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HYDRA PROGRAM - THEORETICAL AND EXPERIMENTAL DETERMINATION OF ENERGY PARTITION OF SELECTED UNDERWATER EXPLOSIVES

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ADMINISTRATIVE INFORMATION

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ABSTRACT

A summary is presented of the derivation of the equation of motion of the bubble due to an underwater spherical explosion. Migration and surface effects are included. The formulae for the first maximum radius and the first oscillation period, including surface effects, are given.

From the experimentally determined bubble period data and the theoretical period formula:

$$T_2 = Kw^{1/3}z^{-5/6}[1.0 + Kw^{1/3}z^{-1/3}HF/(D+B)]$$

the energy partitions of three explosives, Pentolite, RDX + Alum (50/50), and $2H_2 + O_2$ are calculated.
SUMMARY

The Problem
To compare underwater explosions producing steam bubbles and non-condensable gas bubbles on the basis of their computed energy partition values. To determine which of three selected chemical explosives best simulate an underwater nuclear explosion.

Findings
Using Friedman's formula for the bubble oscillation period and the experimentally determined bubble periods from Hydra studies, the energy partition values (i.e., fraction of charge energy left for bubble oscillation after shock passage) of three explosives were found to be:

<table>
<thead>
<tr>
<th>Explosive</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentolite</td>
<td>0.45 ± 0.12</td>
</tr>
<tr>
<td>RDX + Alum</td>
<td>0.50 ± 0.14</td>
</tr>
<tr>
<td>ZR₂ + O₂</td>
<td>0.41 ± 0.11</td>
</tr>
</tbody>
</table>

Since the minimum error variation in energy partition is much greater than the variation due to type of explosive, it makes little difference which explosive (steam or non-condensable gas) is selected to simulate the nuclear case.
INTRODUCTION

Underwater nuclear explosions form steam bubbles whose dynamics determine the early dispersion of the radioactivity produced. To simulate nuclear explosions, the Hydra program had been using chemical high explosives (e.g. Pentolite) which form non-condensable gas bubbles. In an attempt to more accurately simulate these detonations, two steam producing explosives were developed: an equal mixture by weight of RDX + Alum (i.e., AlNH$_4$(SO$_4$)$_2$$\cdot$12H$_2$O), and a pressurized stoichiometric mixture of hydrogen and oxygen. Using these two explosives and Pentolite, a series of experiments was undertaken during the summer of 1960 to determine the variation of bubble parameters between chemical explosives producing condensable steam bubbles and non-condensable gas bubbles.

The bubble parameter usually compared is the energy partition value (i.e., the fraction of charge energy left for bubble oscillation after shock passage). This is best determined from the theoretical formula for the bubble oscillation period using experimental data.

This report reviews the bubble dynamics theory that has been developed over the past several decades including surface effects. Using these theories and the bubble period data from Hydra studies, the energy partition values for selected explosives of interest have been determined.

In the literature, the first order approximation, relating the bubble period constant $K$ to the cube root of the bubble energy is well known. However, the second order effects of fixed and free surfaces on the bubble period are generally ignored. Where these effects are
mentioned, the approaches are difficult to follow because of inconsistencies or incompleteness. After a critical review of the literature, Friedman's papers were found to be the first and only rigorous qualitative and quantitative determination of surface effects.

Because no papers were found to use Friedman's method to its full advantage, and because Friedman's original work is more detailed than necessary for the field engineer, the following section on bubble dynamics was undertaken with effort to present a comprehensive and coherent outline of the equations of motion using a consistent system of symbols in a concise package permitting bubble period prediction.
SECTION 1

DYNAMICS

1.1 EQUATION OF MOTION

The earliest derivation of the equation of motion of the gas bubble is that of Ramsauer.\(^1\) He used the energy balance:

\[ E_0 = E + WD + KE \]  \hspace{1cm} (1.1)

which translated to words says that the initial internal bubble energy \( E_0 \), equals the internal energy at a later time \( E \), plus the work of displacement against the water \( WD \), plus the kinetic energy imparted to the surrounding water at any later time \( KE \). Here

\[ E_0 = P_0 V_0/(\gamma - 1) \]

\[ E = PV/(\gamma - 1) \]

\[ WD = P_h(V - V_o) \]

\[ KE = 2\pi \rho \lambda^2 \]

Assuming the bubble expands adiabatically, then

\[ PV' = P_o V_o' \]  \hspace{1cm} (1.2)

Note: See Appendix 7 for definition of symbols.
and Eq. 1.1 can be rewritten as

\[
P_0/(\gamma - 1) \left[ \left( A_0/A \right)^3 - \left( A_0/A \right)^{3\gamma} \right] + P_h \left[ \left( A_0/A \right)^3 - 1 \right] = 3\alpha^2/2 \tag{1.3}
\]

In the existing literature these equations are usually written in dimensionless variables a and t, by using the unit of length L, and the unit of time C:

\[
L = (3r\bar{W}/\mu g \gamma Z)^{1/3} \tag{1.4}
\]

\[
C = (3/2gZ)^{1/2} L \tag{1.5}
\]

Thus a = A/L, t = T/C, and c = Ct/L. Subscripts 0, 1, and 2, refer to variables at detonation, first maximum, and first minimum.

Since \( E_0 = r\bar{W} = L^3 \mu nP_h/3 \), then

\[
P_0/(\gamma - 1) = P_h a_0^{-3} = P_h k^{-1/(\gamma - 1)} \tag{1.6}
\]

where k is a convenient notation and is defined by

\[
k = a_0^{-3(\gamma - 1)} = (\mu g a_0^{-3} Z/3r\bar{W})^{(\gamma - 1)} \tag{1.7}
\]

and Eq. 1.3 now reads

\[
1.0 - ka^{-3(\gamma - 1)} + a_0^{-3} - a^3 = a^3 a^2 \tag{1.8}
\]

Since \( P_0/(\gamma - 1) \gg P_h \), that is, \( 1.0 \gg a_0^{-3} \), the \( a_0^{-3} \) term is usually dropped and the equation is written:

\[
1.0 - ka^{-3(\gamma - 1)} - a^3 = a^3 a^2 \tag{1.8}
\]

This approach does not include the effects of fixed and free surfaces,
and assumes no migration.

1.2 MIGRATION

During the first half oscillation, the migration is found from experiments to be very small. However, including migration, the kinetic energy term would be

$$K_e = 2\pi \rho a^3 (A^2 + B^2/6)$$

Equation 1.8 would now read

$$1.0 - k a^{-3(\gamma-1)} - a^3 = a^3(A^2 + B^2/6)$$

The migration is usually written in terms of a non-dimensional vertical momentum, $s$. Since

$$s = a^3 b/3 \quad (1.9)$$

then the equation of motion, including migration, is

$$1.0 - k a^{-3(\gamma-1)} - a^3 - 3s^2/2a^3 = a^3 a^2 \quad (1.10)$$

At the maximum and minimum radius, $\dot{a} = 0$. Thus $a_1$ and $a_2$ are the largest and smallest roots of

$$1.0 - k a^{-3(\gamma-1)} - a^3 - 3s^2/2a^3 = 0$$

(Note: $a_0$ is also a root of the equation retaining the $a_0^3$ term since $s_0 = 0$.)

The maximum limit of $a_1$ approaches 1.0 as $k$ (i.e. $a_0$) and $s$ approach zero. Thus a good approximation to the maximum bubble radius is given by
\[
\alpha_1 = 1.0 - k/3 - (3\gamma-2)k^2/9 - s_1^2/2
\]  
(1.11)

(Reference 2 gives \(3s_1^2/2\). This is a typographical error.)

In determining the oscillation period, the momentum term is negligible (as shown by calculations in Appendix 2), and therefore is ignored in all further discussion. Thus, the time to first maximum radius is found from Eq. 1.8 to be:

\[
t_1 = \frac{s_1}{s_0} \frac{a^{3/2}}{1.0 - \alpha^3 - k\alpha^{3(\gamma-1)}} \frac{1}{a^{1/2}} \text{ da}
\]

(1.12)

Making use of Friedman's I functions (see Appendix 1), \(t_1 = I_1\). Assuming \(t_2 = 2t_1\), i.e. assuming the integrations from \(a_0\) to \(a_1\) and from \(a_1\) to \(a_2\) are identical, then

\[
T_2 = Ct_2 = 2CI_1
\]

(1.13)

1.3 SURFACE EFFECTS

The bubble motion is affected by surfaces, i.e. the bottom, targets, and the air-water interface. Free surfaces repel while fixed surfaces attract the bubble. The oscillation period is reduced by the free surface and increased by the fixed surface. These effects do not cancel; the free surface tends to be stronger. Thus the equation of motion must include surface corrections. The only rigorous surface correction method available is that of Friedman in reference 2 and summarized below. A comparison of Friedman's final equation with those of Kennard is also included.

1.3.1 Friedman's equation

In determining the effect of plane (or spherical) surfaces, the above approach to the equation of motion is valid with one exception. The kinetic energy of the water is found using a velocity potential function which is evaluated using the method of images. This method
is detailed in reference 2. The resulting kinetic energy expression, ignoring second and third order terms and migration, is

\[ K = 2 \eta p A^3 a^2 \left( 1.0 + AF/(D+B) \right) \]

where

\[ F = 2xf(x) - \ln2 \]

\[ x = (d-b)/(d+b) \]

\[ f(x) = \sum_{n=0}^{\infty} (-1.0)^n \left( \frac{1}{n+1} \right)^2 \]

For tabulated values of \( x \) versus \( F(x) \), see Appendix 3.

The non-dimensional equation of motion is now

\[ 1.0 - ka^{-3} (\gamma-1) - a^3 = \left[ 1.0 + aF/(d+b) \right] a^3 a^2 \]

Since \( \left[ 1.0 + aF/(d+b) \right]^{1/2} = 1.0 + aF/2(d+b) + \ldots \), then ignoring higher order terms, the time to first maximum radius is

\[ t_1 = a^{-3/2} \int_0^1 a^{-3} - ka^{-3} (\gamma-1)^{-1/2} (1 + aF/2(d+b)) \]}

Or in terms of the I functions,

\[ t_1 = I_1 + I_2 F/2(d+b). \]

And the oscillation period is

\[ T_2 = 2I_1 \left[ 1.0 + I_2 F/2I_1 (D+B) \right] \]

If the explosive were detonated at a depth such that \( F = 0 \) (i.e. at \( D = 2B \)), then \( T_2 = 2G I_1 \). Since this must agree with the well known
formula

\[ T = K W^{1/3} z^{-5/6}, \]  

then the period constant \( K \) is defined by

\[ K = 2I_1 (3/2g)^{1/2} (3\pi Q/4\pi \rho g)^{1/3} \]  

For convenience, a constant is defined:

\[ H = I_2 (2g/3)^{1/2} / 4I_1^2 \]  

Now Eq. 1.19 reads

\[ T = K W^{1/3} z^{-5/6} \left[ 1.0 + K W^{1/3} z^{-1/3} \right] \]  

This is the form of Friedman's theoretical period formula which will be used later to compute energy partition values (see eq. 2.3).

Next consider the first maximum bubble radius:

\[ A_{\text{max}} = L (1.0 - k/3 - (37-2)k^2/9) \]  

From Eq. 1.7, \( k \) does not equal zero, since \( A_o \) cannot be zero. However, in experiments, the error in the measured maximum radius is much greater than the second order correction due to \( k \). Thus \( A_{\text{max}} = L \) must agree with the well known formula

\[ A_{\text{max}} = J W^{1/3} z^{-1/3} \]  

This defines the explosive's radius constant \( J \) to be

\[ J = (3\pi Q/4\pi \rho g)^{1/3} \]
The commonly used $K/J$ ratio is seen to be

$$K/J = 2I_1(3/2^5)^{1/2} \quad (1.27)$$

All the $I$ functions depend on $k$ and $\gamma$, but from the graphs of $I(k,\gamma)$, in reference 2, $I_1 \geq 0.7405$. Thus,

$$K/J \geq 0.320 \quad (1.28)$$

From the same set of graphs, $I_1^2/I_2 \geq 0.92$, and

$$H \leq 1.26 \quad (1.29)$$

1.3.2 Kennard's equation

Kennard's reports of the early 1940's often give the formula

$$T = kw^{1/3}z^{-5/6}(1.0 + 0.2A_{\text{max}}/R)$$

where $R$ is the distance from the charge to the fixed or free surface. The $+$ sign is associated with the attractive force of the fixed (bottom) surface, and the $-$ sign with the repulsive force of the free (air-water) surface. Thus the period would be

$$T = kw^{1/3}z^{-5/6}(1.0 + 0.2A_{\text{max}}(D-B)/DB) \quad (1.30)$$

Usually Eq. 1.30 overcorrects the bottom effect, and also the surface effect in deep shots.

In reference 3, Kennard says 1.30 is an approximate empirical expression and that the theoretical formula, using Friedman's method, is

$$T = kw^{1/3}z^{-5/6}(1.0 + 0.2P(x)A_{\text{max}}(D+B)/DB) \quad (1.31)$$
where $x$ is defined by eq. 1.15 and values of $P(x)$ are calculated from reference 2. Comparing Kennard's graph of $P(x)$ with Friedman's tabulation of $F(x)$, we find

$$P(x) = (1-x^2)F/2$$  \hspace{1cm} (1.32)

Since $A_{\text{max}} = L_{\text{a}1}$, Eq. 1.31 can be written as

$$T_2 = Kw^{1/3} z^{-5/6} \left[ 1.0 + 0.4a_{11} LF/(D+B) \right]$$  \hspace{1cm} (1.33)

This agrees with Friedman's expression (Eq. 1.19) if $I_2/2I_1 = 0.4a_{11}$.

Thus Kennard's eq. 1.33 is a special case of Friedman's eq. 1.23.

A choice should be made between Friedman's theoretical period eq. 1.23 involving the complicated $F(x)$ function, and the much simpler eq. 1.30 of Kennard. Friedman's eq. 1.23 is preferred not only because of its theoretical foundation but also because it better fits the experimental data (see page 15).
2.1 ENERGY YIELD OF EXPLOSIVES

Detonation of an explosive releases an amount of energy QW. An amount \((1-r)QW\) is carried off by the shock wave, leaving the bubble with \(rQW\) for the first oscillation.

Reported values of Q for any one explosive are varied. For example, for the standard explosive, TNT, they range from around 500 gram calories per gram weight to over 1000. Keeping in mind that the accuracy of Q may be improved, Table 2.1 was used in reducing the experimental data of this report.

<table>
<thead>
<tr>
<th>Explosive</th>
<th>(Q^*) (cal/gm)</th>
<th>Q (ft-lb/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TNT</td>
<td>1000</td>
<td>1.40 (10^6)</td>
</tr>
<tr>
<td>Pentolite</td>
<td>1220</td>
<td>1.71 (10^6)</td>
</tr>
<tr>
<td>RDX + Alum (50/50)</td>
<td>1335</td>
<td>1.87 (10^6)</td>
</tr>
<tr>
<td>(2H_2 + O_2)</td>
<td>3794</td>
<td>5.31 (10^6)</td>
</tr>
</tbody>
</table>

Q values are usually reported in calories per gram weight. These have been converted to the fps system using the conversion factors:

453.6 grams = 1.0 pound  
1.0 gram calorie = 3.087 foot-pounds

*Private communication: W. W. Perkins, USNRDL.*
The Q value for $2H_2 + O_2$ is based on the change in enthalpy.

2.2 EVALUATION OF $K$ AND $J$

The fraction of charge energy left in the bubble for the first oscillation, that is $r$, can be found using Eqs. 1.21 or 1.26. Thus

$$r = (\ln \gamma Q/3Q)K^3(2g/3)^{3/2}/8I_1^3$$  \hspace{1cm} (2.1)

or

$$r = (\ln \gamma Q/3Q)J^3$$  \hspace{1cm} (2.2)

where $I_1$ is a function of $k$ and $\gamma$, $Q$ is the total charge energy per unit weight, and $K$ and $J$ are constants of the explosive to be determined from experimental data.

In section 1.3, the general form of the equation for the oscillation period was found to be Eq. 1.23. Written as

$$y = K + HK^2z,$$  \hspace{1cm} (2.3)

where

$$y = T_z^{5/6}w^{-1/3}$$

and

$$z = w^{1/3}z^{-1/3}F/(D+B),$$

this is the equation for a straight line with slope $HK^2$ and intercept $K$. Using the method of least squares (Appendix 5) to fit the explosive's experimental data (for varying charge depth, pond depth, and perhaps charge weight), the values of $K$ and $H$ can be computed for the explosive.

Using a least squares fit, it is possible to minimize the error in $y$ or in $z$. From the experimental data of Appendix 4, the first order variable $y$ is at least a factor of 100 times the second order variable $z$. Therefore the following values of $K$ and $H$ were computed based on minimizing the error in $y$. 
This needs some explanation. The K values are based on actual charge weight (except for RDX + Alum where 0.8 pounds was used; that is, the Alum weight was excluded). However, if, instead of the actual weight, the equivalent weight TNT \((i.e. \frac{Q_W}{Q_{TNT}})\) is used, the \(K'\) values compare with those found in the literature which vary from 4.19 to 4.37 for TNT and Pentolite.

From Eq. 1.29, the largest possible value of \(H\) is 1.26. Thus except for Pentolite the above \(H\) values are too high, but these high values can be explained by experimental error (see Appendix 6). If the error in \(z\) were minimized, the \(H\) values would be still higher.

\(J\) can be determined from Eq. 1.27 if the value of \(I_1(k, \gamma)\) is known. The ratio of specific heats of the bubble's gas is not always known, but it is standard procedure to let

\[
\gamma = 1.25
\]

Thus to determine \(J\), \(k\) must be evaluated. But from Eq. 1.7, \(k\) is a function of the energy partition \(r\).

### 2.3 ENERGY PARTITION VALUES

As shown in section 2.2, the energy partition can be determined from Eq. 2.1 by a trial and error method since \(r = f(I_1) = f(k) = f(r)\). In other words, it is a circular relationship. To simplify things, we solve for \(k(I_1)\) by eliminating \(r\) between Eqs. 1.7 and 1.21. Thus
From the experiments, the charge radii in feet are: Pentolite, 0.1354; RDX + Alum (50/50), 0.1458; $2H_2 + O_2$, 0.50. Equation 2.5 gives one relationship between $k$ and $I_1$. From reference 2, the graph of $I_1$ versus $k$ for $\gamma = 1.25$ gives a second relationship. The intersection of the two curves gives the sought for values of $k$ and $I_1$. $J$ is found from $K$ and $I_1$. Knowing $k$ yields $I_1^2/I_2$, and thus $H$ from Eq. 1.22. Finally, $r$ is found from Eq. 2.1. The results are below.

**TABLE 2.3.1**

Energy Partition Values ($\gamma = 5/4$)

<table>
<thead>
<tr>
<th>Explosive</th>
<th>$k$</th>
<th>$I_1$</th>
<th>$J$</th>
<th>$J'$</th>
<th>$H$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentolite</td>
<td>0.075 ± .001</td>
<td>.7423</td>
<td>14.32</td>
<td>13.42</td>
<td>1.23</td>
<td>0.45</td>
</tr>
<tr>
<td>RDX + Alum</td>
<td>0.081 ± .001</td>
<td>.7420</td>
<td>15.26</td>
<td>13.86</td>
<td>1.23</td>
<td>0.50</td>
</tr>
<tr>
<td>$2H_2 + O_2$</td>
<td>0.213 ± .008</td>
<td>.7410</td>
<td>20.32</td>
<td>13.03</td>
<td>1.14</td>
<td>0.41</td>
</tr>
</tbody>
</table>

where the ± values are due to the depth ($Z$) variation.

For sea water, i.e. $\rho g = 65$ lb/ft$^3$, the respective $r$ values are 0.46, 0.51 and 0.42.

The effect of increasing $\gamma$ from 5/4 to 4/3 is shown below.

**TABLE 2.3.2**

Energy Partition Values ($\gamma = 4/3$)

<table>
<thead>
<tr>
<th>Explosive</th>
<th>$k = a_0$</th>
<th>$I_1$</th>
<th>$J$</th>
<th>$J'$</th>
<th>$H$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentolite</td>
<td>0.032 ± .001</td>
<td>.7480</td>
<td>14.21</td>
<td>13.31</td>
<td>1.24</td>
<td>0.44</td>
</tr>
<tr>
<td>RDX + Alum</td>
<td>0.035 ± .001</td>
<td>.7482</td>
<td>15.14</td>
<td>13.74</td>
<td>1.24</td>
<td>0.49</td>
</tr>
<tr>
<td>$2H_2 + O_2$</td>
<td>0.127 ± .006</td>
<td>.7550</td>
<td>19.94</td>
<td>12.79</td>
<td>1.17</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Again, for sea water, the respective $r$ values are .45, .50, .40.
The ratio of internal energies at maximum radius and initial radius is \( \frac{E_1}{E_0} = \left( \frac{a_0}{a_1} \right)^{3(\gamma - 1)} \). For \( \gamma = 5/4 \), these ratios are 0.077, 0.083, and 0.227 for Pentolite, RDX + Alum (50/50), and \( 2H_2 + O_2 \). For \( \gamma = 4/3 \), the ratios decrease to 0.032, 0.035, and 0.113.

All of the above calculations are based on Friedman's method. If we use a least squares fit of the experimental data to Kennard's eq. 1.30, we find the intercept \( K \) and slope \( KJa_1 \) of the straight line. Using the calculated \( K \) values, and assuming \( \gamma_1 = 0.741 \), the respective energy partition values are 0.38, 0.41, and 0.33. Using the calculated \( KJa_1 \) values, and assuming \( a_1 = 1.0 \), the respective energy partition values are 0.49, 0.67, and 0.83. (Assuming \( a_1 \) is some value less than 1.0 would increase this second set.)

If Kennard's eq. 1.30 were a better fit of the experimental data than Friedman's eq. 1.23, then the two sets of data should agree. Since they differ radically, Friedman's eq. 1.23 is a better fit.
CONCLUSIONS

The theoretical equation for the bubble oscillation period, including the effects of surfaces, was found to be Friedman's:

\[ T_2 = k M^{1/3} z^{-5/6} \left( 1.0 + k M^{1/3} z^{-1/3} H/(D+B) \right) \]

Using the method of least squares to fit the experimental periods of three selected explosives, the period constants \( k \) were determined. The error in \( k \) was calculated to be less than 5.5% based on maximum probable variations in the experimental variables. The energy partition values of the explosives were then found using the method discussed in Section 2.3. This involved assuming that \( \gamma \), the ratio of specific heats of the bubble gas, was 1.25. The resulting \( r \) values for Pentolite, RDX + Alum, and \( 2H_2 + O_2 \) were 0.45, 0.50, and 0.41, respectively.

Increasing \( \gamma \) would decrease the resulting energy partition values. For example, for \( 2H_2 + O_2 \), if \( \gamma \) is increased from 1.25 to 1.50, \( r \) is decreased from 0.41 to 0.38.

From Eq. 2.1, the accuracy of \( r \) is directly proportional to the accuracy of \( Q \) (probably \( \pm 10\% \) at best) and to the cube of \( k \)'s accuracy (\( \pm 17\% \)). Thus the minimum probable experimental accuracy for \( r \) is \( \pm 27\% \), and the energy partition values, including error variation, are 0.45 \( \pm 0.12 \), 0.50 \( \pm 0.14 \), and 0.41 \( \pm 0.11 \), respectively. Although the best available experimental data was used, the minimum probable error variation in \( r \) is greater than the variation due to type of explosive.

There is a tendency for the steam bubble to retain slightly less energy than non-condensable gas bubbles for the bubble oscillation. But, based on energy partition considerations, it makes little difference which explosive is selected to simulate the nuclear case.
APPENDIX I

THE I FUNCTIONS

The I functions are used in determining the oscillation period ($I_1$ and $I_2$), the vertical momentum ($I_3$, $I_4$, $I_5$, and $I_6$), and the vertical displacement of the bubble ($I_7$ and $I_8$). They are:

\[ I_1 = \int_{a_0}^{a_1} u^{-1} a^{3/2} \, da \]
\[ I_2 = \int_{a_0}^{a_1} u^{-1} a^{5/2} \, da \]
\[ I_3 = \int_{a_0}^{a_1} u a^{5/2} \, da \]
\[ I_4 = \int_{a_0}^{a_1} u a^{7/2} \, da \]
\[ I_5 = \int_{a_0}^{a_1} u^{-1} a^{9/2} \, da \]
\[ I_6 = \int_{a_0}^{a_1} u^{-1} a^{11/2} \, da \]
\[ I_7 = \int_{a_2}^{a_3} (u^2 - 3s^2/2a^3)^{-1/2} a^{-3/2} \, da \]
\[ I_8 = \int_{a_2}^{a_3} (u^2 - 3s^2/2a^3)^{-1/2} a^{-1/2} \, da \]

where $u = (1 - a^3 - ka^{-3(y-1)})^{1/2}$.

Graphs of the first six I functions for $\gamma = 1.25, 1.33, 1.40, 1.50,$
and $0 \leq k \leq 0.3$, are given in reference 2. A check on their limit values (at $k = 0$), using the beta function

$$\beta(m,n) = \int_0^1 v^{m-1} (1-v)^{n-1} \, dv \quad \text{for } m > 0, n > 0,$$

where $v = a^3$, agrees with the graphs and gives: $I_1 = 0.7468$, $I_2 = 0.6072$, $I_3 = 0.1821$, $I_4 = 0.1309$, $I_5 = 0.4668$, $I_6 = 0.4250$, and $I_1^2/I_2 = 0.9186$. Except for $I_1$ and $I_1^2/I_2$, these are all upper limit values.

This method cannot be used to determine the limit values of $I_7$ and $I_8$ since they are integrated from $a_2$ to $a_1$, and $a_2$ is not zero. Evaluation of $I_7$ and $I_8$, using numerical integration, is possible but very involved because of the $s^2$ term (see Appendix 2).
APPENDIX 2

VERTICAL MOMENTUM

The vertical momentum of the bubble at its maximum radius, \( s_1 = a_1^3 a_1/3 \), is reported in reference 2 to be:

\[
s_1 = \frac{(L/Z)\left[I_5 + LFI_6/2(D+B)\right]}{2L^2/2(D+B)^2}\left[I_3 - LFI_4/2(D+B)\right]
\]

where \( \dot{F} = dF/dx \). The first term is the upward momentum due to gravity, and the second term is the downward momentum due to the rigid surface. The vertical momentum at the bubble minimum is twice this value.

Usually the migration at maximum radius is considered negligible. However, to determine the effect of \( s_1 \) in Eq. 1.11, the calculations in Table A.2 were made.

TABLE A.2
Vertical Momentum

<table>
<thead>
<tr>
<th>Explosive</th>
<th>( D = 4.05 \text{ ft} )</th>
<th></th>
<th></th>
<th>( D = 9.0 \text{ ft} )</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s_1 )</td>
<td>( s_1^2/2 )</td>
<td>( a_1 )</td>
<td>( s_1 )</td>
<td>( s_1^2/2 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>Pentaolite</td>
<td>-.0750</td>
<td>+.0028</td>
<td>0.9711</td>
<td>+.0064</td>
<td>+.00002</td>
<td>0.9739</td>
</tr>
<tr>
<td>RDX + Alum</td>
<td>-.0696</td>
<td>+.0024</td>
<td>0.9693</td>
<td>+.0072</td>
<td>+.00003</td>
<td>0.9717</td>
</tr>
<tr>
<td>( 2H_2 + O_2 )</td>
<td>-.0200</td>
<td>+.0002</td>
<td>0.9200</td>
<td>+.0120</td>
<td>+.00007</td>
<td>0.9202</td>
</tr>
</tbody>
</table>

Increasing depth increases \( a_1 \). Since \( s_1 \) is less than zero at shallow depth, \( s_1^2/2 \) decreases to a negligible value at mid-depth.
### APPENDIX 3

#### Table A.3

<table>
<thead>
<tr>
<th>x</th>
<th>F(x)</th>
<th>x</th>
<th>F(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.693</td>
<td>+0.05</td>
<td>-0.601</td>
</tr>
<tr>
<td>-0.05</td>
<td>-0.785</td>
<td>-0.10</td>
<td>-0.878</td>
</tr>
<tr>
<td>-0.10</td>
<td>-0.878</td>
<td>-0.15</td>
<td>-0.975</td>
</tr>
<tr>
<td>-0.15</td>
<td>-0.975</td>
<td>-0.20</td>
<td>-1.076</td>
</tr>
<tr>
<td>-0.20</td>
<td>-1.076</td>
<td>-0.25</td>
<td>-1.184</td>
</tr>
<tr>
<td>-0.25</td>
<td>-1.184</td>
<td>-0.30</td>
<td>-1.307</td>
</tr>
<tr>
<td>-0.30</td>
<td>-1.307</td>
<td>-0.35</td>
<td>-1.431</td>
</tr>
<tr>
<td>-0.35</td>
<td>-1.431</td>
<td>-0.40</td>
<td>-1.577</td>
</tr>
<tr>
<td>-0.40</td>
<td>-1.577</td>
<td>-0.45</td>
<td>-1.744</td>
</tr>
<tr>
<td>-0.45</td>
<td>-1.744</td>
<td>-0.50</td>
<td>-1.939</td>
</tr>
<tr>
<td>-0.50</td>
<td>-1.939</td>
<td>-0.55</td>
<td>-2.174</td>
</tr>
<tr>
<td>-0.55</td>
<td>-2.174</td>
<td>-0.60</td>
<td>-2.462</td>
</tr>
<tr>
<td>-0.60</td>
<td>-2.462</td>
<td>-0.65</td>
<td>-2.829</td>
</tr>
<tr>
<td>-0.65</td>
<td>-2.829</td>
<td>-0.70</td>
<td>-3.312</td>
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<tr>
<td>-0.70</td>
<td>-3.312</td>
<td>-0.75</td>
<td>-3.985</td>
</tr>
<tr>
<td>-0.75</td>
<td>-3.985</td>
<td>-0.80</td>
<td>-4.991</td>
</tr>
<tr>
<td>-0.80</td>
<td>-4.991</td>
<td>-0.85</td>
<td>-6.661</td>
</tr>
<tr>
<td>-0.85</td>
<td>-6.661</td>
<td>-0.90</td>
<td>-9.998</td>
</tr>
<tr>
<td>-0.90</td>
<td>-9.998</td>
<td>-0.95</td>
<td>-20.000</td>
</tr>
<tr>
<td>-0.95</td>
<td>-20.000</td>
<td>-1.00</td>
<td>-∞</td>
</tr>
</tbody>
</table>

The following series expression was found to agree with the above tabulation:

\[
F = -0.693 + 1.832x + 1.978x^3 + 2x^5(1-x^2)^{-1}
\]
APPENDIX 4

EXPERIMENTAL DATA

Three explosives were studied in the Hydra test pond during the summer of 1960. They were: Pentolite, the control; an equal mixture by weight of HMX + Alum \((\text{Al NH}_4\text{(SO}_4)_2 \cdot 12\text{H}_2\text{O})\); and a pressurized stoichiometric mixture of hydrogen and oxygen in a frangible plastic sphere. All charges were spherical and centrally detonated. They were detonated at various depths in the hemispherical pond of 18-ft radius. Bubble radii and periods were measured.

MAXIMUM BUBBLE RADIUS

A high speed Photosonic camera was mounted underwater, about 25 ft from the charge (in a camera bay). A six inch wire grid was mounted midway between the camera and the charge. Films showing a well defined first oscillation were reduced using the grid lines, for the first maximum horizontal diameter. The experimental \(J\) values \((J_{\exp} = AZ^{1/3}W^{-1/3})\) are compared with the theoretical \(J_{a_1}\) values in Table A.4.1.

<table>
<thead>
<tr>
<th>Explosive</th>
<th>(J_{\exp})</th>
<th>(J_{a_1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentolite</td>
<td>12.96 ± 0.48</td>
<td>13.07</td>
</tr>
<tr>
<td>RDX + Alum</td>
<td>13.01 ± 0.24</td>
<td>13.47</td>
</tr>
<tr>
<td>(2\text{H}_2 + \text{O}_2)</td>
<td>11.99 ± 0.54</td>
<td>11.97</td>
</tr>
</tbody>
</table>

* Complete experimental details can be obtained from W.W. Perkins.
Bubble periods were measured by recording on an oscilloscope camera, the pressure signal from a tourmaline piezoelectric gauge set about 12 ft from the charge. Initially the scope swept twenty milliseconds per centimeter, recording the time between the shock wave and the bubble pulse. Later in the series, in order to more accurately determine the depth effect on the period, the scope was delayed a set time (150 to 180 msec) from the initiating pulse and then swept slower (from 0.5 to 5.0 msec/cm) to record the bubble pulse only. In some cases the estimated delay time was incorrect and the bubble pulse was partially or entirely missed. But where the period was obtained, it was at least four times more accurate than those from the earlier method.

The following tabulated data are periods obtained using the delay method. Since the same pulse triggered both the explosive and the scope delay circuit, the observed time on the oscilloscope included the time of arrival of the first minimum pressure pulse. This arrival time was found to be 2.4 msec for Pentolite and ADX + Alum (50/50), and 2.8 msec for $2H_2 + O_2$. The periods in Tables A.4.2 - A.4.4 have been corrected.
### TABLE A.4.2

**Experimental Pentolite Periods**

**WEIGHT:** 1.06 lb

**GAUGE:** 0.5 inch diameter, except where noted

**DETONATOR:** Engineer's Special (0.875 g PETN, plus mercury fulminate)

<table>
<thead>
<tr>
<th>Shot</th>
<th>D</th>
<th>D+B</th>
<th>T&lt;sub&gt;exp&lt;/sub&gt;</th>
<th>T&lt;sub&gt;calc&lt;/sub&gt;</th>
<th>ΔT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ft)</td>
<td>(ft)</td>
<td>(sec)</td>
<td>(sec)</td>
<td>(sec)</td>
</tr>
<tr>
<td>66</td>
<td>4.3</td>
<td>18</td>
<td>0.1906</td>
<td>0.1850</td>
<td>-0.0056</td>
</tr>
<tr>
<td>67</td>
<td>4.3</td>
<td>18</td>
<td>0.1896</td>
<td>0.1850</td>
<td>-0.0046</td>
</tr>
<tr>
<td>50</td>
<td>5.0</td>
<td>18</td>
<td>0.1866</td>
<td>0.1892</td>
<td>+0.0026</td>
</tr>
<tr>
<td>51</td>
<td>5.0</td>
<td>18</td>
<td>0.1875</td>
<td>0.1892</td>
<td>+0.0017</td>
</tr>
<tr>
<td>901</td>
<td>5.0</td>
<td>18</td>
<td>0.1859</td>
<td>0.1892</td>
<td>+0.0033</td>
</tr>
<tr>
<td>901*</td>
<td>5.0</td>
<td>18</td>
<td>0.1854</td>
<td>0.1892</td>
<td>+0.0038</td>
</tr>
<tr>
<td>902</td>
<td>5.0</td>
<td>18</td>
<td>0.1851</td>
<td>0.1892</td>
<td>+0.0041</td>
</tr>
<tr>
<td>902*</td>
<td>5.0</td>
<td>18</td>
<td>0.1842</td>
<td>0.1892</td>
<td>+0.0050</td>
</tr>
<tr>
<td>75</td>
<td>7.0</td>
<td>19</td>
<td>0.1956</td>
<td>0.1936</td>
<td>-0.0020</td>
</tr>
<tr>
<td>74</td>
<td>9.5</td>
<td>19</td>
<td>0.1966</td>
<td>0.1937</td>
<td>-0.0029</td>
</tr>
</tbody>
</table>

*1.0 inch diameter gauge.
TABLE A.4.3
Experimental RDX + Alum (50/50) Periods

WEIGHT: 0.8 lb of RDX, plus 0.8 lb of Alum
GAUGE: 0.5 inch diameter, except where noted
DETONATOR: Modified Engineer's Special (0.875 grams tetryl, plus lead azide, plus a 2.5 gram tetryl booster)

<table>
<thead>
<tr>
<th>Shot</th>
<th>D (ft)</th>
<th>D+B (ft)</th>
<th>$T_{exp}$ (sec)</th>
<th>$T_{calc}$ (sec)</th>
<th>$\Delta T$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90*</td>
<td>4.0</td>
<td>18</td>
<td>.1746</td>
<td>.1749</td>
<td>+.0003</td>
</tr>
<tr>
<td>96*</td>
<td>4.0</td>
<td>18</td>
<td>.1770</td>
<td>.1749</td>
<td>-.0021</td>
</tr>
<tr>
<td>96</td>
<td>4.0</td>
<td>18</td>
<td>.1772</td>
<td>.1749</td>
<td>-.0023</td>
</tr>
<tr>
<td>94*</td>
<td>5.0</td>
<td>18</td>
<td>.1792</td>
<td>.1818</td>
<td>+.0026</td>
</tr>
<tr>
<td>94</td>
<td>5.0</td>
<td>18</td>
<td>.1804</td>
<td>.1818</td>
<td>+.0016</td>
</tr>
<tr>
<td>93*</td>
<td>6.0</td>
<td>18</td>
<td>.1826</td>
<td>.1854</td>
<td>+.0028</td>
</tr>
<tr>
<td>56</td>
<td>9.5</td>
<td>19</td>
<td>.1882</td>
<td>.1871</td>
<td>-.0011</td>
</tr>
<tr>
<td>57</td>
<td>9.5</td>
<td>19</td>
<td>.1886</td>
<td>.1871</td>
<td>-.0015</td>
</tr>
</tbody>
</table>

*1.0 inch diameter gauge.
TABLE A.4.4
Experimental $2H_2 + O_2$ Periods

GAUGE: 0.5 inch diameter, except where noted

DETONATOR: Pyrofuze (Palladium and Aluminum alloy wire)

<table>
<thead>
<tr>
<th>Shot</th>
<th>W (lb)</th>
<th>D (ft)</th>
<th>D+B (ft)</th>
<th>$T_{exp}$ (sec)</th>
<th>$T_{calc}$ (sec)</th>
<th>ΔT (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>0.3175</td>
<td>4.0</td>
<td>19</td>
<td>0.1712</td>
<td>0.1649</td>
<td>-.0063</td>
</tr>
<tr>
<td>70</td>
<td>0.2635</td>
<td>4.0</td>
<td>18</td>
<td>0.1572</td>
<td>0.1581</td>
<td>+.0009</td>
</tr>
<tr>
<td>77</td>
<td>0.2635</td>
<td>6.0</td>
<td>19</td>
<td>0.1582</td>
<td>0.1683</td>
<td>+.0101</td>
</tr>
<tr>
<td>87*</td>
<td>0.2635</td>
<td>6.0</td>
<td>18</td>
<td>0.1672</td>
<td>0.1686</td>
<td>+.0014</td>
</tr>
<tr>
<td>92*</td>
<td>0.2635</td>
<td>6.0</td>
<td>18</td>
<td>0.1678</td>
<td>0.1686</td>
<td>+.0008</td>
</tr>
<tr>
<td>92</td>
<td>0.2635</td>
<td>6.0</td>
<td>18</td>
<td>0.1687</td>
<td>0.1686</td>
<td>-.0001</td>
</tr>
<tr>
<td>76</td>
<td>0.2635</td>
<td>8.0</td>
<td>19</td>
<td>0.1732</td>
<td>0.1707</td>
<td>-.0025</td>
</tr>
<tr>
<td>61</td>
<td>0.3175</td>
<td>9.5</td>
<td>19</td>
<td>0.1862</td>
<td>0.1812</td>
<td>+.0050</td>
</tr>
</tbody>
</table>

*1.0 inch diameter gauge.
APPENDIX 5

METHOD OF LEAST SQUARES

The best linear fit to a series of values is a line about which the sum of the squares of the deviations is a minimum. Applying this principal to $y = K + HK^2z$, and minimizing the error in $y$, we find:

$$K = (\Sigma yz^2 - \Sigma z\Sigma y)/\left(\Sigma z^2 - \Sigma z^2\right)$$

$$HK^2 = (\Sigma yz - \Sigma yz)/\left(\Sigma z^2 - \Sigma z^2\right)$$

where $N$ is the number of data sets $(y, z)$.

If the error in $z$ were minimized, then:

$$K = (\Sigma yz - \Sigma z\Sigma y^2)/\left(\Sigma z^2 - \Sigma z^2\right)$$

$$HK^2 = (\Sigma yz - \Sigma zy^2)/\left(\Sigma z^2 - \Sigma z^2\right)$$
APPENDIX 6

ERRORS IN K AND H

From the method of least squares, K is proportional to \( y \) which equals \( T^{5/6} W^{-1/3} \), and H is proportional to \( 1/yz \) which equals \( (D+B)/TFZ^{1/2} \). Thus any errors in the measurement of the period and or the depths will affect the accuracy of K and H. K will also be affected by weight errors. From the experimental data, the period errors and the weight errors are each less than 2%. To determine the effect of the depth error, the following assumptions were made: (1) the error in charge depth \( D \) due to water surface wave motion is less than 0.18 feet, and (2) the pond depth error, due to water blown out by preceding shots, is less than 0.5 feet.

Then for this data, the 0.5 foot variation in pond depth will cause less than 4.7% error in \( (D+B)/F \). The 0.18 foot variation in charge depth will cause less than 0.4% error in \( z^{5/6} \) and less than 5.6% error in \( F^{1/2} \). The hydrostatic charge depth in sea water is \( D+33 \) feet while in fresh water it is \( D+34 \) feet. The variation is less than 2.3% in \( z^{5/6} \) and less than 1.4% in \( F^{1/2} \).

Thus, from the above assumptions, the total error in K is less than 5.5% and the total error in H is less than 1.4%.

An additional possible error in the experimentally determined K should be mentioned here. Rudlin's work* indicates that for spherical explosives, the fraction of the charge consumed is a function of the

---

charge size. Assuming Rudlin's theory for TNT applies for Pentolite, the following values were calculated. Here N is the fraction of explosive consumed.

<table>
<thead>
<tr>
<th>W(lbs)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.759</td>
</tr>
<tr>
<td>1.06</td>
<td>0.880</td>
</tr>
<tr>
<td>1.16</td>
<td>0.942</td>
</tr>
</tbody>
</table>

The experimental value of $K'$ was found to be 4.30 for a 1.06 pound Pentolite charge. But if only 88% of the explosive was consumed, the actual weight was 88 x 1.06 pounds. Since $K W^{1/3}$ is constant, the experimentally determined $K'$ value can be predicted to vary with the charge weight: $K'_{exp} = 4.49 N^{1/3}$. In other words, the $K'$ value found with a 1.06 pound Pentolite charge should be 2% less than the $K'$ value found using a 1.16 pound charge and 5% higher than the $K'$ value found using a 0.05 pound charge. This effect should not apply to the $2H_2$ + $O_2$ explosive.
APPENDIX 7

SYMBOLS

All dimensions are in the fps system

A  bubble radius
a  non-dimensional radius
B  charge distance from bottom
b  B/L
C  unit of time
D  charge depth
d  D/L
E  internal bubble energy
F, F(x) function giving surface effects on period
f(x) function giving surface effects on period
g  acceleration of gravity
H  second order bubble period constant
I  see Appendix I
J  bubble radius constant
K  bubble period constant
KE  kinetic energy of water
k  \(a_o^3(\gamma-1)\)
L unit of length
P bubble pressure
Ph hydrostatic pressure at charge depth; = \rho g Z
Q charge energy per unit weight
r fraction of charge energy in bubble
s non-dimensional vertical momentum of bubble
T time variable
T2 time to first minimum, i.e. bubble period
t non-dimensional time variable; = T/C
V bubble volume
W charge weight
WD work of displacement
x \( \frac{(D-B)}{(D+B)} \)
Z D+33 ft (34 for fresh water)
\gamma ratio of specific heats of bubble gas
\rho mass density of water
REFERENCES


DASA-HYDRA IIA DIST

INITIAL DISTRIBUTION

NAVY

<table>
<thead>
<tr>
<th>NO.</th>
<th>CPY</th>
<th>Name and Position</th>
</tr>
</thead>
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<tr>
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<td></td>
<td>CHIEF, BUREAU OF SHIPS (CODE 320)</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>CHIEF, BUREAU OF SHIPS (CODE 210L)</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>CHIEF, BUREAU OF SHIPS (CODE 3628)</td>
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<td>4</td>
<td></td>
<td>CHIEF, BUREAU OF NAVAL WEAPONS (RRMA-11)</td>
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<td></td>
<td>CHIEF, BUREAU OF NAVAL WEAPONS (RRE-5)</td>
</tr>
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<td></td>
<td>CHIEF, BUREAU OF YARDS AND DOCKS (CODE 423)</td>
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<td>7</td>
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<td>CHIEF, BUREAU OF YARDS AND DOCKS (CODE 42)</td>
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<tr>
<td>8</td>
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<td>DIRECTOR, BUREAU OF YARDS AND DOCKS (NORTHWEST DIV)</td>
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<td></td>
<td>CHIEF OF NAVAL OPERATIONS (OP-07T)</td>
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<td>DIRECTOR, NAVAL RESEARCH LABORATORY (CODE 220)</td>
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<td>15</td>
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<td>CO, OFFICE OF NAVAL RESEARCH, FPO, NEW YORK</td>
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<tr>
<td>16</td>
<td></td>
<td>CO, U.S. NAVAL CIVIL ENGINEERING LABORATORY</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>CO, NAVAL AIR DEVELOPMENT CENTER</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>COMM, U.S. NAVAL ORDNANCE LAB, WHITE OAK (CODE EU)</td>
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<td>19</td>
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<td>20</td>
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<td>COMM, U.S. NAVAL ORDNANCE LAB, WHITE OAK (CODE EA)</td>
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1
A summary is presented of the derivation of the equation of motion of the bubble due to an underwater spherical explosion. Migration and surface effects are included. The formulas for the first maximum radius and the first oscillation period, including surface effects, are given.

From the experimentally determined bubble period data and the theoretical period formula:

\[ T_2 = \frac{KW^{1/3}Z^{-5/6}}{1.0 + KW^{1/3}Z^{-1/3}HF(D+R)} \]

the energy partition of three explosives, Pentolite, RDX + Alum (50/50), and \( 2H_2 + O_2 \) are calculated.