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RESEARCH INVESTIGATION OF HYDRAULIC PULSATION CONCEPTS

Third Quarterly Progress Report

RAC 933-3

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REPUBLIC AVIATION CORPORATION
Farmingdale, L.I., N.Y.

NOTICE

The work covered by this report was accomplished under Air Force Contract AF 33(657)-10622, but this report is being published and distributed prior to Air Force review. The publication of this report, therefore, does not constitute approval by the Air Force of the findings or conclusions contained herein. It is published for the exchange and stimulation of ideas.

ABSTRACT

This is the Third Quarterly Progress Report under Contract AF33(657)-10622. During the period of this report, designs of a piston type transformer, fluid level control unit, and rectifier valve were studied. Analysis of transmission lines considered as distributed parameters, together with other transmission line effects and parameters were made. Over-all efficiency of the miniaturized pulsating system received preliminary study. Materials for use with liquid metal fluids were also studied.

FOREWORD

This Third Quarterly Progress Report on the program for Research Investigation of Hydraulic Pulsation Concepts, covering the period from 1 September 1963 to 30 November 1963, was prepared by Republic Aviation Corporation, Farmingdale, New York, under USAF Contract AF 33(657)-10622 with the Flight Vehicle Power Branch of the Aero Propulsion Laboratory, Research and Technology Division. The program is scheduled for a period of 20 months, beginning March 1963 and ending October 1964.

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SECTION 1 - INTRODUCTION

The fundamental objective of this program is to establish the technical feasibility of the transmission of hydraulic power through the use of pulsating flow and/or pulsating pressure concepts. To accomplish this end, research is proceeding along the following lines:

(1) Analyses and experimental component designs will be made to establish the hydraulic functional equivalents of the following electrical components: generators, transformers, rectifiers, meters, controls, and transmission and distribution lines.

(2) Analyses of pulsating flow and pulsating pressure hydraulic parameters will be performed. These analyses will at least include the following parameters: pressure attenuation, transmission line impedance, frequencies, line sizes and lengths, fluid effects, and efficiency of lines and components.

(3) Analytical investigations of a pulsating flow or pulsating pressure subsystem will be utilized for comparison of this subsystem with a representative continuous flow hydraulic subsystem which delivers power to a simulated aileron control. From the analyses will be determined the relative weight, efficiency, undamped natural frequency, and frequency response.

SECTION II - SUMMARY

Designs for a piston type transformer and a fluid replenishment unit were completed.

The transformer piston is double ended with a 1:1 area ratio. A center land is incorporated to separate any fluid leaking past the seals at both ends of the piston. The cylinder barrel incorporates two separate drain ports to conduct this leakage away from the unit.

The transformer was designed to be packaged with the fluid replenishment unit. The piston is provided with a stud at one end to actuate the thermal expansion relief valve incorporated in the replenishment unit. The piston is provided with a stud at one end to actuate the thermal expansion relief valve incorporated in the replenishment unit.

The fluid replenishment unit was designed to contain 0.25 gal of fluid pressurized to 100 psi. Two methods of pressurization were investigated: bootstrap pressure and mechanical springs. The bootstrap design showed a weight saving of 9.42 pounds over the spring loaded design.

A study of rectifier design has been completed and the advantages of cone or ball type poppets compared with the face seating type. It was concluded that due to the very high cyclic life required, the face seating type will be more satisfactory.

In a 3-phase, 10 gpm pulsating system, it is found that the average line flow is 6.37 gpm.

A standard stock size stainless steel tubing, 0.5-inch OD x 0.042-inch is found adequate to carry the above flow at an average fluid velocity of 15 fps.

In the analytical program a study of the losses in hydraulic transmission lines treated as distributed parameters has been completed. A laboratory setup for the study of line losses as distributed parameters has been completed and is being instrumented. Study has also been made of the resonance conditions of transmission lines treated as distributed parameters with and without line losses. Also the effect of a series hydraulic condenser on the distributed line pressure has been analyzed. Experimental data from the miniaturized pulsating system setup relative to over-all system efficiency has been studied and a breakdown made.

One of the most serious problems in liquid metals systems is corrosion. Careful selection of material combinations and system design will help reduce this problem. The refractory metals, nickel-base and cobalt-base alloys, and the austenitic and ferritic stainless steels are sufficiently resistant to NaK for use to 1400°F.

SECTION III - DESIGN

Designs for piston transformers, piston pressurized fluid replenishment units, and poppet type rectifiers were completed during this reporting period.

The transformed and replenishment unit were designed as units of a single package which also incorporates a check valve for supplying leakage make-up fluid and a mechanically actuated poppet valve for relieving thermally expanded fluid. Design and operation of these components are discussed in the following sections.

A. TRANSFORMER

The transformer was sized for a 3-phase, 3-line, pulsating hydraulic system operating at a frequency of 8 cps and delivering a rectified flow of 10 gpm at 4000 psi.

Assuming the generator in this system produces sinusoidal flow having a peak value of 10 gpm in each of the three lines, then by integrating the sine curve an average flow of 6.37 gpm is obtained in each of the three lines.

The transformer piston displacement was determined as follows:

$$6.37 \text{ gal/min} = 6.37 \text{ gal/min} \times 231 \text{ in}^3\text{/gal} \times \frac{1}{60} \text{ min/sec} = 24.5 \text{ in}^3\text{/sec}$$

For a pulsation frequency of 8 cps, the period is $1/f = 1/8$ cps = .125 sec/cyc. Since the average flow for each half cycle is 24.5 in^3/sec, the amount of fluid displaced in each half cycle is 24.5 x .125/.2 = 1.535 in^3.

Based on tests already conducted on the miniaturized system, a piston stroke of 1/2 inch was tentatively selected as being compatible with the assumed pulsation frequency of 8 cps. The piston diameter was then determined from the known displacement and stroke.

$$\text{Piston area} = \frac{\text{displacement}}{\text{stroke}} = \sqrt{\frac{1.535}{.5}} = 3.07 \text{ sq. in.}$$

$$\text{Piston diameter} = 1.128 \quad 3.07 = 1.975 \text{ in.}$$

The piston diameter was rounded out to a nominal diameter of 2.00 inches for which piston rings and seals are available.

The piston is a double ended design with a one to one ratio. A center land is provided to prevent mixing of any leakage flow past the seals at the two ends of the piston. A drain is provided on each side of the center land to segregate the leakage.

For a 3-phase, 3-line, 2-fluid system, three transformers and one fluid replenishment unit will be required. For a 3-phase, 3-line, 3-fluid system, six transformers and four fluid replenishment units will be required as shown in Figure 1. The four piston transformers shown joined to the fluid replenishment units will incorporate a slightly modified piston. These pistons will be provided with a stud at one end to enable actuation of the thermal expansion check valve as explained below.

B. FLUID REPLENISHMENT UNIT

In the underlying philosphy guiding the design of the fluid replenishment unit, it is assumed that if this component is to be of any practical value then fluid replenishment must be accomplished while the system is in operation.

To transfer fluid from a reservoir to a closed system, some pressure must be applied to the fluid in the reservoir. Pressurizing the fluid by means of an inert gas was ruled out because of the following problems:

1) Additional system complexity
2) Additional system weight
3) Reduced reliability

Such a system would require the use of stored pressurized gas to maintain the fluid at constant pressure. A replenishment unit designed along the lines of a precharged bladder or piston type accumulator would be impractical because accumulators cannot maintain constant gas pressure with varying ambient temperatures.

The logical solution was to design the replenishment unit similar to a piston type reservoir if the problems of the accumulator configuration are to be avoided.

In the piston type replenishment unit two methods are available for applying a force to the piston. One method is the use of mechanical springs and the other is the use of bootstrap pressure.

A replenishing fluid pressure of 100 psi and 0.25 gallon of fluid were tentatively selected to enable initial sizing of the unit. With this relatively small pressure and reasonable quantity of fluid (based on the amount of fluid contained in the F-105 primary system reservoir) the spring loaded piston unit appeared worthy of investigation.

Design layouts, on both the spring loaded and bootstrap pressure loaded piston units, were made to determine the weight difference of the two configurations.

The cylinder dimensions to enclose 0.25 gallon (57.75 in.^{3}) of fluid were determined from the following formulae which gave the minimum surface area (and therefore weight) of a cylinder to enclose a given volume:

$$\text{DIAMETER} = 2\sqrt[3]{\frac{\text{Volume}}{2\pi}} = 2\sqrt[3]{\frac{57.75}{6.28}} = 4.19 \text{ inches}$$

$$\text{LENGTH} \cong 1.1\sqrt[3]{\text{Volume}} = 1.1\sqrt[3]{57.75} = 4.26 \text{ inches}$$

The piston diameter was reduced to a nominal 4.00 inches for which piston rings are readily available. The cylinder length was proportionately increased to 4.60 inches to maintain the same volume.

For the spring loaded piston configuration, a pair of nested springs were designed to provide the required piston force. Several nested spring combinations as well as single springs were investigated from the standpoints of compactness and weight before the final nested combination was arrived at.

The nested spring design afforded an over-all weight saving of 2.27 pounds over the best single spring design. Solid height of the nested spring combination is 11.6 inches while that of the single spring design is 15.15 inches. The difference, 3.55 inches, represents the reduction in housing length for the nested design. This reduced housing length alone afforded a weight saving of 1.71 pounds. The combined weight of the nested springs is 10.67 pounds while that of the single spring is 11.23 pounds, resulting in a weight saving of .56 pounds for the nested design.

Efficient use of spring material in the nested design was achieved by designing the inner and outer springs to have identical index values and solid height stresses.

Inconel-X was chosen as the spring material, since the replenishment unit was designed for operation at 500°F. With a full reservoir pressurized to 100 psi, the springs are stressed to 90,000 psi. No relaxation of the spring occurs when operating at this stress level at 500°F.

In the bootstrap pressure-loaded piston design, the over-all length of the cylinder is 3.75 inches less than that for the spring loaded piston design. This reduced cylinder length affords a weight saving of 1.8 pounds. Elimination of the springs and substitution of the bootstrap pressure piston net an additional saving of 9.4 pounds.

As shown in Figure 1, use of the bootstrap pressurized replenishment unit requires the addition of only two pressurizing lines for a 3-phase, 3-fluid, pulsating system. The constant pressure of system "A" is utilized to pressurize the replenishment unit for system "B". The rectified pressure of system "C" pressurizes its own replenishment unit, in a real "bootstrap" sense.

The design studies of the two types of replenishment units point out the big weight advantage of the bootstrap version over the spring loaded version. Another advantage of the bootstrap version is that it will maintain constant pressure on the fluid regardless of the amount of fluid lost through leakage.

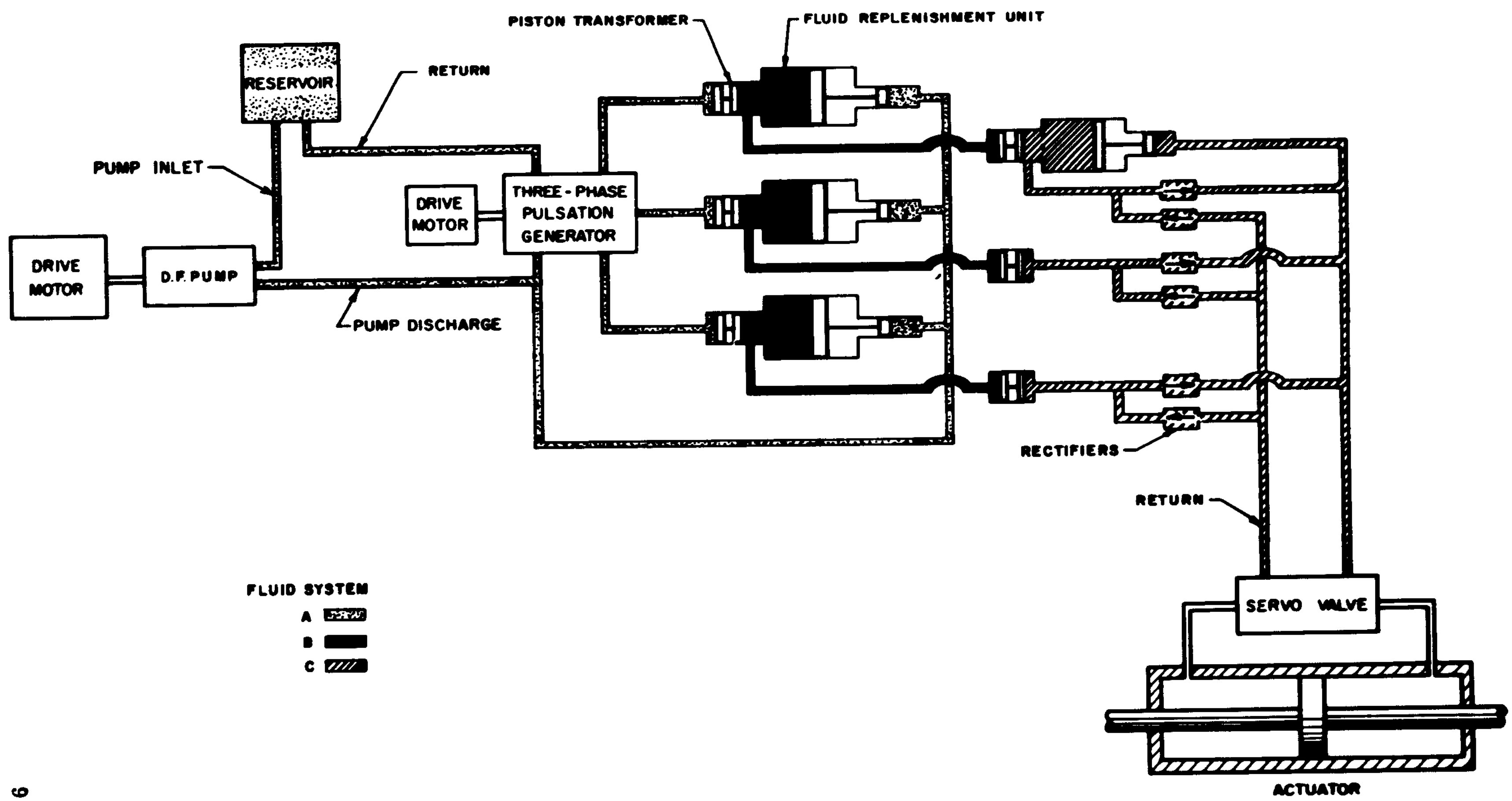


Figure 1. Three-Phase, Three-Fluid Pulsating Hydraulic System

In the spring loaded version, the pressure of the fluid decreases as the volume decreases due to leakage. When nearly all of the fluid is lost through leakage (short of piston bottoming) the fluid pressure decreases to 16.27 psi. The total load exerted by both springs at this point would be 204.26 pounds. This minimum spring load is also depended upon to overcome the piston seal friction.

The only disadvantage of the bootstrap replenishment unit is the requirement of additional lines, fittings, and piston seals which tend to lessen the over-all reliability of the unit.

The fluid replenishment unit incorporates two check valves to control fluid level. One of the check valves is a ball type and operates automatically to replenish fluid during the return pulse of the cycle. The second check valve utilizes a conical poppet and relieves thermally expanded fluid. It is mechanically lifted off its seat by the transformer piston when the piston oscillates about a center other than its normal center of travel. Off-center travel is induced by unequal thermal expansion of the pulsed fluids.

C. RECTIFIER

In order to rectify pulsating flow to continuous flow, a hydraulic check valve provides a very convenient tool since it is the mechanical equivalent of the electrical diode. Two check valves are used in each pulsating line; one with free flow leading from the pulsating line to the continuous flow pressure circuit, and the other with free flow leading toward the pulsating line from the continuous flow return circuit. For a two line pulsating transmission system (180° interline phase shift), two pairs of check valves will be required to produce full wave rectified continuous flow. For a three line pulsating transmission system (120° interline phase shift) three pairs of check valves will be required for rectification to continuous flow.

During the reporting period a design study was made to determine the advisability of combining two or more check valves in a single housing. In general, it is believed that for the rectification of a single pulsating line, there is little or nothing to be gained by the provision of a combined or dual unit. The number of plumbing connections necessary will be reduced from four to three (pulsating line continuous pressure, and continuous return). Even with a single line there will be an elimination of the possibility of reversed assembly of the individual check valves when a dual unit is provided, however, such a unit must accept some slight penalty of installation convenience. For the rectification of a pulsating system consisting of two or more lines, there appears to be a definite advantage in providing for a combined unit. The space requirement for the rectifier is considerably reduced from that taken by individual check valves. The elimination of a possible reversed connection still applies. The system reliability will be enhanced, due to the reduction in the number of plumbing connections (in a three line system 5 connections will be required for a combined unit, versus 12 connections for individual units), and the system weight will thus be decreased.

The conventional check valve design currently being used in hydraulic components utilizes a conical or a ball type poppet which is seated against a sharp edge by a spring. Since in a pulsating circuit a rectifying check valve will unseat and seat for each cycle of pulsation, the resultant requirement of cyclic life will be in excess of that which can be expected with an edge-seating poppet. Therefore, a design study was made of a rectifier valve utilizing check valves with face seating poppets. This type poppet is ground with a flat surface which seats against a similar surface. Such a seat has a much higher leakage than an edge seating poppet, which can brinell a full seating ring, but the unit bearing force is so drastically reduced that its endurance will be greatly increased and should result in a unit which will be satisfactory for the high cyclic life necessary for a pulsating system.

D. LINES

A study was made of the line size required for a 3-phase, 10 gpm pulsating system. As stated previously, the average flow rate in the lines of the above system is 6.37 gpm. Specification MIL-H-5440D specifies that 0.5-inch OD shall be used for an average flow rate of 6.0 gpm. The specification tube sizes are based on the use of aluminum alloy tubing which requires a thicker wall than steel tubing and thereby results in a smaller ID.

Since this study was based on an operating fluid temperature of 500°F, the use of a high temperature material, such as stainless steel, was automatically dictated. The higher tensile strength of stainless steel permits the use of a thinner wall tubing. This results in a larger ID and therefore decreases fluid velocity and pressure drop. The larger ID afforded by the use of stainless steel justifies the use of 0.5-inch OD tubing for the average flow rate of 6.37 gpm obtained in a 3-phase, 10 gpm system.

Use of 0.5-inch OD tubing is further justified by the following computation:

Given a flow rate of 6.37 gpm (24.5 in^3/sec) and an average flow velocity of 15 ft/sec (Spec. MIL-H-5440D), the required tube flow area was obtained from the following equation

$$A = \frac{Q}{V} = \frac{24.5}{15 \times 12} = .1361 \text{ in}^2$$

The wall thickness of a corrosion resistant steel tubing having a minimum ultimate tensile strength of 95,000 psi was computed from Barlow's formula

$$t = \frac{DP}{2S} = \frac{.5 \times 16{,}000}{2 \times 95{,}000} = .0422 \text{ inches}$$

Stainless steel tubing 0.5-inch OD x .042-inch wall is a standard stock size. The nominal ID of this tube, .416 inches, results in a flow area of .136 in^2, which exactly meets the required area.

SECTION IV - ANALYSIS

A. SYSTEM COMPONENTS AND THEIR ELECTRICAL EQUIVALENTS (Continued from the Second Quarterly Progress Report) *

1. Distributed Transmission Line with Friction Loss and Low Average Flow Velocity

In the Second Quarterly Progress Report, only transmission lines with lumped parameters were discussed. In this subsection transmission lines with distributed parameters are discussed. For an unsteady, compressible, low average velocity, and one dimensional flow in an uniform tube, the following pair of wave equations can be used to describe its wave propagation. (See Appendix A for the derivation of the equations in this subsection.)

$$\frac{\partial^2 P}{\partial X^2} - \gamma^2 P = 0 \tag{1}$$

$$\frac{\partial^2 Q}{\partial X^2} - \gamma^2 Q = 0 \tag{2}$$

where γ, propagation constant $= \sqrt{ZY}$

The solutions of the above wave equations are

$$P = P_1 e^{jwt} e^{\gamma X} + P_2 \; e^{j\omega t} \, e^{-\gamma X} \tag{3}$$

$$Q = - P_1 \sqrt{\frac{Y}{Z}} \; e^{j\omega t} \; e^{\gamma X} + P_2 \sqrt{\frac{Y}{Z}} \; e^{j\omega t} \, e^{-\gamma X} \tag{4}$$

* A cumulative nomenclature (First, Second, and Third Quarterly Progress Report) may be found in Section IV — D.

The characteristic impedance of the line, which is the ratio of Equation 3 and 4, is

$$Z_s = \sqrt{\frac{Z}{Y}} = \frac{1}{Y_s} \tag{5}$$

The general impedance expression of the line at a distance X looking toward the load is found to be

$$Z_X = Z_s \frac{Z_\ell + Z_s \tanh \gamma X}{Z_s + Z_\ell \tanh \gamma X} \tag{6}$$

where Z_ℓ is load impedance.

Equation 6 shows that the hydraulic line impedance is not only a function of the line characteristic impedance Z_s, but also is a function of load impedance, Z_ℓ, and the propagation constant, γ.

Replacing the hydraulic line characteristic impedance, load impedance, and propagation constant of Equation 6 by that of the electric transmission line produces the electrical transmission line impedance.

Equation 6 and its important parameters are discussed in more detail here for both loss-less and linear loss hydraulic lines.

a. Line Characteristic Impedance:

The line characteristic impedance is defined previously as

$$Z_s = \sqrt{\frac{Z}{Y}} \tag{7}$$

For mathematical convenience, Z and Y are represented by the following forms

$$Z = r_1 e^{j\theta_1} = \sqrt{\alpha_1^2 + \beta_1^2}\; e^{j\left(\tan^{-1} \frac{\beta_1}{\alpha_1}\right)} \tag{8}$$

$$Y = r_2 e^{j\theta_2} = \sqrt{\alpha_2^{\,2} + \beta_2^{\,2}}\; e^{j\left(\tan^{-1} \frac{\beta_2}{\alpha_2}\right)} \tag{9}$$

where

$$\alpha_1 = \frac{R_h}{A}, \quad \beta_1 = \frac{\rho\omega}{A}, \quad \alpha_2 = 0, \quad \beta_2 = \frac{A\omega}{\beta_e} \tag{10}$$

Equation 7 can be expanded into the following form

$$Z_s = \frac{1}{2r_2}\left\{\left[\sqrt{(r_1+\alpha_1)(r_2+\alpha_2)} + \sqrt{(r_1-\alpha_1)(r_2-\alpha_2)}\right] + j\left[\sqrt{(r_1-\alpha_1)(r_2+\alpha_2)} - \sqrt{(r_1+\alpha_1)(r_2-\alpha_2)}\right]\right\} \tag{11}$$

When α_1 and α_2 are smaller than r_1 and r_2 respectively, but are not negligible, Equation 11 reduces to

$$Z_s = \sqrt{\frac{r_1}{r_2}}\left[1 - j\,\frac{1}{2}\left(\frac{\alpha_2}{r_2} - \frac{\alpha_1}{r_1}\right)\right] \tag{12}$$

When $\alpha_1 << r_1$ and $\alpha_2 << r_2$, Equation 11 becomes

$$Z_s = \sqrt{\frac{r_1}{r_2}} \tag{13}$$

Substituting Equation 10 into Equations 12 and 13 and neglecting second order terms, the line characteristic impedance further reduces to the following two cases:

(1) Line With Low Average Velocity Flow (but not loss-less)

$$\begin{aligned} Z_s &\simeq \frac{1}{A}\sqrt{\rho\beta_e}\left(1 - j\,\frac{R_h}{2\rho\omega}\right) \\ &= \frac{\rho\epsilon}{A}\left(1 - j\,\frac{R_h}{2\rho\omega}\right) \end{aligned} \tag{12a}$$

(2) **Loss-Less Line:**

$$Z_s \simeq \frac{1}{A}\sqrt{\rho\beta_e} = \frac{\rho c}{A} \tag{13a}$$

b. Propagation Constant:

The propagation constant is defined previously as

$$\gamma = \sqrt{ZY} \tag{14}$$

Equation 14 can be expanded into the following form

$$\gamma = \frac{1}{2}\left\{\left[\sqrt{(r_1+\alpha_1)(r_2+\alpha_2)} - \sqrt{(r_1-\alpha_1)(r_2-\alpha_2)}\right] + j\left[\sqrt{(r_1-\alpha_1)(r_2+\alpha_2)} - \sqrt{(r_1+\alpha_1)(r_2-\alpha_2)}\right]\right\} \tag{15}$$

When α_1 and α_2 are smaller than r_1 and r_2 respectively, but are not negligible, Equation 15 reduces to

$$\gamma = \sqrt{\frac{r_1 r_2}{2}}\left[\left(\frac{\alpha_1}{r_1} + \frac{\alpha_2}{r_2}\right) + j\left(2 - \frac{\alpha_1\alpha_2}{r_1 r_2}\right)\right] \tag{16}$$

when $\alpha_1 << \gamma_1$ and $\alpha_2 << r_2$. Equation 15 reduces to

$$\gamma = j\sqrt{\frac{r_1 r_2}{2}} \tag{17}$$

Substituting Equation 10 into Equations 16 and 17 and neglecting second order terms, the propagation constant becomes for line with low average velocity flow (but not loss-less):

$$\gamma \simeq \omega\sqrt{\frac{\rho}{\beta_e}}\left(\frac{R_h}{2\rho\omega} + j\right) = \frac{\omega T_e}{\ell}\left(\frac{R_h}{2\rho\omega} + j\right) \tag{18}$$

For loss-less line:

$$\gamma \simeq j\,\omega\sqrt{\frac{\rho}{\beta_e}} = j\,\frac{T_e\,\omega}{\ell} \tag{19}$$

c. Hydraulic Line Impedance:

For line with low average flow velocity and friction loss, the line impedance is

$$Z_X = Z_s \frac{Z_\ell + Z_s \tanh \gamma X}{Z_s + Z_\ell \tanh \gamma X} \tag{20}$$

The input impedance is obtained by replacing X by total line length, ℓ

$$Z_o = Z_s \frac{Z_\ell + Z_s \tanh \gamma \ell}{Z_s + Z_\ell \tanh \gamma \ell} \tag{21}$$

For loss-less line the line impedance is:

$$Z_X = Z_s \frac{Z_\ell + j\, Z_s \tan \gamma X}{Z_s + j\, Z_\ell \tan \gamma X} \tag{22}$$

The input impedance is obtained by replacing X by total line length, ℓ

$$Z_o = Z_s \frac{Z_\ell + j\, Z_s \tan \gamma \ell}{Z_s + j\, Z_\ell \tan \gamma \ell} \tag{23}$$

where:

$$\gamma = \omega \sqrt{\frac{\rho}{\beta_e}} = \omega \frac{T_e}{\ell}$$

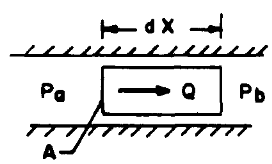
X = the distance from the load impedance

Z_ℓ = the line imput impedance. This expression is very useful in finding the line resonance condition.

In order to establish an equivalence between the hydraulic line (with low average flow velocity) and the electric transmission line, the basic properties of the fluid in a line are represented by their equivalent electric properties.

Inertance (inductance):

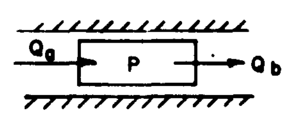
$$P_a - P_b = \left(\frac{P}{A} \, dX \right) \frac{dQ}{dt} \tag{24}$$



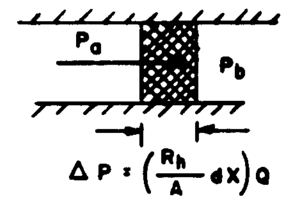
Where m, inertance per unit length of line = $\frac{\rho}{A}$

Capacitance:

$$Q_a - Q_b = \left(\frac{A}{\beta_e} \, dX \right) \frac{dP}{dt} \tag{25}$$



where C_h, capacitance unit length of line $= \frac{A}{\beta_e}$



Resistance:

$$\left(P_a - P_b\right) = \left(\frac{R_h}{A}\, dX\right) Q \qquad (26)$$

Where $\frac{R_h}{A}$, fluid resistance per unit length of line.

The equivalents between the characteristic impedance and propagation constant of the hydraulic and electric line are shown in Table I.

TABLE I

THE CHARACTERISTIC IMPEDANCE AND PROPAGATION CONSTANT OF THE HYDRAULIC AND ELECTRIC LINE

		Characteristic Impedance, Z_s	Propagation Constant, γ
Hydraulic Line		$\sqrt{\frac{m}{C_h}}\left[1 + j\left(\frac{S}{2\omega C_h} - \frac{R_h/A}{2\omega m}\right)\right]^{*}$ $= \frac{1}{A}\sqrt{\rho\beta_e}\left(1 - j\,\frac{R_h}{2\rho\omega}\right)$	$\omega\sqrt{m\,C_h}\left[\frac{1}{2\omega}\left(\frac{R_h/A}{m} + \frac{S}{C_h}\right) + j\right]^{*}$ $= \omega\sqrt{\frac{\rho}{\beta_e}}\left(\frac{R_h}{2\rho\omega} + j\right)$
Hydraulic Line	Loss-Less	$\sqrt{\frac{m}{C_h}} = \frac{1}{A}\sqrt{\rho\beta_e}$	$j\,\omega\sqrt{m\,C_h} = j\,\omega\sqrt{\frac{\rho}{\beta_e}}$
Electric Line	With Small Loss	$\sqrt{\frac{L}{C}}\left[1 + j\left(\frac{G}{2\omega C} - \frac{R}{2\omega L}\right)\right]$	$\omega\sqrt{LC}\left[\frac{1}{2\omega}\left(\frac{R}{L} + \frac{G}{C}\right) + j\right]$
Electric Line	Loss-Less	$\sqrt{\frac{L}{C}}$	$j\,\omega\sqrt{LC}$

* The leakage per unit length of hydraulic line, s, is assumed to be zero.

B. THE EFFECT OF FLOW PARAMETERS TO THE SYSTEM COMPONENT PERFORMANCES

1. Resonance in a Distributed and Loss-Less Hydraulic Line

The resonant frequency of a hydraulic line is defined as that at which the reactive component of the input impedance reduces to zero.

Let the load impedance be

$$Z_\ell = R_\ell + j\,X_\ell \tag{27}$$

For convenience the load impedance is replaced by

$$Z_\ell = Z_s\,(\alpha + j\,\beta) \tag{28}$$

where

$$\alpha = \frac{R_\ell}{Z_s}, \quad \beta = \frac{X_\ell}{Z_s}, \text{ and } Z_s = \frac{\rho \epsilon}{A}$$

Substituting Equation 28 in the input impedance

$$Z_o = Z_s \frac{Z_\ell + j\,Z_s \tan \frac{\omega}{\epsilon}\ell}{Z_s + j\,Z_\ell \tan \frac{\omega}{\epsilon}\ell} \tag{29}$$

and letting the reactive component be zero, produces the resonant condition

$$\beta \tan^2 \frac{\omega}{\epsilon}\ell + (\beta^2 + \alpha^2 - 1)\tan \frac{\omega}{\epsilon}\ell - \beta = 0 \tag{30}$$

If α and β are much smaller than unity, Equation 30 can be simplified to

$$\tan \frac{\omega}{\epsilon}\ell = -\beta \tag{31}$$

The resonant frequency can be predicated by Equation 30 or Equation 31 when the fluid properties, load impedance and line length are known.

2. Resonance in a Distributed Line with Friction Loss and Low Average Flow Velocity

The input and line characteristic impedance of a distributed line with friction loss and low flow velocity are

$$Z_o = Z_s \frac{Z_\ell + Z_s \tanh \gamma\,\ell}{Z_s + Z_\ell \tanh \gamma\,\ell} \tag{32}$$

and

$$Z_s = \frac{\rho \epsilon}{A}\left(1 + j\frac{R_h}{2\rho\omega}\right) \quad , \tag{33}$$

respectively.

Where the propagation constant

$$\gamma = \frac{\omega}{\epsilon}\left(\frac{R_h}{2\rho\omega} + j\right) \tag{34}$$

The load impedance is

$$Z_\ell = R_\ell + j X_\ell = Z_s(\alpha + j\beta) \tag{35}$$

where:

$$\alpha = \frac{R_\ell - \dfrac{X_\ell R_h}{2\rho\omega}}{\dfrac{\rho\epsilon}{A}\left[1 + \left(\dfrac{R_h}{2\rho\omega}\right)^2\right]} \tag{36}$$

$$\beta = \frac{X_\ell + \dfrac{R_\ell R_h}{2\rho\omega}}{\dfrac{\rho\epsilon}{A}\left[1 + \left(\dfrac{R_h}{2\rho\omega}\right)^2\right]}$$

Substituting Equation 35 in Equation 32 and letting the reactive component be equal to zero, produces the resonant condition.

$$(1 - \alpha^2 - \beta^2)\tan\frac{2\omega}{\epsilon}\ell + 2\beta = 0 \tag{37}$$

If α and β are much smaller than unity, Equation 37 reduces to

$$\tan\frac{2\omega}{\epsilon}\ell = -2\beta \tag{38}$$

The resonant frequency can, therefore, be predicted once the fluid properties, load impedance, and line length are known.

3. The Effect of Series Condensers on the Pressure Output of the Distributed and Loss-less Line

As the fluid acceleration increases due to the increase of pulsating frequency and stroke, the inertia of the fluid will not only reduce the magnitude but also cause a phase shift of the pressure output. To reduce (or eliminate if possible) the inertia effect, two identical hydraulic condensers can be connected in series with the hydraulic line.

The wave Equations 34 and 40 are solved here by different manner to show the effect of the series condensers. The wave equations are rewritten here for convenience:

$$\frac{\partial^2 P}{\partial X^2} - \gamma^2 P = 0 \tag{39}$$

$$\frac{\partial^2 Q}{\partial X^2} - \gamma^2 Q = 0 \tag{40}$$

where γ, propagation constant $= \sqrt{Zy}$

For loss-less line, $\gamma = j\omega \frac{\rho}{\beta_e} = j\frac{\omega}{\epsilon}$

Equation 30 and 40 reduce to

$$\frac{\partial^2 P}{\partial X^2} + \frac{\omega^2}{\epsilon^2} P = 0 \tag{39a}$$

$$\frac{\partial^2 Q}{\partial X^2} + \frac{\omega^2}{\epsilon^2} Q = 0 \tag{40a}$$

The general solutions of Equations 39a and 40a are

$$P = \Psi_1 \cos\frac{\omega}{\epsilon} X + \Psi_2 \sin\frac{\omega}{\epsilon} X \tag{41}$$

$$Q = \Psi_3 \cos\frac{\omega}{\epsilon} X + \Psi_4 \sin\frac{\omega}{\epsilon} X \tag{42}$$

Using boundary conditions

$$\left.\begin{aligned} P &= P_o \\ Q &= Q_o \end{aligned}\right\} \text{at } X = 0$$

$$\left.\begin{aligned} \frac{\partial P}{\partial X} &= -j\frac{\omega\rho}{A} Q_o \\ \frac{\partial Q}{\partial X} &= -j\omega\frac{A}{\beta_e} P_o \end{aligned}\right. \qquad \text{at } X = 0$$

the constants reduce to

$$\Psi_1 = P_o, \quad \Psi_3 = Q_o$$

$$-j\frac{\omega\rho}{A} Q_o = \frac{\omega}{C}\left(-C_1 \sin\frac{\omega}{\epsilon}X + C_2 \cos\frac{\omega}{\epsilon}X\right)$$

$$-j\omega\frac{A}{\beta_e} P_o = \frac{\omega}{\epsilon}\left(-C_3 \sin\frac{\omega}{\epsilon}X + C_4 \cos\frac{\omega}{\epsilon}X\right)$$

$$\Psi_2 = -j\frac{\rho\epsilon}{A} Q_o = -j\frac{1}{A}\sqrt{\rho\beta_e}\, Q_o$$

$$\Psi_4 = -j\frac{A\epsilon}{\beta_e} P_o = -j\frac{A}{\sqrt{\rho\beta_e}} P_o$$

Substituting the constants in Equation 41 and 42, produces

$$P = P_o \cos\frac{\omega}{\epsilon}X - j\frac{\rho\epsilon}{A} Q_o \sin\frac{\omega}{\epsilon}X$$

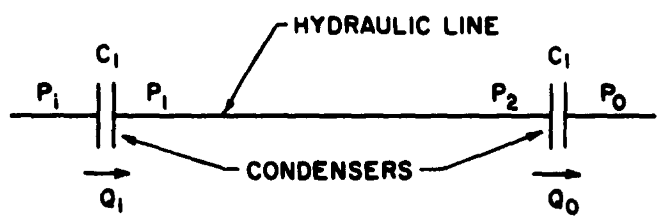
$$Q = Q_o \cos\frac{\omega}{\epsilon}X - j\frac{A\epsilon}{\beta_e} P_o \sin\frac{\omega}{\epsilon}X$$

At $X = \ell$, $P = P_i$ and $Q = Q_i$, therefore the above equations reduce finally to

$$P_i = P_o \cos\frac{\omega}{\epsilon}\ell - j\frac{\rho\epsilon}{A} Q_o \sin\frac{\omega}{\epsilon}\ell \tag{43}$$

$$Q_i = Q_o \cos\frac{\omega}{\epsilon}\ell - j\frac{A\epsilon}{\beta_e} P_o \sin\frac{\omega}{\epsilon}\ell \tag{44}$$

The sketch below shows two condensers of equal size connected in series with a hydraulic distributed line.



Applying Equation 43 and 44 to the above sketch

$$P_1 = P_2 \cos\frac{\omega}{\epsilon}\ell - j\frac{\sqrt{\rho\beta_e}}{A} Q_o \sin\frac{\omega}{\epsilon}\ell \tag{45}$$

$$Q_j = Q_o \cos\frac{\omega}{\epsilon}\ell - j\frac{A}{\sqrt{\rho\beta_e}} P_2 \sin\frac{\omega}{\epsilon}\ell \tag{46}$$

The flow and pressure relation across the condenser is

$$j\omega C_1 (P_i - P_1) = Q_i \quad \text{or} \quad P_j = P_1 - j\,\frac{Q_i}{\omega C_1} \tag{47}$$

$$j\,\omega C_1 (P_2 - P_o) = Q_o \quad \text{or} \quad P_2 = P_o - j\,\frac{Q_o}{\omega C_1} \tag{48}$$

Combining Equations 45, 46, 47, and 48 produce

$$\begin{aligned} P_i &= P_o \left(\cos\frac{\omega}{\epsilon}\ell - \frac{1}{\omega C_1}\frac{A}{\sqrt{\rho\beta_e}}\sin\frac{\omega}{\epsilon}\ell\right) \\ &\quad - j\,Q_o\,\frac{\sqrt{\rho\beta_e}}{A}\left[\left(1 - \frac{A^2}{\rho\beta_e}\,\frac{1}{\omega^2 C_1{}^2}\right)\sin\frac{\omega}{\epsilon}\ell \right. \\ &\quad \left. + \frac{2}{\omega C_1}\,\frac{A}{\sqrt{\rho\beta_e}}\cos\frac{\omega}{\epsilon}\ell\right] \end{aligned} \tag{49}$$

$$\begin{aligned} Q_i &= Q_o\left(\cos\frac{\omega}{\epsilon}\ell - \frac{A}{\sqrt{\rho\beta_e}}\,\frac{1}{\omega C_1}\sin\frac{\omega}{\epsilon}\ell\right) \\ &\quad - j\,P_o\,\frac{A}{\sqrt{\rho\beta_e}}\sin\frac{\omega}{\epsilon}\ell \end{aligned} \tag{50}$$

If the following relation is true

$$\frac{-A}{\sqrt{\rho\beta_e}}\,\frac{1}{\omega C_1} = \frac{1 - \cos\frac{\omega}{\epsilon}\ell}{\sin\frac{\omega}{\epsilon}\ell} = \tan\frac{\omega\ell}{2\epsilon} \tag{51}$$

then Equations 49 and 50 reduce to

$$P_i = P_o \tag{52}$$

$$Q_i = Q_o - j\,P_o\,\frac{A}{\sqrt{\rho\beta_e}}\sin\frac{\omega}{\epsilon}\ell \tag{53}$$

Equations 52 and 53 show that the magnitude and phase of the output pressure are maintained the same as that of the input pressure regardless of the magnitude and phase of the output flow rate.

If the following relation is true

$$\frac{A}{\sqrt{\rho\beta_e}}\,\frac{1}{\omega C_1} = \frac{1 + \cos\frac{\omega}{\epsilon}\ell}{\sin\frac{\omega}{\epsilon}\ell} = \cot\frac{\omega\ell}{2\epsilon} \tag{54}$$

then Equations 49 and 50 reduce to

$$P_i = -P_o \tag{55}$$

$$Q_i = -Q_o - j P_o \frac{A}{\sqrt{\rho \beta_e}} \sin \frac{\omega}{\epsilon} \ell \tag{56}$$

Equations 55 and 56 show that the magnitude of the output pressure is maintained the same as that of the input pressure, while the phase of the output pressure is maintained 180 degrees different from that of the input pressure, regardless of the magnitude and phase of the output flow rate.

Equations 51 and 54 can further be reduced to

$$C_1 = \frac{A}{\sqrt{\rho \beta_e}} \frac{1}{\omega} \cot \left(\pi - \frac{\omega \ell}{2\epsilon}\right) \tag{51a}$$

$$\begin{aligned} C_1 &= \frac{A}{\sqrt{\rho \beta_e}} \frac{1}{\omega} \tan \frac{\omega \ell}{2\epsilon} \\ &= \frac{A}{\sqrt{\rho \beta_e}} \frac{1}{\omega} \cot \left(\frac{\pi}{2} - \frac{\omega \ell}{2\epsilon}\right) \end{aligned} \tag{54a}$$

The size of the condensers that should be used in order that Equations 52, 53, 55 and 56 are satisfied are shown in Table 2.

TABLE 2

AN ANALYSIS OF CONDENSER SIZE WITH RESPECT TO HYDRAULIC LINE LENGTH

Hydraulic Line Length	Relations to be Satisfied	
	$P_i = P_o$ Q_i $Q_i = Q_o - j P_o \frac{A}{\sqrt{\rho\beta_e}} \sin \frac{\omega \ell}{\epsilon}$	$P_i = P_o$ $Q_i = -Q_o - j P_o \frac{A}{\sqrt{\rho\beta_e}} \sin \frac{\omega \ell}{\epsilon}$
$\ell < \frac{\lambda}{2}$		$C_1 = \frac{A}{\sqrt{\rho\beta_e}} \frac{1}{\omega} \tan \frac{\omega \ell}{2\epsilon}$
$\ell > \frac{\lambda}{2}$	$C_1 = \frac{A}{\sqrt{\rho\beta_e}} \frac{1}{\omega} \cot \pi - \frac{\omega \ell}{2\epsilon}$	
$\ell = \frac{\lambda}{2}$		$C_1 = 0$
$\ell = \lambda$	$C_1 = 0$	

where λ, wave length $= \frac{2\pi\epsilon}{\omega}$

4. Representation of Composite Distributed Loss-less Lines by a Single Distributed Loss-Less Line

The transmission lines in a pulsating hydraulic system may not be continuous but are made up of a series of lines with condenser inertias and leakages in between the lines (such as the hydraulic transformer and the leakage through the connecting seals). Therefore, it is important to know that the analytical study performed on a continuous distributed hydraulic line can also be applied to a composite distributed hydraulic line. It is found that by proper distribution of the apparatus along the transmission line, the interruption of the wave (of certain frequency) traveling by the apparatus can be eliminated. The composite distributed hydraulic line can then be treated the same as the continuous distributed hydraulic line. The schematic of the composite distributed hydraulic line is shown below in Figure 2

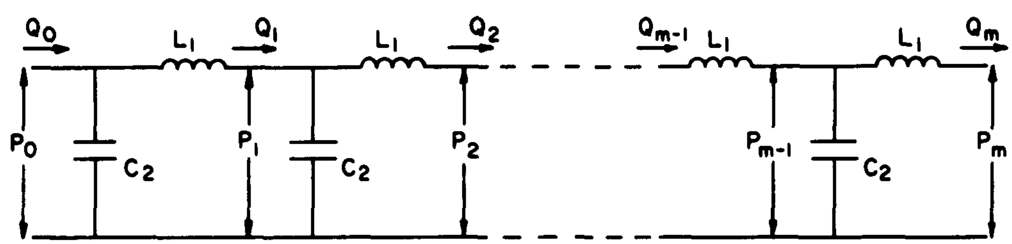


Figure 2. Composite Distributed Loss-Less Hydraulic Line

For simplification of analysis, the flow leakage and line loss are assumed to be zero at the present time. The basic pressure drop and flow relation of each individual section of the line can be expressed as

$$P_{n-1} - P_n = L_1 S Q_n \qquad (57)$$

and

$$Q_n - Q_{n+1} = C_2 S P_n \tag{58}$$

The condition that the composite distributed line can be treated the same as the continuous distributed line is obtained by the following:

The pressure drop and flow relationship for two adjacent sections of the line are

$$P_{n-1} - P_n = L_1 S Q_n \tag{59}$$

$$P_n - P_{n+1} = L_1 S Q_{n+1} \tag{60}$$

Equation 52 and Equation 59 produce

$$P_{n-1} - 2 P_n + P_{n+1} = L_1 S (Q_n - Q_{n+1}) \tag{61}$$

pressure and flow relation across the condenser is

$$Q_n - Q_{n+1} = C_2 S P_n \tag{62}$$

Substituting Equation 62 into Equation 61, obtains

$$P_{n-1} - 2 P_n + P_{n+1} = L_1 C_2 S^2 P_n \tag{63}$$

Let

$$-L_1 C_2 S^2 = 2 \sin \frac{\theta}{2}$$

Equation 63 becomes

$$P_{n-1} - 2 P_n \cos \theta + P_{n+1} = 0 \tag{64}$$

Equation 65 is a possible solution of Equation 64

$$P_n = A \cos (m-n) \theta + B \sin (m-n) \theta \tag{65}$$

From Equation 65, we can write

$$\begin{aligned} P_n - P_{n+1} = {} & A \cos (m-n) \theta + B \sin (m-n) \theta \\ & - A \cos (m-n-1) \theta - B \sin (m-n-1) \theta \end{aligned} \tag{66}$$

Substituting Equation 60 in Equation 66, produces

$$\begin{aligned} L_1 S Q_{n+1} = {} & A \cos (m-n) \theta + B \sin (m-n) \theta \\ & - A \cos (m-n-1) \theta - B \sin (m-n-1) \theta \end{aligned} \tag{67}$$

If $n = m - 1$, than Equation 67 becomes

$$L_1 \, S \, Q_m = A \, (\cos \theta - 1) + B \sin \theta \tag{68}$$

Substituting boundary condition

$$P = P_a \quad \text{at } n = 0$$

and $$P = P_m \quad \text{at } n = m$$

into Equation 68 and replacing the Laplace operator by $j\omega$, obtains

$$P_o = P_m \frac{\cos\left(m - \frac{1}{2}\right)\theta}{\cos \frac{\theta}{2}} + j \, Q_m \sqrt{\frac{L_1}{C_2}} \; \frac{\sin m \theta}{\cos \frac{\theta}{2}} \tag{69}$$

By the same method the following flow and pressure relation can also be obtained

$$Q_o = Q_m \frac{\cos\left(m - \frac{1}{2}\right)\theta}{\cos \frac{\theta}{2}} + j \; P_m \sqrt{\frac{C_2}{L_1}} \; \frac{\sin m \theta}{\cos \frac{\theta}{2}} \tag{70}$$

The pressure and flow relation of a continuous distributed line had been derived previously and is written here

$$P_o = P_m \cos a \, \ell + j \, Q_m \sqrt{\frac{L}{C}} \sin a \, \ell \tag{71}$$

where:

$a = \omega\sqrt{LC}$

L = uniform inductance $= \dfrac{m \, L_1}{\ell}$

C = capacitance of continuous distributed line $= \dfrac{m \, C_2}{\ell}$

Replacing L and C of Equation 71 by $\dfrac{m \, L_1}{\ell}$ and $\dfrac{m \, C_2}{\ell}$, produces

$$P_o = P_m \cos a_1 + j \, Q_m \sqrt{\frac{L_1}{C_1}} \; \sin a_1 \, \ell \tag{72}$$

and $$a_1 = \frac{m \omega}{\ell} \sqrt{L_1 \, C_1} = 2 \, \frac{m}{\ell} \sin \frac{\theta}{2}$$

Comparing Equations 69 and 70 with Equations 71 and 72, respectively, obtains

$$\cos a_1 \, \ell = \frac{\cos \;\; m - \frac{1}{2} \, \theta}{\cos \frac{\theta}{2}} = \cos \left(2 \, m \sin \frac{\theta}{2}\right) \tag{73}$$

$$\sin a_1 \ell = \frac{\sin m\,\theta}{\cos\frac{\theta}{2}} = \sin\left(2\,m\,\sin\frac{\theta}{2}\right) \qquad (74)$$

If θ is a very small number, then Equation 73 reduces to

$$\theta \sin m\,\theta = 0$$

or $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (75)

$$\sin m\,\theta = 0$$

Equation 74 results in a trivial solution

$$\sin m\,\theta = m\theta$$

In order for Equation 75 to be satisfied, the following relation must hold

$$\theta = \pm\frac{K\pi}{m} \qquad (76)$$

where K = 0, 1, 2, ...

From Equation 74

$$a_1 \ell = 2\,m\,\sin\frac{\theta}{2} \approx 2\,m\theta \qquad (\text{for } \theta \rightarrow \text{small})$$

Because

$$a_1 = \frac{2\pi}{\lambda} \qquad (\lambda = \text{wave length})$$

We have

$$\frac{2\pi\ell}{\lambda} = m\,\theta = \pm K\,\pi$$

or $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (77)

$$\ell = \frac{K\lambda}{2}$$

Therefore, in order that the composite distributed line can be treated the same as the continuous distributed line, the following two relations must hold

$$\sin\frac{\theta}{2} \approx \frac{\theta}{2}$$

and the length of the composite distributed line should be a multiple of half wave length of the continuous distributed line.

C. TEST SET-UP AND EXPERIMENTAL RESULTS

1. Experimental Set-up and Test Results of the Over-all System Efficiency Study

The miniaturized pulsating system schematic shown in Figure 26 (of Second Quarterly Progress Report, RAC 933-2) was used for the over-all system efficiency study. A modification was made on the test set-up by cementing two strain gauges on the pump piston rod. The strain gauges were used to measure the harmonic force acting on the fluid by the piston.

During the test the load was simulated by a needle valve which was connected to the output and return terminals of the system down stream of the accumulator. The pressure drop across and the flow through the needle valve are in phase with each other. However, the force and velocity of the pump piston rod are usually not in phase. Therefore, during the calculation of experimental power input, the effect of phase difference was included.

Test data analyzed to date from recorded results are shown in Figures 3 and 4. The data shown in the figures indicate that with the pulsating frequencies, ω, and strokes, X_p, tested, the over-all efficiency, η peaks at ω = 6 cps and X_p = 0.34 inch.

The efficiency is defined as

$$\eta = \frac{P_m \quad Q_m}{\frac{1}{2} (P_p \, A_p \, X_p \, \omega \cos \theta)} \qquad (78)$$

The maximum over-all system efficiency of the miniaturized pulsating system so far recorded was 47 percent. This occurred when the fluid Reynold's number was 1,570. This is by no means the maximum over-all efficiency that is obtainable with a pulsating hydraulic system. Further efficiency determinations will be made during later testing.

2. Distributed Line Loss Study Test Set-up

In Figure 5 below is shown the distributed line loss test setup which is near completion. The hydraulic linear resistance is matched with the line characteristic impedance so that the reflected pressure waves are entirely eliminated or at least reduced to an extremely small amplitude. Several hydraulic resistors of different magnitudes were made. Before future test data is taken, the pulsating frequency will be adjusted so that the linear resistance and the line characteristic impedance are properly matched. The air loaded accumulator is used to prevent fluid cavitation during the return half cycle. An alternating flow meter will be

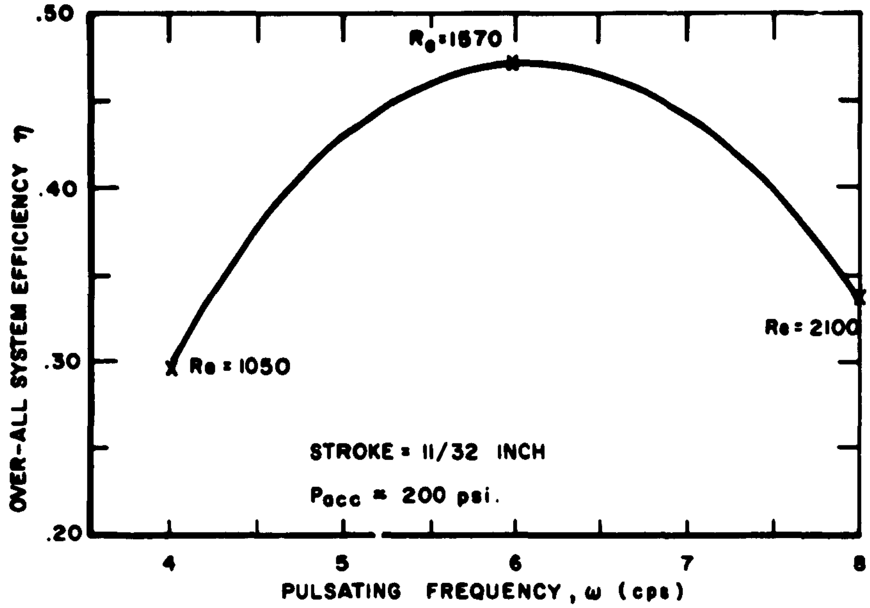


Figure 3. System Over-All Efficiency vs Pulsating Frequency for Constant Pulsating Strokes

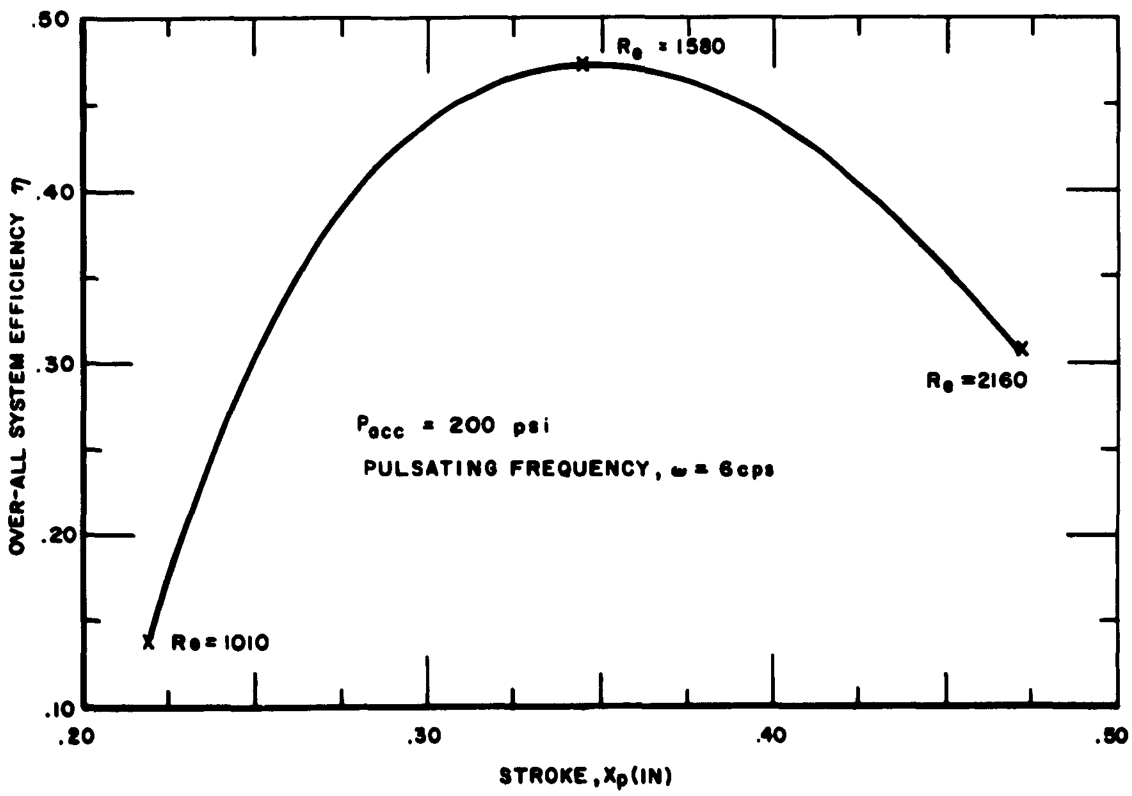


Figure 4. System Over-All Efficiency vs Pulsating Stroke for Constant Pulsating Frequency

installed between the linear resistor and the accumulator to measure alternating flow rate. However, the flow rate can also be calculated from the pressure drop across the linear resistor. Because the physical length of the linear resistance unit is much shorter than the expected shortest pressure wave length, the flow through the linear resistance can be considered as a slug flow and is essentially in phase with the pressure drop. Several pressure pickups were installed along the hydraulic line to measure the flow pressure.

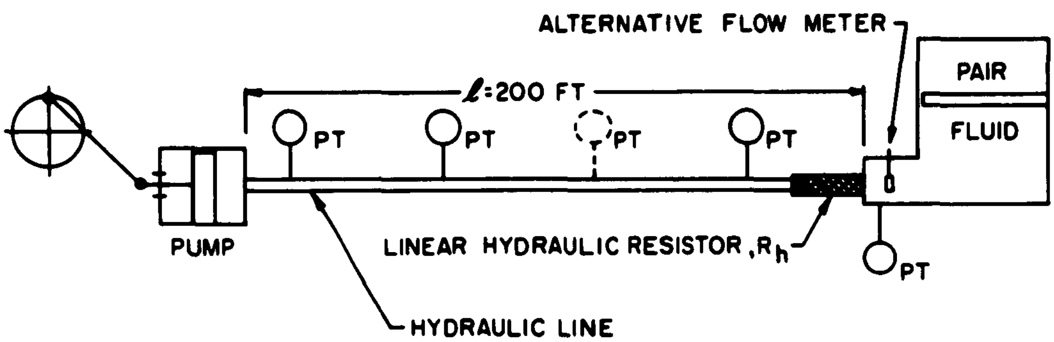


Figure 5 . Distributed Hydraulic Line Loss Study Test Set-up

Future tests to be performed will include:

1) Line loss vs. pulsating frequency and stroke (or Reynold's number)
2) Gain and phase change of pulsating pressure and flow vs. pulsating frequency and stroke
3) Input impedance of the line
4) Minimum precharged accumulator pressure to avoid flow cavitation.

D. NOMENCLATURE

This nomenclature is cumulative; it includes symbols from the First, Second, and Third Quarterly Progress Report.

A = Area

b = Damping coefficient

C = Capacitance

C_d = Discharge Coefficient

c = Sonic velocity

$D = \frac{d}{dt}$

e = Voltage

E = Amplitude of sinusoidal voltage

f = Force

G = Conductance

$g = C_d W \sqrt{\frac{2}{\rho}}$

I = Current

K_h = Compliance (Hydraulics)

K = Spring constant

K_r = Ratio of specific heat of air

K_a = Amplifier gain

K_m = Compliance (Mechanics)

ℓ = Length of line

L = Inductance

M = Mass

m = Inertance per unit length of line

N = Number of coil turns

P = Pressure

Q = Flow rate

R = Resistance

R_h = Resistance (Hydraulics)

R_e = Reynolds number

S = Laplace operation

s = Hydraulic line leakage per unit line length

T_e = Wave traveling time through a line of length ℓ

U = Velocity

V = Voltage

W = Valve part width

X = Displacement

X_v = Spool displacement

X_p = Pulsating pump stroke

Y = Admittance

Z = Impedance

β = Bulk modulus

β_e = Equivalent bulk modulus

γ = Propagation constant

Δ = Increment

ρ = Fluid density

ω = Frequency

υ = Volume

$\frac{\partial}{\partial X}$ = Partial differentiation with respect to displacement X

SECTION V - MATERIALS

As presently designed, the hydraulic system will probably use a fluid such as a mixed polyphenyl ether for operation to 700°F. Since Republic has previous experience with this fluid during developmental work on its 1000°F hydraulic system (Contract AF33(616)-7454), it expects to apply this knowledge in the selection of materials for use in the 700°F zone.

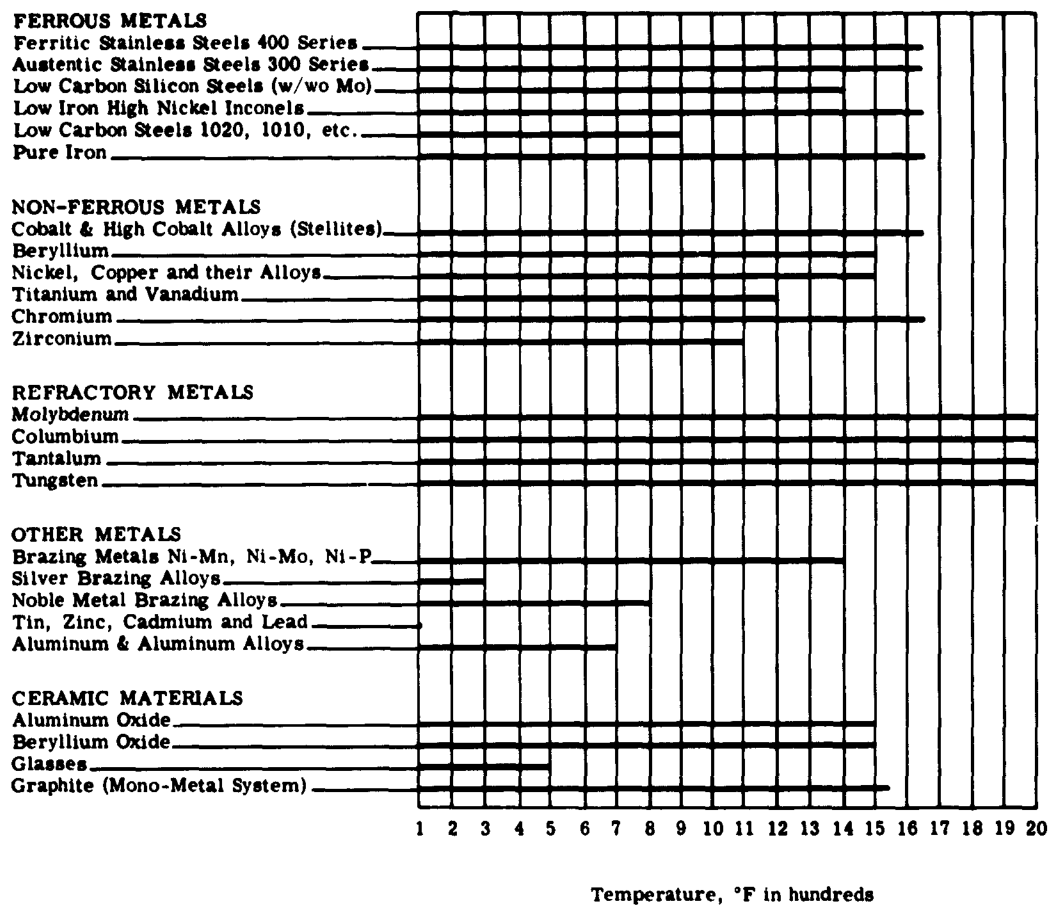
For operation to 1400°F the most suitable fluids appear to be the liquid metals, with NaK_{77} (Sodium-Potassium Eutectic) receiving primary consideration. The use of liquid metals will have an effect on the containment materials with respect to corrosion, erosion, deterioration of mechanical properties, and sliding and bearing properties.

A number of materials that are compatible with NaK for operation at 1400°F are shown in Table 3. The austenitic stainless steels as well as the cobalt-base and nickel-base alloys are among those materials with good corrosion resistance. The refractory metals, including columbium, molybdenum, tantalum, tungsten, and chromium are recommended for use in NaK to 1690°F.

There are many factors which affect liquid metal corrosion, among which are the following:

(1) Temperature
(2) Temperature gradient
(3) Cyclic Temperature fluctuation
(4) Surface-area to volume ratio
(5) Purity of liquid metal
(6) Flow velocity or Reynolds number
(7) Surface condition of containment material
(8) The number of materials in contact with the same liquid metal

TABLE 3. MATERIALS COMPATIBILITY WITH LIQUID NaK_{77}



(9) Condition of the material (the presence of a grain boundary precipitate, the presence of a second phase, the state of stress of the material, and the grain size).

Liquid metal attack may take place by several fairly common mechanisms. One is a relatively uniform solution attack on the solid surface by the liquid corrodent. Another common method of attack is direct alloying, the interaction between liquid and solid to form surface films or typical layers of intermetallic compounds and solid solutions. These films may form a loosely adherent scale or, if held tightly, may serve as a barrier to slow down additional diffusion.

Selective reaction of the liquid metal with minor constituents of the solid may result in intergranular penetration or in the depletion of a dissolved component of a solid. Selective grain-boundary attack can drastically alter the physical properties of a material without appreciably changing its weight or appearance. This type of attack is often accelerated by the application of stress to the solid during exposure to the liquid metal. Particular care will be exercised in designing any part which will be subjected to stresses; a minimum of stress parts will be used.

Attack also results from corrosion by contaminants rather than by the liquid itself. Such contaminants may include oxygen, nitrogen, or carbon, dissolved in the liquid or present as part of a compound in suspension. For example, in oxygen-contaminated systems, the container metal may become coated with a layer of its own oxide, if its oxide is more stable than the oxide formed by the liquid metal. The oxide layer, as with alloy layers, is sometimes tenacious and adherent. Hence, the layer acts as a diffusion barrier, thereby inhibiting further attack. On the other hand, the oxide layer may be non-adherent, in which case drastic weight loss ensues, especially under dynamic conditions. There remains the possibility that a film, once formed, can be removed by the fluxing action of other oxides.

One of the major problems inherent in the use of NaK is the formation of potassium and sodium oxides. The most harmful effects of this oxide contamination are increased corrositivity and cold zone precipitation. The precipitation on close fitting parts is of primary concern; but of almost equal concern is the fact that in a flowing system the oxides may accumulate in a cold zone and plug the system. This will be avoided by keeping NaK flow out of cooler areas (< 700°F).

One of the most serious manifestations of liquid metal behavior (in which actual corrosion plays only a small part) is diffusion bonding or welding of solid metal surfaces to each other. Such self-welding takes place when contact with an alkali metal occurs, and is intensified if the metals are held together under pressure. This effect becomes increasingly serious with increasing temperature.

Of particular interest in most liquid metal systems is a type of mass-transport, thermal-gradient transfer. This transfer is a result of the coexistence of a temperature differential and an appreciable thermal coefficient of solubility. Even though the actual solubility may be low, large amounts of a solid component may be dissolved from the zone of higher solubility and precipitated in the zone of lower solubility. This continued removal of the dissolved component from the system accelerates corrosion attack in some areas, eventually causing plugging of flow channels. Certain impurities, especially oxygen, may accelerate solution and the thermal-gradient transfer effect.

The selection of materials for the seal surfaces and bearings will be of major importance. Bearings which use hydrodynamic films for lubrication will have to be carefully designed in view of the exceptionally low viscosities of the liquid metals. Normally, oil lubricants form strong adherent surface films which reduce metal-to-metal contact or remove it entirely. However, with but a few exceptions, liquid metals tend to promote metal-to-metal contact by reacting with and dissolving protective surface films.

Data on the bearing properties of metals in NaK is very limited. Compatibility tests have been performed on a variety of materials at 350°F using a fixed specimen bearing against a rotating sleeve. Material pairs showing the best wear properties include a tungsten carbide-cobalt cermet, a 6-6-2 high strength tool steel, white cast iron, and 17-4PH stainless steel against the tungsten carbide-cobalt cermet (carboloy 779). Austenitic and ferritic stainless steels, low-carbon steel, cobalt-base alloys, and a copper alloy showed poor compatibility in NaK. Bearing-material compatibility tests in NaK at temperatures to 950°F have been performed with the following results:

(1) Chromium carbide cermets are unsatisfactory bearing materials.

(2) Porous tungsten carbide cermets gave excellent results and are considered the best for oscillating bearing materials.

(3) A combination of titanium carbide cermet against tungsten carbide cermet has excellent compatibility characteristics and low coefficients of friction.

(4) The lower the percentage of cobalt in the tungsten carbide cermet, the less is the tendency for superficial damage. This is attributed to the increase in hardness and finer grain structure, the latter being considered more important.

(5) There is less tendency for surface damage to titanium carbide cermets if they do not contain a solid-solution type of carbide.

(6) Titanium carbide cermets tested with similar cermets tend to have less superficial surface damage when tested at temperatures below 850°F.

(7) Cermets with equivalent compositions tend to have similar compatibility characteristics.

(8) The compatibility of nickel-bonded cermets compares favorably with the compatibility characteristics of cobalt-bonded cermets.

(9) Metal-cermet and cermet-cermet combinations have fractional compatibility characteristics which are directly proportional to temperature.

(10) Copper and some copper alloys are most compatible with chromium or cermets at temperature up to 600°F.

(11) Nickel and nickel alloys, with the possible exception of Colmonoy 6, do not have good compatibility characteristics at high temperatures.

(12) The chromium-tungsten-cobalt alloys showed fair compatibility characteristics at 600°F; however, above this temperature performance was poor.

(13) Iron-base alloys are also unsatisfactory at temperatures above 600°F.

(14) Surface-treated materials behaved according to their substrate materials.

(15) Surface finish is suspected as being an important factor in the compatibility of low-shear-strength materials which do not weld readily to the complementary material.

SECTION VI - FUTURE WORK

Design studies will be made on a diaphragm type transformer. The objective of the design will be to incorporate the same functions provided by the piston-type transformer, namely, to provide for fluid leakage and fluid thermal expansion. It is anticipated that due to the very limited stroke which can be extracted from a diaphragm, considerable difficulty will be experienced in deriving a position signal for fluid level control.

A derivation will be made of an approximate expression of the over-all system efficiency and other system parameters by using a simplified pulsating hydraulic system model. Means of estimating the maximum obtainable over-all efficiency of a typical system and comparing theoretical and experimental data will be studied. Tests will be performed and data derived on the distributed line loss set up.

APPENDIX

DISTRIBUTED TRANSMISSION LINE CHARACTERISTIC EQUATIONS DERIVATION

UNSTEADY, COMPRESSIBLE AND LOW AVERAGE VELOCITY FLOW IN A UNIFORM PASSAGE

One Dimensional Flow

In Figure A-1 is shown the control volume of an unsteady, compressible and frictionless flow in a uniform passage. The average velocity is assumed to be small, and the flow is one dimensional.

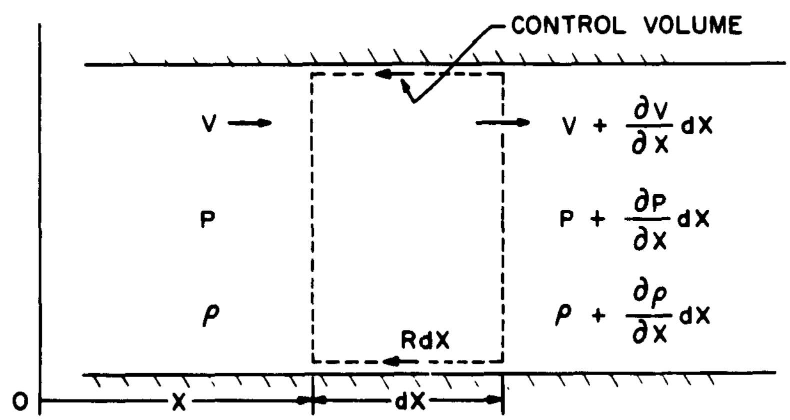


Figure A-1. Control Volume of Unsteady, Compressible and One Dimensional Flow in a Uniform Hydraulic Line

Continuity equation:

$$\rho VA - \left[\rho VA + \frac{\partial(\rho VA)}{\partial r} \, dX \right] = AdX \, \frac{\partial \rho}{dt} \qquad \text{(A-1)}$$

Equation of state of liquid:

$$d\rho = \frac{\rho}{\beta_e} dP \tag{A-2}$$

where

$$\frac{1}{\beta_e} = \frac{1}{\beta} + \frac{1}{E\left(\frac{D_o}{D_i} - 1\right)}$$

Combining Equation A-1 and A-2, results in

$$\rho \frac{\partial V}{\partial X} + \frac{V}{\beta_e} \frac{\partial \rho}{\partial X} = - \frac{\rho}{\beta_e} \frac{\partial P}{\partial t} \tag{A-3}$$

When the velocity is low enough we may neglect the second term on the left side of Equation A-3. The simplified continuity equation is, therefore, obtained.

$$\frac{\partial V}{\partial X} = - \frac{1}{\beta_e} \frac{\partial P}{\partial t}$$

or

$$\frac{\partial Q}{\partial X} = - \frac{A}{\beta_e} DP \tag{A-4}$$

where $D \equiv \frac{\partial}{\partial t} \equiv j\omega$

The momentum equation is:

$$PA - \left[PA + \frac{\partial PA}{\partial X} dX \right] - QRdX = \left[\rho V^2 A + \frac{\partial(\rho V^2 A)}{\partial X} dX \right] - \rho V^2 A + \frac{\partial}{\partial t} \int_{cv} \rho VdA \ dX \tag{A-5}$$

Carrying Equation A-5 out, and again neglecting the terms that have V as a coefficient, the momentum equation is reduced to

$$-\frac{\partial P}{\partial X} = \rho \frac{\partial V}{\partial t} + \frac{RQ}{A}$$

or

$$-\frac{\partial P}{\partial X} = \frac{1}{A} (R + \rho D) Q \qquad \text{(A-6)}$$

Let $\frac{A}{\beta_e} D$ and $\frac{1}{A} (R + \rho D)$ be represented by Y and Z, respectively, Equation A-4 and A-6 can be rewritten as

$$-\frac{\partial Q}{\partial X} = YP \qquad \text{(A-7)}$$

$$-\frac{\partial P}{\partial X} = ZQ \qquad \text{(A-8)}$$

Equation A-7 and A-8 are the pair of wave equations that describe the wave propagation in the distributed hydraulic line.

Differentiating Equation A-7 and A-8 with respect to X and substituting Equation A-7 and A-8 into the proper terms of the differentiated equations, the following pair of equations can be obtained.

$$\frac{d^2P}{dX^2} - \gamma^2 P = 0 \qquad \text{(A-9)}$$

$$\frac{d^2Q}{dX^2} - \gamma^2 Q = 0 \qquad \text{(A-10)}$$

where γ, propagation constant $= \sqrt{Z\,Y}$

The proper solution form of Equation A-9 is

$$P = C_1 e^{\gamma X} + C_2 e^{-\gamma X} \tag{A-11}$$

Differentiating A-11 with respect to X

$$\frac{dP}{dX} = C_1 \gamma e^{\gamma X} - C_2 \gamma e^{-\gamma X} = -ZQ \tag{A-12}$$

Therefore, the solution of Equation A-10 must be in the following form

$$Q = C_1 \frac{\gamma}{Z} e^{\gamma X} + C_2 \frac{\gamma}{Z} e^{-\gamma X}$$

or

$$Q = \frac{-C_1}{Z_s} e^{\gamma X} + \frac{C_2}{Z_s} e^{-\gamma X} \tag{A-13}$$

where: Z_s, hydraulic line characteristic impedance $= \sqrt{\frac{Z}{Y}}$

Letting X = 0, Equation A-13 reduces to the following expression which is the instantaneous pressure at the section X = 0 on the hydraulic line

$$P = C_1 + C_2 \tag{A-14}$$

It should be noticed from the above expression that the quantities C_1 and C_2 are constant with respect to X but vary harmonically with respect to time. Therefore the term C_1 and C_2 may be of the following form

$$C_1 = P_1 e^{j\omega t}$$

and

$$C_2 = P_2 e^{j\omega t} \tag{A-15}$$

Substituting A-15 into Equation A-14, Equation A-14 becomes

$$P = P_1 e^{j\omega t} + P_2 e^{j\omega t} \tag{A-16}$$

Substituting Equation A-16 into A-11 and A-13 results

$$P = P_1 e^{j\omega t} e^{\gamma X} + P_2 e^{j\omega t} e^{-\gamma X} \tag{A-17}$$

and

$$Q = \frac{-P_1}{Z_s} e^{j\omega t} e^{\gamma X} + \frac{P_2}{Z_s} e^{j\omega t} e^{-\gamma X} \tag{A-18}$$

Let us consider the wave traveling in the positive X direction only. The characteristic (or surge) impedance of the line is obtained by taking the ratio of the pressure and flow rate.

$$Z_s = \sqrt{\frac{Z}{Y}} = \frac{1}{Y_s} \tag{A-19}$$

The phase velocity of the traveling wave is

$$v = \frac{\omega}{\operatorname{Im} \gamma}$$

which is the sonic velocity of the fluid in the line. This velocity may be affected both by the line wall rigidity and the amount of air dissolved in the fluid.

Let us consider a line of characteristic impedance, Z_s, which is terminated at a load impedance, Z_ℓ as shown in Figure A-2 .

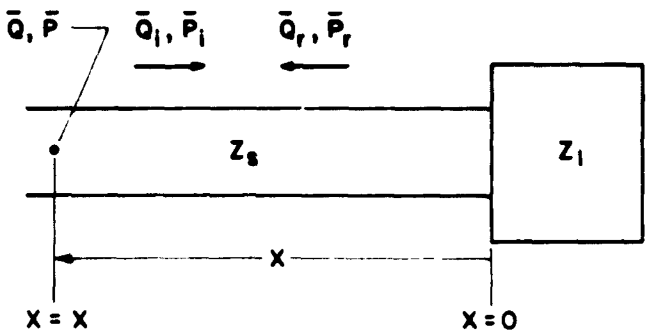


Figure A-2. Terminated Transmission Line

$\bar{Q}_i$ and $\bar{P}_i$ are the incident flow and pressure wave. $\bar{Q}_r$ and $\bar{P}_r$ are the reflected flow and pressure wave. They are complex functions of position. The resultant flow and pressure wave are the sum of the incident and reflected components

$$\bar{P} = \bar{P}_i + \bar{P}_r \tag{A-20}$$

$$\bar{Q} = \bar{Q}_i + \bar{Q}_r \tag{A-21}$$

where $\bar{P}_i = P_i e^{\gamma X}$

$$\bar{P}_r = P_r e^{-\gamma X + j\zeta}$$

$$\bar{Q}_i = Q_i e^{\gamma X - j\delta}$$

$$\bar{Q}_r = Q_r e^{-\gamma X + j(\zeta - \delta)}$$

ζ is phase shift at load

δ is phase difference between the pressure and flow

At the load (X = 0)

$$\frac{\bar{P}_r}{\bar{P}_i} = \frac{P_r}{P_i} e^{j\zeta} = \bar{\rho}_p$$

$$\frac{\overline{Q}_r}{\overline{Q}_i} = \frac{Q_r}{Q_i} e^{j\kappa} = \overline{\rho}_q$$

where $\overline{\rho}_p$ = reflection coefficient of pressure wave

$\overline{\rho}_q$ = reflection coefficient of flow wave

It follows that

$$\overline{P} = P_i \left(e^{\gamma X} + \overline{\rho}_p \, e^{-\gamma X} \right)$$

and

$$\overline{Q} = Q_i \, e^{-j\sigma} \left(e^{\gamma X} + \overline{\rho}_q \, e^{-\gamma X} \right)$$

By the definition of characteristic impedance we have the following relationship

$$Z_s = \frac{\overline{P}_i}{\overline{Q}_i} = \frac{P_i}{Q_i} e^{j\sigma} = - \frac{\overline{P}_r}{\overline{Q}_r} = - \frac{P_r}{Q_r} e^{j\sigma} \tag{A-24}$$

At the load

$$Z_s = \frac{\overline{P}}{\overline{Q}} \tag{A-25}$$

Substituting Equation A-24 and A-25 into Equation A-21 we have

$$\frac{\overline{P}}{Z_s} = \frac{\overline{P}_i}{Z_s} - \frac{\overline{P}_r}{Z_s} = \frac{\overline{P}_i - \overline{P}_r}{Z_s} \tag{A 26}$$

Substituting $\bar{P}$ by Equation A-20 we have

$$\frac{\bar{P}_i + \bar{P}_r}{Z_\ell} = \frac{\bar{P}_i - \bar{P}_r}{Z_o}$$

Solving for $\dfrac{\bar{P}_r}{\bar{P}_i}$ yields

$$\frac{\bar{P}_r}{\bar{P}} = \frac{Z_\ell - Z_s}{Z_\ell - Z_s} = \bar{\rho}_p \qquad \text{(A-27)}$$

By a similar method we can obtain

$$\bar{\rho}_q = -\frac{Z_\ell - Z_s}{Z_\ell + Z_s} = -\bar{\rho}_p \qquad \text{(A-28)}$$

The impedance Z_x at any point X of the line looking toward the load is defined as

$$Z_x = \frac{\bar{P}}{\bar{Q}} \qquad \text{(A-29)}$$

Substituting the appropriate equations into Equation A-29, the impedance of a line with linear friction loss is obtained as

$$Z_X = Z_s \left(\frac{Z_\ell + Z_s \tanh \gamma X}{Z_s + Z_\ell \tanh \gamma X} \right) \qquad \text{(A 30)}$$

For a frictionless line (γ has imaginary part only) Equation A-30 reduces to

$$Z_X = Z_s \left(\frac{Z_\ell + j\, Z_s \tan \frac{\omega}{c} X}{Z_s + j\, Z_\ell \tan \frac{\omega}{c} X} \right) \qquad \text{(A-31)}$$

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