maneuvers to Ensure the Avoidance of Collision", <u>Journal of the Institute of Navigation</u>, Vol. XIII, April 1960.

Altitude, Feet	Guard Band, Feet
0	±90
10,000	±125
20,000	±175
30,000	±250
40,000	±375
50,000	±610
60,000	±980

Implementation would require a special barometric sensor having an output proportional to pressure and an accuracy, including uncorrected static source errors, compatible with the widths of the guard band shown. The feasibility of obtaining a practical sensor meeting these requirements has not been investigated.

2. Performance Relaxation

The preceding section discussed modifications to improve CAS performance, if required. However, it has not been firmly established that even the predicted performance is required. Advantages in the form of reduced size, cost, and complexity will result if operational considerations indicate that performance requirements can be relaxed. Some of the more promising areas for simplification through performance relaxation are summarized here.

The lowest proposed velocity scale factor might be eliminated, leaving only two scales: subsonic (zero to 800 knots) and supersonic (zero to 1700 knots). This change would effect performance of the propeller class systems only, by raising their normal velocity threshold (degrading discrimination) to that of the subsonic jet class. Permissible errors at this higher threshold level would allow indicated airspeed to be employed in the low performance class systems in place of true airspeed. Some simplification in the data processing circuits, particularly in the transponder, would also result.

If the recent trend to segregate IFR and VFR traffic in plan area continues, simplification of the proposed altitude code structure might also become operationally feasible. The limiting case would be represented by a continuous, non-segmented pulse position code covering all altitudes. Using such a code, together with the pressure coding previously described, a summary examination of system and input data errors indicates that a guard bandwidth of ±0.3 inches of mercury might be feasible. The corresponding widths of the guard bands in feet of altitude are as follows:

Altitude, Feet	Guard Band, Feet	Altitude, Feet	Guard Band, Feet
0	±270	30,000	±750
10,000	±375	40,000	±1125
20,000	±525	50,000	±1830

These guard bands will provide discrimination between one IFR flight level and another at the most heavily travelled flight levels.

The coarse altitude code employed in the proposed CAS is instrumental in rejecting interference as well as in providing accurate relative altitude data. To retain the benefits of this interference rejection property, it would probably be desirable to retain some coarse altitude discrimination code even though the intelligence contained therein is redundant with that of the continuous data. A four segment coarse code is suggested, subdivided as follows:

Segment	Pressure Range, Inches of Hg	Altitude Range
1	32 to 24	-2000 to 6000
2	24 to 16	6000 to 16000
3	16 to 8	16000 to 32000
4	8 to 0	above 32000

This subdivision would result in a fairly equal subdivision of traffic between segments, each segment having about the same percent of all traffic as the more crowded coarse altitude bands (-1000 to 4000 feet and 4000 to 9000 feet) in the present configuration. This coarse code would be employed only to inhibit the triggering of the transponder continuous altitude decoder unless the interrogator was in or near the same segment. The interrogator might transmit a dual segment code when near the borderline between segments to permit detection by a transponder in the adjacent segment without requiring dual decoding equipment in the transponder.

Adoption of this simplified altitude code would reduce the size of the transponder about 15 percent.

Elimination of the automatic provisions to cope with encounters involving climb and descent, along with the previous simplifications, would reduce the size and complexity of the transponder about 30 percent. As an alternate to the automatic climb and descent provisions, the following measures might be adopted:

- eliminate the "maneuver relay" from the interrogator to the transponder, assuming that the PWI or CAS-equipped aircraft will have a sufficiently higher performance capability to avoid the aircraft equipped with transponder only
- employ a manual altitude search prior to and/or during climb or descent. The search knob could be calibrated in terms of rate of climb, if desired, to simplify operation
- eliminate provisions to vary the width of the transponder's altitude guard band. The interrogator might transmit multiple vernier altitude groups instead, spaced to produce a similar effect.

Alternate levels of transponders might also be considered. For instance, for some portions of the airspace, a transponder capable of supplying cooperating signals for PWI operation only might have utility. The resulting elimination of the need for velocity information would reduce transponder size and complexity by about another 30 percent of the present configuration, and would also eliminate the need for airspeed and heading input data.

SECTION III

TEST METHODS

A. GENERAL

The breadboard equipment consists of the coding, decoding, and computing components of the PWI/CAS. The equipment corresponds to a system that would be used in a propeller-driven aircraft (e.g. DC-7). Its maximum speed capability is 300 knots, maximum altitude 31,500 feet, and maximum rate of climb or descent 3000 feet per minute. A block diagram of the simulation is shown in figure 3-1. One set of transponder equipment and one set of interrogator equipment is used. It is assumed that the interrogator is in aircraft A and the transponder is in aircraft B. A set of sensor inputs representing the magnetic heading, airspeed, altitude and altitude rate of aircraft A is provided for the interrogator. A separate set of sensor inputs is provided for the transponder. The two-way microwave link between aircraft is simulated by two coaxial cables, one connecting the interrogator encoder to the transponder decoder (interrogation message) and the other connecting the transponder encoder to the interrogator decoder (reply message). The scanning of the interrogator's directional antenna past a transponder is simulated by allowing interrogations to pass only during intervals representing the beamwidth. These intervals recur at the antenna scan rate (30 rpm). The length of the interval, that is the beamwidth, can be varied to simulate the change in beamwidth with range. The range between aircraft A and B is simulated by delaying the transponder's reply by approximately 12 microseconds per mile. (The seven-pulse reply is delayed by delaying the reply trigger generated by the transponder decoder).

The operation of the PWI/CAS in a noisy environment produced by large numbers of other PWI/CAS systems as well as other microwave equipment, is simulated by introducing noise into the system as shown in the block diagram. Noise pulses which are identical to message pulses are introduced in the transponder decoder along with the interrogation message, and into the interrogator decoder along with the reply message. Overheard replies from other transponders are simulated by introducing randomly generated reply triggers into the transponder encoder. The characteristics of the noise will be described later. The simulation is a static one; that is, the situations set up represent particular instants during an encounter between aircraft.

B. SENSOR SIMULATORS

The sensor inputs to the interrogator and transponder were simulated by manually controlled synchros, potentiometers and switches. A list of the sensor inputs, the type of signal required, and the signal source is shown in table 3-1.

TABLE 3-1

SENSOR SIMULATORS

	Input	Signal	Source
Interrogator	Relative Bearing	3-wire, 400-cps synchro	Kearfott RS-911-4A
	Heading	3-wire, 400-cps synchro	(Kearfott RS-911-4A)
	Airspeed	0 to 30v dc	1K ten-turn pot
	Coarse Altitude	one of six wires grounded	rotary switch
	Vernier Altitude	0 to 30v dc	5K ten-turn pot
	Altitude Rate	12v dc on one of three wires	switches
	Altitude Rate Polarity	0 or 12v dc on one wire	switch
Transponder	Heading	3-wire, 400-cps synchro	Kearfott RS-911-4A
	Airspeed	0 to 30v dc	1K ten-turn pot
	Altitude	3-wire 400-cps synchro	Kearfott RS-911-4A
	Altitude Rate	2-wire, 400-cps, 0.25v/1000 fpm	100-ohm pot

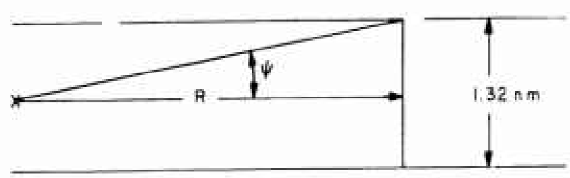
During the false alarm rate tests, the transponder heading and altitude inputs were varied continuously. The altitude input was varied by driving the altitude simulator pot at a rate of 500 ft/min. over a range of 5000 feet. The heading was varied by driving the heading synchro at one revolution per hour by a timing motor. For all other tests both inputs were set manually.

C. RANGE DELAY SIMULATOR

The range delay between interrogation and reply was simulated by delaying the reply triggers produced by the transponder decoder with a standard laboratory fixture. A Rutherford Pulse Generator Model B-7 was used. The delay was variable from 0 to 150 microseconds.

D. ANTENNA SCAN SIMULATOR

The function of the antenna scan simulator is to permit communications between the interrogator and transponder only during the time interval representing the angle subtended by the directional beam as it passes a target at a particular range. The length of the interval is manually controllable to simulate the effect of range on this angle. The coverage pattern of the directional beam in the horizontal plane is assumed to be rectangular with a width of 1.32 nautical miles for the head-on case, as indicated in the following sketch.



For a scan speed of 30 rpm, the relation between the time interval and the range is

$$t = \frac{1}{90} \tan^{-1} \left[\frac{0.66}{R} \right]$$

where

t = the time interval, in seconds

R = range, in nautical miles

$\tan^{-1}(0.66/R)$ is measured in degrees

A block diagram of the antenna scan simulator is shown in figure 3-2.

The cam-operated antenna switch is used to trigger the one-shot multivibrators mv-1 and mv-2. The switch cam and relative bearing synchro are on a common shaft and are driven by a 30-rpm motor. Multivibrator mv-3 is triggered on by the trailing edge of the output voltage from mv-2, while mv-3 produces a gating output to the AND circuit which allows the interrogation pulses to pass through. The period of mv-3 represents the time an intruding aircraft is within the antenna beam at a given range. The timing diagram in figure 3-2 shows the dwell time of mv-2 and mv-3. The center of mv-3 gate T_o represents the center of the antenna beam. The beam width is controlled by varying the period of mv-3 while the beam center T_o is maintained constant by simultaneously controlling the period of mv-2 such that

$$T_2 + \frac{T_3}{2} = T_o = \text{constant}$$

The function of the multivibrator mv-1 is to prevent multiple triggering of mv-2 caused by contact bounce of the microswitch. The period of mv-1 is made longer than the dwell time of the microswitch and since the input collector of mv-1 and mv-2 are in parallel (collector triggering is used) mv-1 will effectively eliminate triggering by switch bounce.

E. BACKGROUND NOISE GENERATOR

A block diagram of the background noise generator is shown in figure 3-3. In addition to random single pulses, the background noise for the interrogation and reply message includes random pairs (doublets) and triplets of pulses. The interpulse spacing in these groups corresponds to the spacings used in the two messages. In the interrogation background noise, two triplets are generated simultaneously representing the two coarse altitude codes the transponder will accept. The spacings of the last two pulses of the triplets are set to correspond to the two altitude bands to which the transponder is sensitive under the test conditions.

The density of singlets, doublets, and triplets is controlled by means of thresholds in the pulse generators. The values used were generated in the Task 1 interference analysis.[4] The relative density of singlets, doublets, and triplets is determined by the message structure as follows:

The total reply background noise density in pulses per second is

$$D_R = (7x + \text{man-made noise}) \text{ pulses/second}$$

where x is the total number of replies per second from all aircraft at all altitudes. Man-made noise consists of stray pulses from other types of equipment such as radars, etc. The doublet density is equal to 4x pulses per second (2 doublets per reply times two pulses per doublet). The two types of doublets generated represent velocity polarities, and since both are equally likely the doublet density is divided equally between them. Similarly, the triplet density is equal to 3x pulses per second and is divided equally between the two types of triplets generated which differ in the relative altitude polarity codes. Since the doublet and triplet total is equal to 7x, the singlet density represents the man-made noise density.

The total interrogation background noise density in pulses per second is

$$D_I = (5Y + \text{man-made noise}) \text{ pulses/second}$$

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[4] Design Study Report - Experimental Evaluation of Compatible PWI/CAS Interrogator-Transponder Techniques - Volume I, furnished by Sperry Gyroscope Company to Federal Aviation Agency under Contract No. FAA/ARDS-444.

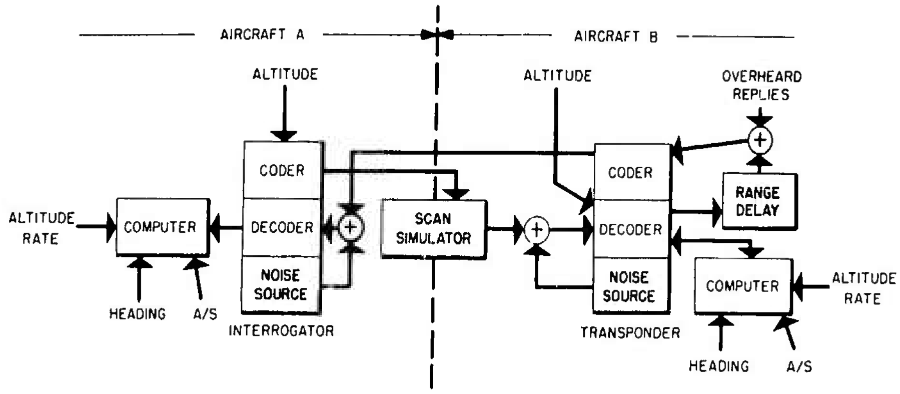


FIGURE 3-1. TEST METHOD

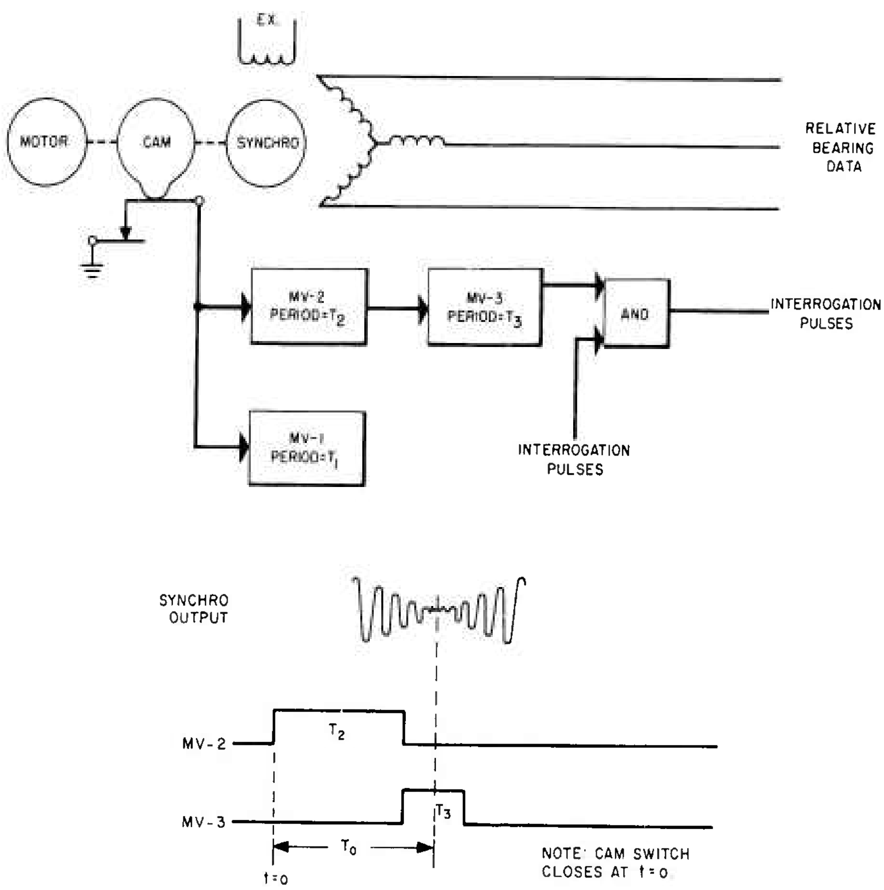


FIGURE 3-2. SCAN SIMULATOR BLOCK DIAGRAM

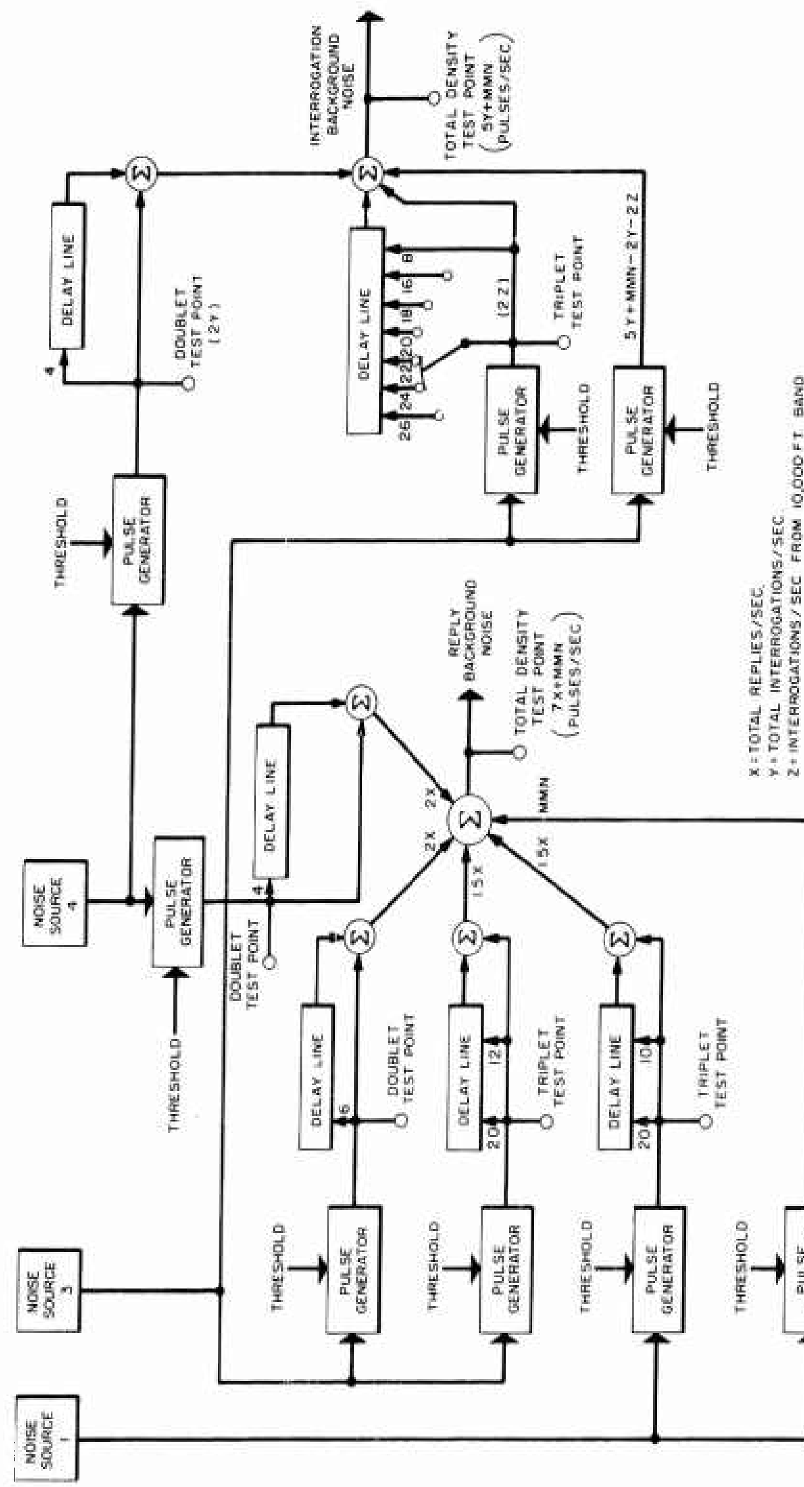


FIGURE 3-3. BACKGROUND NOISE GENERATOR

where Y is the total number of interrogations per second from all aircraft at all altitudes. The doublet density is equal to 2Y. The triplet density is equal to 2Z pulses per second, where Z is the number of interrogations per second from the two 5,000 foot altitude bands from which the transponder will accept coarse altitude codes. The factor of two results from the fact that the two types of triplets generated simultaneously contain a total of four pulses (two overlap). However, due to the decoding technique used in the transponder, only half of these triplets can cause interference. The singlet density is adjusted to make up the total density required.

As shown in figure 3-3, separate noise sources are used so that there will be no correlation between pulse groups. Where a common noise source is used, the succeeding pulse generators are made to trigger on opposite polarity noise thresholds to prevent correlation. Constant amplitude one-microsecond pulses are generated.

F. SECOND TRANSPONDER SIMULATOR

The range blanking test using the second transponder simulator was not performed owing to the failure in adequately simulating the second transponder. The failure in making the simulation resulted from the poor recovery time of the magneto-striction delay lines used. The range blanking circuit in the interrogator decoder is operative however and the range blanking test can be performed if the magneto-striction delay lines are replaced by the electromagnetic type.

The purpose of this circuit is to simulate a second transponder in the interrogator beam at the same relative bearing as the first but at a greater range. The second transponder will represent a collision threat, while the first and closer transponder will not. This circuit will be used to test the ability of the interrogator to decode two overlapping replies when the time-to-close corresponding to the first reply received is greater than 40 seconds.

The second transponder simulator consists of two magneto-striction delay lines (see figure 3-4). The overlapping reply is generated by delaying the first transponder reply by an interval representing the range difference between transponders and then adding the two together. This results in two identical overlapping seven-pulse replies. The range delay line is not continuously variable but has taps to simulate several range separations. The delayed reply message is further delayed by four microseconds and added in forming a third overlapping reply. The interrogator and transponder sensor inputs are set so that the two delayed replies interfere with each other in such a way that the interrogator interprets the combination as a collision threat, while the undelayed reply corresponds to a time-to-go greater than 40 seconds. Figure 3-4 shows one encounter that can be simulated to accomplish this and the overlapping replies that result. Both interrogator and transponder are heading due west along the same track. The spacing between pulses 6 and 7 in the undelayed reply will therefore be 6 microseconds. The interaction between the two delayed replies will be interpreted by the interrogator as a reply from an aircraft heading due east on a collision course.

G. OVERHEARD REPLY GENERATOR

A block diagram of the overheard reply generator is shown in figure 3-5. The purpose of this circuit is to generate groups of replies with a nominal spacing of 5 milliseconds between triggers. The number of triggers in a group and the spacing between groups will vary at random. The triggers generated will be mixed in with those generated by the transponder decoder, to produce interfering false replies.

An oscillator and pulse generator produces one-microsecond pulses at a repetition rate of 200 pps. The pulses are gated on and off by the output of a bistable multivibrator. A single noise source and two pulse generators produce pulses to set and reset the multivibrator at random. The output of one pulse generator inhibits the output of the other to prevent the multivibrator from being turned off as soon as it is turned on. The average on-time, off-time, and PRF of the multivibrator are controlled by the thresholds on the two pulse generators, permitting the average density of reply triggers and gates to be controlled. A counter is used to measure the densities.

H. TEST INSTRUMENTATION

The primary item of test instrumentation to be used is a pulse counter. The counter will be used to set the background noise densities and the overheard reply trigger density. A Hewlett-Packard Model 524B Counter has been selected. A digital voltmeter is used to monitor the power supply. Standard laboratory equipment employed in conjunction with the breadboard equipment is listed in table 3-2.

TABLE 3-2

TEST EQUIPMENT AND POWER SUPPLIES EMPLOYED AS PART OF PWI/CAS SIMULATION

Description	Make	Model No.	Serial No.	Purpose
D-C Supply	Power Designs	1515	14A200 (10970)	+12v Transponder
D-C Supply	Kepco Labs	SC36-05	AF204410	+30v Transponder
D-C Supply	Electronic Research	110	14A115 (2148)	-30v Transponder -20v
Pulse Gen.	Rutherford	87	AF308507	Range Delay
D-C Supply	Electronic Research	110	14R115 (2265)	-30v Interrogator -12v Interrogator
D-C Supply	Kepco Labs	SC36-05	(C-14757)	+30v Interrogator
D-C Supply	Consolidated Corp. Electro Dynamics	3-132	14H136 (22436)	+12v Interrogator
Power Supply	Kepco Labs	815	14A95 (A-1730)	-60, 6.3v ac Interrogator and Transponder
Power Supply	Sorensen	500BB	14BP25	+160v, 6.3v ac Interrogator, Transponder and Antenna Simulator

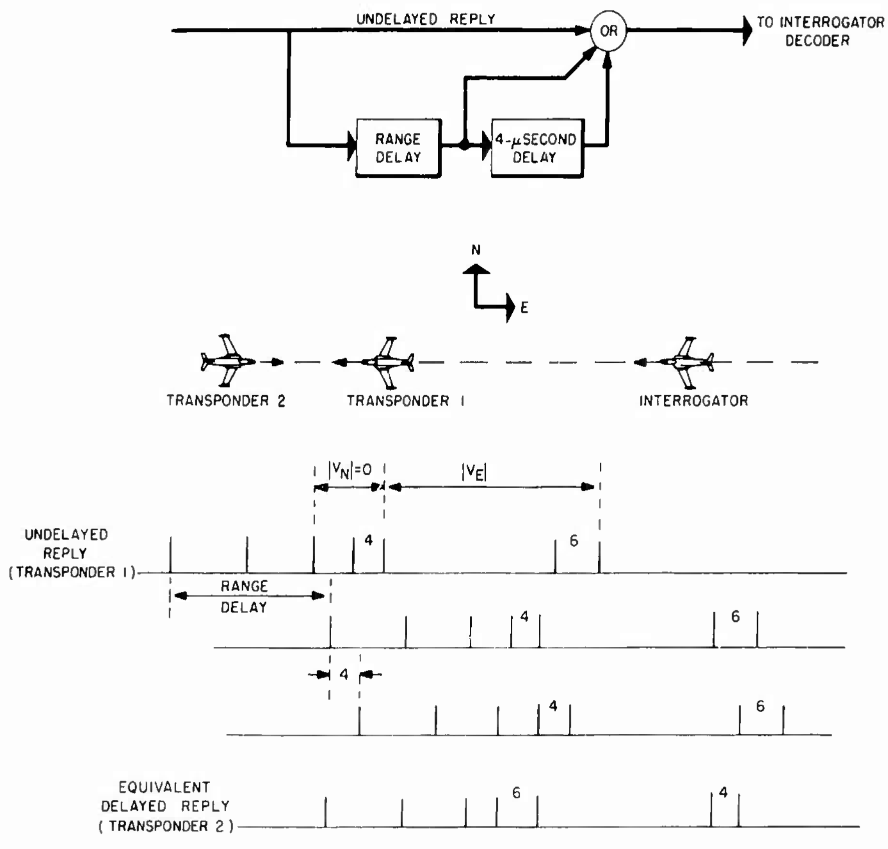


FIGURE 3-4. SECOND TRANSPONDER SIMULATOR

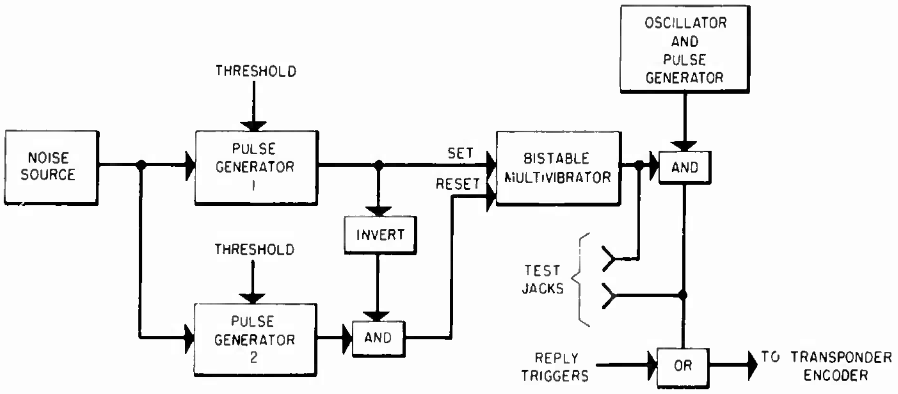


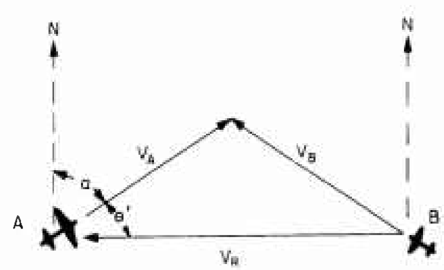
FIGURE 3-5. OVERHEARD REPLY GENERATOR

SECTION IV

TEST PROCEDURES AND RESULTS

A. WARNING TIME TEST

A warning time test was performed to verify that collisions are predicted under proper nominal conditions and to determine the accuracy of time-to-go computations. The warning time test was made with the interrogator altitude slightly greater than the transponder altitude, and with zero rates of climb, requiring an "up" maneuver command. Collision courses were employed. The horizontal plane geometry is shown in the following sketch.



The various flight conditions used in this test are given in the test data sheet. The test is performed by putting in a set of flight conditions (setting up a collision situation) and then increasing the delay of the range delay generator until the immediate threat indicator goes out. At this time the interval between the fifth interrogation pulse and the third reply pulse is measured and recorded in the immediate column of the data sheet. The range delay is then further increased until the threat indicator goes out. The time interval between the fifth interrogation pulse and the third reply pulse is again measured and recorded in the threat column of the data sheet. The same procedure is repeated for each collision situation covered.

The expected time interval is found by use of equations 4-1 and 4-2 given on the data sheet.

1. Discussion of Test Results

The error in measured range delay for the threat and immediate threat computation tests is plotted as points in figure 4-1. The distribution of errors above about 50 μseconds appears to be random but below 50 μseconds nearly all of the plotted points are below the ideal locus, indicating a systematic computational error at close range.

The average error, RMS error and standard deviation calculated from the data are as follows:

Threat Test

Average error. . . . . -0.114μseconds = 0.14 percent maximum range
RMS error. . . . . 1.385μseconds = 1.69 percent maximum range
Standard deviation. . . . . . . 1.38 μseconds = 1.68 percent maximum range
Maximum threat range = 6.67 nautical miles

Immediate Threat Test

Average error. . . . . -0.465 μseconds = 0.85 percent maximum range
RMS error. . . . . 1.01 μseconds = 1.85 percent maximum range
Standard deviation. . . . . . . 0.89 μseconds = 1.62 percent maximum range
Maximum threat range = 4.45 nautical miles

The standard deviations computed above are reflected as standard deviations of error in the collision warning time in figure 4-2. The error is plotted as a function of the maximum range at which the warning is displayed. The dashed vertical line is the range at which a warning indication is given due to proximity only.

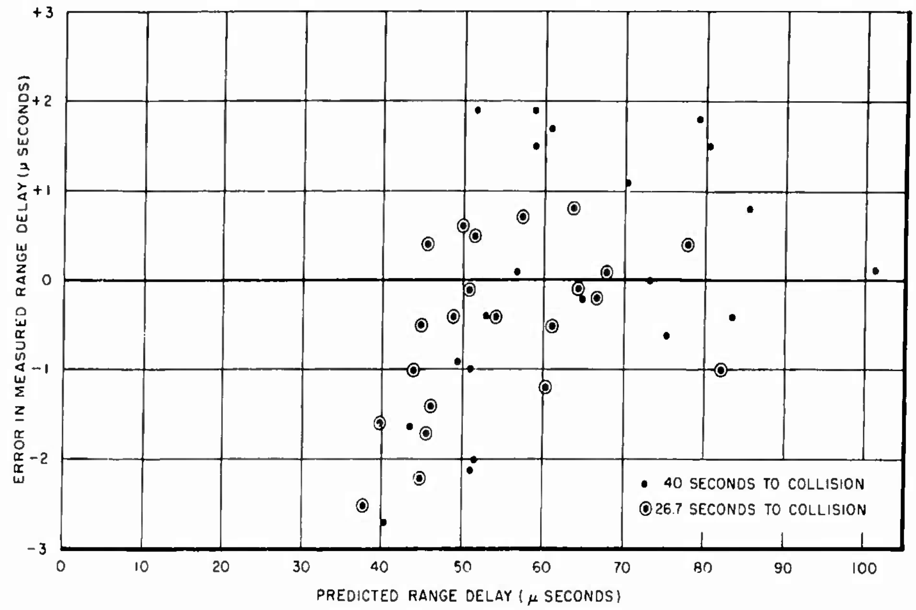


FIGURE 4-1. RESULTS OF COLLISION COMPUTATION TEST (TIME-TO-GO)

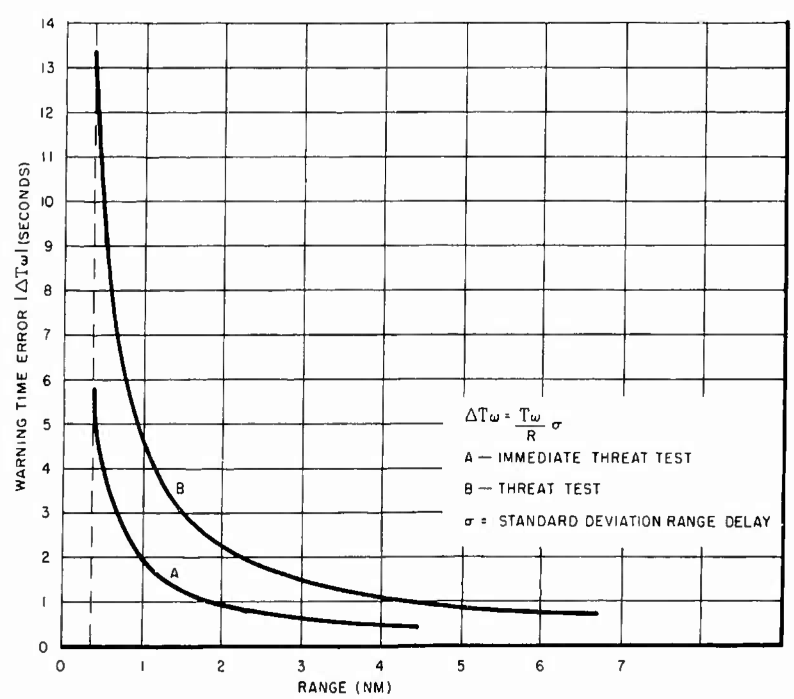


FIGURE 4-2. STANDARD DEVIATION OF RANGE DELAY MEASUREMENTS REFLECTED AS WARNING TIME ERROR

2. <u>Collision Computation and Time-To-Close Measurements Data Sheet</u>

Test Equipment

Description	Serial No.	Date Calibrated	Purpose
Digital Voltmeter Hewlett-Packard Model 405AR	203405 (USAF)	4/18/63	Voltage Monitoring
Electronic Counter Hewlett-Packard Model 524B	3975	4/10/63	Used for Range Delay Measurements
Time Interval Unit Hewlett-Packard Model 526B	12B80-C (L-302)	2/18/63	Used for Range Delay Measurements

Fixed Test Conditions

Zero Noise Densities	
Zero Altitude Rates	
Interrogator Altitude	16,300 feet
Transponder Altitude	16,000 feet
Beamwidth	21 degrees

Note: The measurement made in this test is the time between the fifth interrogation pulse and the third reply pulse. The predicted times are found from the following equations.

$$T_{threat} = \left[V_R \times \frac{1}{90}\ hr \times 12.36\ \frac{\mu sec}{nm}\right] + \left[\text{minimum range} \times 12.36\ \frac{\mu sec}{nm}\right] + 28\ \mu sec \quad (4-1)$$

$$T_{immediate} = \left[\frac{2}{3} V_R \times \frac{1}{90}\ hr \times 12.36\ \frac{\mu sec}{nm}\right] + \left[\text{minimum range} \times 12.36\ \frac{\mu sec}{nm}\right] + 28\ \mu sec \quad (4-2)$$

Minimum Range = 1/3 nautical mile
V_R is the line of sight component of relative velocity

Note: The measurement made in this test is the time between the fifth interrogation pulse and the third reply pulse. The predicted times are found from the following equations.

$$T_{threat} = \left[V_R \times \frac{1}{90}\ \text{hr} \times 12.36\ \frac{\mu sec}{nm}\right] + \left[\text{minimum range} \times 12.36\ \frac{\mu sec}{nm}\right] + 28\ \mu sec \qquad (4\text{-}1)$$

$$T_{immediate} = \left[\frac{2}{3} V_R \times \frac{1}{90}\ \text{hr} \times 12.36\ \frac{\mu sec}{nm}\right] + \left[\text{minimum range} \times 12.36\ \frac{\mu sec}{nm}\right] + 28\ \mu sec \qquad (4\text{-}2)$$

Minimum Range = 1/3 nautical mile

V_R is the line of sight component of relative velocity

Results

Variable Test Conditions					Predicted		Measured	
Airspeed (knots)		Heading (degrees)		Bearing	Delay (μ sec.)		Delay (μ sec.)	
Interrog.	Trans.	Interrog.	Trans.	(degrees)	Threat	Immediate	Threat	Immediate
125	250	45	225	0	83.5	66.3	83.1	66.1
125	250	45	270	30.4	80.4	64.2	81.9	64.1
125	250	45	315	63.4	70.1	57.4	71.2	58.1
125	250	45	0	106.3	56.7	48.5	56.8	48.1
125	250	45	15	126.2	52.7	45.8	52.3	44.4
125	250	45	30	150.9	50.6	44.5	49.6	44.0
125	250	45	45	180.0	49.3	43.6	48.4	42.6
200	141	120	345	18.4	75.2	60.8	74.6	60.3
200	141	120	30	35.3	64.9	53.9	64.7	53.5
200	141	120	75	45.0	51.6	45.1	49.6	43.4
200	141	120	120	0	40.2	37.5	37.5	35.0
200	283	210	120	54.7	79.3	63.5	81.1	64.3
200	283	210	165	90.0	58.8	50.4	60.3	50.3
200	283	210	210	180.0	43.5	39.7	42.4	38.1
275	137.5	330	240	26.6	74.3	60.2	73.3	59.0
275	137.5	330	285	28.7	58.8	49.9	60.7	50.5
275	137.5	330	330	0	50.9	44.6	48.8	42.4
275	275	0	180	0	107.1	82.0	105.3	81.0
275	275	0	225	22.5	101.0	77.8	101.1	78.2
275	275	0	270	45.0	85.5	67.7	86.3	67.8
275	275	0	315	67.5	60.8	51.2	62.5	51.7
275	275	0	330	75.0	51.5	45.1	52.9	45.5

B. NORMAL VELOCITY DISCRIMINATION TEST

A series of tests were performed to test the ability of the system to distinguish between a collision course and a non-collision course.

The variable employed to detect collisions is normal velocity:

$$V_0 = V_A \sin \theta' - V_B \cos \beta \sin (\theta' + \alpha) + V_B \sin \beta \cos (\theta' + \alpha) \qquad (4\text{-}3)$$

In theory, a collision course exists when normal velocity is zero. In practice, a threshold value greater than zero is chosen since computational and sensor errors make a zero threshold impractical. The threshold used in this test was 30 knots.

Equations 4-4 through 4-6 are used to predict sensitivity of normal velocity to changes in sensed data. The data sheet lists the predicted changes in the variables necessary to inhibit the warning together with the measured values for a number of collision situations.

$$\Delta V_A = \frac{V_T}{\sin \theta'} \qquad (4\text{-}4)$$

$$\Delta V_B = \frac{V_T}{\sin (\beta - \theta' - \alpha)} \qquad (4\text{-}5)$$

$$\Delta\beta = \sin^{-1}\left[\frac{V_T - V_A \sin \theta'}{V_B}\right] - \sin^{-1}\left[-\frac{V_A \sin \theta'}{V_B}\right] \qquad (4\text{-}6)$$

$$\Delta\alpha = -\sin^{-1}\left[\frac{V_T - V_A \sin \theta'}{V_B}\right] + \sin^{-1}\left[-\frac{V_A \sin \theta'}{V_B}\right] \qquad (4\text{-}7)$$

$$\Delta\alpha = -\Delta\beta$$

$$\Delta\theta' = \cos^{-1}\left[\frac{V_T}{\left[[V_B \sin (\beta - \alpha)]^2 + [V_A - V_B \cos (\beta - \alpha)]^2\right]^{\frac{1}{2}}}\right] - \frac{\pi}{2} \qquad (4\text{-}8)$$

The definitions of the symbols used in the previous equations 4-3 through 4-8 are as follows:

θ'	Relative bearing of Aircraft B with respect to Aircraft A.
α	Heading of Aircraft A
β	Heading of Aircraft B
V_R	Velocity along line of sight
V_θ	Velocity normal to line of sight
V_A	Velocity of Aircraft A
V_B	Velocity of Aircraft B
R	Range
τ_W	Warning time
V_T	Threshold velocity
h_A	Altitude of Aircraft A
h_B	Altitude of Aircraft B
$\dot{h}_A$	Altitude rate of Aircraft A
$\dot{h}_B$	Altitude rate of Aircraft B

This test is performed by simulating a collision situation and then changing the sensor settings (one at a time) until the collision indicator light is extinguished. The change in the simulator setting is then recorded. This procedure is repeated for a number of collision simulations with varying velocity, headings and bearing angles. The settings of the five variables which determine the collision situation together with the expected and measured differences in the variable are shown in the data sheet for this test. The range and altitudes are set to produce a go-up threat indication for each test.

1. Discussion of Test Results

The statistics of the errors in the bearing change, $\Delta\theta'$, are as follows:

TABLE 4-1

ERROR STATISTICS IN BEARING CHANGE ($\Delta\theta'$)

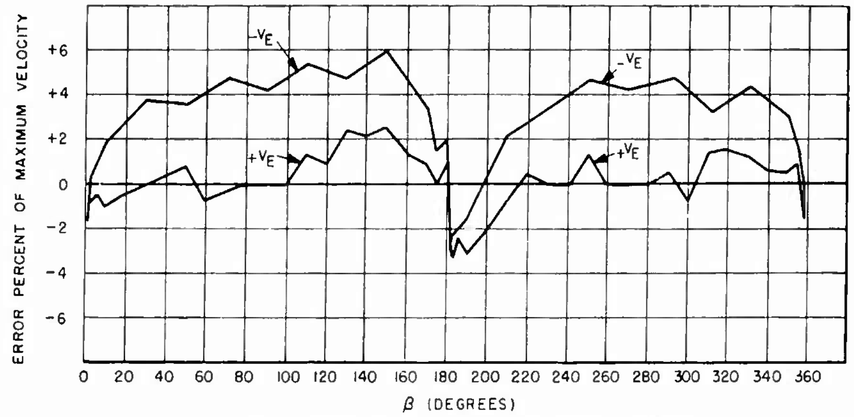
	Overall, Percent	Center Shift, Degrees	Width Changes, Degrees
Mean Square	980.8	3.326	1.661
Square of Mean	91.4	0.106	0.632
Variance	889.4	3.220	1.029
Standard Deviation	29.8%	1.795°	1.014°

This particular variable was selected for analysis because it can be most readily related to normal velocity. The first column gives the overall errors in bearing change in percent of calculated change. The resulting standard deviation due to system errors alone is about 30 percent of the threshold value. This exceeds the design goal of 25 percent due to both system and sensor errors.

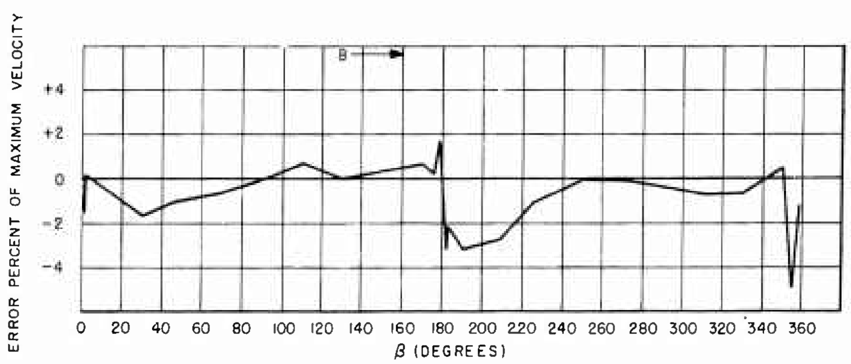
From the data on the two end limits of the collision zone, the bearing change errors were also reflected as effective shifts in the collision bearing and effective changes in the width of the collision zone. These statistics are shown in degrees in the last two columns of table 4-1. These data show that the effective shift in the collision bearing is the more significant contributor to the errors, but not by an order of magnitude. In no case is the mean error (systematic error) a major contributor.

A brief investigation of the possible cause for the excesses in the measured errors indicates that the discrepancy is primarily attributable to errors in the computing, encoding, and decoding of transponder velocity data. Figure 4-3 is a plot of the errors in computation of the east component of transponder velocity and in the encoding and decoding of the transponder east velocity component. The error in the computation of $V_B \sin \beta$ (produced principally by the resolver potentiometer) was expected to be less than one percent of maximum velocity. As can be seen in figure 4-3 this error exceeds the expected value in part of the operating range. The error in the two decoded values of the east component of transponder velocity was expected to be less than about 1.7 percent of maximum velocity at β = 90 degrees and 270 degrees, and less than about 1.4 percent of maximum velocity at β = 45 degrees, 135 degrees, 225 degrees and 315 degrees. It is apparent from figure 4-3 that the errors that exist far exceed the anticipated values.

The north velocity component computations have not been separately examined but it is expected that since the equipment in both north and east velocity component channels is the same the errors will be similar.



ERROR IN DECODED VALUES OF THE EAST VELOCITY COMPONENT OF THE TRANSPONDER VELOCITY



ERROR IN TRANSPONDER VELOCITY RESOLVER

FIGURE 4-3. ERROR PLOTS FOR $V_E = V_B$ SIN β AND DECODED VALUES FOR $+V_E$ AND $-V_E$ AT INTERROGATOR DECODER

2. Normal Velocity Discrimination Test Data Sheet

Test Equipment

Description	Serial No.	Date Calibrated	Purpose
Electronic Counter Hewlett-Packard Model 524B	12B80	4/10/63	Time Measure- ments
Time Interval Unit Hewlett-Packard Model 526B	12B80-C	2/18/63	

Fixed Test Conditions

Zero Noise Densities	
Zero Altitude Rates	
Interrogator Altitude	16,300 feet
Transponder Altitude	16,000 feet
Beamwidth	21 degrees
V_T = 30 Knots	

Results

Fixed Test Conditions

Zero Noise Densities	
Zero Altitude Rates	
Interrogator Altitude	16,300 feet
Transponder Altitude	16,000 feet
Beamwidth	21 degrees
V_T = 30 Knots	

Results

Variables					Predicted					Measured				
V_A (knots)	V_B	α (degrees)	B	θ′	ΔV_A (knots)	ΔV_B	Δα (degrees)	ΔB	Δθ′	ΔV_A (knots)	ΔV_B	Δα (degrees)	ΔB	Δθ′
125	250	45	225	0	-	-	-6.9	6.9	-4.6	-	-	-9	8.2	-5.5
					-	-	6.9	-6.9	4.6	-	-	4.5	-4.5	2.7
125	250	45	0	106.3	31.3	-62.4	-7.6	7.6	-9.4	32	-57.5	-6.5	7	-8.2
					-31.3	62.4	9.3	-9.3	9.4	-30.5	-	7	-5.7	9.2
125	250	45	45	180	-	-	-6.9	6.9	-13.9	-	-	-8.1	8.0	-14.8
					-	-	6.9	-6.9	13.9	-	-	5.1	-5.1	9.5
200	141	120	75	45	42.4	-30	-37.8	37.8	-12.2	45	-30.5	-28.5	30.5	-11.5
					-42.4	30	38.0	-38	12.2	-36	24.5	37	-35.5	13.4
200	283	210	165	90	30	-42.4	-8.1	8.1	-8.6	27.5	-39	-8.0	8.2	-7.8
					-30	42.4	9.5	-9.5	8.6	-30	-	6.9	-7.2	7.2
275	137.5	330	330	0	-	-	-12.6	12.6	-12.6	-	-	-10.8	11	-13.4
					-	-	12.6	-12.6	12.6	-	-	11.9	-12.0	12.1
275	275	0	180	0	-	-	-6.3	6.3	-3.1	-	-	-8.8	9	-5
					-	-	6.3	-6.3	3.1	-	-	2.5	-1.5	+0.8
275	275	0	270	45	42.4	-42.4	-8.3	8.3	-4.4	-	-45	-8.0	10.8	-4.2
					-42.4	42.4	9.5	-9.5	4.4	-40.5	-	8.0	-8.2	4.2
275	275	0	330	75	31.1	-31.1	-15.9	15.9	-12.2	-	-38	-17.5	17.2	-8.2
					-31.1	31.1	16.5	-16.5	12.2	-13	19	17.5	-17.5	14.9
300	240	135	15	26	68.4	-53.6	-8.2	8.2	-6.3	-	-51.5	-8.0	7.8	-3.6
					-68.4	53.6	9.1	-9.1	6.3	-61.5	52	7.5	-7.5	3.6

C. ALTITUDE DISCRIMINATION TEST

The purpose of these tests is to check the coarse-vernier altitude coding and decoding circuits, and the maneuver command logic in level, climbing, and descending flight. The combinations of interrogator and transponder altitude and altitude rate are listed in the data sheet. All runs are made with the same horizontal plane collision situation.

The interrogator altitudes that were used in this test (11.8K ft., 13.8K ft. and 16.3K ft.) were deliberately selected such that the transponder decoder would have to decode at the middle as well as at the upper and lower ends of its fine altitude decoding equipment. This was done to test altitude decoding accuracy over the full 5000-ft altitude band.

The decoding accuracy is determined in the test labelled "Level Flight" on the data sheet. The remaining tests with interrogator and or transponder having a rate of climb are used principally to demonstrate the ability of the data processing equipment to select the proper avoidance maneuver.

The test in the level flight condition is performed by setting in the fixed and variable conditions and then varying the transponder altitude to

- just extinguish the up light
- just cause a change from up to down indication
- just extinguish the down light.

At each of these transition points the transponder altitude is recorded. The data processing tests (with altitude rate) are performed in the same way except that the maneuver indications include all of the avoidance maneuvers available. The avoidance maneuvers to be expected under the various conditions are tabulated on the data sheet in the "maneuver" column.

1. Discussion of Results

The test results are given in the test data sheet. For each test the data sheet gives the expected avoidance maneuver indication together with the expected and measured transponder altitude limits within which such indication is given.

Calculations made from the test data show that the errors made in detection of the center and of the upper and lower edges of the altitude guard band are as follows:

- average error at band center	111 feet
- rms error at band center	116 feet
- rms error at lower band edge	95 feet
- rms error at upper band edge	68 feet
- standard deviation at band center	33 feet

- standard deviation at lower band edge 58 feet
- standard deviation at upper band edge 88 feet

The Phase I study report goals for the maximum allowable error at the band center and at the band edges are 100 feet and 150 feet respectively.

2. Altitude Discrimination Test Data Sheet

Test Equipment

Description	Serial No.	Date Calibrated	Purpose
Electronic Counter Hewlett-Packard Model 524B	12B80	4/10/63	Pulse Time Measure-ment
Time Interval Unit Hewlett-Packard Model 525B	12B80C	2/18/63	Pulse Time Measure-ment

Fixed Test Conditions

V_A = 275 knots θ = 270

V_B = 275 knots θ' = 45 degrees

α = 0 Beamwidth = 21 degrees

V_T = 30 knots

Results

Level Flight						
Variables h_A (K feet)	$\dot{h}_A$ (ft/min.)	$\dot{h}_B$ (ft/min.)	Range (miles)	Interrogator Maneuver	Predicted h_B (K feet)	Measured h_B (K feet)
16.3	< 1350	0	4	UP	16.3 - 15.7	16.425-15.810
				DN	16.3 - 16.9	16.425-16.960
13.8	< 1350	0	4	UP	13.8 - 13.2	13.915-13.295
				DN	13.8 - 14.4	13.915-14.565
11.8	< 1350	0	4	UP	11.8 - 11.2	11.900-11.280
				DN	11.8 - 12.4	11.900-12.465

Note: Altitudes given in tables are projected altitudes when rate of climb is greater than 1350 ft/min.

Interrogator Climbing						
Variables					Predicted	Measured

(K feet)	(ft/min.)	(ft/min.)	(miles)	Maneuver	(K feet)	(K feet)
16.3	< 1350	0	4	UP	16.3 - 15.7	16.425 - 15.810
				DN	16.3 - 16.9	16.425 - 16.960
13.8	< 1350	0	4	UP	13.8 - 13.2	13.915 - 13.295
				DN	13.8 - 14.4	13.915 - 14.565
11.8	< 1350	0	4	UP	11.8 - 11.2	11.900 - 11.280
				DN	11.8 - 12.4	11.900 - 12.465

Note: Altitudes given in tables are projected altitudes when rate of climb is greater than 1350 ft/min.

Interrogator Climbing

Variables h_A (K feet)	$\dot{h}_A$ (ft/min.)	$\dot{h}_B$ (ft/min.)	Range (miles)	Interrogator Maneuver	Predicted h_B (K feet)	Measured h_B (K feet)
16.3	1350 to 3000	0	4	DN	16.9 - 15.7	16.925 - 15.805
16.3	-3000 to -1350	0	4	UP	16.9 - 15.7	16.940 - 15.795
16.3	-3000 to -1350	0	2	UP	16.9 - 15.7	16.955 - 15.800
16.3	> 3000	0	4	HOLD	16.3 - 16.9	16.940 - 16.410
16.3	> 3000	0	4	HOLD	16.3 - 15.7	16.410 - 15.790
16.3	< -3000	0	4	HOLD	16.3 - 15.7	16.415 - 15.790
16.3	< -3000	0	4	HOLD	16.3 - 16.9	16.415 - 16.940
16.3	< -3000	0	2	HOLD	16.3 - 15.7	16.415 - 15.800
16.3	< -3000	0	2	HOLD	16.3 - 16.9	16.415 - 16.945

Transponder Climbing

Variables h_A (K feet)	$\dot{h}_A$ (ft/min.)	$\dot{h}_B$ (ft/min.)	Range (miles)	Transponder Maneuver	Predicted h_B (K feet)	Measured h_B (K feet)
16.3	< 1350	> 1350	4	Hold	16.3 - 15.7	16.395 - 15.800
16.3	< 1350	> 1350	4	Immed. Hold	16.3 - 16.9	16.395 - 16.965
16.3	< 1350	< -1350	4	Hold	16.3 - 16.9	16.420 - 16.965
16.3	< 1350	< -1350	4	Immed. Hold	16.3 - 15.7	16.420 - 15.790

Interrogator Climbing - Transponder Descending

Variables h_A (K feet)	$\dot{h}_A$ (ft/min.)	$\dot{h}_B$ (ft/min.)	Range (miles)	Transponder Maneuver	Interrogator Maneuver	Predicted h_B (K feet)	Measured h_B (K feet)
16.3	1350 to 3000	< -1350	4	Hold	Remote DN	16.3 - 16.9	16.430 - 16.960
16.3	1350 to 3000	< -1350	4	Imm. Hold	Remote DN	16.3 - 15.7	16.430 - 15.805
16.3	> 3000	< -1350	4	Hold	Remote Hold	16.3 - 16.9	16.425 - 16.955
16.3	> 3000	< -1350	4	Imm. Hold	Immed. Hold	16.3 - 15.7	16.425 - 15.805

D. CO-ALTITUDE INTERFERENCE TESTS

1. Description

Co-altitude interference tests were conducted to determine the effect of interference on system operation when an intruder is within an interrogator's altitude guard band and thus replying to his interrogations. The following two effects were considered:

- miss probability - defined as the probability that a warning will not be displayed (for a two-scan, or four-second, period) in the presence of interference, under conditions which would produce a warning without interference
- false alarm rate - defined as the average rate at which alarms will be displayed (for a single scan, or two-second period) in the presence of interference, under conditions which would not result in a warning without interference. The difference in the period in the two cases is due to the system rule that collision decisions must be reached on two successive scans before a warning is displayed (scan-to-scan redundancy).

Previous predictions of interference effects[5] were not considered sufficiently specific to serve as a good basis for comparison of test results. Specifically,

- miss probabilities were restricted to encounters involving zero miss distance
- the only beamwidth considered was the nominal 6-degree half-power beamwidth
- the effect of "range blanking" on miss probabilities was not considered
- false alarm rates were not predicted for the specific case where an intruder is properly replying, but does not constitute a threat.

As a result, interference effects were predicted in Task 2 for the specific conditions of the tests, as described later in this section.

These latter predictions disclosed two deficiencies in the system. For the first deficiency, excessive miss probabilities were disclosed at wide beamwidths. However, it was also disclosed that this condition could be corrected for all practical combinations of closing rate and beamwidth by

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[5] Design Study Report - Experimental Evaluation of Compatible PWI/CAS Interrogator - Transponder Techniques, Volume I and II furnished by Sperry Gyroscope Company to the Federal Aviation Agency under Contract No. FAA/ARDS-444.

increasing the normal velocity threshold from 30 to 45 knots. This increased threshold level was employed during the tests.

In the second deficiency, excessive miss probabilities were predicted due to initiating "range blanking" based on one sample of time-to-close greater than 40 seconds. However, it was also disclosed that requiring two sequential samples would reduce the contribution of "range blanking" to miss probability to negligible proportions. This change was not implemented for the tests. Instead, interference tests were performed with the range blanking circuit deactivated. However, one check was also made with this circuit in operation, which verified predicted results.

Test conditions were selected to demonstrate the effect of the variables which have the greatest influence on error rates. These variables were miss distance, and closing rate and beamwidth (these latter variables are related because of the increased detection range at high closing rates). A curve of error (miss or false alarm) rate as a function of bearing offset (the difference between an intruder's bearing and the bearing corresponding to a true collision) was predicted and checked for a high closing rate (head-on) case. A single condition (zero miss distance) was checked for a low closing rate (90-degree bearing) case, for comparison.

The interference levels employed for both the predictions and the test represent the levels predicted[6] in Task 1 for a propeller class system in the altitude range from 9000 to 14000 feet in a random traffic situation in sector 9 with all aircraft equipped with at least transponders, the most severe situation for this class of system. The maximum projected extra-system noise level was also employed. The specific parameters employed are shown on the test data sheets.

2. Predicted Effects

The primary assumptions employed in predicting miss probabilities and false alarm rates were as follows:

- fault probabilities are independent from interrogation to interrogation, and from scan to scan
- the probability of a fault occuring in a number of trails is approximated by the sum of the probabilities for the individual trials
- the probability of a random pulse or pulse group falling within a given time period is approximated by the product of the average pulse or pulse group density and the length of the time interval.

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[6] Design Study Report - Experimental Evaluation of Compatible PWI/CAS Interrogator-Transponder Techniques, Volume I, furnished by Sperry Gyroscope Company to the Federal Aviation Agency under Contract No. FAA/ARDS-444.

In predicting interference effects, single interrogation-reply cycle miss and error probabilities were first predicted for the test conditions,. subdivided in accordance with their effect on the collision computer. Next, their effect on a single scan basis was examined in terms of the effective shift in target bearing data caused by the miss or error. Bearing shift was employed as opposed to, say, normal velocity shift, because its use minimized the number of independent variables involved. These results were then reflected as the probability of a positive bearing shift in excess of a given amount due to interference. This result is readily interpreted as the probability of a miss or false alarm as a function of the intruder's bearing relative to the collision bearing. (The collision bearing is defined as the bearing an intruder would have to have, for a given set of velocity vectors, for a true collision situation to exist.)

As an example of the derivation of single interrogation-reply cycle miss and error probabilities, the probability of missing a reply to a given interrogation was derived as

$$P_m = D_T T_I + 2 \times 10^{-6} S_I \qquad (4\text{-}9)$$

where

P_m = the probability of missing a reply to a given interrogation

D_T = transponder decoder recovery time in seconds

T_I = the total number of interrogation triplets received per second coded in the two coarse altitude bands to which the transponder is sensitive

S_I = the total number of pulses per second received by the transponder.

In the previous equations, $D_T T_I$ is the probability that a random triplet to which a transponder is sensitive arrives close enough before the interrogation message that the transponder decoders have not recovered.

The term $2 \times 10^{-6} S_I$ is the probability that a random pulse falls from 1 to 3 microseconds after the third interrogation pulse causing the transponder to erroneously interpret the interrogator's coarse code. The latter term is applicable only if the interrogator is in the higher of the two coarse bands to which the transponder is sensitive.

There are no provisions in the breadboard equipment to simulate suppression of receivers when the transmitters in that aircraft are operating (however, the excessive recovery time of the breadboard transponder decoder raises the total single interrogation-reply cycle miss probability to a similar level). Also, a brief examination shows that contributions to single interrogation-reply cycle miss and error probabilities due to random pulses combining to synthesize a pulse group to which the equipment is sensitive is negligible compared to the corresponding probabilities due to the critical groups generated by other interrogators and transponders.

Similar expressions derived for the single interrogation-reply cycle probability of a range error and a velocity data error are

$$P_r = 12 \times 10^{-6} RT_R \tag{4-10}$$

$$P_v = (R_{37} - 8 \times 10^{-6}) D_R + (8 \times 2 \times 10^{-6}) S_R \tag{4-11}$$

where

P_r = the probability of incorrectly interpreting the range data associated with a given reply

P_v = the probability of incorrectly interpreting the velocity data associated with a given reply

R = the range between interrogator and transponder in nautical miles

T_R = the total number of reply triplets received per second

R_{37} = the interval between the third and seventh pulses in the proper reply message in seconds

D_R = the total number of reply doublets received per second

S_R = the total number of reply pulses received per second.

The probability of a relative altitude data error is omitted because the effect of this type of error on the system is negligible compared to the contributors considered herein. This minimal effect is due to the high degree of redundancy provided in the interrogator computer, by using the average of the relative altitude samples received from a given transponder in a given scan.

Probability distribution functions of effective bearing shifts which might cause a miss on a given scan, depending on the transponder's "bearing offset", were then derived. Only one or two single interrogation-reply cycle misses or errors on a given scan were considered. The probabilities associated with higher order terms (more misses or errors) decay rapidly, although the bearing shifts associated with them tend to increase. The expressions derived for the probability of a bearing shift greater than some number, δ, due to various fault combinations, are tabulated in table 4-2. The symbols used therein are tabulated in table 4-3.

TABLE 4-2

PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS APPLICABLE TO MISSES

Cause	Probability of a Bearing Shift Greater Than δ for the Indicated Limits on δ	Equation Number
Single Miss	$P_M(>\delta) = P_m \left[\frac{W}{1.8^\circ} - \left[\frac{W}{0.9^\circ} - 1 \right] \frac{\delta}{0.9^\circ} \right], \quad 0 \le \frac{\delta}{0.9^\circ} \le \frac{1}{2(1 - 0.9^\circ/W)}$	(4-12)
Single Range Error	$P_R(>\delta) = P_r \left[4 - 2 \frac{\delta}{0.9^\circ} \right], \quad 0 \le \frac{\delta}{0.9^\circ} \le 2$	(4-13)
Single Velocity Error	$P_V(>\delta) = \frac{P_v}{2} \left[\frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{0.9^\circ} + \frac{W}{0.9^\circ} \left[\frac{1}{2} - \frac{\delta}{0.9^\circ} \right] \right],$ $-\frac{1}{2} + \frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{W} \le \frac{\delta}{0.9^\circ} \le \frac{1}{2} + \frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{W}$	(4-14)
Two Non-consecutive Misses	$P_{MM1}(>\delta) = P_m^2 \sum_{n=2}^{W/0.9^\circ} \left[\frac{W}{1.8^\circ} - \frac{n}{2} - \left[\frac{W}{1.8^\circ} - 1 \right] \frac{\delta}{0.9^\circ} \right],$ $0 \le \frac{\delta}{0.9^\circ} \le \frac{W - 0.9^\circ n}{W - 1.8^\circ}$	(4-15)

TABLE 4-2, Continued

PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS APPLICABLE TO MISSES

Cause	Probability of a Bearing Shift Greater Than δ for the Indicated Limits on δ	Equation Number
Two Consecutive Misses	$P_{MM2}\ (>\delta) = P_m^2 \left[5 - 2 \frac{\delta}{0.9°} \right], \quad 0 \leq \frac{\delta}{0.9°} \leq \frac{5}{2}$	(4-16)
Two Range Errors	$P_{RR}\ (>\delta) = P_r^2 \sum_{n=1}^{3} \left[n + 4 - 2 \frac{\delta}{0.9°} \right], \quad \frac{n}{2} + \frac{1}{4} \leq \frac{\delta}{0.9°} \leq \frac{n}{2} + 2$	(4-17)
Two Velocity Errors with Same Polarity	$P_{VV1}\ (>\delta) = \frac{P_v^2}{4} \sum_{n=1}^{W/0.9°} \left[\frac{W}{0.9°} \left[1 - \frac{\delta}{0.9°} \right] + \frac{2 \sin^{-1} \left[\frac{V_L}{V} \right]}{0.9°} - n \right],$ $\frac{2 \sin^{-1} \left[\frac{V_L}{V} \right]}{W} - \left[1 - \frac{0.9° n}{W} \right] \leq \frac{\delta}{0.9°} \leq \frac{2 \sin^{-1} \left[\frac{V_L}{V} \right]}{W} + \left[1 - \frac{0.9° n}{W} \right]$	(4-18)
Two Velocity Errors with Opposite Polarity	$P_{VV2}\ (>\delta) = \frac{P_v^2}{2} \sum_{n=1}^{W/0.9°} \left[\frac{W}{0.9°} \left[1 - \frac{\delta}{0.9°} \right] - n \right],$ $0 \leq \frac{\delta}{0.9°} \leq 1 - \frac{0.9°}{W} n$	(4-19)

TABLE 4-2. Continued

PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS APPLICABLE TO MISSES

Cause	Probability of a Bearing Shift Greater Than δ for the Indicated Limits on δ	Equation Number
One Miss and One Velocity Error	$P_{MV}\ (>\delta) = \frac{P_m P_v}{2} \sum_{n=1}^{W/0.9^\circ} \left[\frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{0.9^\circ} + \frac{W}{0.9^\circ} - \left[\frac{W}{0.9^\circ} - 1\right] \frac{\delta}{0.9^\circ} - n \right],$ $\frac{\sin^{-1}\left[\frac{V_L}{V}\right] - W + 0.9^\circ n}{W - 0.9^\circ} \le \frac{\delta}{0.9^\circ} \le \frac{\sin^{-1}\left[\frac{V_L}{V}\right] + W - 0.9^\circ n}{W - 0.9^\circ}$	(4-20)
One Range Error and One Velocity Error	$P_{RV}\ (>\delta) = \frac{P_r P_v}{2} \sum_{n=1}^{3} \left[\frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{0.9^\circ} + \frac{W}{2 \times 0.9^\circ} + \right.$ $\left. \left[\frac{W}{0.9^\circ} - n - \frac{1}{2}\right] \left[\frac{n + \frac{1}{2}}{2} - \frac{\delta}{0.9^\circ}\right] - n - 1 \right], \quad \frac{n + \frac{1}{2}}{2} +$ $\frac{\sin^{-1}\left[\frac{V_L}{V}\right] - \frac{W}{2}}{W - 0.9^\circ\left[n + \frac{1}{2}\right]} \le \frac{\delta}{0.9^\circ} \le \frac{n + \frac{1}{2}}{2} + \frac{\sin^{-1}\left[\frac{V_L}{V}\right] + \frac{W}{2} - 0.9^\circ\ (n + 1)}{W - 0.9^\circ\left[n + \frac{1}{2}\right]}$	(4-21)

TABLE 4-2. Concluded

PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS APPLICABLE TO MISSES

Cause	Probability of a Bearing Shift Greater Than δ for the Indicated Limits on δ	Equation Number
One Range Error and One Miss	$P_{RM}\ (<\delta) = P_r P_m \sum_{n=1}^{3} \left[\frac{W}{2 \times 0.9^\circ} + \left[\frac{W}{0.9^\circ} - n - \frac{1}{2} \right] \left[\frac{n + \frac{1}{2}}{2} - \frac{\delta}{0.9^\circ} \right] - n - 1 \right]$, $\frac{n + \frac{1}{2}}{2} - \frac{\frac{W}{2}}{W - 0.9^\circ \left[n + \frac{1}{2} \right]} \leq \frac{\delta}{0.9^\circ} \leq \frac{n + \frac{1}{2}}{2} + \frac{\frac{W}{2} - 0.9^\circ\ (n + 1)}{W - 0.9^\circ \left[n + \frac{1}{2} \right]}$	(4-22)

TABLE 4-3

SYMBOLS IN PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS

$P_x(> \delta)$ = The probability of a positive bearing shift greater than δ due to cause X.

P_m = Single interrogation-reply cycle miss probability.

P_r = Single interrogation-reply cycle range error probability.

P_v = Single interrogation-reply cycle velocity error probability.

W = Antenna beamwidth at range in question in degrees.

V_L = The level at which normal velocity samples are limited in knots.

V = The absolute value of the relative velocity between aircraft in knots.

0.9^o = The bearing increment between adjacent interrogations.

The derivation of the probability distribution functions is summarized as follows:

Single Miss

A single miss causes an error in the derived average of the normal velocity samples, corresponding to a bearing error. The derivation of the expression for the probability distribution function for this case is given in Appendix A as an example.

Single Range Error

For a single range error, if the range error exceeds 0.36 nautical mile, the normal velocity accumulator will be read out and a new average started. Because most range errors will exceed this value, it is assumed for simplicity that all range errors exceed this value. Collision computations will be made on the remaining series of samples; however, their average bearing will differ from that of the original series, again resulting in an equivalent bearing shift. The reset function will in general result in two collision zones in bearing each $[2 \sin^{-1} (V_T/V)]$ wide, with centers separated by $[1/2(W + 0.9^o)]$, where

V_T = the normal velocity threshold, knots

V = the absolute value of relative velocity, knots

W = effective beamwidth, degrees.

These two zones will overlap when $[2 \sin^{-1}(V_T/V)] > 1/2 (W + 0.9^o)$ (which applies in general with the revised normal velocity threshold, V_T), and no misses can result as long as two zones exist. Two zones will exist unless the error occurs close enough to one edge of the beam (less than about 3.2 degrees) that one of the computations will be based on less than three samples, and thus be rejected. Under the latter conditions, bearing shifts which can cause misses will occur, as tabulated.

Single Velocity Error

Most velocity errors will result in normal velocity samples of the limiting value (±133 knots). For simplicity, it was assumed that all velocity errors result in limit samples, with either polarity equally likely. In the case of a velocity error, the bearing shift is a function of the actual bearing offset of the target from the collision bearing. For simplicity, this offset was assumed to be zero, an assumption which results in the maximum possible adverse bearing shift. The balance of the derivation is similar to that shown for a single miss.

Two Non-Consecutive Misses

The derivation for two non-consecutive misses is also similar to that for a single miss. The series expression, rather than continuous function, results from the fact that only discrete values (multiples of 0.9 degree) of the spacing between the two misses can occur.

Two Consecutive Misses

The effect of two consecutive misses is very similar to that of a single range error, previously described.

Two Range Errors

The case of two range errors is similar to that of one range error, except that three collision computations will be performed instead of two. If all three or any two adjacent series contain sufficient samples to prevent inhibiting a warning, the collision zones will overlap as described in connection with a single error, and no miss can result. Moreover, if the center series is too short to cause a warning, then the maximum spacing between the centers of the non-adjacent series will be less than $1/2\ (W + 5 \times 0.9^{o})$, which is again less than the width of the collision zone, $2 \sin^{-1} (V_T/V)$, and no misses will result. Therefore, misses can occur only if only one of the residual series is of sufficient length to result in a warning, which yields the expression tabulated.

Two Velocity Errors

When considering two velocity errors (assumed limiting), they may have the same polarity (with cumulative effect on bearing shift) or opposite polarity (with compensating effects). The two cases were assumed equally likely. The derivation of the expressions for the two cases is otherwise similar to that for a single velocity error. As in the case of two non-consecutive misses, series expressions result due to the system restriction to certain discrete spacings between the two faults.

One Miss and One Velocity Error

The derivation of the probability distribution resulting from one miss and one velocity error is very similar to that for two velocity errors.

One Range Error and One Miss or One Velocity Error

In the cases involving a range error and one other type of error a rigorous derivation is quite tedious because the relationship between the position of the error within the beamwidth and the resulting bearing shift is non-linear. The non-linear relationship results from the fact that the number of samples over which the effect of the second error is averaged is a function of this position. For simplicity, a less rigorous approach was used in these cases, in which the spacing between the two errors was assumed to be continuous but the position of the range error within the beam was assigned discrete increments. This simplification results in a series expression with less terms than the rigorous expression, but should otherwise approximate the rigorous one.

Table 4-4 is a numerical summary of these probability distribution functions applicable to the conditions of the laboratory tests. Contributing parameters are listed in the upper portion of the table. Also shown in this portion is the "bearing threshold", $\sin^{-1}(V_T/V)$, and the maximum separation between collision zone centers where two or more zones exist, $(W + 5 \times 0.9^o)/2$. This worst-case expression applies for two nonadjacent zones in the two range-error cases. It can be seen that this maximum separation remains less than twice the bearing threshold, preventing large bearing shifts as previously discussed.

TABLE 4-4

PARAMETERS AFFECTING MISS PROBABILITIES FOR THE LABORATORY TEST CONDITIONS

Parameters of the Probability Distribution Functions

Parameter	High closing rate case	Low closing rate case
$P_m \times 10^3$	11.3	11.3
$P_r \times 10^3$	1.44	0.48
$P_v \times 10^3$	4.32	4.32
W	12^o	21^o
$\sin^{-1}(V_L/V)$	12.8^o	41.7^o
$\sin^{-1}(V_T/V)$	4.3^o	14^o
$(W + 5 \times 0.9^o)/2$	8.25^o	12.75^o

End Points of the Probability Distribution Functions

X	High closing rate case			Low closing rate case
	P_X max X 10^3	δ max (degrees)	δ min (degrees)	δ max (degrees)
M	75.0	0.50	0	0.47
R	5.80	1.80	0	1.80
V	28.8	1.40	0.50	2.24
MM1	4.50	0.90	0	0.90
MM2	0.640	2.25	0	2.25
RR	0.037	3.15	0	3.15
VV1	0.770	2.74	1.08	4.44
VV2	0.770	0.83	0	0.86
MV	4.00	1.94	0.15	2.77
RV	0.096	3.12	1.40	4.03
RM	0.505	1.82	0.19	1.93

The end points of the various distribution functions are shown in the lower portion of table 4-4 for the high closing rate case. The functions will, of course, have maximum probability, P_x max, at δ min, decaying to zero at δ max. The cumulative distribution function can be determined by adding the probabilities due to the various contributions at each angle of interest. This cumulative function is approximated in figure 4-4, where each contributor is approximated by a straight line joining the end points tabulated. This cumulative probability of a bearing shift greater than δ is equal to the single scan miss probability for a target which is δ degrees from the bearing threshold. For the static situation in the laboratory, the corresponding hourly miss rate predicted is 1800 scans/hour X the probability of a single scan miss.

Also shown in table 4-4 are the maximum bearing shifts for the low closing rate case. It can be seen that the greatest shift is considerably less than the bearing threshold, $\sin^{-1}(V_T/V)$, resulting in a predicted miss probability of zero for zero bearing offset, (collision situation) in this case also.

The probability of a single scan miss due to the system range blanking provisions is somewhat unique in that this probability will be independent of bearing offset. Range blanking, as presently implemented, will cause a miss if an error in the first reply of a series results in a derived range within about 1/3 of a nautical mile of the proper range, but results erroneously in a derived time-to-close greater than 40 seconds. A range error will always be in the negative direction, and thus cannot increase the apparent time-to-close. However, because the situation simulated in the laboratory results in a maximum closing rate, there is a good chance that a velocity error on the first reply will result in a miss. For simplicity, it will be assumed that any velocity error in the first reply will result in a miss. Then the probability of occurrence due to this source will be $P_v = 4.32 \times 10^{-3}$, an excessive number for the propeller class system. Because of this high resulting miss rate, it is recommended that the system be modified to require two successive samples with derived times-to-close exceeding 40 seconds before range blanking is activated. Then the single-scan miss probability due to this source will be $P_v^2 = 0.02 \times 10^{-3}$, a negligible value.

Probability distribution functions of effective bearing shifts due to interference which may cause a false alarm were derived in a manner similar to those applicable to misses, and are listed in table 4-5. The expressions differ somewhat from those applicable to misses as follows:

- One miss, two non-consecutive misses, one or two velocity errors, and one miss and one velocity error: These faults result in only one collision zone, and thus yield the same expressions for false alarms as for misses.
- One range error, two consecutive misses, and one range error and one velocity error: These faults result in two residual collision zones. The fact that these zones overlap, although helpful in preventing misses, has no beneficial effect on false alarms. Thus faults in any positions will contribute to false alarms, as reflected in the expressions listed.
- Two range errors and one range error, and one miss: These faults will also result in more than one residual collision zone, but will not increase the adverse bearing shift over that obtained for a single range error. Thus these combinations are omitted from table 4-5.

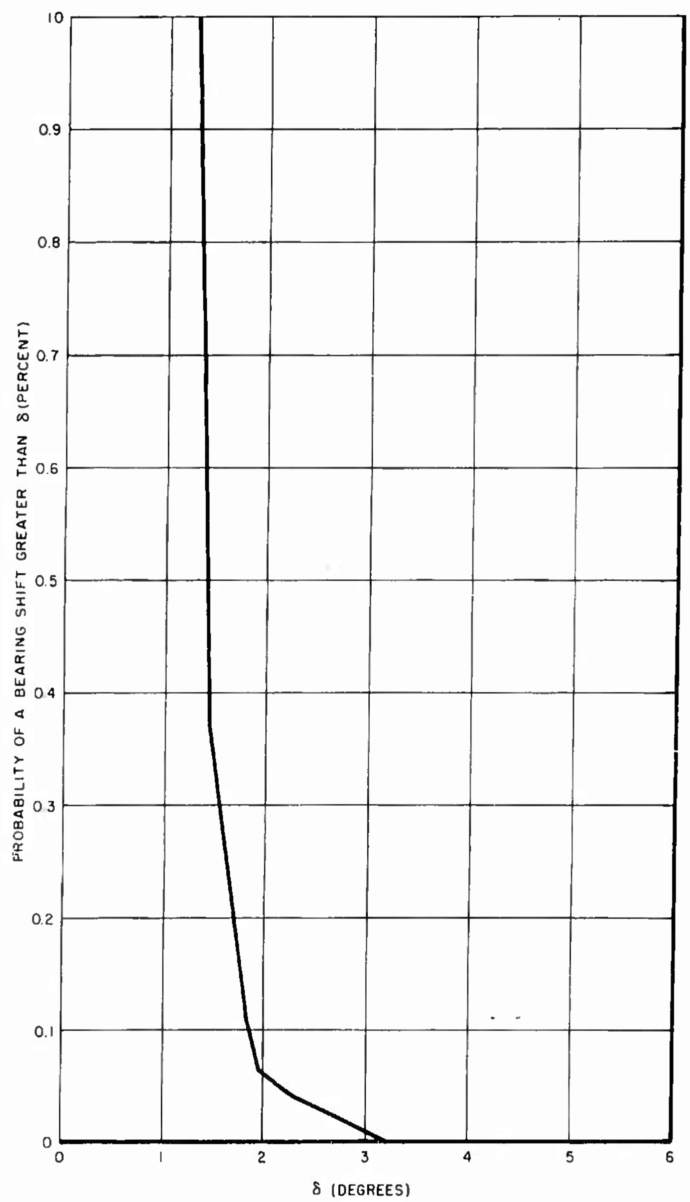

FIGURE 4-4. COMBINED PROBABILITY DISTRIBUTION FUNCTION OF EFFECTIVE BEARING SHIFT DUE TO INTERFERENCE APPLICABLE TO MISSES

TABLE 4-5

PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS APPLICABLE TO FALSE ALARMS

Cause	Probability of a Bearing Shift Greater Than δ for the Indicated Limits on δ	Equation Number
Single Miss	$P_M(>\delta) = P_m\left[\frac{W}{1.8°} - \left[\frac{W}{0.9°} - 1\right]\frac{\delta}{0.9°}\right], \quad 0 \le \frac{\delta}{0.9°} \le \frac{1}{2(1 - 0.9°/W)}$	(4-23)
Single Range Error	$P_R(>\delta) = P_r\left[\frac{W}{0.9°} - 3 - \frac{2\delta}{0.9°}\right], \quad \frac{1}{4} \le \frac{\delta}{0.9°} \le \frac{1}{2}\left[\frac{W}{0.9°} - 3\right]$	(4-24)
Single Velocity Error	$P_V(>\delta) = \frac{P_v}{2}\left[\frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{0.9°} + \frac{W}{0.9°}\left[\frac{1}{2} - \frac{\delta}{0.9°}\right]\right],$ $-\frac{1}{2} + \frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{W} \le \frac{\delta}{0.9°} \le \frac{1}{2} + \frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{W}$	(4-25)
Two Non-consecutive Misses	$P_{MM1}(>\delta) = P_m^2 \sum_{n=2}^{W/0.9°}\left[\frac{W}{1.8°} - \frac{n}{2} - \left[\frac{W}{1.8°} - 1\right]\frac{\delta}{0.9°}\right],$ $0 \le \frac{\delta}{0.9°} \le \frac{W - 0.9°n}{W - 1.8°}$	(4-26)

TABLE 4-5, Continued

PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS APPLICABLE TO FALSE ALARMS

Cause	Probability of a Bearing Shift Greater Than δ for the Indicated Limits on δ	Equation Number
Two Consecutive Misses	$P_{MM2}(>\delta) = P_m^2 \left[\frac{W}{0.9°} - 3 - \frac{2\delta}{0.9°}\right], \frac{3}{4} \leq \frac{\delta}{0.9°} \leq \frac{1}{2}\left[\frac{W}{0.9°} - 3\right]$	(4-27)
Two Velocity Errors with Same Polarity	$P_{VV1}(>\delta) = \frac{P_v^2}{4} \sum_{n=1}^{W/0.9°} \left[\frac{W}{0.9°}\left[1 - \frac{\delta}{0.9°}\right] + \frac{2\sin^{-1}\left[\frac{V_L}{V}\right]}{0.9°} - n\right]$ $\frac{2\sin^{-1}\left[\frac{V_L}{V}\right]}{W} - \left[1 - \frac{0.9° n}{W}\right] \leq \frac{\delta}{0.9°} \leq \frac{2\sin^{-1}\left[\frac{V_L}{V}\right]}{W} + \left[1 - \frac{0.9° n}{W}\right]$	(4-28)
Two Velocity Errors with Opposite Polarity	$P_{VV2}(>\delta) = \frac{P_v^2}{2} \sum_{n=1}^{W/0.9°} \left[\frac{W}{0.9°}\left[1 - \frac{\delta}{0.9°}\right] - n\right]$. $0 \leq \frac{\delta}{0.9°} \leq 1 - \frac{0.9°}{W} n$	(4-29)

TABLE 4-5. Concluded

PROBABILITY DISTRIBUTION FUNCTIONS OF EFFECTIVE BEARING SHIFTS APPLICABLE TO FALSE ALARMS

Cause	Probability of a Bearing Shift Greater Than δ for the Indicated Limits on δ	Equation Number
One Miss and One Velocity Error	$P_{MV}(>\delta) = \frac{P_m P_v}{2} \sum_{n=1}^{W/0.9^\circ} \left[\frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{0.9^\circ} + \frac{W}{0.9^\circ} - \left[\frac{W}{0.9^\circ} - 1\right] \frac{\delta}{0.9^\circ} - n \right]$. $\frac{\sin^{-1}\left[\frac{V_L}{V}\right] - W + 0.9^\circ n}{W - 0.9^\circ} \leq \frac{\delta}{0.9^\circ} \leq \frac{\sin^{-1}\left[\frac{V_L}{V}\right] + W - 0.9^\circ n}{W - 0.9^\circ}$	(4-30)
One Range Error and One Velocity Error	$P_{RV}(>\delta) = \frac{P_r P_v}{2} \sum_{n=0}^{\frac{W}{0.9^\circ} - \frac{7}{2}} \left[\frac{\sin^{-1}\left[\frac{V_L}{V}\right]}{0.9^\circ} + \frac{W}{2 \times 0.9^\circ} + \right.$ $\left. \left[\frac{W}{0.9^\circ} - n - \frac{1}{2}\right] \left[\frac{n + \frac{1}{2}}{2} - \frac{\delta}{0.9^\circ}\right] - n - 1 \right]$, $\frac{n + \frac{1}{2}}{2} + \frac{\sin^{-1}\left[\frac{V_L}{V}\right] - \frac{W}{2}}{W - 0.9^\circ \left[n + \frac{1}{2}\right]} \leq \frac{\delta}{0.9^\circ} \leq \frac{n + \frac{1}{2}}{2} + \frac{\sin^{-1}\left[\frac{V_L}{V}\right] + \frac{W}{2} - 0.9^\circ (n + 1)}{W - 0.9^\circ \left[n + \frac{1}{2}\right]}$	(4-31)

The end points of the probability distribution functions pertaining to false alarms are listed in table 4-6, and the cumulative function is approximated in figure 4-5, in a manner similar to that described in connection with misses. This cumulative probability function can be interpreted as the probability that a single scan collision decision will result, in the presence of interference, when a transponder is properly replying but does not constitute a collision threat, being δ degrees beyond the edge of the collision zone in bearing. Because of scan-to-scan redundancy, however, two sequential decisions of this type will be required before a warning is displayed. The resulting hourly false alarm rate under the static conditions of the laboratory test equals,

1800 scans/hour x (the single scan false collision decision probability)^2.

TABLE 4-6

PARAMETERS AFFECTING FALSE ALARM RATES
FOR THE LABORATORY TEST CONDITIONS

X	P_X max x 10^{-3}	δ max (degrees)	δ min (degrees)
M	75.0	0.50	0
R	14.15	4.65	0.22
V	28.8	1.40	0.50
MM1	4.5	0.90	0
MM2	1.13	4.65	0.68
VV1	0.77	3.16	1.20
VV2	0.77	0.83	0
MV	4.00	1.94	0.15
RV	0.25	7.66	0.83

The resulting overall hourly fault rate predicted (misses or false alarms) for the high closing rate encounters simulated in the laboratory tests is shown as the solid line in figure 4-6 as a function of bearing offset from the collision bearing. This predicted error rate has the very desirable characteristic of remaining near zero except within a few degrees of the threshold point.

3. Laboratory Tests

For the high closing rate case, spot checks of the analytic curve were made at bearing offsets of 0, 3, 6, and 12 degrees. Each test was run for three hours in an attempt to get a statistically significant number of faults. The measured fault rates are tabulated in the test data sheet and rates are shown as circles in figure 4-6.

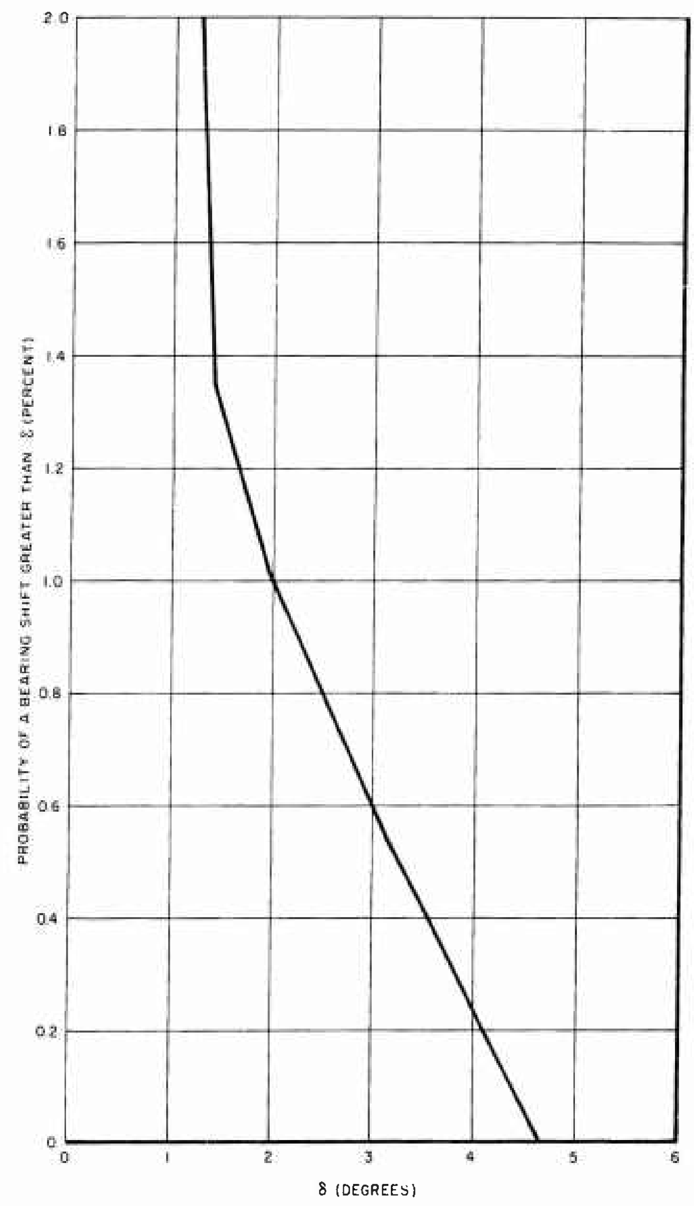

FIGURE 4-5. COMBINED PROBABILITY DISTRIBUTION FUNCTION OF EFFECTIVE BEARING SHIFT DUE TO INTERFERENCE APPLICABLE TO FALSE ALARMS

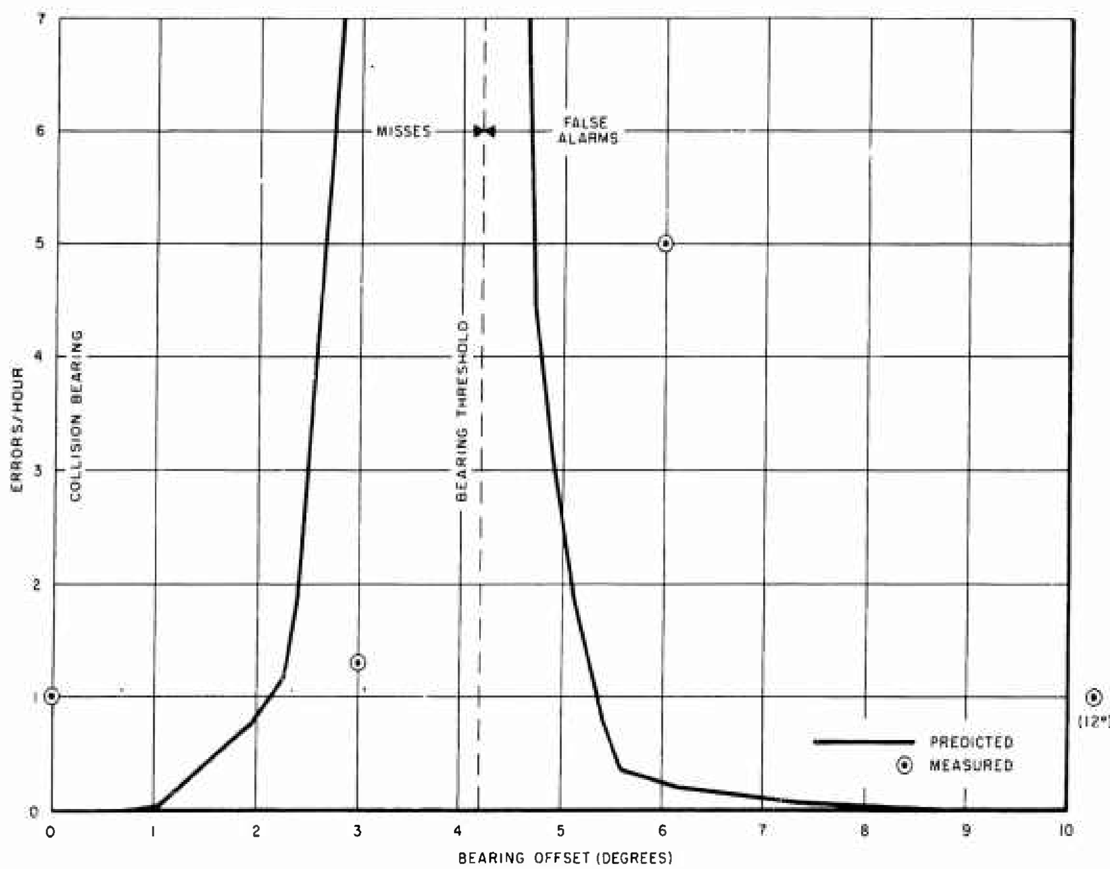

FIGURE 4-6. HOURLY ERROR RATE AS A FUNCTION OF BEARING OFFSET FOR THE STATIC CONDITIONS OF LABORATORY TEST

4. Evaluation of Results

Comparison of the measured and predicted results for the 3-degree and 6-degree bearing offsets in figure 4-6 indicates that the bearing employed as the collision bearing was in error by about one degree. This magnitude of error is understandable based on the results of the normal velocity discrimination tests, reviewed in paragraph B of this section. Shifting the test data laterally to compensate for this error brings the measured and predicted results for these bearing offsets into reasonably good agreement.

For the 0- and 12-degree bearing offsets, a measured error rate of one per hour was obtained as compared to a prediction of zero. This apparently irreducible minimum fault rate for the co-altitude situation, although of an acceptable magnitude (about a 0.5×10^{-3} single-scan miss probability), is too high to be ignored as resulting from higher order effects than those predicted. This cause has not yet been explained. About a one-per-hour fault rate for the zero-offset low-closing-rate case was also obtained as compared to a zero rate predicted.

The spot check made with the range blanking circuit activated (high closing rate, zero bearing offset) resulted in a measured error rate of 11 per hour, in reasonably good agreement with the predicted rate of 8 per hour.

5. Co-altitude Interference Test Data Sheet

Test Equipment

Description	Serial No.	Date Calibrated	Purpose
Oscilloscope Tektronix Model 535A	SN020143 AF305795	9/18/63	Monitor, visual check, delay sweep time. Check of coded pulses, etc.
Oscilloscope Tektronix Model 535	SN7816 IL #11Q59	9/18/63	Delay sweep, check of gates, pulses and random gated noise pulses, etc. (dual trace unit).
Scope Preamp Unit Tektronix CA plug-in unit	SN007810 IL #15E55	9/18/63	
Scope Preamp Unit Tektronix Model 53/54C	SN15119 IL #15C54	9/18/63	
Digital Voltmeter DC Hewlett-Packard Model 405AR	SN349-00696 AF203405	7/18/63	D-c supply voltage check and test voltage monitoring.
Dual Channel Recorder Sanborn Co. Model 60-1300	SN162 IL #29C34	12/6/63	Monitoring false and miss counts.
DC Amplifier Sanborn Co. Model 64-300A	SN1553 IL #15V36	6/18/63	Recorder level amp. and centering control.
DC Amplifier Sanborn Co. Model 64-300A	SN849 IL #15E36	6/18/63	

Fixed Test Conditions

Interrogator Altitude	7100 feet
Transponder Altitude	6800 feet
Reply Background Noise*	
Triplet Noise Density	10 triplets/sec
Doublet Noise Density	19 doublets/sec
Singlet Noise Density	28 singlets/sec

Dual Channel Recorder Sanborn Co. Model 60-1300	[illegible] IL #29C34	[illegible]	Monitoring false and miss counts.
DC Amplifier Sanborn Co. Model 64-300A	SN1553 IL #15V36	6/18/63	Recorder level amp. and centering control.
DC Amplifier Sanborn Co. Model 64-300A	SN849 IL #15E36	6/18/63	

Fixed Test Conditions

Interrogator Altitude	7100 feet
Transponder Altitude	6800 feet
Reply Background Noise*	
Triplet Noise Density	10 triplets/sec
Doublet Noise Density	19 doublets/sec
Singlet Noise Density	28 singlets/sec
Interrogation Background Noise**	
Triplet Noise Density No. 1	49 triplets/sec
Triplet Noise Density No. 2	44 triplets/sec
Doublet Noise Density	124 doublets/sec
Singlet Noise Density	2226 singlets/sec
Test Interval for Each Test	3 hours

*Reply background noise is noise introduced into the interrogator decoder.
**Interrogation background noise is introduced into the transponder decoder.

Misses Due to Range Blanking (Range Blanking On)

V_A (knots)	V_B (knots)	α (degrees)	β (degrees)	θ' (degrees)	Range (nm)	Beam Width (degrees)	Test Period (hours)	No. of Misses
300	300	150	330	0	6	12	3	33

Misses or False Alarms with Range Blanking Disabled

V_A (knots)	V_B (knots)	α (degrees)	β (degrees)	θ' (degrees)	Range (nm)	Beam Width (degrees)	Test Period (hours)	No. of Misses	No. Of False Alarms
200	283	15	330	90	2	21	3	4	
300	300	150	330	0	6	12	3	3	
300	300	150	330	3	6	12	3	4	
300	300	150	330	6	6	12	3		15
300	300	150	330	12	6	12	3		3

E. FALSE REPLY TEST

1. Description

When no transponder is near enough in altitude to an interrogating aircraft to generate legitimate replies, there is still a possibility that false replies will result in false alarms. For convenience, false replies are grouped into the following three categories

- Overheard Replies - If aircraft B is in the interrogator beam of aircraft A, but not in its guard band, and is in both the interrogator beam and guard band of aircraft C, then the interrogator of aircraft A may overhear some of the replies made by aircraft B to the interrogations of aircraft C.
- Random Reply Faults - If aircraft B is in the interrogator beam of aircraft A, but not in its guard band, the interrogator of aircraft A may detect some reply faults from B due to random signals arriving at B with relative timing such that they appear to constitute a valid interrogation. This source of false replies is negligible compared to the other two sources.
- Correlated Reply Faults - If aircraft B is in the interrogator beam of aircraft A, but not in its guard band, and is receiving pulses from any source, then one of the interrogations from A may be interfered with so as to cause B to reply improperly to A.

In a busy sector in the predicted 1975 air traffic environment, an interrogator will be interrogating a large number of transponders within range of its equipment. From the traffic model derived in the Task 1 design study, it is estimated that there will be from 0.5 to 5.0 transponders in the interrogator beam at a time depending on aircraft class and location (sector). Most of these transponders will not reply because they are at a different altitude than the interrogator. However some replies will be received due to correlated and random reply faults. The velocity components and range of the replies received will be randomly distributed resulting in a random distribution of predicted miss distance and time-to-go. Some of these correlated and random reply faults will therefore cause false alarms.

The situation just described is simulated by increasing the beamwidth in the antenna scan simulator to 360 degrees permitting continuous interrogation of the transponder equipment. This simulates an average of one transponder at a time in the interrogator beam. The simulated antenna rotation is continued, however, so that a reply will be associated with the relative bearing input existing at the time of its arrival. At the same time the heading input to the transponder equipment is rotated at a rate which is slow compared with the antenna scan rate. With a fixed transponder airspeed and a fixed range delay between interrogator and transponder this will simulate a random distribution of miss distance and time-to-go.

Since the probability of a correlated reply fault is influenced by the relative altitude, the transponder altitude input is varied linearly over one coarse altitude band at a rate which is slow compared to the antenna scan rate. For the measurement of false alarms due to false replies, the simulated altitude of the interrogator is set outside this range so that legitimate replies do not occur.

The other main source of false replies is overheard replies, i.e., replies from transponders which are triggered by other interrogators. These replies arrive in groups where the number of replies in a group depends on the common dwell time of the two interrogator beams on a transponder. Overheard replies are simulated by introducing groups of reply triggers into the transponder from a separate source at the interrogation repetition rate of 200 per second. The number of triggers in a group and the spacing between groups vary at random. The relative on-time and off-time of the triggers and the total density of triggers are controllable.

The maximum expected rate of false alarms due to false replies for a propeller class system in the anticipated operating environment was quite low. As a result, the interference levels used in this test were not the anticipated levels, but were set as high as possible within the limitations of the noise generators. Those limitations are generally based on staying far enough below the maximum possible repetition rate of the generators to assure that intergroup spacings would be random rather than periodic. The resulting total density of interrogation background noise pulses, itemized by groups in the test data sheet, is about 2.5 times the highest level anticipated. And the tabulated overhead reply density is about 4 times the maximum predicted.

2. Predicted Results

Using these escalated density figures, the false alarm rate due to false replies was predicted by applying the formulas developed by the design study report. The predicted rate was about one every 1200 hours of operation, still an unmeasurable rate for an evaluation program of practical scope.

3. Test Results

A 60-hour test was conducted with the objective of establishing whether the false alarm rate under the conditions described was within the design goal of one every 24 hours of operation. No false alarms occurred during this interval, indicating fairly conclusively that the rate is less than the one in 24-hour design goal. The result, of course, neither confirms nor contradicts the predicted rate of one in 1200 hours.

4. False Reply Test Data Sheet

Test Equipment

Description	Serial No.	Date Calibrated	Purpose
Recorder Twin Viso	SN162	6/18/63	Monitor Noise Densities
Recorder Sanborn Model 60-1300	IL #20C34	-	
DC Amplifier Sanborn Model 60-300A	SN1553 IL #15V36	6/18/63	
DC Amplifier Sanborn Model 60-300A	SN869 IL #15E36	6/18/63	

Test Conditions

Interrogator Altitude: 18,000 feet

Transponder Altitude: Varying linearly from 11,500 to 16,500 feet at 6 cycles per hour

Interrogator Airspeed: 300 knots

Transponder Airspeed: 300 knots

Interrogator Heading: 150 degrees

Transponder Heading: Varying linearly through 360 degrees at one cycle per hour

Beamwidth: 360°

Intruder Bearing: not applicable

Range: 6 nautical miles

1

Sanborn Model 60-300A	IL #15E36			

Test Conditions

Interrogator Altitude: 18,000 feet

Transponder Altitude: Varying linearly from 11,500 to 16,500 feet at 6 cycles per hour

Interrogator Airspeed: 300 knots

Transponder Airspeed: 300 knots

Interrogator Heading: 150 degrees

Transponder Heading: Varying linearly through 360 degrees at one cycle per hour

Beamwidth: 360°

Intruder Bearing: not applicable

Range: 6 nautical miles

Interrogation Background Noise Densities:

Triplets, 8 usec. Coarse code:	50/second
Triplets, 10 usec. Coarse code:	50/second
Doublet:	600/second
Singlets:	5500/second

Overhead Reply Trigger Rate:

Average number of series per second: 8
Average number of triggers per second: 50

Test Data

Date	Hours Logged	Cumulative Hours	False Alarms
6/14/63	5:12	5:12	0
6/17/63	8:22	13:34	0
6/18/63	8:12	21:46	0
6/19/63	7:52	29:38	0
6/20/63	8:20	37:58	0
6/21/63	8:20	46:18	0
6/24/63	8:17	54:35	0
6/25/63	7:22	61:57	0

APPENDIX A

DERIVATION OF THE PROBABILITY DISTRIBUTION FUNCTION OF EFFECTIVE BEARING SHIFTS DUE TO ONE SINGLE INTERROGATION-REPLY CYCLE MISS

Figure A-1 shows the envelope of the samples of normal velocity derived for a given target as the interrogator antenna beam sweeps by. ε is the offset of the target from the collision bearing, and w is the effective beamwidth for the case in question. The envelope of the samples is:

$$V \sin(\Delta\theta - \varepsilon) \simeq \frac{\pi V}{180^\circ} \cdot (\Delta\theta - \varepsilon),$$

where

V = the absolute value of the relative velocity between aircraft

$\Delta\theta$ = the instantaneous angle from the antenna beam center, degrees.

Neglecting the fact that normal velocity samples occur at discrete intervals (every 0.9 degree) rather than being a continuous function, the average value of normal velocity without errors will be

$$\bar{V}_\theta = \frac{1}{w} \int_{-w/2}^{w/2} \frac{\pi V}{180^\circ} (\Delta\theta - \varepsilon)\, d(\Delta\theta) = \frac{\pi V}{180^\circ} \varepsilon,$$

corresponding to a bearing offset of

$$\sin^{-1} \frac{\bar{V}_\theta}{V} \simeq \frac{180^\circ}{\pi} \frac{\bar{V}_\theta}{V} = \varepsilon \text{ as desired.}$$

A single miss, occurring at an angle ψ, as illustrated, will delete an element of area of width 0.9 degree and height $V \sin(\psi - \varepsilon) \simeq \pi V/180^\circ (\psi - \varepsilon)$ from the numerator and will decrease the denominator by 0.9 degree, resulting in an effective bearing shift of:

$$\delta = \varepsilon - \frac{180^\circ}{\pi V} \cdot \frac{1}{(w-0.9^\circ)} \left[\frac{\pi V \varepsilon}{180^\circ} - 0.9^\circ \frac{\pi V}{180^\circ} (\psi - \varepsilon) \right]$$

$$= \frac{0.9^\circ}{w-0.9^\circ} \psi .$$

All positions of the single miss between -w/2 and w/2 are assumed equally likely and, because the probability of a miss per increment (0.9 degree of bearing) is P_m, the probability density function of a miss at position ψ is:

$$P(\psi) = P_m/0.9^o \text{ for } -w/2 \leq \psi \leq w/2 \text{ and}$$

$$P(\psi) = 0 \text{ elsewhere.}$$

Then the probability of a bearing shift greater than δ due to this source is the probability that $\psi > (w-0.9^o)\ \delta/0.9^o$, or

$$P_M(>\delta) = \int_{(w-0.9^o)\frac{\delta}{0.9^o}}^{\infty} P(\psi)\,\partial\psi = \frac{P_m}{0.9^o}\int_{(w-0.9^o)\frac{\delta}{0.9^o}}^{w/2} \partial\psi$$

$$= P_m\left[\frac{w}{1.8^o} - \left[\frac{w}{0.9^o} - 1\right]\frac{\delta}{0.9^o}\right]$$

$$\text{for } -\frac{w}{2} \leq \frac{\delta}{0.9^o}(w-0.9^o) \leq \frac{w}{2}$$

$$\text{or } -\frac{1}{2(1-0.9^o/w)} \leq \frac{\delta}{0.9^o} \leq \frac{1}{2(1-0.9^o/w)}$$

However, we need only be concerned here with bearing shifts in one direction; that direction which, for a given initial bearing offset, will increase the miss probability. Thus the range of significant values of $\delta/0.9^o$ can be restricted to

$$0 \leq \frac{\delta}{0.9^o} \leq \frac{1}{2(1-0.9^o/w)}\ .$$

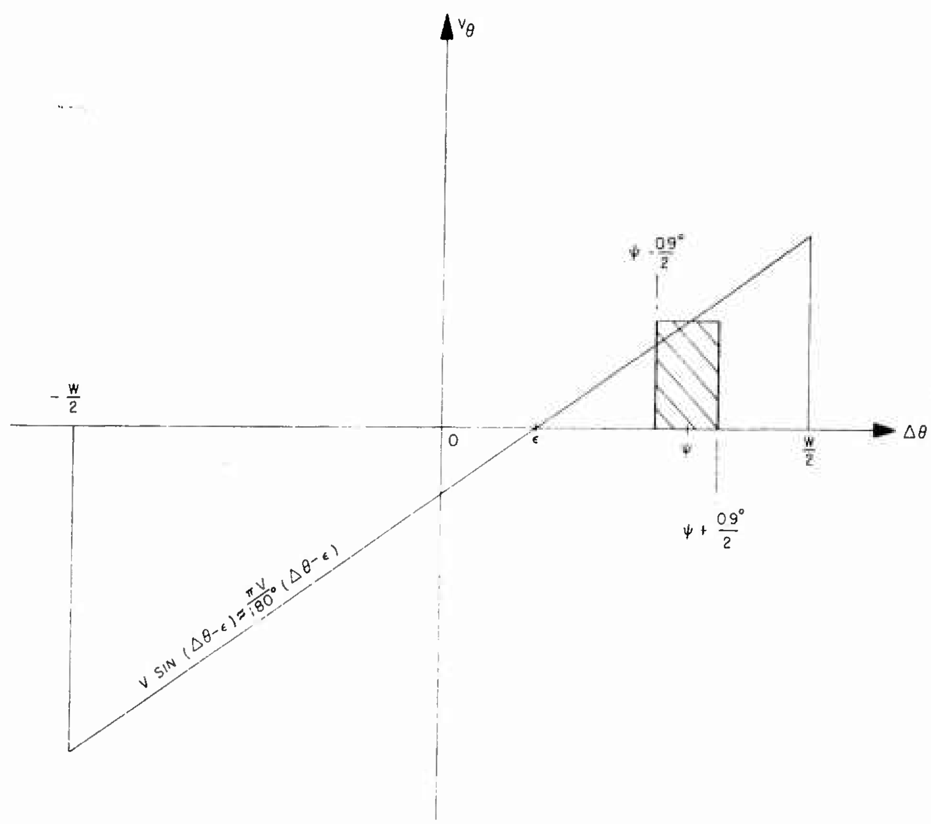

FIGURE A-1. NORMAL VELOCITY AS A FUNCTION OF INSTANTANEOUS DISPLACEMENT FROM BEAM CENTER